Functional Formulation of Wilson's RG and Holographic RG

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Outline

1. Introduction and Motivation
2. Wilson's Exact RG
3. Obtaining AdS Action
4. Green Function
5. Non Trivial Fixed Point CFT
6. Conclusions

Holography

This talk is about an attempt to understand AdS/CFT or Holography in terms of Wilsonian RG.
Based on arXiv:1706.03371 (Nucl.Phys. B) with Hidenori Sonoda (Kobe University).

Holography

- Holography is a conjectured correspondence between a $d+1$-dimensional (gravity) bulk theory and a d-dimensional boundary field theory.
- Lot of evidence by now.
- Main Feature (for our purposes): The extra dimension of the bulk corresponds to the scale of the boundary field theory.

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- Duality between boundary conformal field theory in d-dimensional flat space and bulk gravity theory in AdS_{d+1}
- AdS metric is
$$ds^2 = \frac{dz^2 + dx^i dx^i}{z^2}$$
Boundary is placed at $z = \epsilon$.
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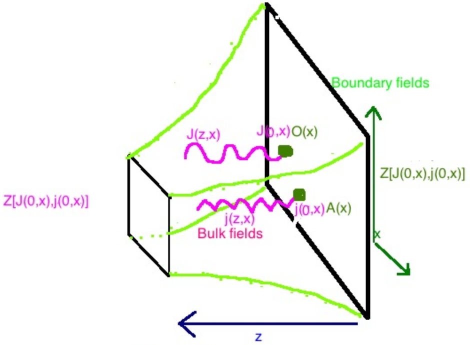
Mathematical Statement of Duality

- Equality of generating functionals: (x - coordinates of boundary and z is radial coordinate of AdS)
- $$Z[J_0]_{bulk} = \int_{J(x,\epsilon)=J_0(x)} \mathcal{D}\, \underbrace{J(x,z)}_{bulk\ \ field}\, e^{-S_{gravity}[J(x,z)]}$$

$$= \int \underbrace{\mathcal{D}\, A(x)}_{boundary\ \ field}\ e^{-S_{field\ \ theory}[A]+\int_{z=\epsilon} dx\ J_0(x)O[A]} = Z[J_0]_{boundary}$$



Holography : Z[J(0,x),j(0,x)] = Z[J(0,x),j(0,x)]

AdS Action

- The AdS action for a scalar field

$$S_{gravity}[J] =$$

$$\int_{z=\epsilon}^{z=\infty} dz \int d^d x \, z^{-d+1}[(\partial_z J \partial_z J + \partial_i J \partial_i J) + \frac{1}{z^2} m^2 J^2 + ...$$

- Semiclassical Prescription: Solve the EOM with boundary conditions at $z = \epsilon$ and $z = \infty$ and evaluate (on-shell) action to get Green function $G(p, \epsilon)$ of the boundary theory:

$$Z[J_0] = e^{-\frac{1}{2}\int_p J_0(p) J_0(-p) G(p,\epsilon)}$$

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- Two steps: 1. Obtain a functional integral representation of Wilson's ERG evolution operator.
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Polchinski's Equation

- Step1: Consider a zero dimensional field variable x (x is going to become a field variable in d dimensions eventually). Let the "Euclidean action" be:

$$S = \underbrace{\frac{1}{2} G^{-1}(t) x^2}_{Kinetic} + \underbrace{S_I(x)}_{Interaction}$$

G is a propagator and t will later be identified with $\ln \frac{\Lambda_0}{\Lambda}$.

- Polchinski's Eqn

$$\frac{\partial S_I}{\partial t} = -\frac{1}{2} \dot{G}(t) [\frac{\partial^2 S_I}{\partial x^2} - (\frac{\partial S_I}{\partial x})^2]$$

Condition for partition function to be invariant as t increases.

