Theory of Poynting dominated jets

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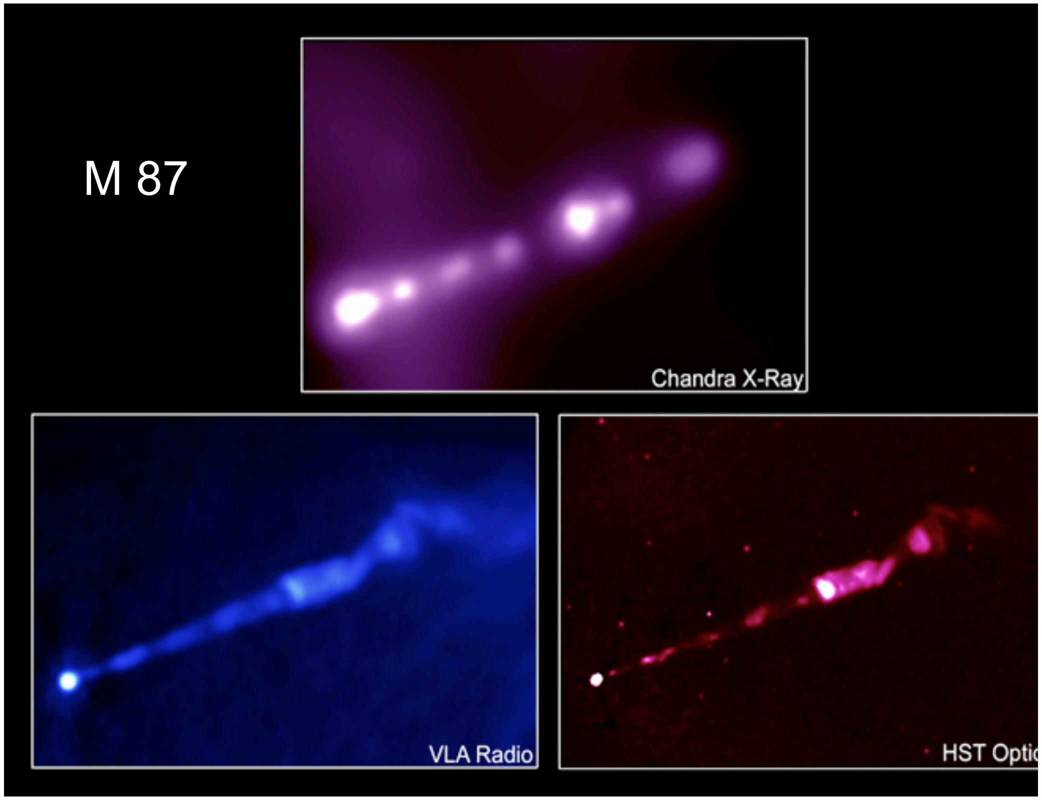
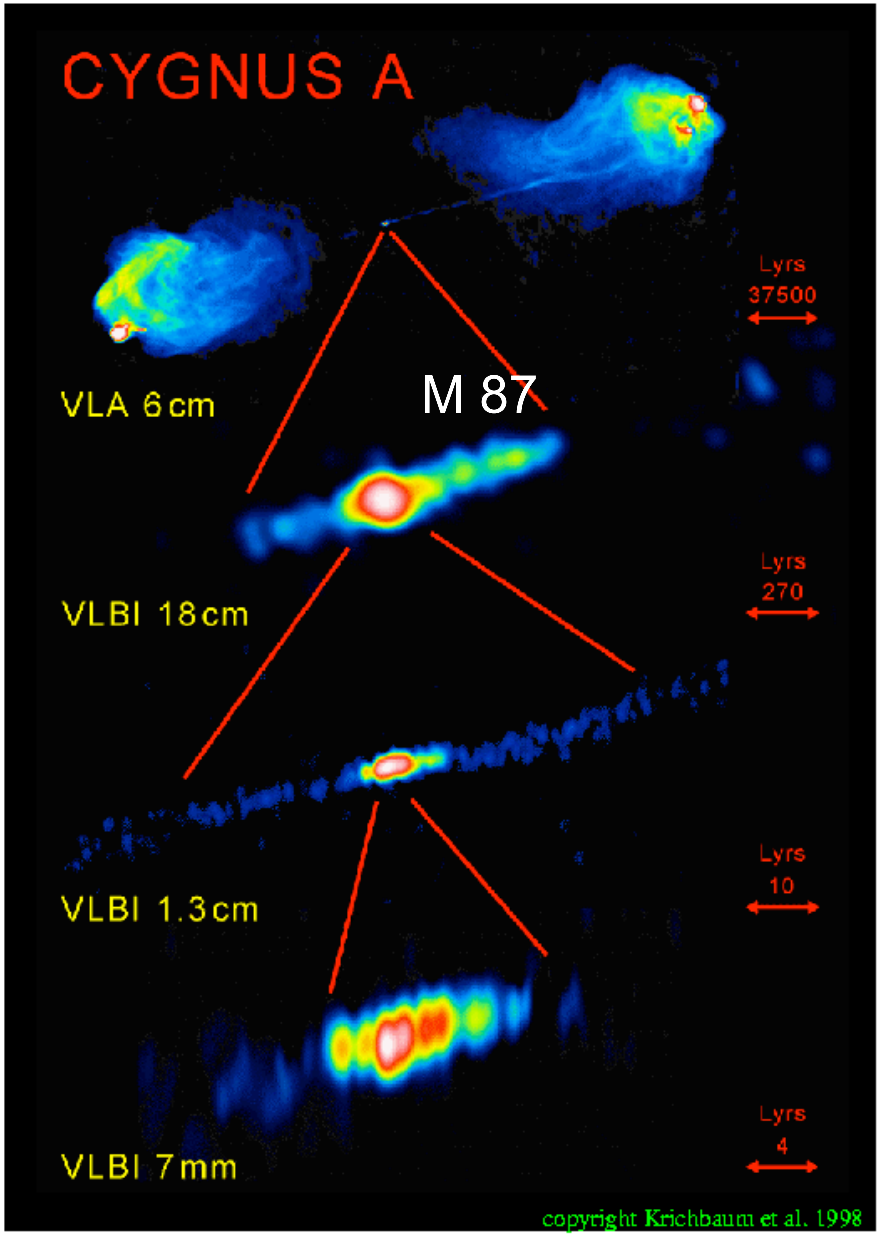
Universality of relativistic jet phenomenon

1. Radio galaxies and quasars





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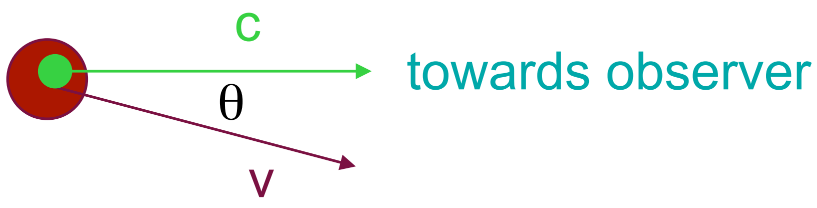
Universality of relativistic jet phenomenon

1. Radio galaxies and quasars (cont)

Apparent superluminal motion (Rees 1967)

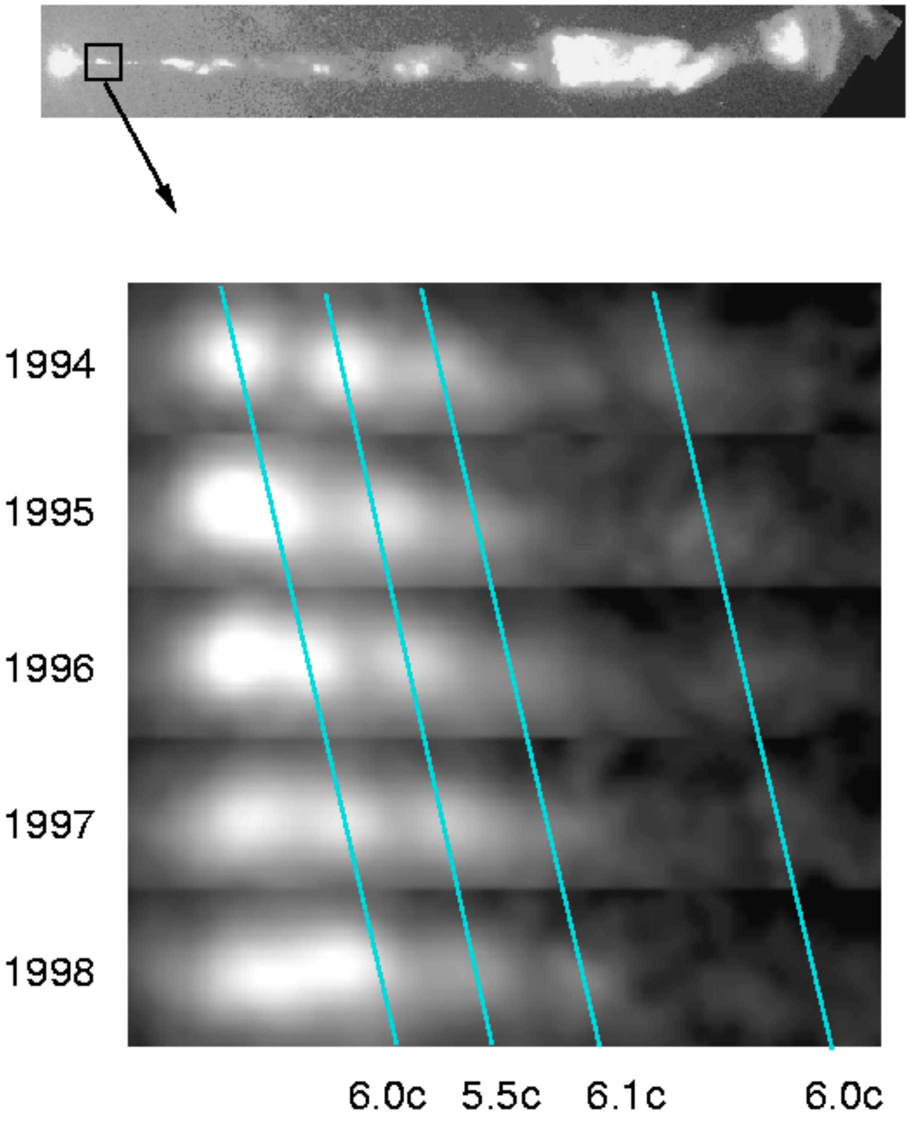


$$\Delta x = ct\sin\theta$$

$$\Delta t = t\left[1-(\mathrm{v}/c)\cos\theta\right]$$

$$\mathrm{V}_{\text{apparent}} = \frac{\Delta x}{\Delta t} = \frac{\sin\theta}{1-(\mathrm{v}/c)\cos\theta}\,c$$

