

NON-INERTIAL FRAMES

IN MINKOWSKI

AND

IN EINSTEIN

SPACE-TIMES

LUCA LUSANNA

SENIOR INFN, FIRENZE

BALFEST 80

CLASSICAL THEORIES → PDE - PREDICTABILITY  
 QUANTUM

↓  
 WELL POSED CAUCHY PROBLEM

↓  
 WELL DEFINED INSTANTANEOUS 3-SPACE FOR CAUCHY DATA

↓  
 CLOCK SYNCHRONIZATION CONVENTION  
 (METROLOGY - TIME : A GIVEN ATOMIC CLOCK)

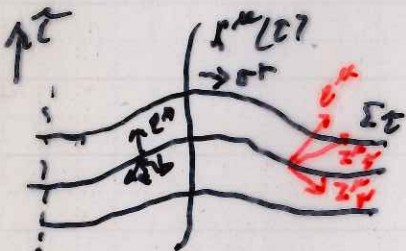
GALILEI SPACETIME - TIME AND 3-SPACE ABSOLUTE

MINKOWSKI SPACETIME - NO NOTION OF 3-SPACE (ONLY CONFORMAL STRUCTURE)  
 INTRINSIC - LIGHT CONE

3+1 POINT OF VIEW

a) WORLDLINE OF A TIMELIKE OBSERVER WITH ATOMIC CLOCK

b) 3+1 SPLITTING WITH A NICE FOLIATION



SCALAR OBSERVER-ADAPTED RADAR 4-COORD.

$$x^\mu \mapsto \sigma^A(x) = (t, \sigma^i)$$

$$\sigma^A \mapsto x^\mu = z^\mu(t, \sigma^i) \text{ EMBEDDING OF 3-SPACES } \Sigma_t$$

→ IT IS A CONVENTION FOR SYNCHRONIZING ALL THE CLOCKS WITH THE CLOCK OF THE OBSERVER

ASYMPTOTIC INERTIAL OBSERVERS

$$\eta_{\mu\nu} = \epsilon(\sigma^i \dots), \epsilon = \pm 1$$

$$\eta_{\mu\nu} \mapsto g_{AB}[z(t, \sigma^i)] = z_A^\mu \eta_{\mu\nu} z_B^\nu \quad z_A^\mu = \frac{\partial z^\mu}{\partial \sigma^A}$$

$g_{AB}$   $z_{Krs}$   
 INDUCED INERTIAL POTENTIALS

$$\frac{\partial z^\mu}{\partial t} = N e^\mu + N^i z^\mu_i \quad \text{LAPSE, SHIFT}$$

$$z_{Krs} = \frac{1}{2N} (N_{[rs]} + N_{s[r} - \partial_t g_{rs})$$

WITH ALBA  
 INT. J. GEOM. METHODS IN  
 PHYS. 7 (2010) 33 and 185  
 0908.0243 and 0245

GLOBAL NON-INERTIAL FRAME CENTERED ON THE OBSERVER

$$N(t, \sigma^i) > 0$$

NO CROSSING OF 3-SPACES (IT HAPPENS WITH FERMI COORD IN 1+3 APPROACH)

$$\epsilon g_{\mu\nu}(t, \sigma^i) > 0$$

NO ROTATING DISK PATHOLOGY (HORIZON PROBLEM:  $\frac{\partial z^\mu}{\partial t}$  BECOMES A NULL VECTOR AT  $WR \rightarrow c$ )

⇒ ONLY DIFFERENTIAL ROTATIONS ALLOWED

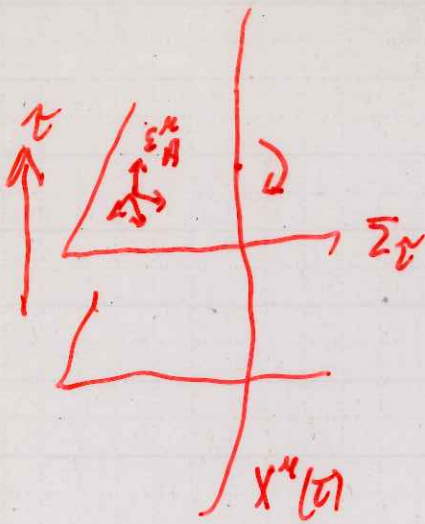
ROLLER

$$\sim \epsilon g_{rs}(t, \sigma^i)$$

WITH 3 NON-DEGENERATE POSITIVE EIGENVALUES

$\Sigma_t \xrightarrow{|\sigma^i| \rightarrow \infty} t$ -INDEPENDENT SPACELIKE HYPERPLANE

# DIFFERENTIALLY ROTATING RADAR 4-COORDINATES



$$Z^M(\sigma, \vec{\sigma}) = X^M(\tau) + \xi_r^M R^r_s(\tau, \vec{\sigma}) \sigma^s$$

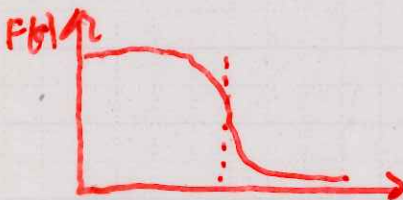
$$R(\tau, \vec{\sigma}) \equiv R(\alpha_i(\tau, \vec{\sigma}))$$

$$\alpha_i(\tau, \vec{\sigma}) \equiv F(\vec{\sigma}) \tilde{\alpha}_i(\tau)$$

NULLER CONDITIONS

$$\left\{ \begin{array}{l} \frac{dF(\vec{\sigma})}{d\sigma} \neq 0 \\ 0 < F(\vec{\sigma}) < \frac{k}{|\vec{\sigma}|} \end{array} \right.$$

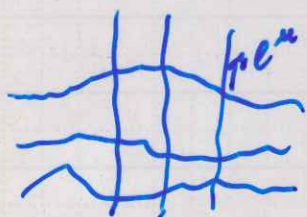
TO SIMULATE RIGID ROTATIONS WITH  $\omega$



$$F(\sigma) = \frac{1}{1 + \frac{\omega^2}{c^2} \sigma^2}$$

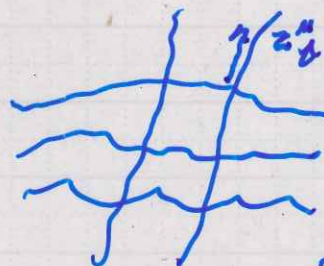
EACH ADMISSIBLE 3+1 SPLITTING HAS 2 ASSOCIATED CONGRUENCES OF TIME-LIKE OBSERVERS

BULERIAN  $u^M(\sigma, \vec{\sigma}) \approx l^M(\sigma, \vec{\sigma})$



VORTICITY  $\approx 0$   
ACCELERATION  $\sim \partial_r \ln(1+a)$   
EXPANSION  $\sim \partial_t k$   
SHEAR  $\tau_{rs}$

ROTATING  $u^M(\sigma, \vec{\sigma}) = \frac{Z^M(\sigma, \vec{\sigma})}{\sqrt{\xi^A \xi_A(\sigma, \vec{\sigma})}}$



