Free Will Theorem as a Bell's Theorem

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Work with

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Two pieces of folklore about quantum mechanics:

- The theoretical framework that can explain everything in the Universe.
- One needs to put in observers by hand into unitary evolution to make any actual predictions with quantum mechanics.

These two claims are in tension with each other.

Because the first claim would require that the dynamics of the observers that show up in the second claim should *also* be understood quantum mechanically.

However, a fully accepted derivation (or even a schematic derivation) of observers and the process of observation, starting from unitary evolution, is unavailable.

Obejective meaure of this non-consensus? Enough Nobel laureates who claim to not understand quantum mechanics.

This is sometimes called the Problem of Interpretation of quantum mechanics.

This talk does not have anything directly to do with the interpretational issues of quantum mechanics.

But it is somehow in the same landscape of ideas, and so it is prossibly the right context to consider it.

Note that interpretational issues arise from a clash of two kinds of time evolution. Roughly- unitary vs collapse. In this talk we will never talk about the former. And only talk about values of measurements and measurements on eigenstates. So only the most rudimentary versions of the Born rule.

Mpore specifically, I think we will only use the fact that-

- quantum states live in Hilbert space,
- observables are Hermitian operators,
- results of observations are eigenvalues of observables, and
- on eigenstates the eigenvalues are guranteed 100% of the time.

In other words, we will never really need probabilities or ensembles.
Without further ado, lets turn to–

Conway-Kochen “Free Will" Theorem:

According to the authors, the theorem shows (under some assumptions inspired by quantum mechanics, locality, etc[1]) that “..if the choice of a particular type of spin 1 experiment is not a function of the information accessible to the experimenters, then its outcome is equally not a function of the information accessible to the particles."

The slogan they extract: “if the experimenter has Free Will, so must the system."

The statement should make more sense when I formulate it in terms of more familiar words.

[1] Small Print: Conway-Kochen also claim to prove some variations of their theorem with stochastic ingredients, but that is not the case that is interesting from a fundamental point of view, so I will not talk about it.

Conway-Kochen “Free Will" Theorem (continued):

I will argue that this theorem has essentially the same physical content (assumptions as well as conclusion) as a form of Bell’s theorem without inequalities proved by Greenberger, Horne, Shimony and Zeilinger.

The form of the latter theorem that I will use will be a cleaned up and simplified version, due to Mermin.

I will call this the Bell-GHSZ theorem, even though Mermin’s writings have been most influenetial in my thinking.

Lets start by partially clarifying the two notions of Free Will involved in the formulation of the Conway-Kochen theorem.

(What we will call) the Free Will of the Experimenter:

This is just the statement that on a quantum mechanical system we typically have the freedom to choose to measure any set of (commuting) operators as a simultaneous measurement.

Conway and Kochen say some words about the precise meaning of experimenter's Free Will (eg: that his actions are not determined deterministically by his past light cone etc etc), but the above is what they need to prove the theorem.

This type of Free Will for the Experimenter is a standard assumption in pretty much any discussion in physics, and in particular in the context of Bell-like theorems, so we will not justify it further.

But it should be emphasized, that this means that the meat of the assumptions underlying the Conway-Kochen theorem is NOT in this free will assumption, even though the statement of the theorem acccording to the authors emphasizes it. The real devil is in the assumptions inspired by quantum mechanics, locality, etc that they also use.

Now lets turn to the Free Will of the System:

By the statement that the system has Free Will, Conway and Kochen merely mean that the result of the experiment is not deterministic.

What is determinism?

Determinism is the statement that the result of an experiment is a function of the state of the system and the observables that comprise the experiment.

In other words, if the state of the system is the same and if one is measuring the same set of (commuting) observables, then the result of the experiment is always the same – that is determinism.

Aside: of course, quantum mechanics is not believed to be deterministic. At least not without adding some bizarre (= non-local and contextual) hidden variables into the description of the state. These words will be clarified.

Clearly, if one buys that quantum mechanics is a non-deterministic theory, there is nothing to prove in the Conway Kochen Free Will theorem, because Free Will of the system is simply **defined** as the absence of determinism in experiments involving that system.

