# *New look: Infrared Divergence, Asymptotic symmetries in QED*

*Is Photon massless?*

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# *PLAN*

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- I gave a talk on ‘Is photon massless?’ at Dublin Inst of Advanced Studies in January 2018. To my surprise found Schrodinger talked about same question..
- Must the Photon Mass be Zero? Author(s): L. Bass and E. Schrodinger Source: Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences,232,1188 (Oct. 11, 1955), pp. 1-6

# *Asymptotic symmetries in QED*

- QED has massless photons, gauge theory, and charged particles. Local gauge invariance and global gauge invariance tied up nicely.
- They get seperated by the asymptotic properties of the gauge transformations. Global gauge transformations lead to current conservation and charge as superselection.
- Local gauge invariance gives redundant degrees of freedom which can be eliminated only by gauge fixing.
- The degrees of freedom describe photons which are massless and is responsible for the long range interactions between charged particles. This interaction affects the freedom of charged particles even at spatial asymptotic infinity. Leads to certain additional global symmetries at asymptotic infinity making the description 'in' and 'out' states dressed.

# *Infrared divergences in QED*

- Masslessness of the photons give propagators which go like $\frac{1}{k^2}$. This leads to a divergence close to large wavelength/low frequency photons in several processes.
- These photons are also tied up with asymptotic dressing of the charged particles.
- Interestingly the divergences are cancelled by using a coherent state of the charged particles along with 'soft photons'
- The above description was the way text books are written and calculations are performed for all the known processes.
- Recently this question is revisited in QED, QCD and gravity theories from some new perspective. We will focus on QED in 3 and 4 dimensions only.
- Strominger and his collaborators, Laddha and Campligia have made studies on the behaviour potentials at null infinity and produced a mapping which has a discontinuity from the past and future.

# *Mass of the photon*

- Interestingly similar things were shown atleast a decade back by Andrzeg Herdegen on the asymptotic structure. Bal, Sachin & co have also relooked at the question with reference to Lorentz symmetry. Bucholz also had a relook at the algebraic formulation of superselections.
- Since the issues are tied up with the mass of the photon we can look at observationally what are the limits on this.
- A S Goldhaber and M M Nieto: Mass limits: Solar magneticfield $10^{-18}eV$.Cosmic magnetic field limit: $10^{-27}eV$.
- Roughly these correspond to Compton wavelength to be $\geq$ AU (astronomical unit) or radius of the galaxy.
- Using the ultimate size of the universe as bound we can get: $m_\gamma \ \leq \ 10^{-33}eV$
- Neutrino was expected to be massless and later established to be massive, but our experimental conclusions cannot be sensitive when we push the barrier at this level.

# *Proca and Stueckelberg theory*

- If we introduce mass term of the photon to the conventional Maxwell action, it breaks local gauge invariance. But global invariance is still there, and current is conserved and charge is still superselected.
- But the massive Proca theory describing massive spin 1 has 3 degrees of freedom unlike Maxwell theory. Hence there is disconinuity in the degrees of freedom.
- But the Massive QED is renormalisable (ultraviolet). If we make the mass to be tiny but nonzero we will find the contribution of the longitudinal photon to several processes are extremely small as used by the text book of Banks! There is no infrared divergence either!
- Also what happens in the $m \longrightarrow 0$ limit, for the infrared divergence and the question of lack of local gauge invariance which has been our guiding principle. Stueckelberg theory avoids discontinuity and gauge invariance question.

# *Massive gauge theory*

- We can preserve local gauge invariance and still give mass to the photon in two ways. (1) Stueckelberg theory (2)topological massive $B \wedge F$ theory.
- The Lagrangian for Stueckelberg theory is:

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + \frac{1}{2}m^2\left(A_\mu - \frac{1}{m}\partial_\mu\phi\right)^2 + \bar{\psi}[\gamma^\mu(i\partial_\mu + eA_\mu) - M]\psi \tag{1}$$

- The gauge fixing: $-\frac{1}{2}(\partial_\mu A^\mu + m\,\phi)^2$. The gauge transformations are:

$$\psi \to e^{i\lambda(x)}\psi, \;\; A_\mu \to A_\mu - \partial_\mu\lambda(x), \;\; \phi \to \phi + m\lambda(x) \tag{2}$$

  where $\phi$ is Stueckelberg scalar field.
- For topological mass theory we use two form $B = B_{\mu\nu}dx^\mu \wedge dx^\nu$ and $H = dB$.