NON-SURFACE  
FORMING  
VORTICITY  $\neq 0$

ACCELERATION  
EXPANSION  
SHEAR

# ISOLATED SYSTEMS IN THE REST-FRAME (INSTANT FORM)

STANDARD DESCRIPTION - POINCARÉ GENERATORS  $P^\mu, J^{AB}$  FINITE

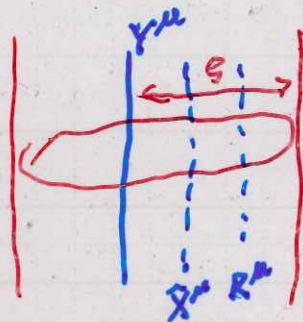
- NON-LOCAL QUANTITIES - THEY KNOW THE WHOLE INSTANTANEOUS 3-SPACE  $P^\mu = \int_{\Sigma_t} d^3x \dots$
- LORENTZ BOOSTS: INTERACTION-DEPENDENT  $[P^\mu, K^\nu] = S^{\mu\nu} P^0 \neq \text{GALILEI BOOSTS}$

$P^\mu, J^{AB} \rightarrow$  3 NOTIONS OF COLLECTIVE VARIABLES

- $\tilde{X}^\mu = (\tilde{X}^0; \tilde{\vec{X}})$  NON-COVARIANT CANONICAL CENTER OF MASS (CENTER OF SPIN)
- $Y^\mu = (\tilde{X}^0; \vec{Y})$  COVARIANT NON-CANONICAL FOKKER-PRYCE CENTER OF INERTIA
- $R^\mu = (\tilde{X}^0; \vec{R} = -\frac{\vec{K}}{\pi c})$  NON-COVARIANT NON-CANONICAL MÖLLER CENTER OF ENERGY

$\rightarrow$  NEWTON CENTER OF MASS

MÖLLER WORLD-TUBE OF NON-COVARIANCE (LORENTZ SIGNATURE)



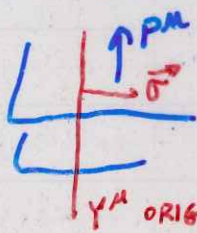
MÖLLER RADIUS

$$S \approx \frac{\sqrt{-W^2}}{P^2 c} = \frac{|\vec{S}|}{\pi c}$$

$S \sim$  COMPTON WAVELENGTH OF ISOLATED SYSTEM  
REMNANT OF ENERGY CONDITIONS OF GENERAL RELATIVITY

## REST-FRAME INSTANT FORM

WIGNER INSTANTANEOUS 3-SPACES



EXACT REST TIME (LORENTZ SCALAR)

$Y^\mu$  ORIGIN OF 3-COORD.

$$Z^\mu(\tau, \vec{\sigma}) = Y^\mu(\tau) + \varepsilon_F^\mu(\vec{h}) \sigma^r$$

$$P^\mu = \pi c (\sqrt{1 + \vec{h}^2}; \vec{h}) = L^\mu_\nu (P, \vec{P}) P^\nu$$

$$\tilde{P}^\mu = \pi c (1; \vec{\sigma}), \pi c = \sqrt{P^2} \quad \downarrow \text{WIGNER BOOST}$$

FOR TITELKUBORBIT

POSITIVE ENERGY PARTICLES

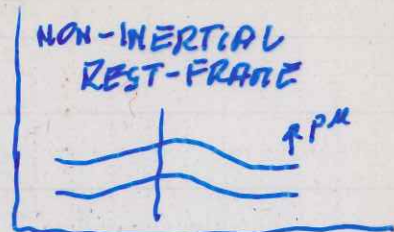
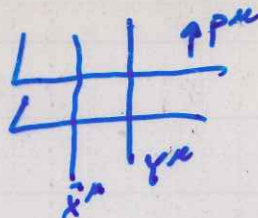
$$X_c^\mu(\tau) = Z^\mu(\tau, \vec{\eta}_c(\tau)) = Y^\mu(\tau) + \varepsilon_F^\mu(\vec{\eta}_c) \eta_c^r(\tau)$$

$\tilde{X}^\mu(\tau), Y^\mu(\tau), R^\mu(\tau)$  KNOWN FUNCTIONS OF  $\tau, \pi, \vec{S}$ , and of the FROZEN

JACOBI DATA  $\vec{Z}, \vec{h}$  OF THE EXTERNAL CENTER OF MASS

$$\tilde{X}_{NW} = \frac{\vec{Z}}{\pi c} \quad \text{NEWTON-WIGNER POSITION}$$

# ISOLATED SYSTEM



- 1) NON-COINVARIANT DECOUPLED EXTERNAL CENTER OF MASS  
(DESCRIBED BY THE FROZEN JACOBI DATA) CARRYING A POLE-DIPOLE STRUCTURE : M INTERNAL MASS,  $\vec{S}$  REST SPIN  
(UNIVERSAL BREAKING OF LORENTZ COVARIANCE BUT DECOUPLED)

EXTERNAL POINCARÉ GENERATORS

$$P^0 = \sqrt{1+h^2} \pi c \sqrt{1+h^2} \quad \vec{P} = \pi c \vec{h}$$

$$\vec{J} = \vec{z} \times \vec{h} + \vec{S} \quad \vec{K} = -\sqrt{1+h^2} \vec{z} + \frac{\vec{S} \times \vec{h}}{1+\sqrt{1+h^2}}$$

- 2) INSIDE THE INSTANTANEOUS WIGNER 3-SPACE THERE ARE THE WIGNER-COHN VARIABLES OF THE ISOLATED SYSTEM (INTERNAL SPACE) AND A UNFAITHFUL INTERNAL REALIZATION OF THE POINCARÉ GROUP

$Z = cT$  LORENTZ-SCALAR REST TIME

$$\left\{ \begin{array}{l} \pi c = c \int d^3\sigma T^{00}(z, \vec{\sigma}) \\ \vec{P} \approx 0 \quad \text{REST-FRAME CONDITION} \\ \vec{J} \approx \vec{S} \\ \vec{K} \approx 0 \quad \text{(INTERACTION-DEPENDENT) ELIMINATION OF THE INTERNAL 3-CENTER OF MASS} \end{array} \right.$$

$$K^i = - \int d^3\sigma \sigma^i T^{i0}(z, \vec{\sigma})$$

POSITIVE-ENERGY SCALAR FREE PARTICLES  $\vec{\eta}_L(t), \vec{K}_L(t)$

$$\pi c = \sum_L \frac{1}{2} \sqrt{m_L^2 c^2 + \vec{K}_L^2}, \quad \vec{P} = \sum_L \vec{K}_L \approx 0, \quad \vec{S} = \sum_L \vec{\eta}_L \times \vec{K}_L, \quad \vec{K} = -\sum_L \vec{\eta}_L \sqrt{m_L^2 c^2 + \vec{K}_L^2} \approx 0$$

$\Rightarrow \vec{\eta}_L, \vec{K}_L, L=1, \dots, N$  FUNCTIONS ONLY OF RELATIVE VARIABLES  $\vec{S}_a, \vec{T}_a, a=1, \dots, N$