So what Conway-Kochen do, is to assume some features of quantum mechanics, and then show that if those features are true, then there cannot exist (certain classes of) deterministic hidden variable theories that can reproduce those features.

The reason why they call it a Free Will theorem is because they view potential hidden variables to include anything that can affect the system, like the past light cone of the appropriate part of the experiment.

In other words, even though they don't say it in so many words, the Free Will Theorem is nothing but a (potentially new) Hidden Variable No Go theorem.

Whether it really is a new hidden variable No Go theorem will depend on the class of hidden vaariable theories that it rules out.

Note that arbitrary hidden variable theories are NOT ruled out by the famous No Go theorems of (Bell-)Kochen-Specker and Bell. Lets inevstigate the nature of these theorems to see where Conway-Kochen fits into it.

(Bell-)Kochen-Specker Theorem

Kochen-Specker theorem rules out the possibility that a class of theories called non-contextual hidden variable theories can simulate quantum mechanics.

A non-contextual theory is a theory where the measured value of an observable A when the system is in a specific state, is independent of which commuting set of observables A is measured with.

For example, the measured value of J^2 in a specific state in the non-contextual theory, will be the same whether J^2 is measured with J_x or J_y. In other words, the result does not depend on the specific experimental set up, aka context.

It is easy to show that such theories cannot reproduce quantum mechanics, and that is the Kochen-Specker theorem. The original proof was in a 3-dimensional state space and involves some complications, but it is trivial to prove it in higher dimensions. Lets consider an 8 dimensional configuration of operators suggested by Mermin:

$$\sigma_{y1}$$

$$\sigma_{x1}\sigma_{x2}\sigma_{x3} \qquad \sigma_{y1}\sigma_{y2}\sigma_{x3} \qquad \sigma_{y1}\sigma_{x2}\sigma_{y3} \qquad \sigma_{x1}\sigma_{y2}\sigma_{y3}$$

$$\sigma_{x3} \qquad \sigma_{y3}$$

$$\sigma_{x1}$$

$$\sigma_{y2} \qquad \sigma_{x2}$$

The 1, 2 and 3 stand for qbit tensor factors in the 8-d state space. The operators in each line commute by direct calculation. This means that each of the five lines stand for a set of simultaneously measurable observables in the 8-d state space.

The product of the operators on the horizontal line, lets calls this product P_1 is easily checked to be -1. The products of all the other lines is seen to give $P_2 = P_3 = P_4 = P_5 = 1$. This means that $P_1P_2P_3P_4P_5 = -1$ as an operator identity in quantum mechanics.

But if we were to assign values to each of these operators, each operator would have to have the value ± 1 and since each operator appears twice in the product $P_1P_2P_3P_4P_5$, the product will have to be $+1$.

Contradiction!

This means that we can't assign non-contextual values to the observables in this arrangement while reproducing quantum mechanics.

What we have shown is that non-contextual deterministic theories cannot simulate quantum mechanics. But demanding complete non-contextuality is perhaps unreasonable. Afterall, why should the result of an experiment not depend on the configuration in which you do the experiment?

Bell's Theorem or equivalently Bell-GHSZ theorem considers a more specialized version of non-contextuality and shows that even the specialized version of non-contextuality is violated by quantum mechanics.

Bell-GHSZ demands non-contextuality, only when it is justifiable by the demand of locality.

We can construct a version of Bell's theorem (or Bell-GHSZ) by adapting the 8-dimensional (Bell-)Kochen-Specker set up.

If we think of the tensor factors 1, 2 and 3 that make up the state space as far (spacelike) separated in spacetime, then one can use locality as a motivation to demand non-contextuality for the operators that live in P_2, P_3, P_4 and P_5. That is, if we want to have a hidden variable theory, then it is reasonable to demand that these observables have non-contextual values.

Note that each of those operators are local operators.

Of course, we cannot make any statements about the non-local observables in P_1.

What we can do instead is to consider an eigenstate of the operators in P_1. This means that those operators have well-defined values in that state, and this works as a proxy for non-contextuality that we used in the Kochen-Specker version of the argument. Thus the previous contradiction goes through here as well.

What have we learnt?