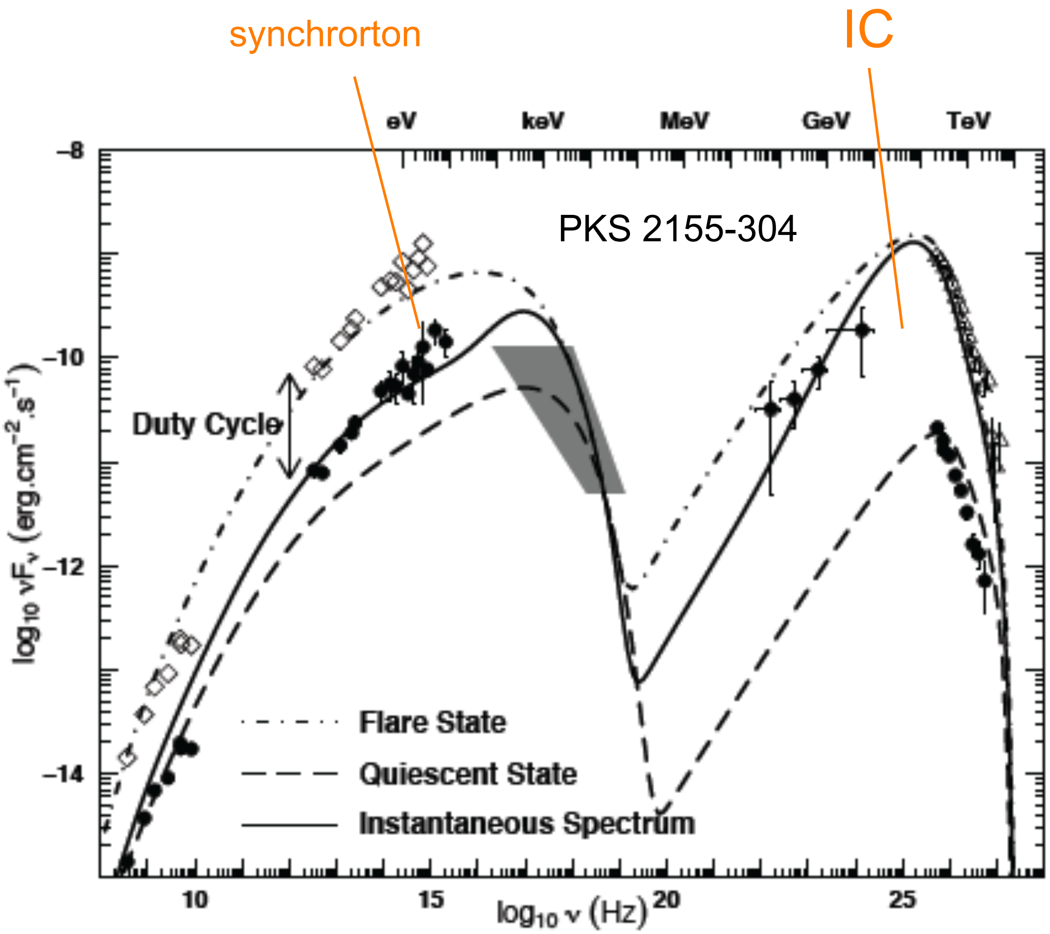
Superluminal Motion in the M87 Jet



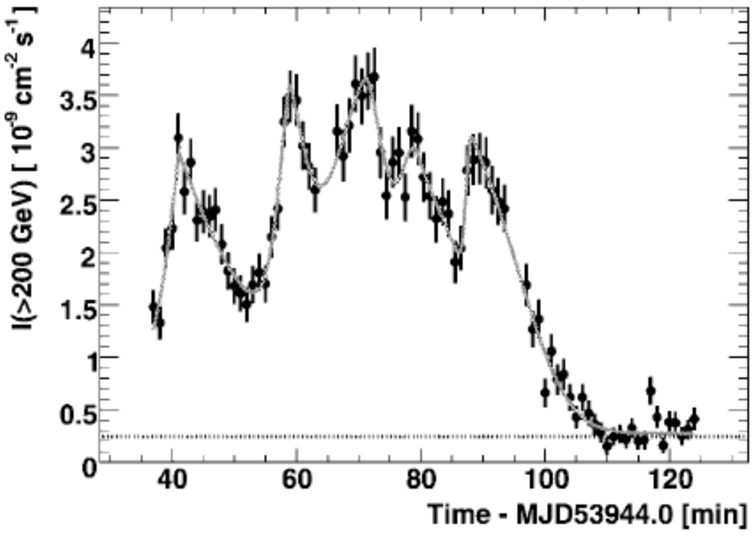
Universality of relativistic jet phenomenon

1. Radio galaxies and quasars (cont)

Blazars - AGNs with jet pointing towards the observer.
The observed emission is dominated by the jet.



Rapid (few-minute) variability in TeV flux of blazars implies $\gamma > 50$ (Levinson 06;Begelman etal 08)



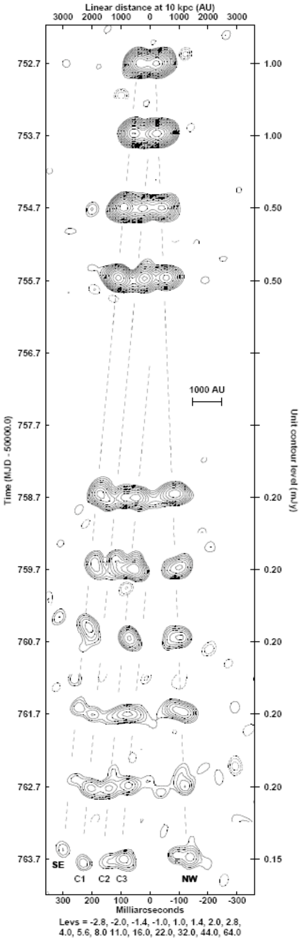
The flux above 200 GeV
(Aharonian et al 07)

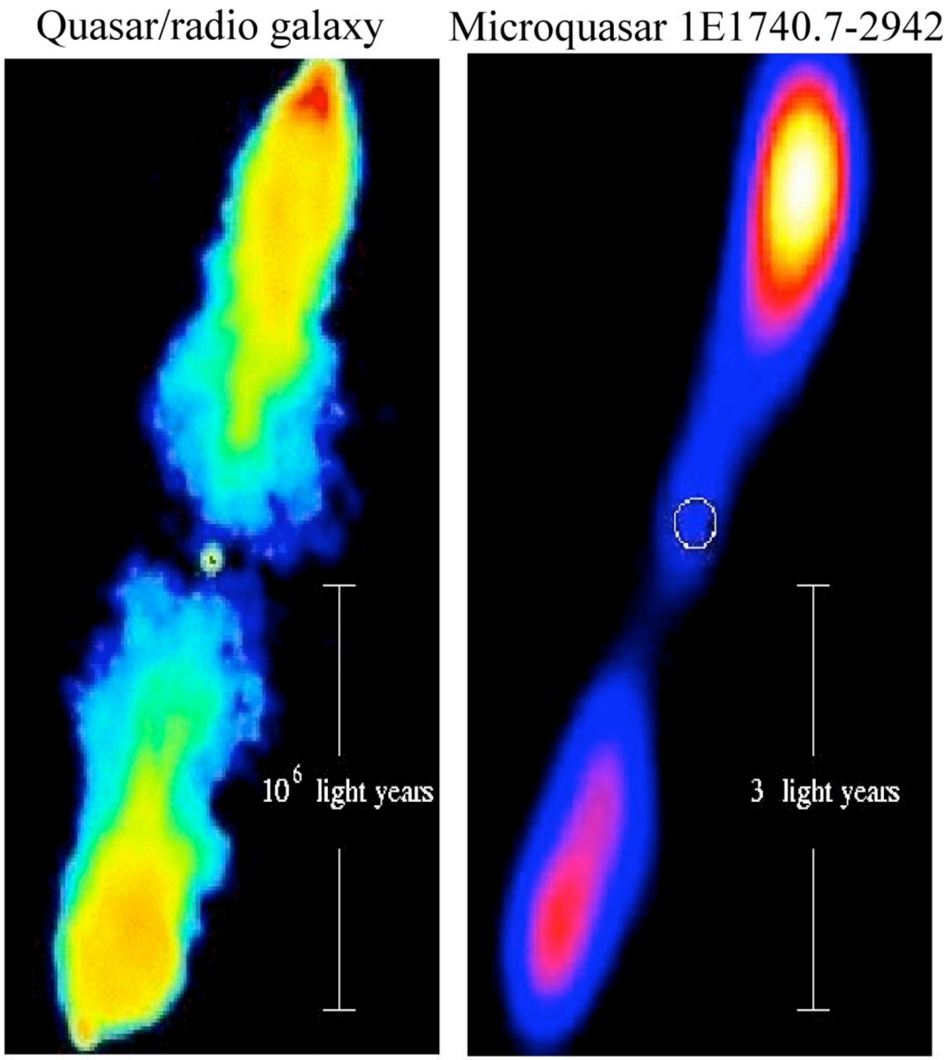
Universality of relativistic jet phenomenon

2. Microquasars

Microquasar is a scaled down (by a factor of 10^6) version of active galactic nuclei

Superluminal motion in microquasar GRS 1915+105; $V_{apparent}$ =1.5c

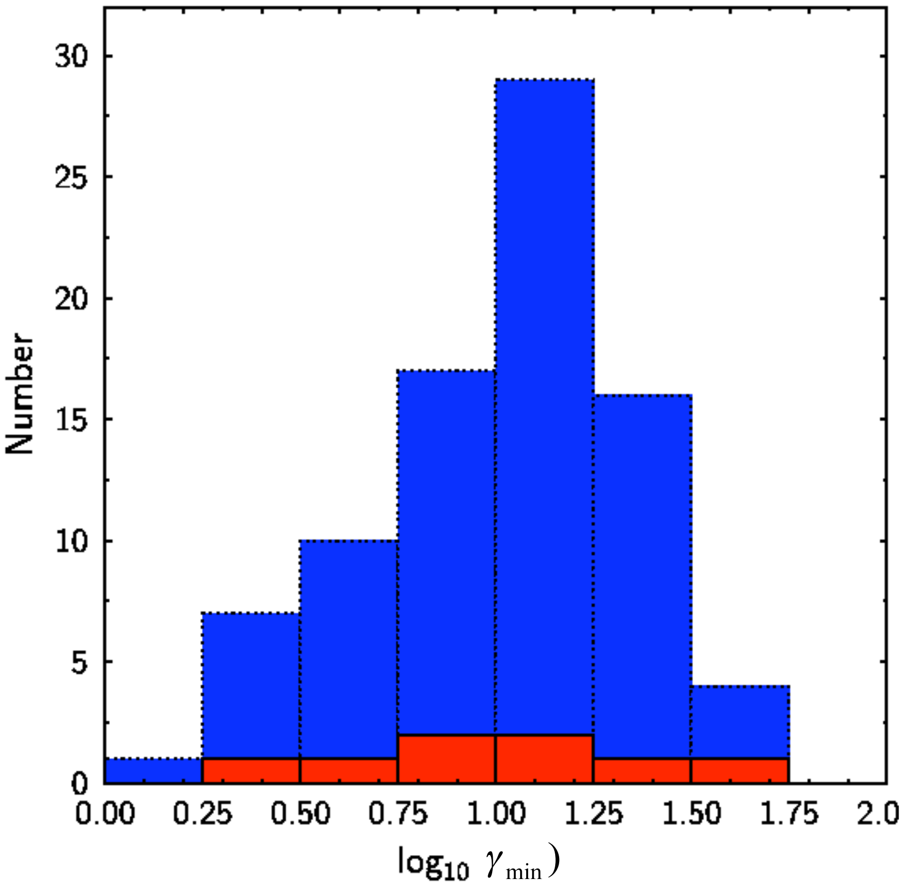



Quasar/radio galaxy

Microquasar 1E1740.7-2942

Lorentz factor

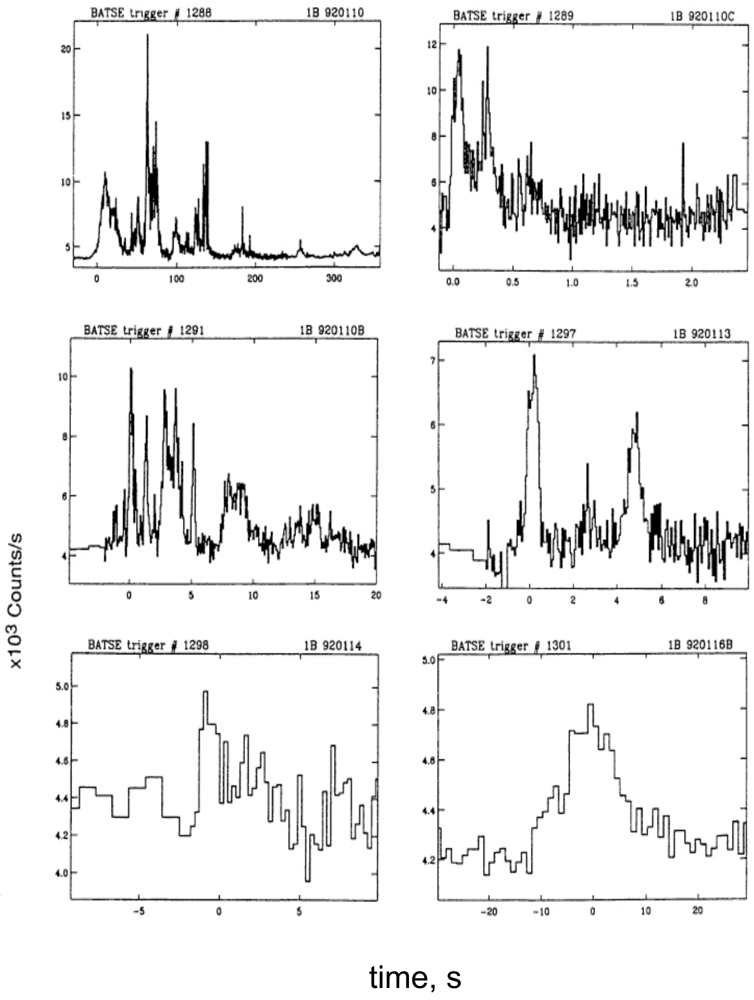
$$\gamma = \frac{1}{\sqrt{1 - (v/c)^2}}$$