J. PHYS. A 40, 9585 (2007) (q=qc/0610200) WITH ALBA, CRATER

2-BODY CASE - ORBIT RECONSTRUCTION

FREE CASE

$$X_L^\mu(t) = Y^\mu(t) + (-)^{L+1} \varepsilon_L^\mu(\vec{h}) \frac{\sqrt{m_L^2 c^2 + \vec{T}_L^2}}{\sum_K \sqrt{m_K^2 c^2 + \vec{T}_K^2}} \vec{S}_{12}(t) \xrightarrow{c \rightarrow \infty} \left\{ \vec{X}(t) + (-)^{L+1} \frac{m_L t + 1}{m_1 + m_2} \vec{T}(t) = \vec{X}_{coll} \right.$$

- COVARIANT WORLDLINES - DERIVED NON-CANONICAL QUANTITIES
  - $\{X_L^\mu, X_S^\nu\} \neq 0$  PREDICTIVE COORDINATES NON-COMMUTATIVE STRUCTURE
  - VIOLATION OF MICROCAUSALITY
- DERIVED  $P_L^\mu$  SATISFY  $P_L^2 = m_L^2 c^2$  ALSO IN PRESENCE OF INTERACTIONS

# THE SIMPLEST INTERACTING 2-BODY SYSTEM

$$\left\{ \begin{aligned} M &= \sqrt{m_1^2 + \vec{k}_1^2 + \phi(\vec{s}^2)} + \sqrt{m_2^2 + \vec{k}_2^2 + \phi(\vec{s}^2)} & \vec{s} &= \vec{\eta}_1 - \vec{\eta}_2 \\ \vec{P} &= \vec{k}_1 + \vec{k}_2 \approx 0 \\ \vec{J} &= \vec{S} = \vec{\eta}_1 \times \vec{k}_1 + \vec{\eta}_2 \times \vec{k}_2 \\ \vec{K} &= -\vec{\eta}_1 \sqrt{m_1^2 + \vec{k}_1^2 + \phi(\vec{s}^2)} - \vec{\eta}_2 \sqrt{m_2^2 + \vec{k}_2^2 + \phi(\vec{s}^2)} \approx 0 \end{aligned} \right.$$

NOW SOME  
N-BODY SYSTEM  
UNDER CONTROL

THE INTERNAL POINCARÉ ALGEBRA IS SATISFIED

$$\Pi \approx M = \sqrt{m_1^2 + \vec{\pi}^2 + \phi(\vec{s}^2)} + \sqrt{m_2^2 + \vec{\pi}^2 + \phi(\vec{s}^2)}$$

$$m_0 = \sqrt{m_1^2 + \vec{\pi}^2} + \sqrt{m_2^2 + \vec{\pi}^2}$$

$$\vec{K} \approx 0$$

$$\Rightarrow \vec{q} \approx \vec{q}(\vec{s}, \vec{\pi}) = \frac{m_1^2 - m_2^2}{2} \left( \frac{1}{m_0^2} - \frac{1}{m^2} \right) \vec{s}$$

$$\Rightarrow \vec{\eta}_L \approx \frac{(-)^{j+1}}{2} \left( 1 - \frac{m_1^2 - m_2^2}{m^2} \right) \vec{s} \xrightarrow{c \rightarrow \infty} (-)^{j+1} \frac{m_i}{m_1 + m_2} \vec{s}$$

HAMILTONIAN FOR THE  
RELATIVE MOTION

EXPLICIT POINCARÉ ALGEBRA  
FOR

MASSLESS PARTICLES

OPEN NANBU STRING

2-LEVEL ATOMS

IN THE PAPER THERE ARE THE ORBITS FOR COULOMB-LIKE POTENTIALS

$$\phi(\vec{s}^2) = -2\mu \frac{e^2}{\sqrt{\vec{s}^2}}$$

N.B. FOR  $\Pi = \sqrt{m_1^2 + \vec{k}_1^2} + \sqrt{m_2^2 + \vec{k}_2^2} + \frac{e^2}{4\pi |\vec{\eta}_1 - \vec{\eta}_2|}$  (REAL COULOMB POTENTIAL)

IT IS NOT KNOWN THE FORM OF THE INTERNAL BOOST  $\vec{K}^0$

(IT IS NOT KNOWN THE LAGRANGIAN, FROM WHICH TO GET THE ENERGY-MOMENTUM TENSOR.)

# ISOLATED SYSTEMS IN MINKOWSKI SPACETIME

TAD

FINITE POINCARÉ GENERATORS  $P^\mu, J^{\mu\nu}$  } NON-LOCAL (THEY KNOW THE WHOLE  $\Sigma_t$ )  
 INTERACTION-DEPENDENT BOOSTS

## A) PARAMETRIZED MINKOWSKI THEORIES

POSS. EN. PARTICLE  $X^\mu(\sigma) = Z^\mu(\sigma, \eta^\nu(\sigma))$   
 KG FIELD  $\hat{\phi}(Z(\sigma, \eta)) = \phi(\sigma, \eta)$

$$S = \int d\sigma d^3\sigma \mathcal{L}(\text{MATTER}, g_{AB}[Z(\sigma, \eta)])$$

→ CONJUGATE MOMENTUM  $S_\mu(\sigma, \eta)$

INVARIANT UNDER FRAME-PRESERVING DIFF  $\sigma \mapsto \beta(\sigma, \eta), \sigma^\mu \mapsto g^\mu(\sigma)$  (PLEBANSKI)

⇒ 4 FIRST CLASS CONSTRAINTS

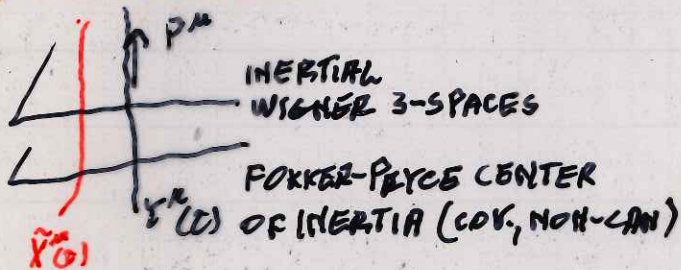
⇒  $Z^\mu(\sigma, \eta)$  GAUGE VARIABLES

$$X_\mu(\sigma, \eta) = S_\mu(\sigma, \eta) - \mathcal{L} \delta \sigma^\mu$$

$$-(L_\mu T^{22} + Z_{\nu\mu} T^{2\nu})(\sigma, \eta) \approx 0$$

↓  
 CHANGE OF CLOCK SYNCHRONIZATION AND/OR OF 3-COORD  $\sigma^\mu$  = GAUGE TRANSF.

## B) INERTIAL REST-FRAME INSTANT FORM OF DYNAMICS

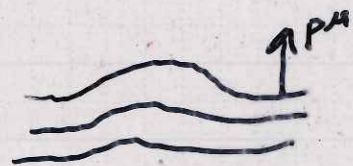


NON-COVARIANT CANONICAL (NEWTON-WIGNER)  
 CENTER OF MASS  $\bar{X}^\mu(\sigma)$

+  
 WIGNER-COVARIANT RELATIVE VARIABLES ON  $\Sigma_t$

⇒ ISOLATED SYSTEM = DECOUPLED NON-COV. (NON-LOCAL ⇒ UNOBSERVABLE) C.O.M. CARRYING A POLE-DIPOLE STRUCTURE  $M, \vec{S}$  (DEPENDING ON RELATIVE VARIABLES INSIDE  $\Sigma_t$ ) WITH EXTERNAL REALIZATION OF POINCARÉ ALGEBRA

## C) NON-INERTIAL REST-FRAME INSTANT FORM



WITH ALBA, CRATER  
 CANAD. J. PHYS 88 (2010) 339 and 425 (0806.2383, 0711.0715)

RELATIVISTIC ATOMIC PHYSICS

POS. EN. PARTICLES + TRANSVERSE EM FIELD (RADIATION GAUGE) - COULOMB POT.