# *Topological massive gauge theory*

- The Lagrangian is:

$$\mathcal{L} \;=\; -\frac{1}{2}F\wedge^{*}F + \frac{1}{2}H\wedge^{*}H + m\,B\wedge F + \bar{\psi}[\gamma^{\mu}(i\partial_{\mu} - eA_{\mu}) + M]\psi \quad (3)$$

- Again the combined gauge transformations leave the Lagrangian upto total divergence invariant. For completness massive $B$ describes a spin 1 field whereas in the massless limit we get scalar theory.
- In $2+1\,D$ we also have Maxwell Chern Simon theory given by the Lagrangian:

$$\mathcal{L} \;=\; -\frac{1}{2}F\wedge^{*}F \;+\; m\,A\wedge\,F \;+\; \bar{\psi}[i\gamma\,D - M]\psi \qquad (4)$$

- Both Maxwell and Maxwell CS theory describes a scalar field. but with different helicity. But we will focus on the 3+1 Stueckelberg QED

# *Stueckelberg QED*

- For perturbative calculation we need to fix gauge. The gauge fixing term is $-\frac{1}{2}(\partial_\mu A^\mu + m\phi)^2$.
- It is known to be renormalizable, and due to the mass infrared divergence is not there. What happens in the limit $m \to 0$ limit? *To facilitate that we will look at a transformed gauge field.*

$$\bar{A}_\mu = A_\mu - \frac{1}{m}\partial_\mu \phi \tag{5}$$

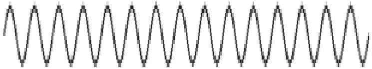
- *This changes the interaction as:*

$$e\bar{\psi}\gamma_\mu A^\mu \psi = e\bar{\psi}\gamma_\mu \bar{A}^\mu \psi + \frac{e}{m}\bar{\psi}\gamma^\mu \psi \, (\partial_\mu \phi) \tag{6}$$

- *In this form $\bar{A}^\mu$ is gauge invariant (transverse component). But the fermion interacts with Stueckelberg field or in effect the longitudinal component!*

# *Stueckelberg QED*

- The gauge field propagator:



$$\frac{-i\, g_{\mu\nu}}{p^2 - m^2} \tag{7}$$

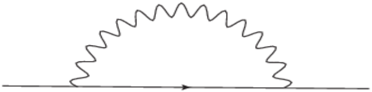
The scalar field propagator:

$$\frac{i}{p^2 - m^2} \tag{8}$$

- Gaugefield vertex: $i\, e\ \ \gamma^\mu$
- scalar field vertex: $-\ \dfrac{e}{m} p^\mu \mathcal{I}$

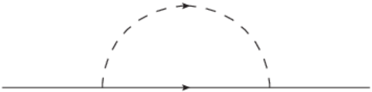
# *Stueckelberg QED*

- The self energy diagram has two components.
- Gauge field part:



$$< \bar{\psi}\psi >_A \;=\; (-i\, e)^2 \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\mu\, (-ig_{\mu\nu})\, (\not{p} + M)\, \gamma^\nu}{[(p-k)^2 - m^2)](p^2 - M^2)} \qquad (9)$$

- scalar field part:

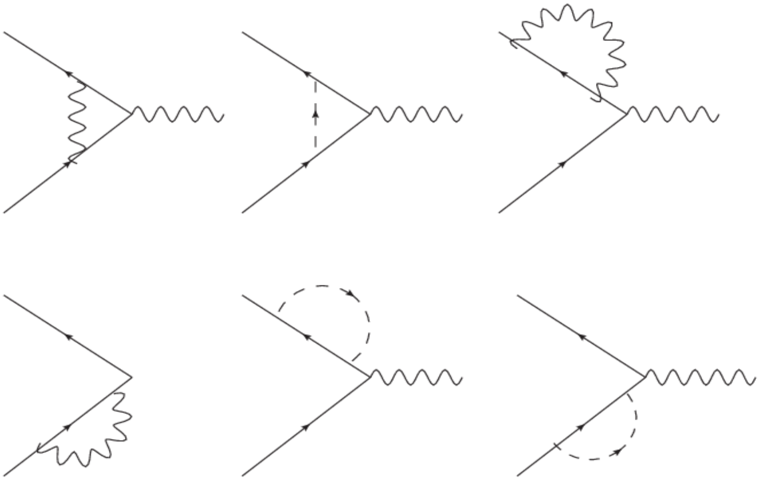


$$< \bar{\psi}\psi >_\phi \;=\; \frac{e^2}{m^2} \int \frac{d^4k}{(2\pi)^4} \frac{(\not{k} - \not{p})[-i(\not{p} \,+\, M)](\not{k} \,-\, \not{p})}{(p^2 - M^2)[(p-k)^2 - m^2]} \qquad (10)$$

- The infrared divergence cancels in the $m \to 0$ limit.

# *Stueckelberg QED*

- For the vertex there are six diagrams to the lowest order. (cancels, caution: verification)



# *Little group analysis-massless limit*

- Since the issue of infrared question and asymptotic symmetries are related to massless particles, one can consider the limit of massive spin 1 representation of Poincare group becoming massless one.
- The little group of massive particle is given by $SO(3)$. That of massless one is $E(2)$.
- $SO(3)$ is described by $L_i = -i\epsilon_{(i)jk}$. But $E(2)$ is given by: $L_3$ and

$$P_1 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & i \\ 0 & 0 & 0 \end{pmatrix} \tag{11}$$

- Def: $P_i = \frac{\epsilon_{ij}}{R} B^{-1} L_j B, \;\; B(R) = Diag(1, 1, R)$ gives the Inonu Wigner contraction of $SO(3) \to E(2)$ in the limit $R \to \infty$.