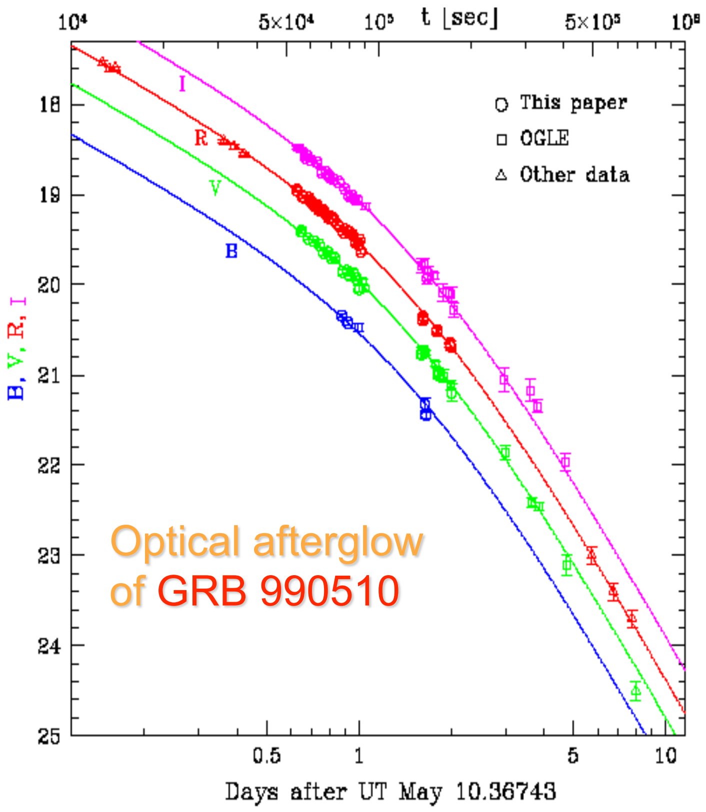


Distribution of Lorentz factors inferred from superluminal motion
Blue – AGNs; red – microquasars
(Fender 2005)

Universality of relativistic jet phenomenon

3. Gamma-ray bursts



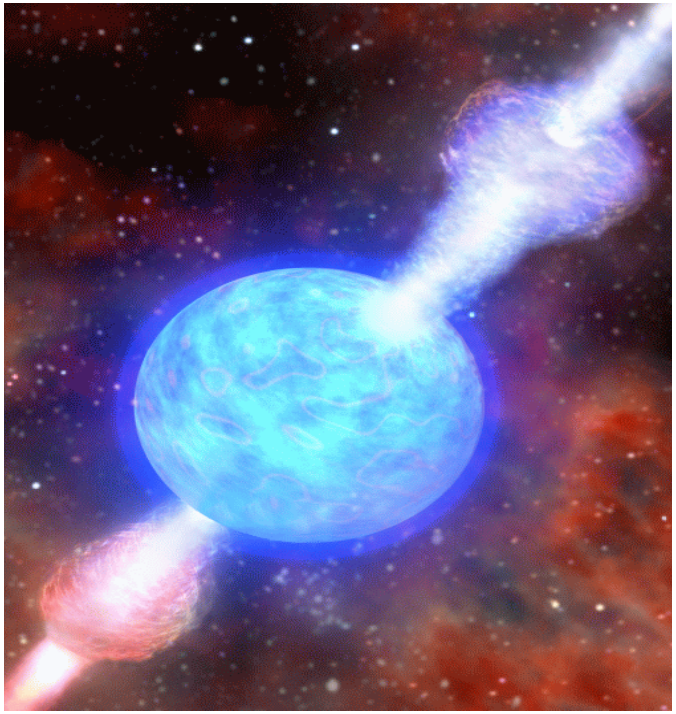


Optical afterglow of GRB 990510

Universality of relativistic jet phenomenon

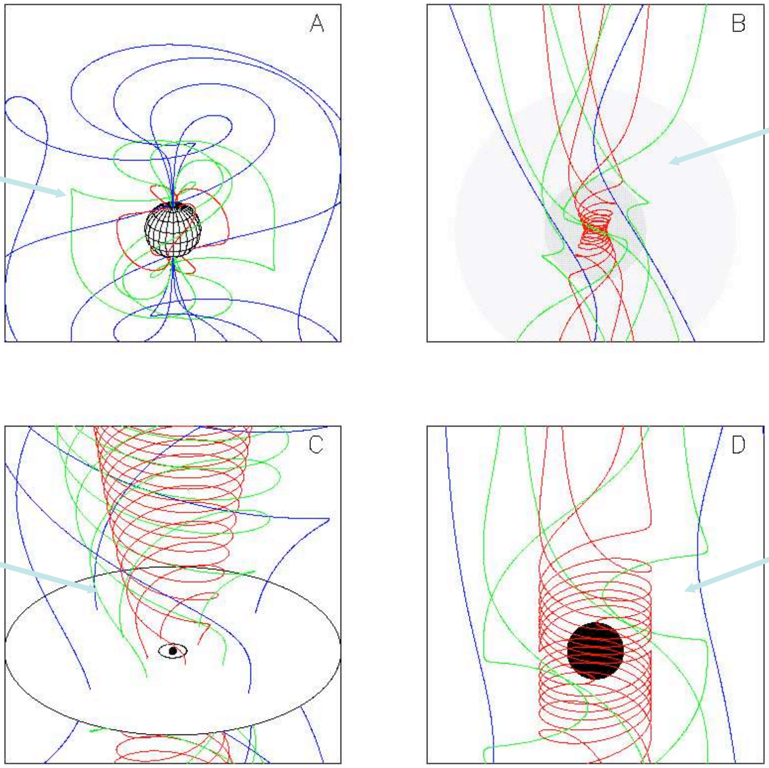
3. Gamma-ray bursts (cont)

GRBs apparently involve ultrarelativistic (γ=100-1000), highly collimated (θ=2^o-5^o) outflows. They likely arise during the collapse of star's core.



All relativistic cosmic jet sources may be connected by a common basic mechanism

A promising model is magnetohydrodynamic acceleration by rotating, twisted magnetic fields



Pulsar magnetosphere

Collapsing, magnetized supernova core

Magnetized accretion disks around neutron stars and black holes

Magnetospheres of Kerr black holes

Picture by David Meier

Rotation twists up the field into toroidal component, converting the rotational energy into the energy of the outflow

Basic principles of ideal (no resistivity) magnetohydrodynamics

1. Plasma shields the electric field; in the proper plasma frame ***E'****=0;* in the lab frame ***E****+(1/c)****vxB****=0.*

2. Magnetic field lines are frozen into the plasma; they move as the plasma moves.

3. Magnetic force could be presented as a superposition of the magnetic pressure and magnetic stress

$$\mathbf{j}\times\mathbf{B} = -\nabla_\perp \frac{B^2}{8\pi} + \frac{B^2}{4\pi}\frac{\mathbf{n}}{R_c}$$

normal to the field line

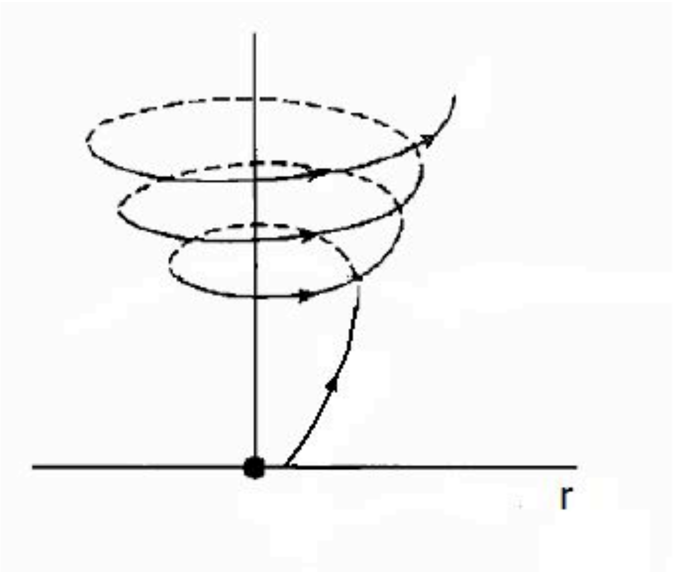
curvature radius of the field line

Basic picture of relativistic magnetohydrodynamic outflows

- Magnetic field lines rotate rigidly at the rate Ω

- Plasma moves along the rotating field lines (beads on a wire)

- At Ωr~c, the field gets wound up



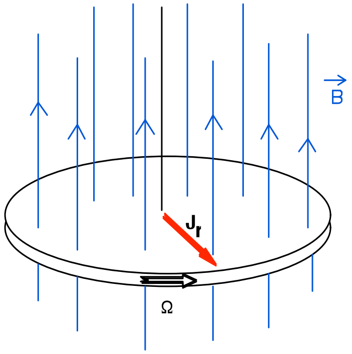
$$R_L = \frac{c}{\Omega}$$ - light cylinder radius

In terms of electrical engineering

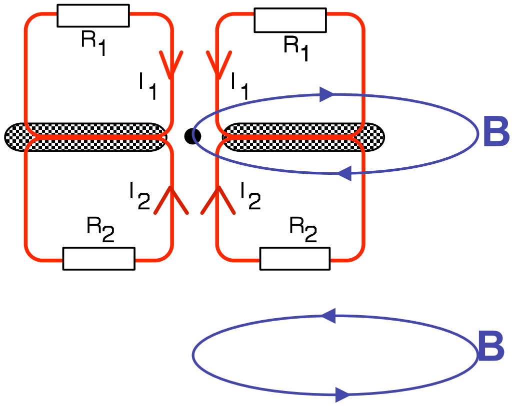
Barlow wheel: unipolar induction effect

1) Rotation + Magnetic Field => e.m.f

$$\varepsilon = \int \Omega r B_z dr$$



2) e.m.f => electric current (2 circuits)

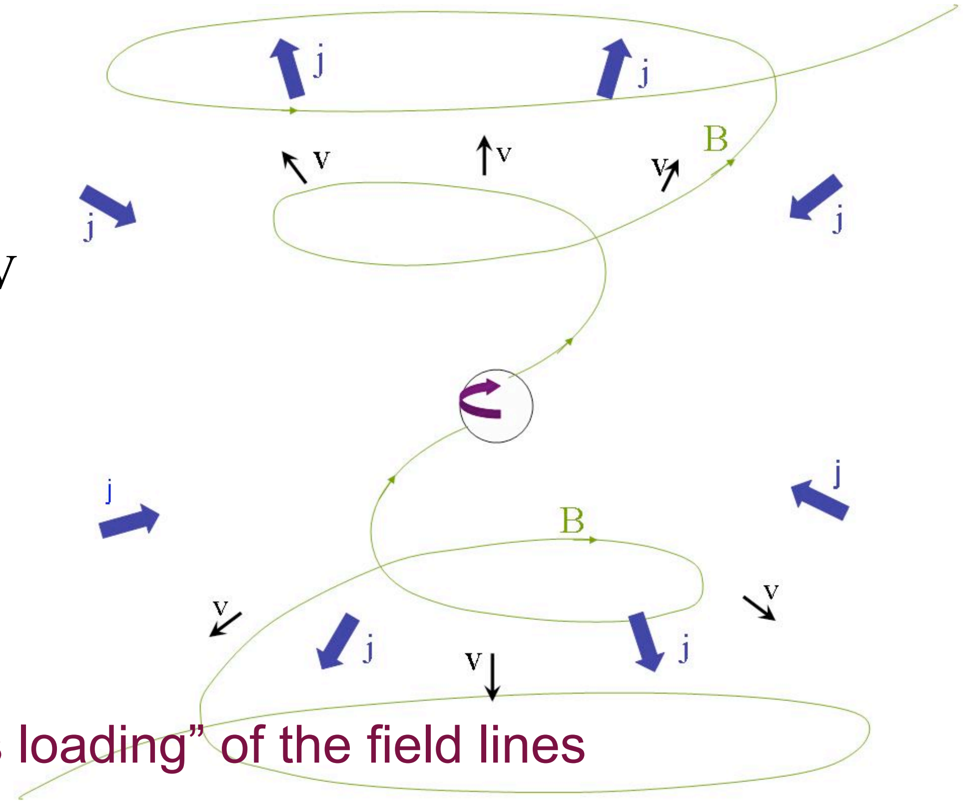


Energy budget of magnetized outflows

In the proper plasma frame $\mathbf{E}' = \mathbf{0}$

In the lab frame $\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B} = \mathbf{0}$

Poynting flux $\mathbf{S} = \frac{c}{4\pi}\mathbf{E} \times \mathbf{B} \approx \dfrac{B^2}{4\pi}\mathrm{v}$



By definition, ultrarelativistic flow implies $E_{kinetic} >> \rho c^2$

Relativistic flow can be produced by having a very strong rotating magnetic field such that $B^2>>4\pi\rho c^2$ ⇒ low “mass loading” of the field lines

The flow starts as Poynting dominated. How could the electro-magnetic energy be transformed into the plasma energy?