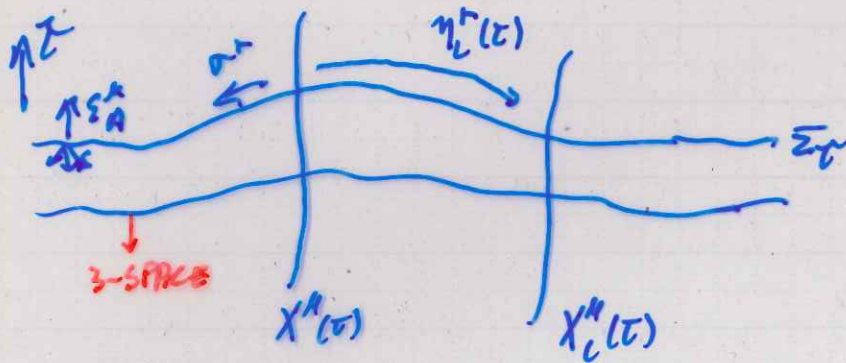
GRASSMANN-VALUED ELECTRIC CHARGES - UV IR REGULARIZATION I

↓  
 NO SELF-ENERGIES (RADIATIVE CORRECTIONS) - HAT. EXPRESSION OF LIENARD-WIECHERT SOLUTION AND ASYMPTOTIC LAMBOR FORMULA

CAN. TO COULOMB-DRESSED PARTICLES WITH (COULOMB + DARWIN POTENTIALS (SALPETER FOR SPINNING P.))  
 TRANS. + FREE RADIATION FIELD (NO CLAS. HARG TH)  
 IN REST-FRAME

⇒ RELATIVISTIC BOUND STATES (TIL NOW DARWIN FROM BETHE-SALPETER EQ)  
 RELATIVISTIC KINETIC THEORY AND RELATIVISTIC MICROCANONICAL ENSEMBLE

# POSITIVE ENERGY SCALAR PARTICLES



$$X^\mu \mapsto \sigma^A = (t, \sigma^r)$$

$$\sigma^A \mapsto X^\mu = Z^\mu(t, \sigma^r)$$

$$X^\mu_L(t) = Z^\mu(t, \eta^r_L(t))$$

$\eta^r_L(t), K_{Lr}(t)$  CANONICAL VARIABLES

WORLD-LINES  $X^\mu_L(t), P^\mu_L(t)$  DERIVED VARIABLES  $P_L^z = \epsilon m_L c^2 z$

$$\{X^\mu_L(t), X^\nu_L(t)\} \neq 0$$

PRELICTIVE 4-COORDINATES  
NON-COMMUTATIVE STRUCTURE

$$m_L \sqrt{\dot{x}_L^2(t)} \mapsto m_L \sqrt{g_{00}(t, \eta^r_L(t)) + 2g_{0r}(t, \eta^r_L(t)) \dot{\eta}^r_L(t) + g_{rs}(t, \eta^r_L(t)) \dot{\eta}^r_L(t) \dot{\eta}^s_L(t)}$$

SR -  $Z^\mu(t, \sigma^r)$  MÖLLER ADMISSIBLE EMBEDDING - 3-SPACE = GAUGE

GR -  $Z^\mu(t, \sigma^r) = X^\mu(t) + \epsilon \sigma^r$  - 3-SPACE  $\Sigma_t$  DYNAMICALLY DETERMINED MODULO GAUGE VARIABLES

GRASSMANN-VALUED ELECTRIC CHARGES  $Q_L^z \neq 0, Q_L Q_J = Q_J Q_L \neq 0 \quad L \neq J$

REGULARIZATION OF EM SELF-ENERGIES

GRASSMANN-VALUED SIGN OF ENERGY  $\eta_L m_L, \eta_L^2 \neq 0, \eta_L \eta_J = \eta_J \eta_L \neq 0 \quad L \neq J$

REGULARIZATION OF GRAVITATIONAL SELF-ENERGIES

THEN GRASSMANN  $\mapsto$  CLIFFORD 2-LEVEL SYSTEM



# LORENTZ SIGNATURE OF SPACETIME AND POINCARÉ GROUP

NON-LOCALITY AND NON-MEASURABILITY OF REL. CENTER OF MASS  
(NON-COVARIANT BUT DECOUPLED)

PREFERRED  $\vec{h}$ -BASIS

COR PLANE WAVE WITH FIXED MOMENTUM  
LINES IN SCATTERING THEORY

SPATIAL NON-SEPARABILITY

$$H \approx H_{\text{com}} \otimes H_{\text{rel}} \neq H_1 \otimes H_2 \otimes \dots$$

↓  
ENTANGLEMENT

$$|\vec{h}_{\text{com}}\rangle \otimes \sum_{\alpha} \beta_{\alpha}(\vec{s}_{\alpha}) |\alpha, \vec{s}_{\alpha}\rangle$$

REL NON-LOCALITY WITH DIFFERENT  
ORIGIN FROM STANDARD ~~REL~~ NON-REL  
QUANTUM NON-LOCALITY (BELL INEQUALITIES,  
EPR, ...)

DECOHERENCE

ROBUST POSITIONAL POINTER  
BASES

IN A DIAGONAL REDUCED  
DENSITY MATRIX

ATOMIC PHYSICS - EM FIELD

⇒ BEYOND GALILEI SPACETIME

REFORMULATE NON-REL ENTANGLEMENT

IN THE  $H_{\text{com}} \otimes H_{\text{rel}}$  PRESENTATION

STANDARD APPROACH  $H_1 \otimes H_2 \dots$

ONLY WITH MANY COMPONENTS WHEN  
THE REST-FRAME CONDITIONS ARE  
NEGLECTIBLE!

PROBLEM OF LOCALIZABILITY OF WORLDLINES

- REL - NEWTON-WIGNER NON SELF-ADJOINT (FLETCHING, BUTTERFIELD, ...)  
ONLY SYMMETRIC (KEMP, ...)
- ⇒ BAD LOCALIZABILITY
- NON-REL - POIN, UNSHARP POSITIONS FROM WIGNER-ARAKI-YONESA TH  
(BUSCH, ... ) - ONLY RELATIVE VARIABLES  
MEASURABLE (DUNNINGHAM, ...)