# *Little group analysis-massless limit*

- Interestingly this is explained better by looking at the rotation and boost generators $L_i,\ K_i$ and considering $N_1 = K_1 - L_2,\ N_2 = K_2 + L_1$. Then $L_3, N_1, N_2$ give the $E(2)$.
- In light cone coordinates, boost generator

$$B(R) = e^{-ilog(R)K_3} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & R & 0 \\ 0 & 0 & 0 & \frac{1}{R} \end{pmatrix}$$

where $R = \sqrt{\frac{(1+\beta)}{(1-\beta)}}$ and $\beta$ is the velocity.

- Using this we can easily write generators of $E(2)$.
- This is to be expected as these are generators in light front coordinators and all finite mass particles behave like massless particles in that limit. The local gauge transformations can be obtained as part of $E(2)$ tself.

# *Maxwell Chern Simons theory*

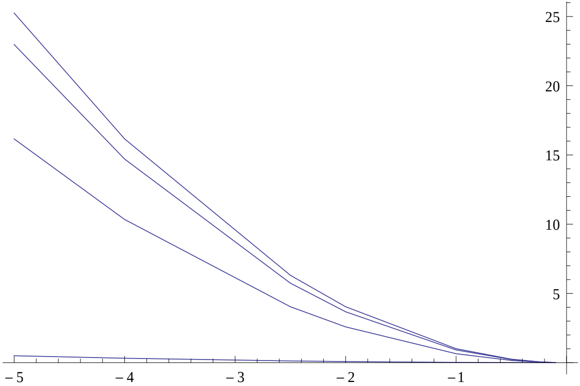
- We now approach the question from the issue of edge modes. For this we consider a simpler model namely 2+1 D Maxwell ED. Since we have a mass for the gauge field is 'm' we consider field modes in a disc of radius Compton wavelength $\frac{1}{m}$. Since the field is a scalar field (Deser, Jackiw, Templeton) we consider the same in the Disc with generic Robin boundary condition.
- We have explored the same (Bal, TRG, ...)in an earliar paper but the sign of the constant $\kappa$ was positive to ensure positive definite 'Laplacian'. We can give up that and get edge states bounded to the edge.
- The edge eigenmodes of Laplacian on the Disc

$$-\nabla^2\psi = \lambda\psi, \quad \kappa\,\psi(R) \;+\; \partial_r\psi(R) \;=\; 0 \tag{12}$$

were computed earlier in a paper by us (TRG, Rakesh Tibrewala),and the edge modes $N \;\equiv\; \kappa R$.

# *Maxwell Chern Simons theory*

- The disc radius is taken to be $R = \dfrac{1}{m}$. We want to scale $R \to \infty, m \to 0$.
- At the same time we take $\kappa \to 0$ so that $N$ is fixed and large. We find as we scale the eigenvalues tend to zero as edge modes.
- The following figures exhibit the same.



# *Maxwell Chern Simons theory*

- If we had taken up Stueckelberg theory, in 2+1 it will correspond to two Maxwell CS theory! with the sign of the mass term opposite.
- The Maxwell CS QED can be studied only in perturbation theory and the limits should be taken including the interaction with edge modes.
- This will be presented elsewhere.
- Further work: There is extension of Stueckelberg theory to Supersymmetric U(1) (P. Nath etal). Again the earlier studies have focussed only on ultraviolet renormalizability. The question of what happens to infrared question in the limit of massless gauge particles.
- Marolf considered earlier BMS symmetries in gravity by considering in a spacetime in a box along with twisting the boundary conditions. Then taking the limit of size of the box to $\infty$ in a suitable prescription one gets the asymptotic behaviour.

# *Holography and Stueckelberg theory*

- Dvali etal: propose holography can be formulated in terms of information capacity of Stueckelberg degrees of freedom.
- These degrees of freedom act as qubits to encode quantum information.
- The capacity is controlled by the inverse Stueckelberg energy gap to the size of the system.
- They relate the scaling of the gap of the boundary Stueckelberg edge modes Bogoliubov modes..
- ideas are not clear but needs further work...

# *Summary and Conclusions*

- Revival Infrared question through asymptotic symmetries is interesting. When we regulate theory through mass maintaining local gauge invariance gives the Stueckelberg scalar a new role. It regulates the divergence, and breaks the asymptotic symmetry. Can the charges due to the new symmetries be observed? Since they are tied up with masslessness of the photon in a limiting process probably they can be observed only to the extent we can measure the mass of the photon.
- What about QCD? Unfortunately there is no Stueckelberg theory for non abelian gauge theory...Speculations about $\frac{1}{N}$?.
- What about gravity? Probably massive gravity theories can shed some information.
- Last speculation? Can it help in dark matter? Stueckelberg field is not coupled to matter but can only gravitate...
- People involved in different parts: Ramadevi, Jai More, Ravindran, Nikhil, Rakesh Tibrewala.