Rotational energy ⟶ Poynting ⟶ ?

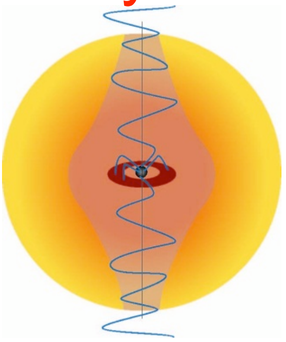
Force balance in Poynting dominated flows

Does the huge tension of wound up magnetic field (hoop stress) compresses the flow towards the axis?

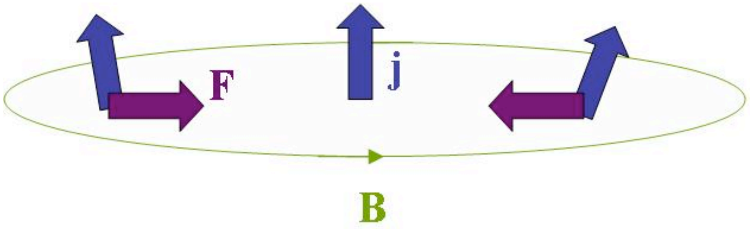
NO!

In the current closure region, the force is decollimating.

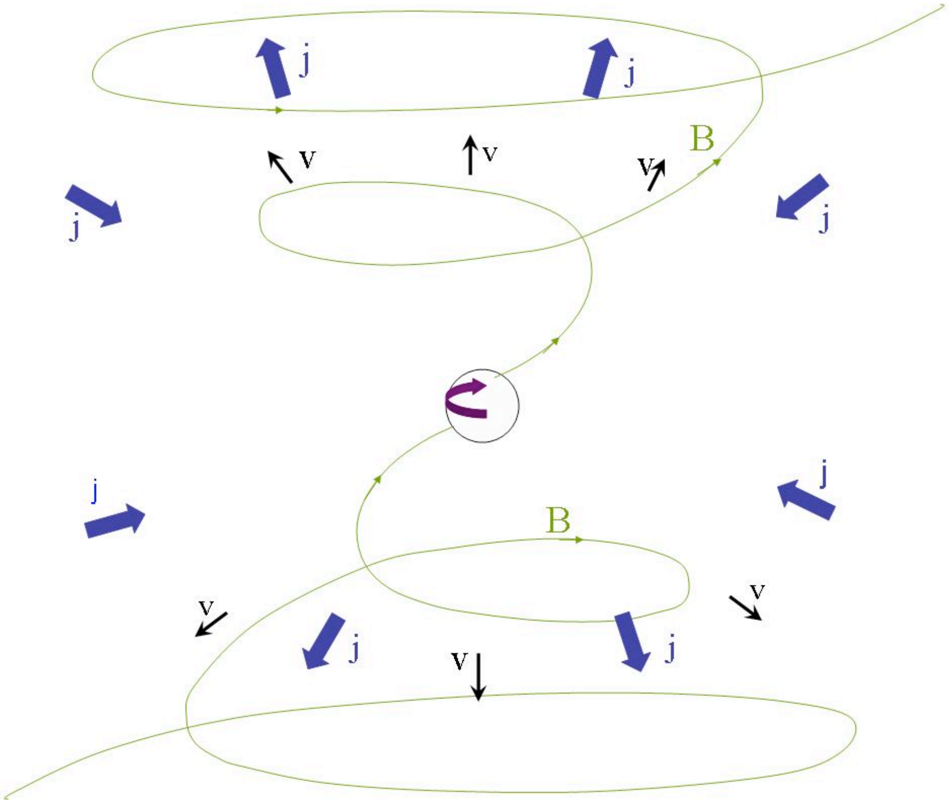
An external confinement is necessary!



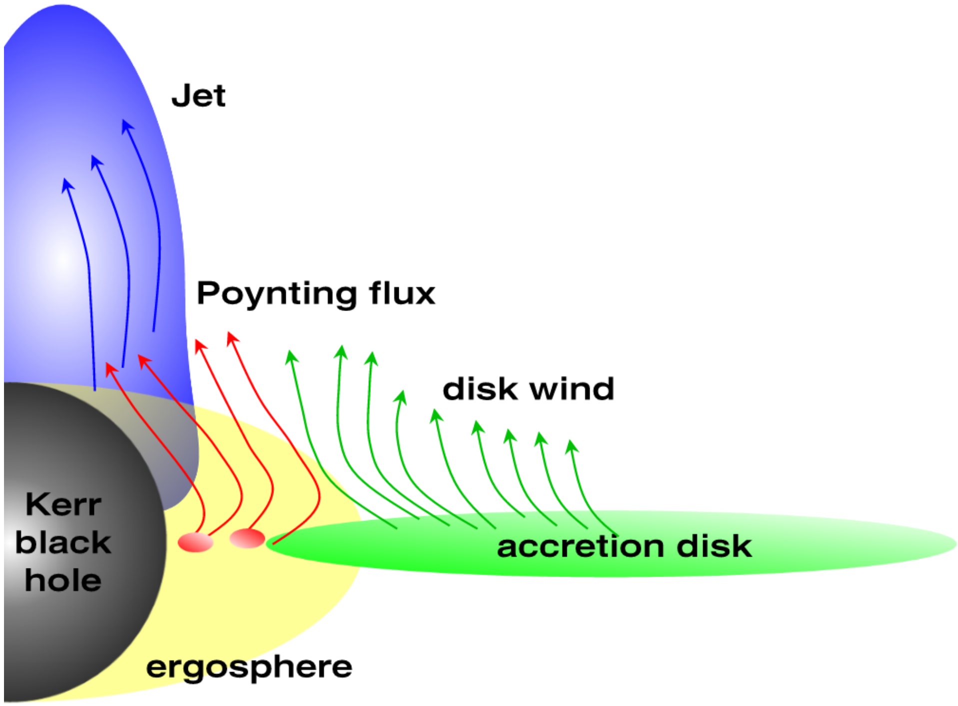
Magnetic hoop stress



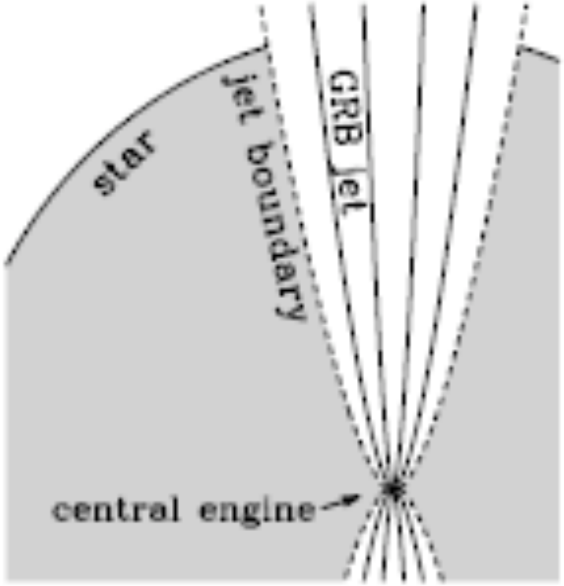
$$\mathbf{F} = \frac{1}{c}\,\mathbf{j}\times\mathbf{B}$$



Externally confined jets



In accreting systems, the relativistic outflows from the black hole and the internal part of the accretion disc could be confined by the (generally magnetized) wind from the outer parts of the disk.



In GRBs, a relativistic jet from the collapsing core pushes its way through the stellar envelope.

Force balance in Poynting dominated flows (cont)

In Poynting dominated flows,
plasma pressure and inertia<< Lorentz force.

Total electromagnetic force $\mathbf{F} = \rho_e \mathbf{E} + \frac{1}{c} \mathbf{j} \times \mathbf{B}$

$$\mathbf{E} + \frac{1}{\mathrm{c}} \mathbf{v} \times \mathbf{B} = 0 \qquad \rho_e = \frac{1}{4\pi} \nabla \cdot \mathbf{E}$$

In the far zone, v $\longrightarrow$ c and E $\longrightarrow$ B.

In highly relativistic flows, the Lorentz force is nearly compensated by the electric force.
Acceleration is only due to a small residual force!

As the jet moves out, acceleration converts EM energy to matter kinetic energy. How efficient is this conversion?

Poynting dominated jets.

What do we want to know?

What are the conditions for acceleration and collimation?

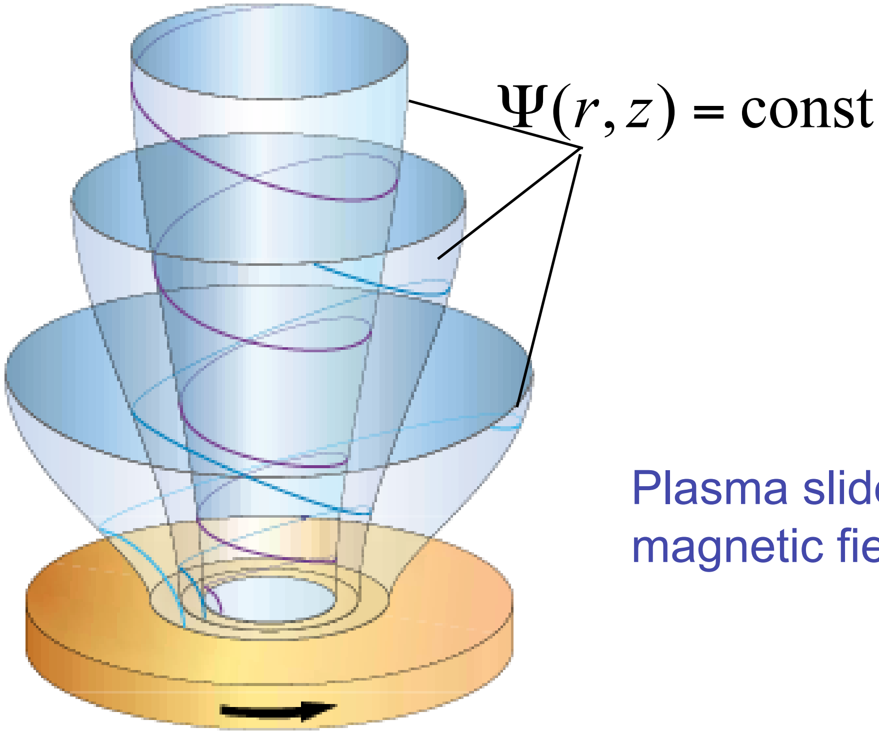
What is the final collimation angle?

Where and how the EM energy is released?
Conversion to the kinetic energy via gradual acceleration?
Or to the thermal and radiation energy via dissipation?

$$\sigma \equiv \frac{\text{Poynting flux}}{\text{plasma energy flux}}$$

How and where σ decreases from >>1 to <<1?

General description of axisymmetric flows



$$\mathbf{B} = \mathbf{B}_p + B_\phi \mathbf{e}_\phi$$

$$\mathbf{B}_p = \frac{1}{r} \nabla \Psi \times \mathbf{e}_\phi$$

Plasma slides along rotating magnetic field lines

$$\mathbf{v} = \boldsymbol{\Omega} \times \mathbf{r} + \kappa \mathbf{B}$$

$$\mathbf{E} = -\tfrac{1}{c} \mathbf{v} \times \mathbf{B} = -\frac{\Omega}{c} \nabla \Psi$$

$$E = \frac{\Omega r}{c} B_p = \frac{r}{R_L} B_p$$

Axisymmetric flows are nested magnetic surfaces of constant magnetic flux. These surfaces are equipotential. Plasma flows along these surfaces.