FOUNDATIONAL PROBLEMS OF QM

REFORMULATION IN RQT

EMERGENCE OF CLASSICAL WORLD

ALSO NEWTON C.O.M.  
NOT MEASURABLE

THEORY OF MEASUREMENT



# ADM TETRAD GRAVITY WITHOUT KILLING SYMMETRIES



$$E_A^{(K)} = L^{(K)}_{(a)}(\varphi_{cal}) \begin{pmatrix} N=1+n \\ 0 \end{pmatrix} + \sum_a^{1,2,3} L^{(K)}_{(a)}(\varphi_{cal}) \begin{pmatrix} n_{cal} \\ 3e_{cal}^r \end{pmatrix}$$

LAPSE
SHIFT

CONSTRAINT
↑ HIGHER BOOST FOR TIMELIKE ORBITS
CONSTRAINTS ON Σ\_t

$$g_{tt} = \epsilon \left[ (1+n)^2 - \sum_a n_{cal}^2 \right], \quad g_{tr} = -\epsilon n_{cal}^3 e_{cal}^r, \quad g_{rs} = -\epsilon g_{rs} = -\epsilon \sum_a^3 e_{cal}^r e_{cal}^s$$

16 FIELDS + 16 MOMENTA

CANONICAL BASIS ADAPTED TO 7 OF THE 14 FIRST CLASS CONSTRAINTS

$\varphi_{cal}$	$N=1+n$	$n_{cal}$	${}^3e_{cal}^r$
$\approx 0$	$\approx 0$	$\approx 0$	${}^3\pi_{cal}^r$

14 GAUGE VARIABLES = INERTIAL EFFECTS

2+2 PHYSICAL D.O.F. = 3 TIDAL EFFECTS

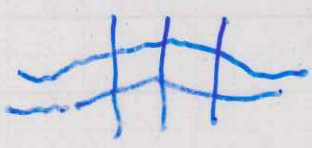
CANONICAL TRANSFORMATION TO THE YORK CANONICAL BASIS ADAPTED TO 10 CONSTRAINTS (NOT SUPER-HAM AND SUPER-MOM)

$\varphi_{cal}$	$d_{cal}$	$N=1+n$	$\tilde{n}_{cal}$	$g^i_j$	$\tilde{\Phi}$	$R_{\tilde{a}}$
$\approx 0$	$\approx 0$	$\approx 0$	$\approx 0$	$\pi_{\tilde{L}}$	$\pi_{\tilde{\Phi}} = \frac{c^3}{128G} {}^3K$	$\pi_{\tilde{a}}$
$o(3)_1$ GENERATORS				FROM SUPER-MOM C.		4-SCALARS
				FROM SUPER-HAM C.		$\tilde{a}_{\tilde{a}}$ TIDAL YORK TIME

$$\begin{cases} {}^3e_{cal}^r = R_{(a)(b)}(d_{cal})^3 \bar{e}_{(b)r} & {}^3\bar{e}_{(a)r} = \tilde{\Phi}^{1/3} V_{ra}(\partial^i) e^{\tilde{a}} \delta_{\tilde{a}a} R_{\tilde{a}} \\ {}^3g_{rs} \approx \sum_a {}^3e_{cal}^r e_{cal}^s = \tilde{\Phi}^{2/3} \sum_a V_{ra}(\partial^i) V_{sa}(\partial^i) e^{\tilde{a}} \delta_{\tilde{a}a} R_{\tilde{a}} & \text{ORTHOGONAL MATRIX} \\ g_{tt} = \epsilon \left[ (1+n)^2 - \sum_a \tilde{n}_{cal}^2 \right] & g_{tr} = -\epsilon \tilde{n}_{cal}^3 \bar{e}_{cal}^r \end{cases}$$

$\tilde{\Phi} = \sqrt{\det {}^3g_{rs}}$   
= VOLUME IN Σ\_t

CONGRUENCE OF EULERIAN OBSERVERS



VORTICITY = 0      EXPANSION  $\theta = -\epsilon {}^3K$

SHEAR  $\sigma_{(a)(b)}$   $[\sigma_{(a)(b)}]_{a \neq b} \leftrightarrow \pi_{\tilde{L}}^a$  ;  $\sigma_{(a)(a)} \leftrightarrow \pi_{\tilde{a}}^a$

ACCELERATION  ${}^3a_r = \partial_t \ln(1+n)$

$$\tilde{\Phi} = \sqrt{\det {}^3g_{rs}}$$

3-VOLUME ELEMENT FROM LICHNEROWICZ EQ.

YORK TIME

${}^3K$

INERTIAL MOMENTUM - REMNANT IN GR OF THE SPECIAL RELATIVISTIC GAUGE FREEDOM IN CLOCK SYNCHRONIZATION

$${}^3K_{rs} = \tilde{\Phi}^{-1/3} \sum_{ab} \left( \frac{1}{2} {}^3K \delta_{ab} + \sigma_{(a)(b)} \right) V_{ra}(\partial^i) V_{sb}(\partial^i) e^{\tilde{\Sigma}_a(\partial^i a + \tilde{K}_{ab})} R_{\tilde{e}}$$

3-SPACE DYNAMICALLY DETERMINED EXCEPT FOR  ${}^3K$

$\tilde{\Sigma}_0$

GRAVITOMAGNETISM

GAUGE FIXING -  $\partial^i$  - FIXATION 3-COORD ON  $\tilde{\Sigma}_0$  - IMPLIES EQS. FOR  $\tilde{\pi}_{(a)}$

${}^3K$  - FORM OF  $\tilde{\Sigma}_0$  AS 3-SUBMANIFOLD - IMPLIES EQ FOR  $N=1$  TH

SCHWINGER TIME GAUGE

$$\psi_{(a)}(\Sigma_{i0^+}) \approx 0, \quad d_{(a)}(\Sigma_{i0^+}) \approx 0$$

FIXATION OF  $\partial^i(\partial^i)$  GAUGE FREEDOM

WEAK ADM ENERGY (INVARIANT MASS OF 3-UNIVERSE)

$$\frac{1}{c} \tilde{E}_{ADM} = \int d^3\sigma \left[ \frac{4\pi G}{c^3} \tilde{\Phi}^{-1} \sum_a \tilde{\pi}_a^2 + \frac{c^3}{16\pi G} \tilde{\Phi} \sum_{a \neq b} \sigma_{(a)(b)}^2 [{}^3K] - \frac{c^3}{4\pi G} \tilde{\Phi} ({}^3K)^2 - \frac{c^3}{16\pi G} \psi + m \right] (\Sigma_{i0^+})$$

POTENTIAL
INERTIAL POTENTIAL

TIDAL KINETIC TERM

GRAVITOMAGNETISM

GAUGE NEGATIVE KINETIC TERM

A) SUPER-HAM AND -MOM CONSTRAINTS - ELLIPTIC EQS

B) 4 CONTRACTED BIRNCHM IDENTITIES  $\partial_b \tilde{\Phi} = \dots, \partial_b \sigma_{(a)(b)} = \dots$

C) 4 EQS FOR PRIMARY GAUGE VARIABLES  $\left\{ \begin{array}{l} \partial_b {}^3K = \dots \text{ (RAYCHAUDHURI)} \\ \partial_b \partial^b = \dots \end{array} \right.$