(Bell-)Kochen-Specker Theorem taught us that non-contextually determisnistic theories cannot reproduce quantum mechanics.

This may make us think that one might be able to reproduce quantum mechanics by making the theory contextual just locally.

Bell's theorem tells us something more stringent: namely that that won't do the job. One must make the theory contextual even at spacelike separated regions.

Bell's theorem says that the kind of contextuality quantum mechanics requires is not just any type of contextuality, it is non-local contextuality.

There were three key ingredients required for the proof of the Bell-GHSZ theorem:

- Observables under consideration containing a Kochen-Specker contradiction configuration.
- Tensor factorization of Hilbert space, viewed as spacelike separation.
- The non-locality being used in the argument not directly through the observables under consideration (as in the Kochen-Specker proof), but by an eigenstate of those observables. (Note that the non-local observables have to commute for this to work, and this is true for the operators in P_1 in our Kochen-Specker configuration).

Now let us consider the set up of the Conway-Kochen Free Will Theorem:

- (SPIN) This is the somewhat complicated original Kochen-Specker contradiction configuration in three (spin-1) dimensions.
- (MIN) Instead of 3 pairs of qbits and 8 dimensions, it uses two pairs of spin-1 states (9 dimensions). The two pairs are thought of as spacelike separated, which is the way locality enters.
- (TWIN) The state one considers is a spin singlet state of the total spin, and therefore non-local.

The 3-d Kochen-Specker Argument (for completeness)

Consider commuting operators given by S_x^2, S_y^2, S_z^2, the squared spin-1 components along three mutually perpendicular directions x, y, z. These operators satisfy:

$$S_x^2, S_y^2, S_z^2 \in \{0, 1\}, \quad S_x^2 + S_y^2 + S_z^2 = 2 \tag{1}$$

These imply that the values assigned to (S_x^2, S_y^2, S_z^2) must be any of $(1, 0, 1)$, $(1, 1, 0)$, $(0, 1, 1)$. Additionally, for any direction w that is neither collinear nor perpendicular to x in the real space, we have

$$[S_x^2, S_w^2] \neq 0$$

Thus, simultaneous measurement of spin-1 squared components is possible only for the observables corresponding to any 3 perpendicular directions in the real space.

The 3-d Kochen-Specker Argument (continued)

Now consider the measurement of S_m^2 for a general direction $\mathbf{m}$ in the real space.

Assuming non-contextual determinism, we need a function, call it $f(m, |\text{state}\rangle)$ representing the result of measuring S_m^2. This function must take the values $\{1, 0, 1\}$ in some order for $m \in x, y, z$, irrespective of the state $|\rangle$ for any mutually orthogonal set of directions (x, y, z).

One can come up with a configuration of directions which contradict this. (Bell gave an indirect argument for this, Kochen-Specker an year later found a concrete configuration with 117 configurations, there is a fairly symmetric 33 direction configuration due to Perez)

The extra ingredient in the Conway-Kochen theorem is the TWIN state:

$$(S \equiv S_1 + S_2)^2 |\mathrm{TWIN}\rangle = 0 \tag{2}$$

From basic angular momentum facts one can show that $S_{w1}^2 = S_{w2}^2$ on the TWIN state. Note that this TWIN is a simultaneous eigenstate of nonlocal operators S^2 and $S_{w1} + S_{w2}, \forall w$ where w stands for direction in physical 3-space.

With these two ingredients one can come up with a contradiction analogous to the 8-d Bell-GHSZ case. The details do not matter, but-

The idea is that we consider all x, y, z (ie., perp directions) measurements in tensor factor 1, and all directions (w) measurements in factor 2.

Beecause the system is in the TWIN state, the value for S^2_{w2}, whenever w is along one of the x, y, z it should also have the same value. This means that if the results of S_{w2} have to be deterministic, it would have to be a 1-0-1 function, which is impossible by the argument that went into the Kochen-Specker paradox.

In any event –

The point is that the non-local TWIN state enables a Kochen-Specker style contradiction, even though we are assuming non-contextuality only at non-local separations.

This suggests that except in detail, the content of the Conway-Kochen theorem is the same as that of Bell's theorem (the version due to GHSZ).

Thank you!

Happy Birthday Bal!