Power of a magnetically launched outflow

At $r \sim R_L = c/\Omega$, $E \sim B_p \sim B_\varphi$.

$$L_{EM} \sim \frac{EB}{4\pi} c \cdot 4\pi R_L^2 \sim B_L^2 c R_L^2 = B_L^2 \frac{c^3}{\Omega^2}$$

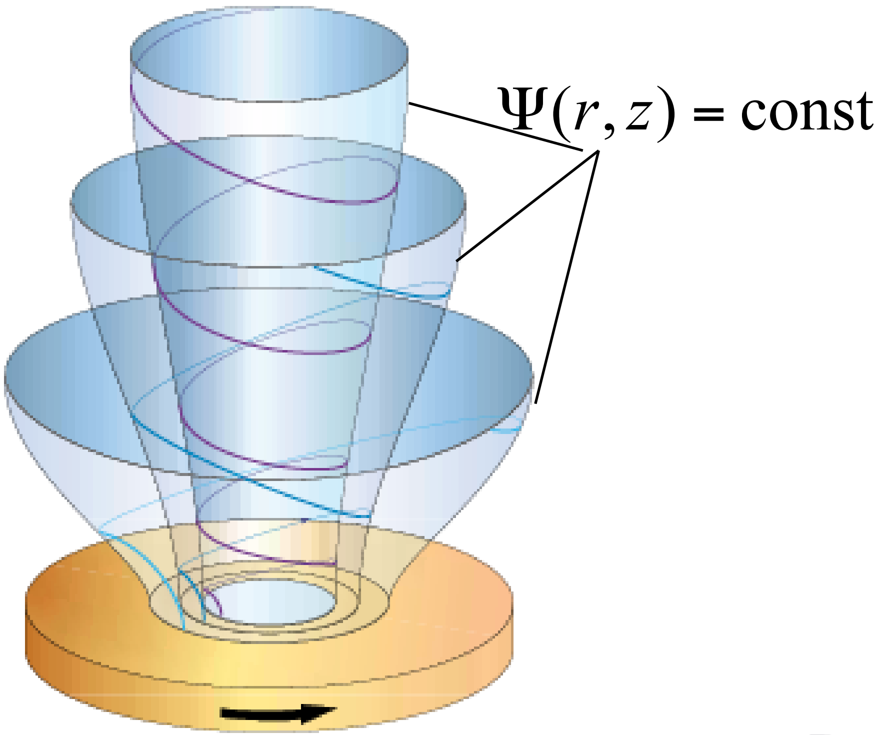
For rapidly rotating BH, $R_L \sim R_g$.

$$L_{EM} \sim B^2 c R_g^2$$

Equipartition: $B^2 \sim \frac{\dot{M} c}{R_g^2}$

$$L_{EM} = \xi \dot{M} c^2$$

The far zone, $r \gg R_L$



At $r \sim R_L$, $E \sim B_p \sim B_\varphi$.

Beyond the light cylinder, each revolution of the source adds to the wind one more magnetic loop, $B_\varphi \approx \frac{r\Omega}{c} B_p = \frac{r}{R_L} B_p$

In the far zone, $E \approx B_\varphi \gg B_p$

$B_p \sim \frac{\Psi}{r^2}$ $E \approx B_\varphi \sim \frac{\Psi}{rR_L}$; $B_\varphi = \frac{2I}{cr}$

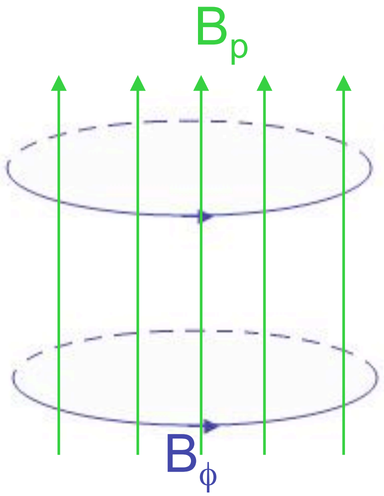
$$\text{Poynting flux} = \frac{EB_\phi}{4\pi} c \sim \frac{B^2}{4\pi} c$$

Simple example: Transverse force balance in cylindrical configuration.

1. v=0; E=0

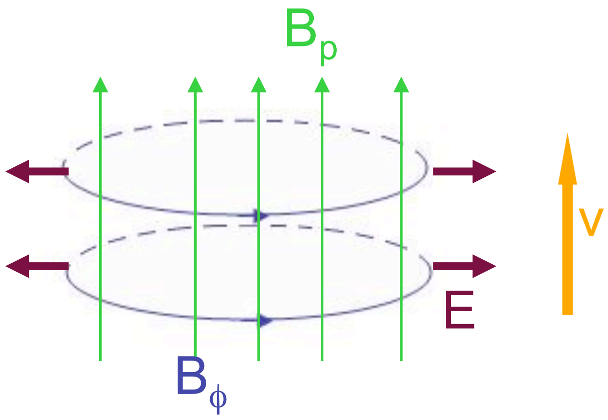
$$\frac{dB_p^2}{dr} + \frac{1}{r^2}\frac{d(r^2 B_\varphi^2)}{dr} = 0$$

$$B_\varphi \sim B_p$$



2. v →c; B_p=B'_p; B_φ=$\gamma B'_\varphi$; E=(v/c)$\gamma B'_p$

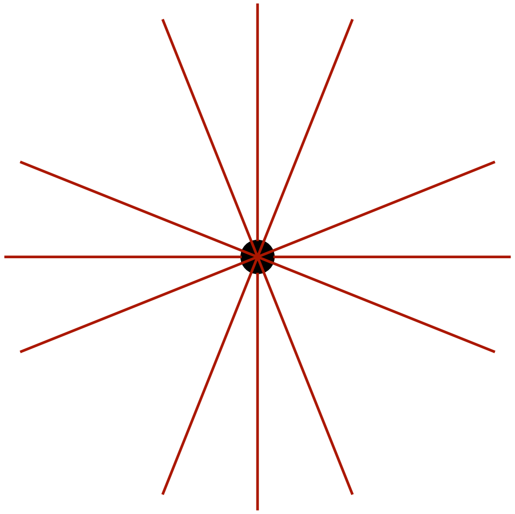
$$E \approx B_\varphi >> B_p$$



Relativistic MHD in the far zone, Ωr>>c

Intimate connection between collimation and acceleration

Without external confinement, the flow is nearly radial; the acceleration stops at an early stage (Tomimatsu94; Beskin et al 98)



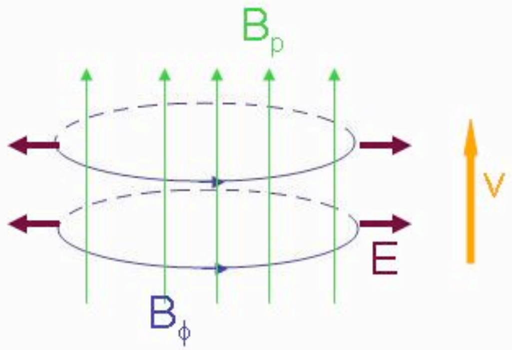
$$\gamma \approx \Omega r / c; \qquad \gamma < \gamma_{\max}^{1/3}$$

$$\gamma \approx \left[\gamma_{\max} \ln\left(\Omega r / c\gamma_{\max}\right)\right]^{1/3}; \; \gamma > \gamma_{\max}^{1/3}$$

$\gamma_{\max}>>1$ - the Lorentz factor achieved when and if the Poynting flux is completely transformed into the kinetic energy

Jet confined by the external pressure: the spatial distribution of the confining pressure determines the shape of the flow boundary and the acceleration rate (Tchekhovskoy et al 08,09; Komissarov et al 09; Lyubarsky 09,10).

Collimation vs acceleration
1. Equilibrium jet



cylindrical equilibrium at any z

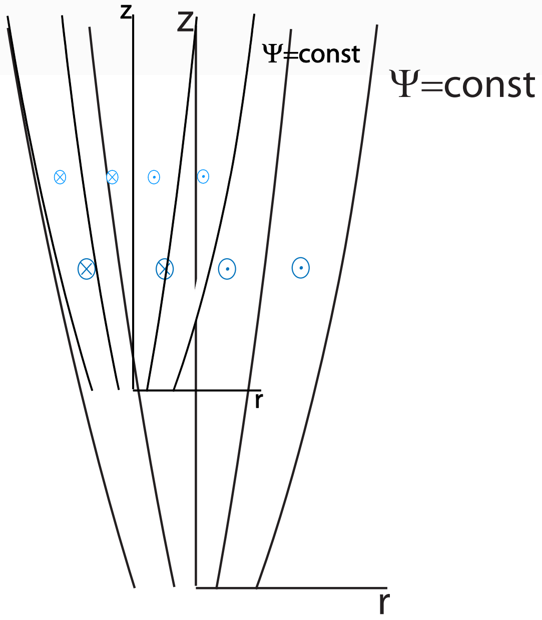
In the proper frame, $B'_{\varphi} \sim B'_{p}$

In the lab frame, $B_{\varphi} = \gamma B'_{\varphi} \sim \gamma B'_{p} = \gamma B_{p}$

At $r > R_L$, each revolution of the source adds to the wind one more magnetic loop: $B_{\varphi.} \approx (r\Omega/c) B_p$

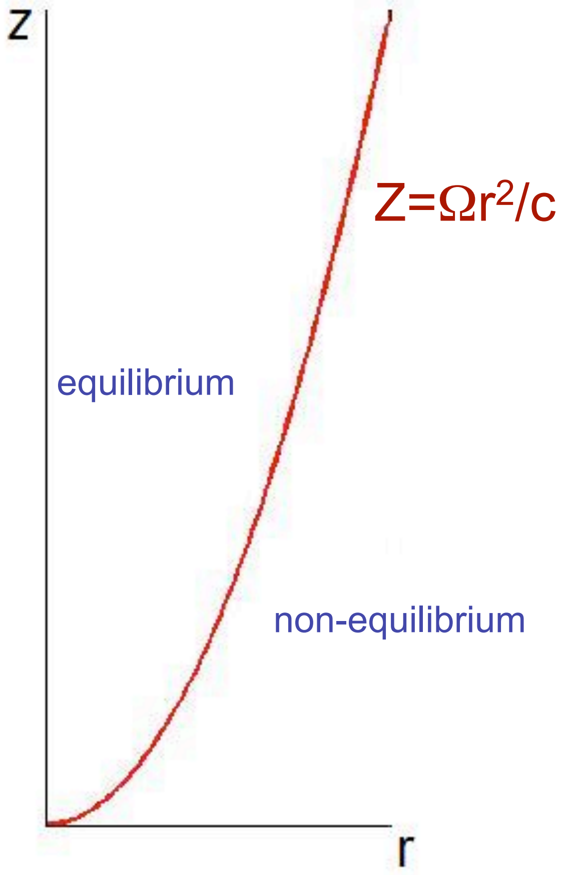
$\gamma \sim r\Omega/c$

The jet is accelerated when expands



Collimation vs acceleration
1. Equilibrium jet (cont)

The flow settles into an equilibrium configuration provided a signal crosses the jet while z varies less than 2 times (strong causality).



Proper propagation time, $z/c\gamma$, >crossing time, r/c

$$z > r\gamma \sim \Omega r^2/c$$

$$\Theta = \frac{dr}{dz} \sim \frac{r}{z} \quad - \text{opening angle}$$

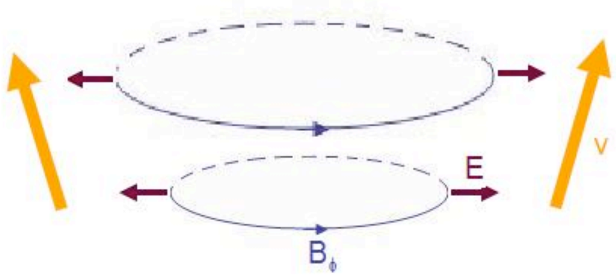
$$\Theta\gamma < 1$$

Equilibrium jets are accelerated up to $\gamma \sim \gamma_{max}$; $\sigma \sim 1$

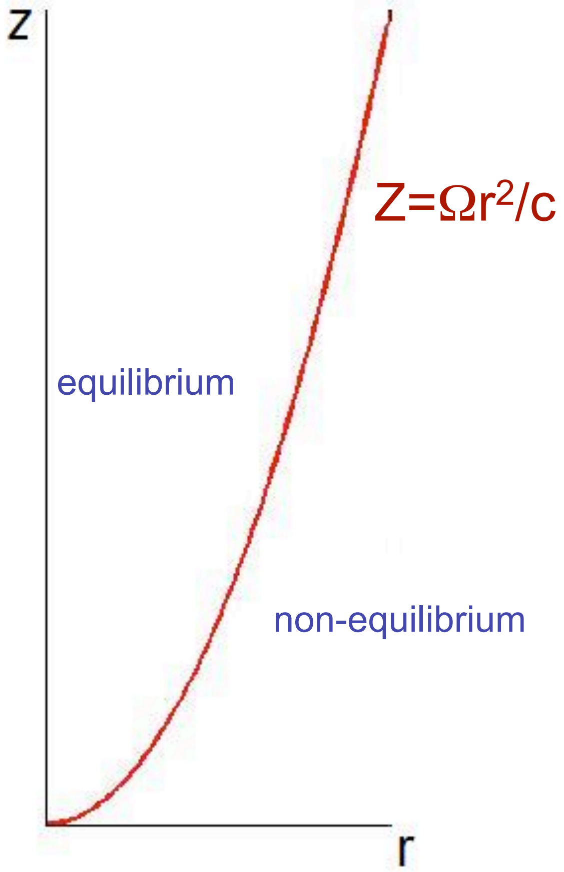
Collimation vs acceleration
2. Non-equilibrium jet

$$z < \Omega r^2/c$$

$$\Theta\gamma > 1$$



Poloidal field is negligible

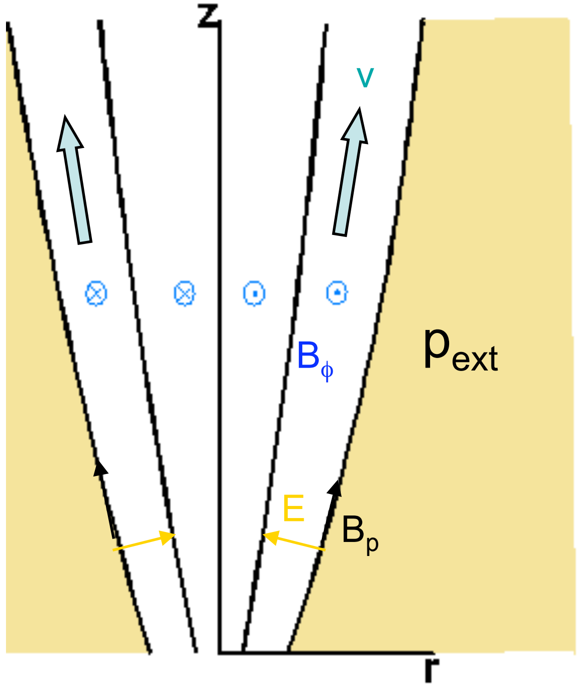


Non-equilibrium jets stop accelerating at

$$\gamma \sim \left(\frac{\gamma_{max}}{\Theta^2}\right)^{1/3} ; \quad \sigma > 1$$