D) 4 HYPERBOLIC EQS FOR TIDAL VARIABLES

E) HAMILTON EQS FOR MATTER

12 ADM EQS

$$\partial_t {}^3g_{rs} = \dots$$

$$\partial_t {}^3K_{rs} = \dots$$

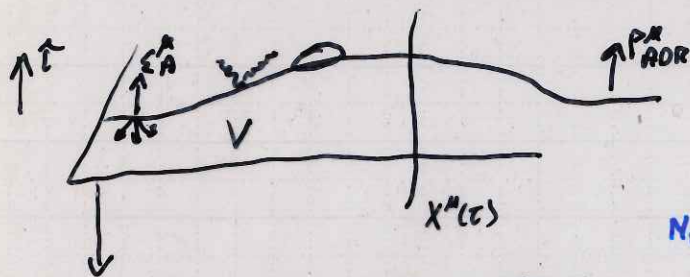
FAMILY OF NON-HARMONIC 3-ORTHOGONAL GAUGES

$$\partial^i(\partial^i) \approx 0 \left\{ \begin{array}{l} {}^3K \text{ FREE} \\ g_{rs} = h_r s_{rs} \end{array} \right.$$

# HAMILTONIAN POST-NEWTONIAN EXPANSION (WEAK FIELD APPROXIMATION)

IN THE FAMILY OF NON-HARMONIC 3-ORTHOGONAL GAUGES  ${}^3K_{(0)}(t, \vec{x})$  FREE

ASYMPTOTIC BACKGROUND - PINK. METRIC AT SPATIAL  $\infty$  FOR  $\Delta, \square, \dots$



ASYMPTOTIC EUCLIDEAN 3-SPACE  
(IN HARMONIC GAUGES IT IS USED IN THE BULK)

$$R_{\bar{a}}(\xi, \sigma^{\bar{a}}) = O(\xi^2) \quad \text{GRAVITATIONAL WAVES (GW)}$$

$${}^4g_{AB}(\xi, \sigma^{\bar{a}}) = {}^4\eta_{AB}^{(ADM)} + {}^4h_{(0)AB}(\xi, \sigma^{\bar{a}}) + O(\xi^2)$$

NON-INERTIAL REST FRAME OF 3-UNIVERSE

$$\tilde{\Phi} = 1 + 6\phi_{(0)} + O(\xi^2)$$

$$\epsilon^4 g_{00} = 1 + 2\psi_{(0)} + O(\xi^2), \quad -\epsilon^4 g_{0i} = \bar{\eta}_{(0)i} + O(\xi^2)$$

$$-\epsilon^4 g_{rs} = {}^3g_{rs} = \delta_{rs} [1 + 2(\Gamma_r^{(0)} + 2\phi_{(0)})] + O(\xi^2)$$

$$\Gamma_r^{(0)} = \sum_{\bar{a}} \kappa_{\bar{a}r} R_{\bar{a}}$$

GRAVITATIONAL WAVES

ULTRAVIOLET CUTOFF  $m$  ON PATTAR TO AVOID POST-NEWTONIAN EXPANSIONS

$$\frac{m_L}{m}, \frac{\vec{K}_L}{m}, \frac{\vec{\pi}}{m} = O(\xi) \Rightarrow T^{AB} = T_{(0)}^{AB} + O(\xi^2)$$

THE APPROXIMATION IS NOT RELIABLE AT DISTANCES FROM POINT PARTICLES  
LESS THAN THE GRAVITATIONAL RADIUS  $\ell_m = \frac{mG}{c^2} \approx 10^{-29} m$  OF CUTOFF

WEAK ADM POINCARÉ GENERATORS  $\approx$  PATTAR TINKOWSKI POINCARÉ GEN.  $+ O(\xi^2)$

ISOLATED 3-UNIVERSE - DECOUPLED CENTER OF MASS +  
RELATIVE VARIABLES

LIKE IN NON-INERTIAL REST-FRAMES OF TINKOWSKI SP.

$$\begin{cases} \frac{1}{2} \hat{E}_{ADM} = M_{(0)} c + \frac{1}{2} \hat{E}_{ADR(2)} \\ \hat{P}_{ADM}^r = P_{(0)}^r + P_{(2)}^r \approx 0 \\ \hat{J}_{ADM}^{rs} = J_{(0)}^{rs} + J_{(2)}^{rs} \\ \hat{K}_{ADM}^r = K_{(0)}^r + K_{(2)}^r \approx 0 \end{cases}$$

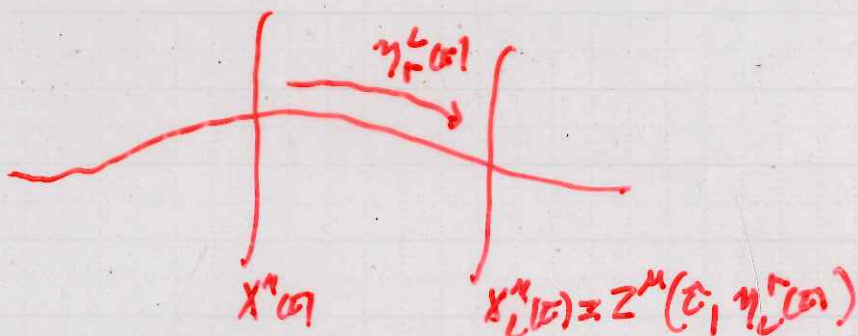
$$\begin{cases} \pi_{(0)}^r = \sum_i \eta_i \sqrt{m_i^2 c^2 + \vec{K}_i^2} \\ P_{(0)}^r = \sum_i \eta_i K_{iL}^r \\ J_{(0)}^{rs} = \sum_i \eta_i (\eta_i^r K_{iL}^s - \eta_i^s K_{iL}^r) \\ K_{(0)}^r = - \sum_i \eta_i \eta_i^r \sqrt{m_i^2 c^2 + \vec{K}_i^2} \end{cases}$$

HPT EXPANSION  
IN NON-HARMONIC  
3-ORTHOGONAL  
SCHWINGER TIRE GAUGES

- ALL CONSTRAINTS SOLVED
- RETARDED ~~GW~~ NON-HARMONIC GW
- RELATIVISTIC PARS QUADROPLE EMISSION FORMULA
- BALANCE IN THE EMISSION OF ENERGY, POTENTIAL, ANGULAR POTENTIAL FROM CONSTANCY OF ADM POINCARÉ GENERATORS WITHOUT GRAVITATIONAL SELF-FORCES

PARTICLES

$$\eta_i^\mu(t), K_{\mu\nu}(t)$$



GRASSMANN-VALUED SIGN OF ENERGY  $\eta_i$  ( $\eta_i^2 = 0, \eta_i \eta_j = \eta_i \eta_j$ )

TO REGULARIZE GRAVITATIONAL SELF-ENERGY

ELECTRIC CHARGE  $q_i$  ( $q_i^2 = 0, q_i q_j = q_i q_j$ )