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- Background: Why this equation? ANS.: Variant of Wilson's eqn
- If $\psi(x) = e^{-S}$, then

$$\psi(y,T) = e^{-\frac{1}{2}y^2} \int dx \; e^{\frac{1}{2}\frac{(y\sqrt{G(T)}-x)^2}{G(T)-G(0)}} e^{\frac{1}{2}G(0)^{-1}x^2} \psi(x,0)$$

  As $T \to 0$, $\psi(y,T) \to \psi(y,0)$ and as $T \to \infty$, $G(T) \to 0$, $\psi(y,T) \to e^{-\frac{1}{2}y^2} \int dx \; \psi(x,0)$. So at $T \to \infty$ we have completely integrated over the variable x - coarse graining. All information about the starting wave function is lost. For finite T, partial coarse graining.
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Diffusion equation

- Can be written as a linear equation in terms of $\psi' = e^{-S_I}$:

$$\frac{\partial \psi'}{\partial t} = -\frac{1}{2}\dot{G}(t)\frac{\partial^2 \psi'}{\partial x^2}$$

$\Longrightarrow$ Diffusion equation.

- The evolution operator is clearly

$$e^{-\frac{1}{2}\int_0^T dt\, \dot{G}\frac{\partial^2}{\partial x^2}} = e^{-\frac{1}{2}(G(T)-G(0))\frac{\partial^2}{\partial x^2}} \equiv e^{\frac{1}{2}F(T)\frac{\partial^2}{\partial x^2}}$$

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Functional Integral

- Thus the path integral representation is obvious:

$$\psi'(x', T) = \int dx \int_{x(0)=x;x(T)=x'} \mathcal{D}x(t)\ e^{\frac{1}{2}\int_0^T dt\ \frac{1}{\dot{G}}(\dot{x})^2} \psi'(x, 0)$$

- If we replace x by $x(p)$ and $G(t)$ to $G(p, t)$ we immediately generalize to higher dimensions.

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Holographic Form

- In field theories, there is a cutoff function $K(p/\Lambda)$ (for e.g. $e^{\frac{-p^2}{\Lambda^2}}$)

$$F(p,T) = G(p,0) - G(p,T) = \frac{K(p/\Lambda_0) - K(p/\Lambda)}{p^2} = \Delta_h$$

  Δ_h is called the "high energy" propagator.

- CONCLUSION: We have a functional form for Polchinski's ERG eqn. The action is $d+1$-dimensional, where d is the dimension of the field theory we started with. $\implies$ Holographic. END OF STEP 1

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- Let $x = yf$ where $f^2 = -\dot{G}$. y is our new variable.
- Let us choose f to satisfy ($z = e^t$):

$$(z\frac{d}{dz})^2 e^{-\ln f} = (z^2p^2 + m^2)e^{-\ln f}$$

- Then the action for y becomes:

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- Equation for $\frac{1}{f}$ is the same as the equation for y!

$$[\frac{d^2}{dz^2} + \frac{1}{z}\frac{d}{dz} - (p^2 + \frac{m^2}{z^2})]\frac{1}{f} = 0$$

  Solutions are Bessel functions -$K_m(pz), I_m(pz)$.

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Higher dimensions

- Multiply and divide by z^d:

$$S_B = -\int dz z^{-d+1} \int \frac{d^d p}{(2\pi)^d} \frac{1}{2} \frac{\partial x_p}{\partial z} \frac{\partial x_{-p}}{\partial z} \frac{1}{z^{-d} \dot{G}(p)}$$

- Let $z^{-d}\dot{G} = -f^2$ and require as before:

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$$S_B = \int dz \int_p \{z^{-d+1}(\frac{\partial y_p}{\partial z}\frac{\partial y_{-p}}{\partial z} + z^{-d+1}(p^2 + \frac{m^2}{z^2})y_p y_{-p}\}$$

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RG evolution

- What have we achieved?