MHD jet confined by the external pressure

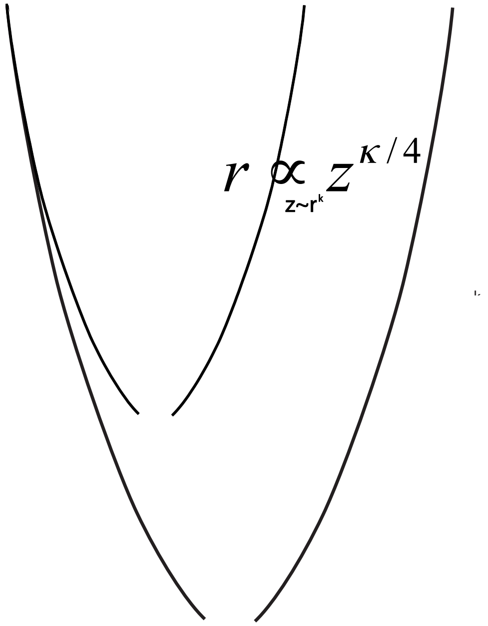
The spatial distribution of the confining pressure determines the shape of the flow boundary and the acceleration rate



Boundary condition: $\frac{B^2}{8\pi\gamma^2} = p_{\mathrm{ext}}$

MHD jet confined by the external pressure (cont)

$$\frac{B^2}{8\pi\gamma^2} = p_{\text{ext}}; \qquad B \sim B_L \frac{R_L}{r} \qquad p_{\text{ext}} = p_0 \left(\frac{R_L}{z} \right)^{\kappa}$$



1. Equilibrium jet; γ~*r/R*$_L$

$$r \propto z^{\kappa/4} \qquad \gamma \propto z^{\kappa/4}$$

Causality condition: $\Theta\gamma < 1$ $\qquad r^2 < R_L z$

Equilibrium jet is formed if κ<2

The fastest acceleration at $\kappa \to 2; \quad \gamma \sim \Omega r / c \sim \sqrt{\Omega z / c}$

MHD jet confined by the external pressure (cont)

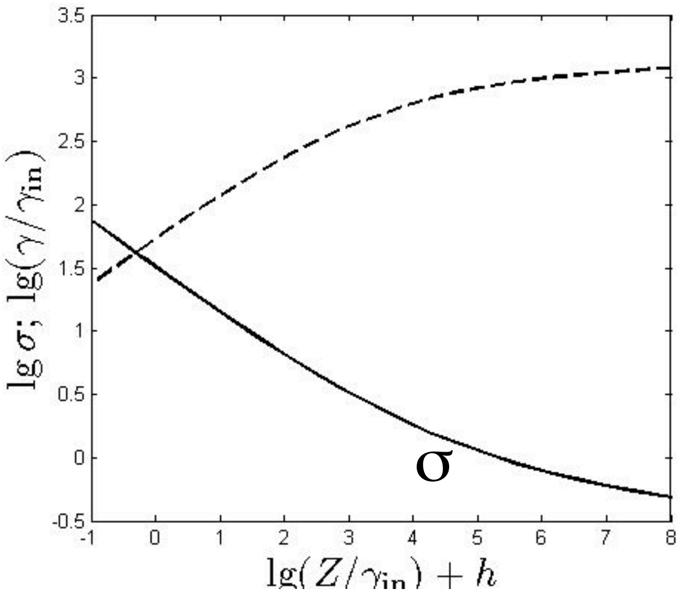
1. Equilibrium jet (cont)

Equipartition, $\sigma \sim 1$; $\gamma \sim \gamma_{\max}$, at $z_0 \sim \gamma_{\max}^{4/\kappa} R_L$

$$\text{At}\ \kappa \to 2, \quad z_0 \sim \gamma_{\max}^{2} R_L$$

Beyond the equipartition:

$$\sigma \sim \frac{1}{\ln\left(z / z_0\right)}$$



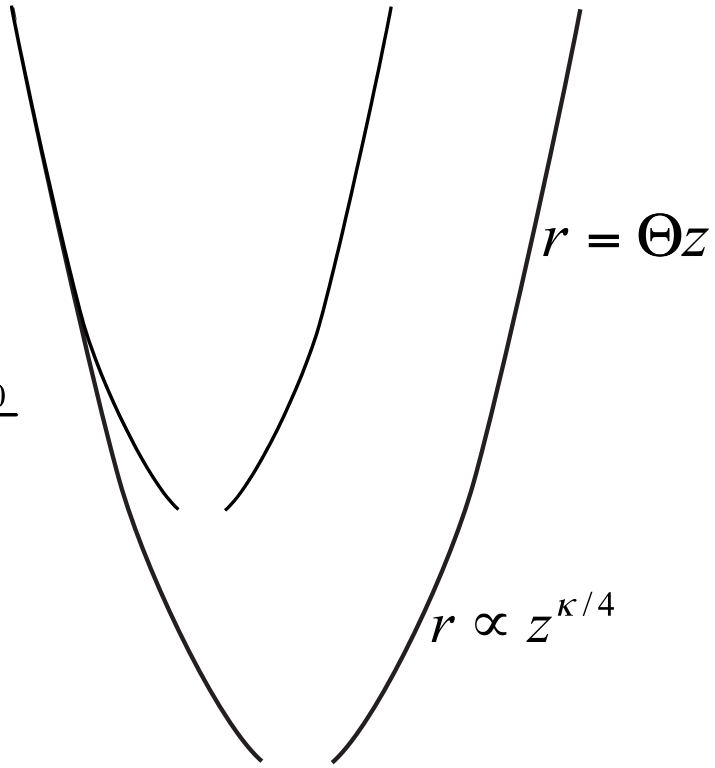
MHD jet confined by the external pressure (cont)

$$p_{\text{ext}} = p_0 \left(\frac{c}{\Omega z} \right)^{\kappa}$$

2. Non-equilibrium jet; κ>2

Jet asymptotically approaches

conical shape $r = \Theta z$



$$\Theta = \frac{1}{\sqrt{\pi}} \Gamma\left(\frac{\kappa-1}{\kappa-2}\right) \left(\frac{(\kappa-2)^{\kappa}}{\beta} \right)^{1/[2(\kappa\text{-}2)]}; \quad \beta = \frac{8\pi p_0}{B_L^2}$$

$\Theta = 0.01/\beta^{2.5}$ at $\kappa = 2.2$

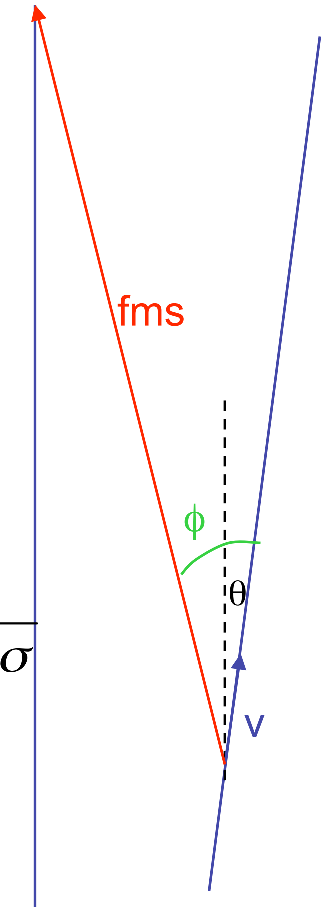
$\Theta = 0.2/\beta$ at $\kappa = 2.5$

$\Theta = 0.56/\sqrt{\beta}$ at $\kappa = 3$

MHD jet confined by the external pressure (cont)

2. Non-equilibrium jet; $\kappa>2$, (cont)

$$\gamma \text{ grows until } \gamma_t \sim \left(\frac{\gamma_{\max}}{\Theta^2}\right)^{1/3}; \qquad \gamma_{\max}\Theta > 1$$



Weak causality condition: $\phi>\theta$

In the comoving frame $v'_{\mathrm{fms}} = \frac{B'c}{\sqrt{4\pi\rho'+B'^2}} = c\sqrt{\frac{\sigma}{1+\sigma}}; \quad \gamma'_{\mathrm{fms}} = \sqrt{1+\sigma}$

In the lab frame $\phi = \frac{\gamma'_{\mathrm{fms}}}{\gamma}$

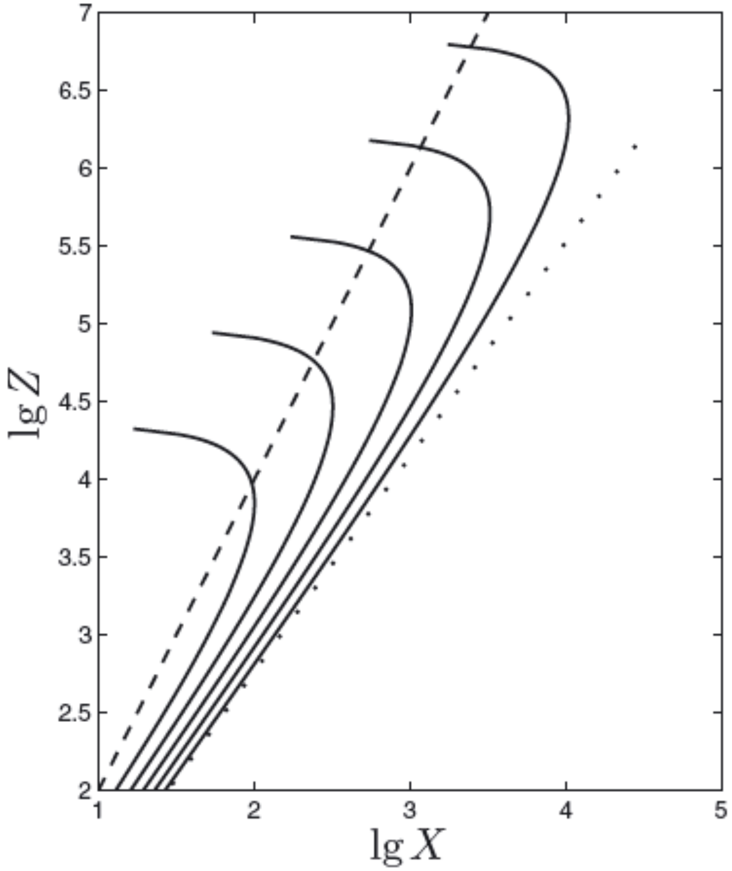
$$\sqrt{1+\sigma} > \gamma\theta; \quad 1+\sigma = \frac{\gamma_{\max}}{\gamma} \quad \Longrightarrow \quad \gamma < \left(\frac{\gamma_{\max}}{\Theta^2}\right)^{1/3}$$

MHD jet confined by the external pressure (cont)

$$p_{\rm ext} = p_0 \left(\frac{c}{\Omega z} \right)^{\kappa} \qquad \beta = \frac{8\pi p_0}{B_L^2}$$

2. A special case; $\kappa=2$

If $\beta<1/4$, the flow is accelerated till $\sigma\sim1$ and then collapses.