TO REGULARIZE EM SELF-ENERGIES

HPT EQS OF MOTION

$$m_i \eta_i \frac{d^2 \eta_i^\mu(t)}{dt^2} = \eta_i F_L^\mu(t)$$

↳ TERM IN  $\partial_t^2 \tilde{3}K$  FROM LAPSE

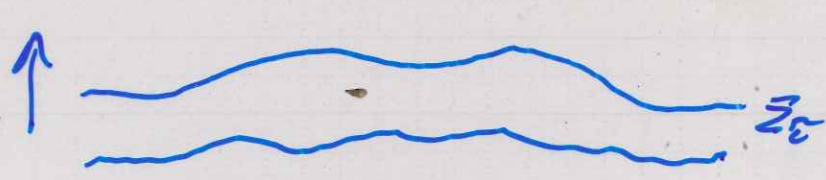
$$\eta_i(t) = \tilde{\eta}_i(t) - q_i \tilde{3}K$$

PN LIMIT  $\tilde{t} = ct \quad \eta_i^\mu(t) = \tilde{\eta}_i^\mu(t)$

$$\tilde{3}K = \frac{1}{8} \tilde{3}K$$

$$m_i \eta_i \frac{d^2 \tilde{\eta}_i^\mu(t)}{dt^2} = \eta_i \tilde{F}_L^\mu(t) = \eta_i \left[ \begin{array}{l} \text{KEPLER} \\ \text{0PN} \end{array} + \frac{1}{2} \frac{d \tilde{\eta}_i^\mu(t)}{dt} \partial_t^2 \tilde{3}K(\partial_i \tilde{\eta}_i^\mu(t)) + \begin{array}{l} \text{0.5 PN} \\ \text{BINARIES} \\ \text{(DARBOUR-DEWELLE)} \end{array} + \dots \right]$$

# DARK MATTER AS A RELATIVISTIC INERTIAL EFFECT



3-SPACE AS 3-SUBMANIFOLD OF SPACETIME

$\Sigma_E$  INTRINSIC CURVATURE SMALL  
EXTRINSIC CURVATURE WITH  $\Sigma_K(t, \vec{x})$ , YORK TIME, ARBITRARY

HPT LAPSE  $1 + \eta(\sigma, \vec{x}) = 1 + \text{NEWTON POTENTIAL} + \text{REL. CORRECTIONS} - \frac{1}{2} \Sigma_K(\sigma, \vec{x})$

THE TIME VARIATION OF 3-SPACE AS A 3-SUBMANIFOLD (I.E. OF CLOCK SYNCHRONIZATION CONVENTION) IS EQUIVALENT TO A POTENTIAL, FOR INSTANCE IT MAY COINCIDE WITH THE YUKAWA-LIKE POTENTIAL OF  $f(R)$  THEORIES, USED TO SIMULATE DARK MATTER  
MATTIA VILLANI - ARXIV 1502.06801

EINSTEIN GR  $M_{GR} \approx M_{IN}$    
 $\left\{ \begin{array}{l} M_{IN} \text{ BARYONIC FROM SPECTROSCOPY} \\ M_{GR} \text{ FROM GALAXY DYNAMICS} \end{array} \right.$

BUT 3-SPACE IS NOT EUCLIDEAN (EQUIVALENCE PRINCIPLE)

IS THE NEWTON VERSION OF  $M_{GR} \approx M_{IN}$  VALID IN THE PN LIMIT OF GR?

NO - O.SPN EROF MOTION OF MATTER CONTAIN TERMS IN  $\frac{1}{2} \Sigma_K(\sigma, \vec{x})$

## SIGNATURES OF DARK MATTER

1) GALAXY PASSES FROM VIRIAL THEOREM  $\Pi_{GR} \approx \frac{A}{G} \langle \vec{v}^2 \rangle \approx M_{IN} + \Pi_{DM}(\Sigma_K)$

2) GALAXY MASS FROM WEAK GRAVITATIONAL LENSING  
EINSTEIN RADIUS  $\sqrt{\frac{4G M_{GR} d}{c^2}}$  (FERMAT PRINC.)  $\Pi_{GR} \approx M_{IN} + \Pi_{DM}(\Sigma_K)$

## 3) ROTATION CURVES OF GALAXIES

$\frac{d^2}{dt^2} (\vec{r}_O + \vec{r}_{CM}) = -GM_{IN} \frac{\vec{r}_O}{|\vec{r}_O|^3} - \frac{A}{G} \frac{d\vec{r}_O}{dt} \gamma(\Sigma_K)$   $\frac{12}{11} \approx 1$   
 KEPLER

CIRCULAR ORBITS  $\vec{r}_O \approx r_O \hat{u}(\phi)$   $\frac{d}{dt} (\vec{r}_O + \vec{r}_{CM}) \approx \sqrt{\frac{G M_{GR}(t, r_O)}{r_O}} \hat{n}(t, r_O)$

$M_{GR} = M_{IN} [1 - \frac{3A}{G} A(\Sigma_K)] \approx M_{IN} + \Pi_{DM}(\Sigma_K)$

NOT  $\Pi_{DM}$ , NOT  $f(R)$ , NOT WIMP 3K IN VOIDS ?

# 0.5 PN EQUATIONS OF MOTION FOR $\vec{\eta}_L(t)$ + PN SOLUTION FOR 4-METRIC

## 1) GALAXY MASSES FROM VIRIAL THEOREM

MEASURED GRAV. MASS  $M_{GR} = \frac{R}{G} \langle v^2 \rangle = M_{IN} + M_{DM}({}^3K_{(u)})$   
↳ BARYONIC (SPECTROSCOPY)

## 2) GALAXY MASS FROM WEAK GRAVITATIONAL LENSING

EINSTEIN RADIUS  $\sqrt{\frac{4G M_{GR}^2}{c^2} \frac{d}{d'}}$  (FERMAT PRINCIPLE REFRACTION INDEX)  
 $M_{GR} = M_{IN} + M_{DM}({}^3K_{(u)})$   
BARYONIC

## 3) ROTATION CURVES OF GALAXIES

RELATIVE VARIABLES  $\frac{d^2}{dt^2} (\vec{r}_{(u)} + \vec{r}_{(s)}) = -G \pi \frac{\vec{r}_{(u)}}{|\vec{r}_{(u)}|^3} - \frac{\kappa}{c} \frac{d\vec{r}_{(u)}}{dt} \gamma({}^3K_{(u)})$   
KEPLER  $\pi$ -BARYONIC MASS  $\equiv M_{IN}$

CIRCULAR ORBITS  $\vec{r}_{(u)} = r_0 \hat{u}(t)$

$\frac{d}{dt} (\vec{r}_{(u)} + \vec{r}_{(s)})^{(t)} = \sqrt{\frac{G M_{EFF}(r_0)}{r_0}} \hat{n}(t, r_0), \hat{n}^2 = 1$

$M_{GR} = \cancel{M_{GR}} M_{EFF} = M_{IN} \left(1 - \frac{\kappa}{c} A({}^3K_{(u)})\right) = M_{IN} + M_{DM}({}^3K_{(u)})$

SINCE 3-SPACE IS NOT EUCLIDEAN

NEWTON  $M_{IN} \approx M_{GR}$  LAW IS NOT VALID

$M_{GR} \stackrel{?}{=} M_{IN} + M_{DM}({}^3K_{(u)})$

↳ DARK MATTER AS A RELATIVISTIC INERTIAL EFFECT

FIT  ${}^3K_{(u)} = \frac{1}{\Delta} {}^3K_{(u)}$  TO DATA

NOT  $\rho_{DM}$   
NOT  $f(R)$   
NOT WIMP

BUT TO FIND  ${}^3K_{(u)} = \Delta {}^3K_{(u)}$  DATA ON VOIDS NEEDED

BUT  $Q_2 {}^3K$

$\uparrow$   
 ${}^3K_{(u)}$

$\uparrow$   
 ${}^3K_{(u)}$



# GAUGE PROBLEM IN GR VERSUS PETROLOGY

4-DIFF - GAUGE FREEDOM IN THE CHOICE OF 4-COORDINATES

4-SCALARS - GAUGE INVARIANTS

ON-SHELL AGREEMENT (INVERSE LEGENDRE, NONKRIEF ...)