$$\underbrace{e^{-S_I[y_f]}}_{IR\ \ theory} = \int dy_i \underbrace{\int \mathcal{D}y(z) e^{-\int_{z_i}^{z_f} dz\ S_B[y(z)]}}_{d+1-dimensional\ \ AdS\ \ \text{``bulk''}\ \ theory} \underbrace{e^{-S_I[y_i]}}_{\text{``boundary''}\ \ UV-theory}$$

- S_I is a perturbation to a d-dimensional CFT. The action S_B depends on the CFT. In our case the CFT is a free field theory. Can be generalized.
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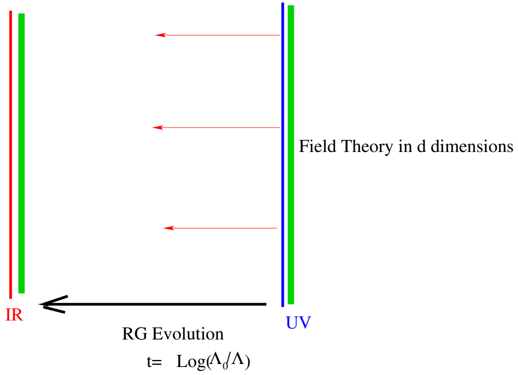
Geometrization of Scale



Geometrization of "scale" as an extra dimension - radial coordinate in AdS space.

Example: Green function

- We can work with $x(p)$ - gives exact result. Or with $y(p) \approx e^{-pz}x(p)$ - gives low energy result.
- For eg. take $S_i[x_i] = kx_i$ and evaluate semiclassically to get

$$S_l[x_f] = \frac{1}{2}k^2(G(T) - G(0)) + kx_f$$

- Field theory language:
$x_i = \phi = \phi_l + \phi_h, \quad x_f = \phi_l, \quad G(T) - G(0) = \Delta_h, \quad k = J.$
We get the expected Wilson action:

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$x_i = \phi = \phi_l + \phi_h, \;\; x_f = \phi_l, \quad G(T) - G(0) = \Delta_h, \;\; k = J.$
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So we have derived the AdS/CFT prescription - but only for for the simplest case - Gaussian theory.
The ERG (in terms of x) has a finite cutoff (but at a fixed point it is conformally invariant). The AdS version in terms of y is a low energy ($p << \Lambda$) “continuum" CFT - which is what is usually studied in the literature.

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Generalization

- Generalize to a non trivial fixed point:

$$S_{Fixed\ Point} = \frac{1}{2}x^2 G^{-1} + S_0(x)$$

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- Both S_0 and $S_0 + S_I$ obey Polchinski equation. Taking the difference we get

$$\frac{\partial S_1}{\partial t} = \frac{1}{2}\dot{G}[\underbrace{-\frac{\partial^2 S_1}{\partial x^2} + (\frac{\partial S_1}{\partial x})^2}_{Gaussian\ part} + 2(\frac{\partial S_0}{\partial x})(\frac{\partial S_1}{\partial x})] .$$

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Non trivial FP

- It can be shown that this gives the following "bulk" action for the evolution operator:

$$S_B[x(p,t)] = \int dt \int \frac{d^d p}{(2\pi)^d} \, [\frac{1}{\dot{G}(p)}(\frac{dx(p,t)}{dt})(\frac{dx(-p,t)}{dt}) + \underbrace{\dot{G}(p)(\frac{\delta S_0[x(p,t),t]}{\delta x(p,t)})(\frac{\delta S_0[x(p,t),t]}{\delta x(-p,t)})]}_{New\ \ term} \, .$$

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Summary

- We have a holographic form of Wilson's exact RG.
- A change of variables maps this to an action in AdS space - makes contact with "AdS/CFT" - without invoking string theory.
- Explicit calculations have been done only for the free theory - Gaussian fixed point.
- In principle it can be done for non trivial fixed point. This is ongoing work.
- This needs to be done for $W[J]$, not just the Wilson action. Ongoing work.
- And most importantly - the significance of dynamical gravity in the bulk needs to be studied.

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THANK YOU!

HAPPY BIRTHDAY BAL!