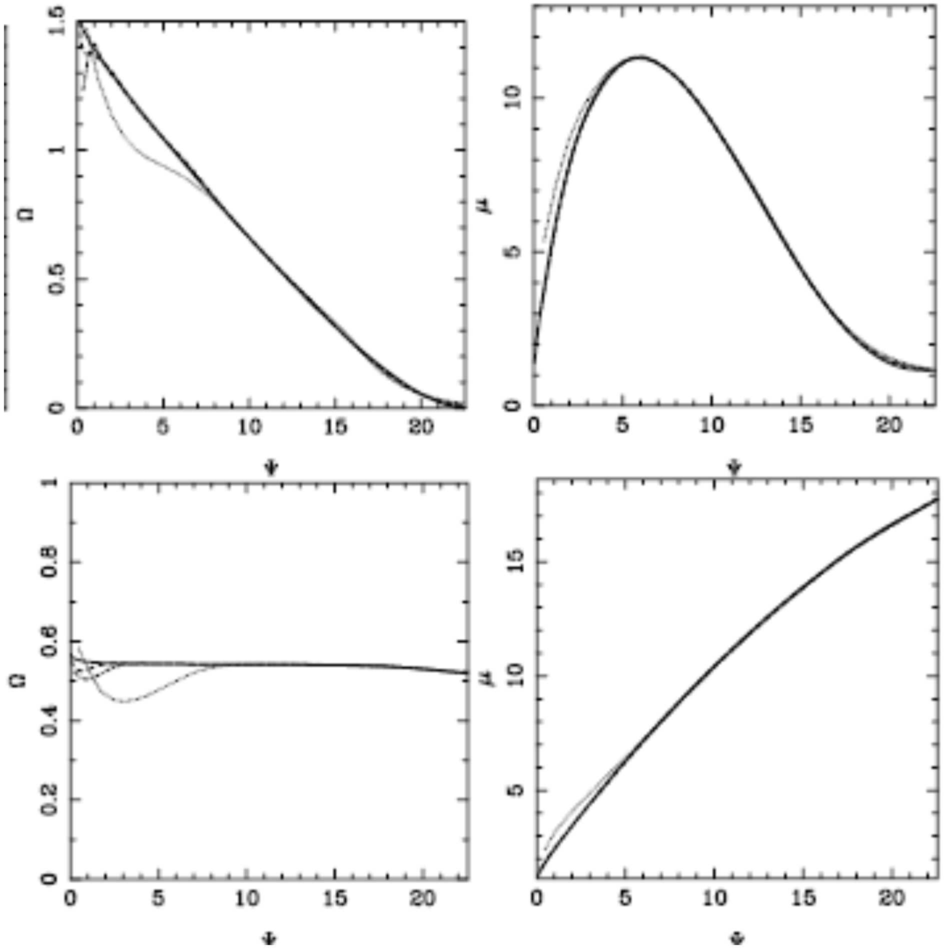
Transverse structure of Poynting dominated jets

Poynting flux goes to 0 at the axis

$$P = \frac{EB_\varphi}{4\pi} c \approx \frac{r\Omega B_p B_\varphi}{4\pi};$$

$$B_\varphi = \frac{2I}{cr} = \frac{2}{cr}\pi\int j\,rdr \longrightarrow \frac{2\pi j(0)}{c} r$$

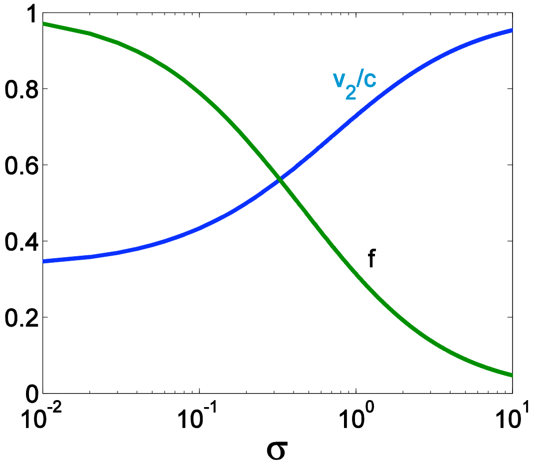
In Poynting dominated outflows, the energy flux has a hollow cone distribution

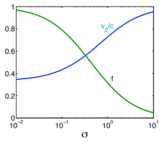


Komissarov et al. 2007

When the flow becomes matter dominated?

The flow could be considered as matter dominated if most of the energy could be released at the termination shock. This condition is fulfilled only if σ<0.1-0.2.



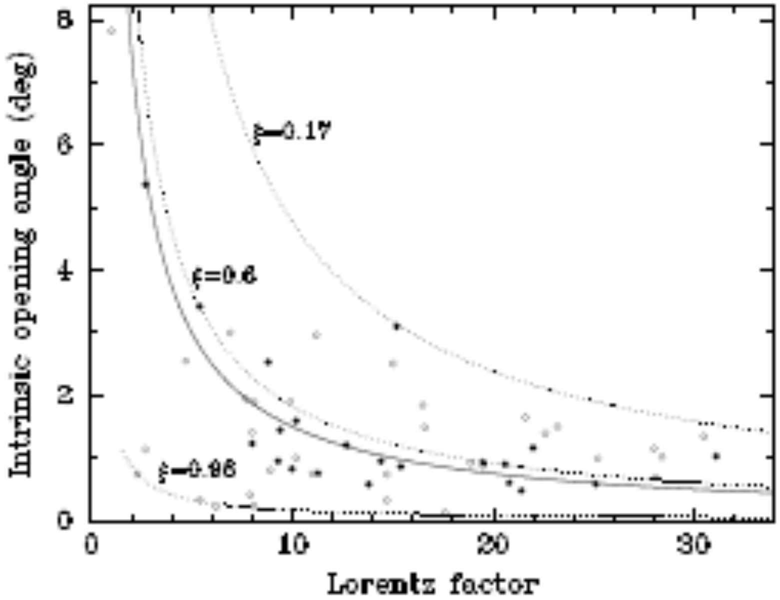


Jump conditions at relativistic shock. f- fraction of energy transferred to the plasma. Compression ratio = c/v_2

Some implications

AGNs: $\gamma \sim 10$ implies the size of the confining zone $z_0 > 100R_g \sim 10^{16}$cm. The condition of efficient acceleration $\gamma\Theta \sim 1$ is fulfilled: $<\theta\gamma> = 0.26$

But according to spectral fitting, jets are already matter dominated at ~1000 R_g (Ghisellini et al 10). Violent dissipation somewhere around1000R_g?



Pushkarev et al 09

GRBs: $\gamma \sim 10^2$ - 10^3; minimal $z_0 \sim 10^{11}$ cm – marginally OK. But achromatic breaks in the afterglow light curves and statistics imply $\gamma\Theta >> 1$, which is fulfilled only if the flow remains Poynting dominated. Magnetic dissipation is necessary.

To summarize the dissipationless MHD acceleration

The flow could be accelerated till $\sigma\sim 1$ provided it is confined by an external pressure, which decreases with distance but not too fast.

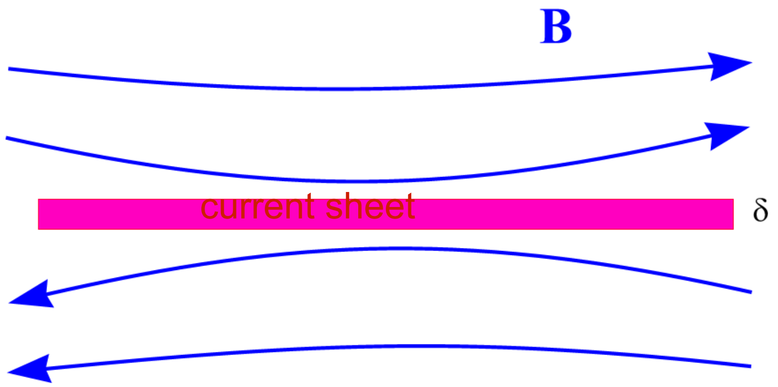
The acceleration zone spans a large range of scales so that one has to ensure that the conditions for acceleration are fulfilled all along the way. Transition to the matter dominated stage, $\sigma\sim 0.1$, could occur only at an unreasonably large scale.

These conditions are rather restrictive. It seems that in real systems, some sort of dissipation (reconnection) is necessary in order to utilize the electromagnetic energy completely.

Beyond the ideal MHD:

magnetic dissipation in Poynting dominated outflows

The magnetic energy could be extracted via anomalous dissipation in narrow current sheets.

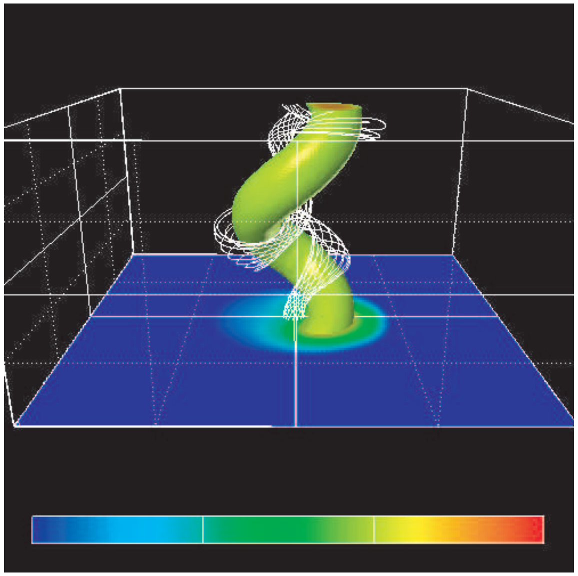


How differently oriented magnetic field lines could come close to each other?

1. Global MHD instabilities could disrupt the regular structure of the magnetic field thus liberating the magnetic energy.
2. Alternating magnetic field could be present in the flow from the very beginning (striped wind).

Global MHD instabilities

The most dangerous is the kink instability (Lyubarsky 92, 99; Eichler 93; Spruit et al. 97; Begelman 98; Giannios&Spruit 06).

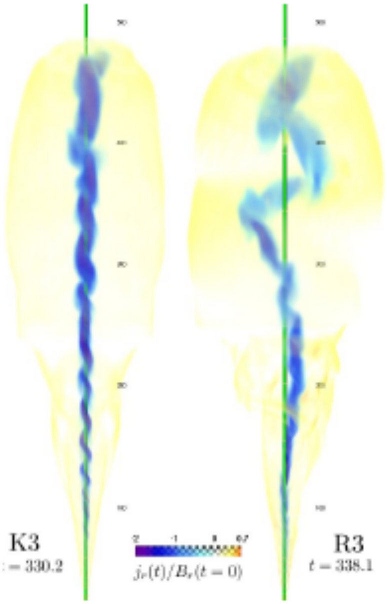


Mizuno et al 09

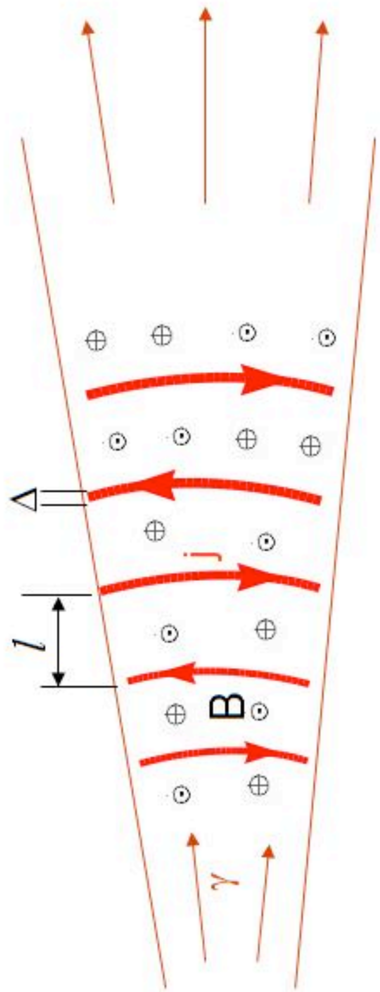
But: The necessary condition for the instability – causal connection, $\gamma\Theta<1$. Not fulfilled in GRBs; may be fulfilled in AGNs.

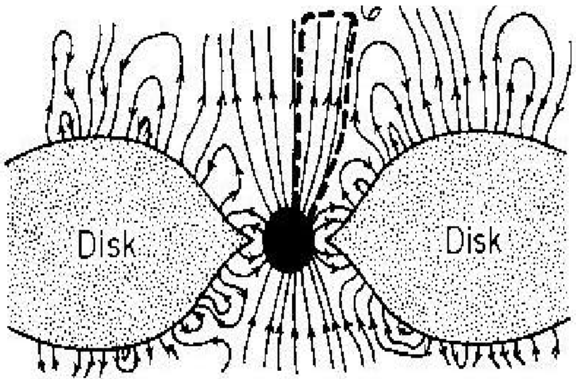
The growth rate is small in relativistic case (Istomin & Pariev '94, '96; L '99).