HAMILTONIAN GAUGE GROUP - DIRAC OBSERVABLES (3-SCALARS)

GAUGE FREEDOM IN 3-COORD. AND  ${}^3K$  OF 3-SPACE

TIDAL VARIABLES  
2+2 (3-SCALARS)

→ 4 EIGENVALUES OF WEYL TENSOR (4-SCALARS)

LAPSE - 4-SCALAR

EM FIELD GRAVITOMAGNETISM (SHIFT)

TETRAD-DEPENDENT 4-SCALARS (NEWTON-PENROSE)

MACROSCOPIC MATTER - ITS DESCRIPTION IS COORDINATE-DEPENDENT

IT DEPENDS ON THE PETROLOGICAL CONVENTIONS OF ATOMIC PHYSICS, NASA ENGINEERS, ASTRONOMERS

EARTH - ITRS (JERS 2003)

GPS, GCRS (IAU 2000)

BCRS (IAU 2000) (SOLAR SYSTEM)

MILKY WAY

ICRS

$g^4 \approx 0$   
 ${}^3K = O(c^2)$   
REL, HARMONIC GAUGES

SATELLITES

SPACECRAFTS, BODIES IN SOLAR SYSTEM

GAIA

NON-REL EUCLIDEAN 3-SPACE  $\Sigma_t$  (CMB, FLRW COSMOLOGY)

AGAINST EQUIVALENCE PRINCIPLE

NECESSITY OF POST-FLINKOWRIAN EXTENSION OF ICRS

TO TAKE INTO ACCOUNT THE EXTRINSIC 3-CURVATURE OF NON-EUCLIDEAN 3-SPACES

BEGIN WITH MILKY WAY WHEN DATA FROM GAIA AVAILABLE

WHICH VALUE OF THE YORK TIME  ${}^3K(\Sigma_t, \sigma^a)$ , I.E. WHICH CLOCK SYNCHRONIZATION CONVENTION, GIVES RISE TO A 3-SPACE IN ACCORD WITH RELATIVISTIC PETROLOGY IN THE SOLAR SYSTEM AND ABLE TO CLARIFY THE DARK SIDE OF UNIVERSE

# OPEN PROBLEMS

NEWTON AND RELATIVISTIC CENTER OF MASS - NON-MEASURABLE

QUANTIZATION? WAVE FUNCTION OF UNIVERSE


QUANTUM METROLOGY - ONLY RELATIVE VARIABLES  
FOUR SPACE ONLY OF RELATIVE NOTIONS

QUANTIZATION IN NON-INERTIAL FRAMES

DEFINITION OF PARTICLES, KILLING VECTORS AND  
BOGOLIUBOV TRANSFORMATIONS

ACCELERATED OBSERVERS: KINEMATICAL OR DYNAMICAL?

UNRUH-DEWITT DETECTORS - RINDLER OBSERVERS

TORRE-VARADARAJAN NO-GO THEOREM - 

- TOMONAGA-SCHWINGER BREAKS UNITARY EVOLUTION  
OF MASSIVE KG FIELD: THE BOGOLIUBOV TRANSFORMATION

$\mathcal{Z}_1 \rightarrow \mathcal{Z}_2$  IS NOT HILBERT-SCHMIDT

⋮

GR - ALSO DARK ENERGY MAY BE A RELATIVISTIC  
INERTIAL EFFECT

FR, RW SPACETIMES - KILLING SYMMETRIES (ISOTROPY, HOMOGENEITY)

IMPLY  $\mathcal{K}_{(t)} = -\frac{\dot{a}(t)}{a(t)} x = -\frac{1}{3} H$  HUBBLE CONSTANT

WITHOUT KILLING SYMMETRIES (SZEKERES, ...)  $\rightarrow$

$\rightarrow \mathcal{K}$  GAUGE VARIABLE

....



BALFEST GO!



H. CRATER and L. LUSANNA - ON RELATIVISTIC ENTANGLEMENT,  
AND LOCALIZATION OF PARTICLES AND THEIR  
COMPARISON WITH THE NON-RELATIVISTIC THEORY  
~~REPRESENTATION~~ arXiv 1306.6524

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arXiv 1207.1248

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arXiv 1310.4465

L. LUSANNA - FROM RELATIVISTIC MECHANICS  
TOWARDS RELATIVISTIC STATISTICAL MECHANICS  
- ENTROPY 2017, 19(9) 436 (SPECIAL ISSUE: ADVANCES  
IN RELATIVISTIC STATISTICAL MECHANICS)

D. ALBA, L. LUSANNA

## THE EINSTEIN-MAXWELL-PARTICLE SYSTEM IN THE YORK CANONICAL BASIS OF ADM TETRAD GRAVITY

I) THE EQUATIONS OF MOTION IN ARBITRARY SCHWINGER TIME GAUGES  
CANAD. J. PHYS. 90 (2012) 1017 (ARXIV 0907.4037)

II) THE WEAK FIELD APPROXIMATION IN THE 3-ORTHOGONAL GAUGES AND HAMILTONIAN POST-MINKOWSKIAN GRAVITY: THE N-BODY PROBLEM AND GRAVITATIONAL WAVES WITH ASYMPTOTIC BACKGROUND  
CANAD. J. PHYS. 90 (2012) 1077 (ARXIV 1003.5143)

III) THE POST-MINKOWSKIAN N-BODY PROBLEM, ITS POST-NEWTONIAN LIMIT IN NON-HARMONIC 3-ORTHOGONAL GAUGES AND DARK MATTER AS AN INERTIAL EFFECT  
CANAD. J. PHYS. 90 (2012) 1131 (ARXIV 1009.1794)

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IN-TECH E-BOOK ROBERT METROLOGY CONCERNS 2012

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REVIEWS INSPIRE ARXIV - TRENDS IN ROBERT METROLOGY 2017

L. LUSANNA, CANONICAL ADM TETRAD GRAVITY: FROM METROLOGICAL INERTIAL GAUGE VARIABLES TO DYNAMICAL TIDAL DIRAC OBSERVABLES

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FROM CLOCK SYNCHRONIZATION TO DARK MATTER AS A RELATIVISTIC INERTIAL EFFECT

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P. VILLANI - CONSTRAINTS ON ADM TETRAD GRAVITY PARAMETER SPACE FROM SZ STAR IN THE CENTER OF GALAXY AND FROM THE SOLAR SYSTEM (ARXIV 1502.06301)

P. VILLANI AND L. LUSANNA - RIEMANN AND WEYL TENSORS IN THE YORK CANONICAL BASIS  
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