Evidence for saturation of the instability (McKinney&Blandford 09)



Moll et al 10





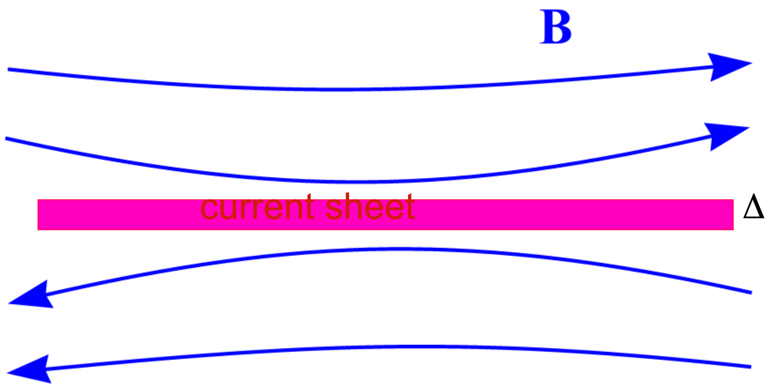
Could alternating magnetic field be presented in the flow from the very beginning?

Let alternating fields preexist in the jet

In an expanding flow, B becomes predominantly toroidal; current sheets are stretched. Local structure: plane current sheet separating oppositely directed fields.

$$nT = \frac{B^2}{8\pi}$$

$$r_B = \frac{T}{eB} = \frac{B}{8\pi e n}$$



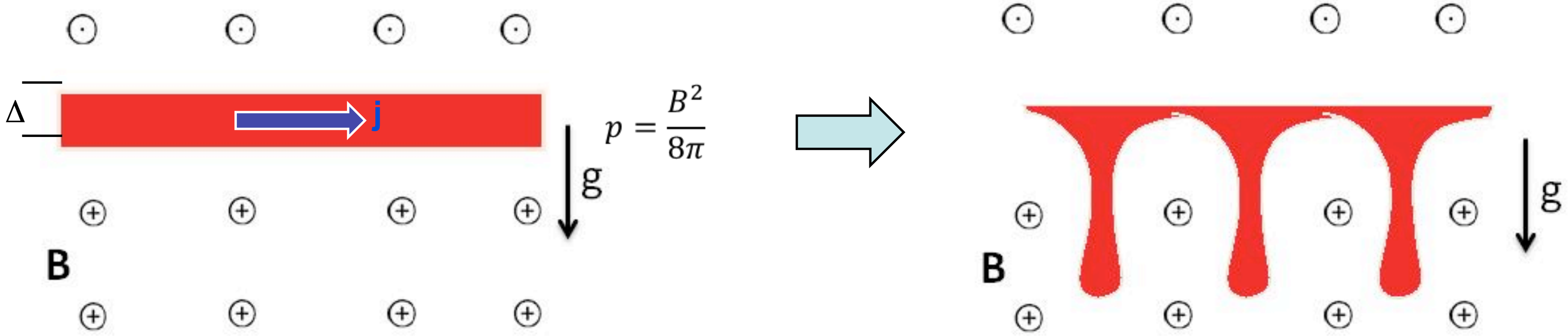
Condition for anomalous resistivity: $\mathrm{v}_{\mathrm{current}} = \dfrac{j}{en} \sim c$

or equivalently: $r_B = \dfrac{kT}{eB} \sim \Delta$

In AGNs and GRBs, $r_B << l \sim R_g$. No conditions for anomalous dissipation.

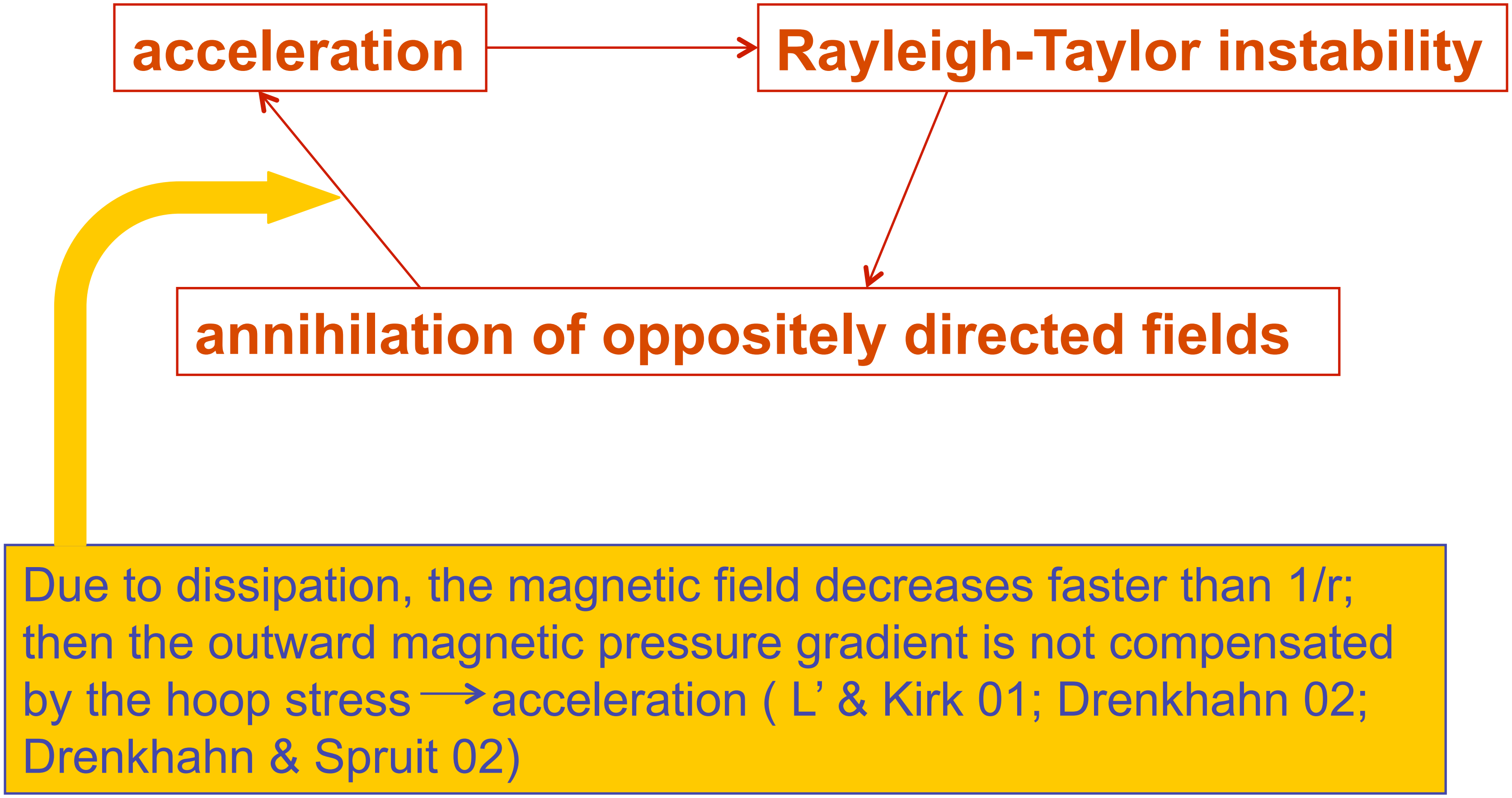
Rayleigh-Taylor instability of currents sheets in accelerating flows (L ‘10)

In an accelerating relativistic flow $g = c^2 \frac{d\gamma}{dr}$



Instability time-scale $\tau = \xi (g\Delta)^{-1/2}$

Dissipation rate $Q = \frac{\Delta}{\tau} \frac{B^2}{8\pi}$



acceleration

Rayleigh-Taylor instability

annihilation of oppositely directed fields

Due to dissipation, the magnetic field decreases faster than 1/r; then the outward magnetic pressure gradient is not compensated by the hoop stress → acceleration (L' & Kirk 01; Drenkhahn 02; Drenkhahn & Spruit 02)

Interplay between acceleration and dissipation; a self-consistent picture

$$r^2\langle nv\rangle = const$$

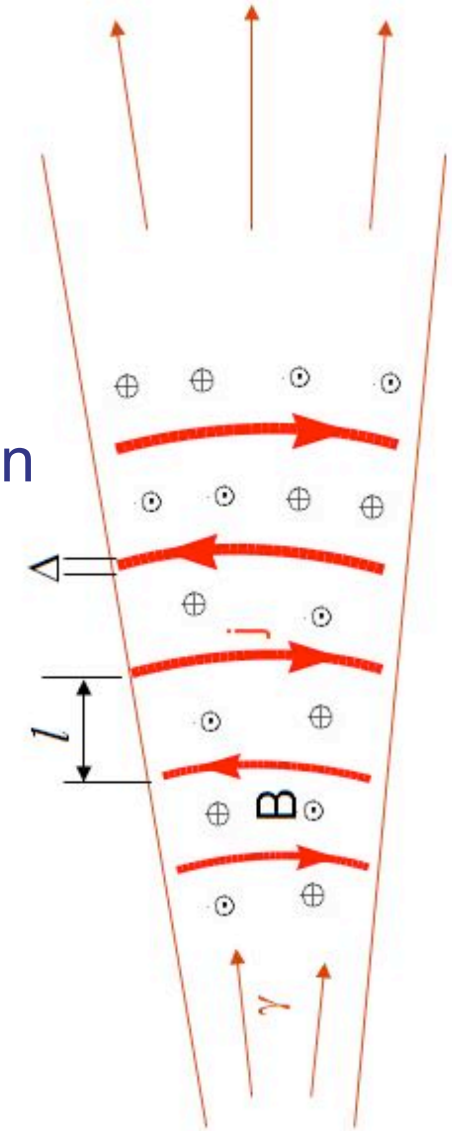
continuity

$$r^2\left\langle h\gamma^2 v + \frac{EB}{4\pi}c\right\rangle = const$$

energy conservation

$$\left\langle \mathrm{T}\frac{dS}{dt}\right\rangle = \frac{Q}{l}$$

heat balance



Interplay between acceleration and dissipation; a self-consistent picture (cont)

$$\gamma = \left(\frac{9\gamma_{\max} z}{4\xi l} \right)^{1/3}; \qquad \frac{\Delta}{l} = \left(\frac{z}{12\xi^2 l \gamma_{\max}^2} \right)^{1/3}$$

Complete dissipation: $\Delta \sim l$; $\gamma \sim \gamma_{\max}$; $z_{diss} \sim (\xi\gamma_{max})^2 l$

In accreting systems: $l \sim R_g$

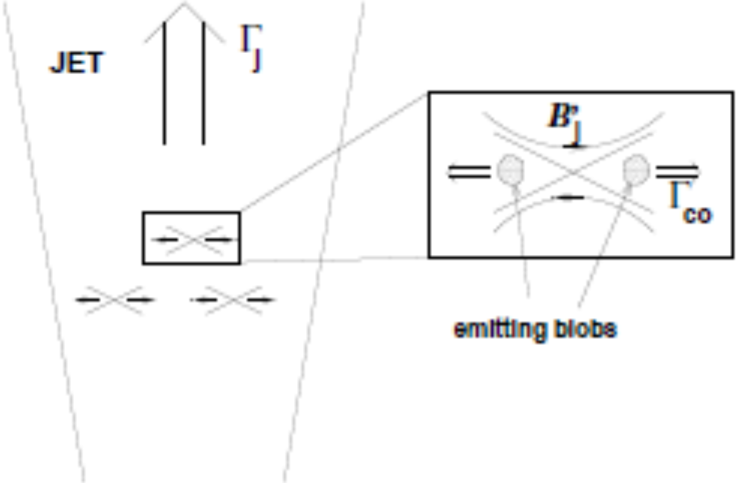
$$z_{diss} \sim \begin{cases} 10^{17} \left(\frac{\xi\gamma}{30} \right)^2 \text{cm} & \text{AGNs} \\ 10^{12} \left(\frac{\xi\gamma}{1000} \right)^2 \text{cm} & \text{GRBs} \end{cases}$$

Advantage of the magnetic dissipation models

1. Dissipation easily provides large radiative efficiencies and strong variability necessary to fit observations

2. Gradual dissipation at the scale ~10^{11}-10^{13} cm could account for the observed spectra of the prompt GRB emission (Giannios & Spruit '07; Giannios '08).

3. Reconnection at high σ produces relativistic "jets in a jet", which could account for the fast TeV variability of blazars (Giannios et al '09,'10)



Conclusions I

1. Magnetic fields are the most likely means of extracting the rotational energy of the source and of producing relativistic, outflows from compact astronomical objects (pulsars,GRBs, AGNs, galactic X-ray binaries)

2. The paradigm of Poynting dominated outflows works in a similar way in all these systems.

3. External confinement is crucial for efficient collimation and acceleration of Poynting dominated outflows.

4. An extended acceleration region is a distinguishing characteristic of the Poyntyng dominated outflows. Jet outflow is initially broad, slowly-collimating and slowly-accelerating

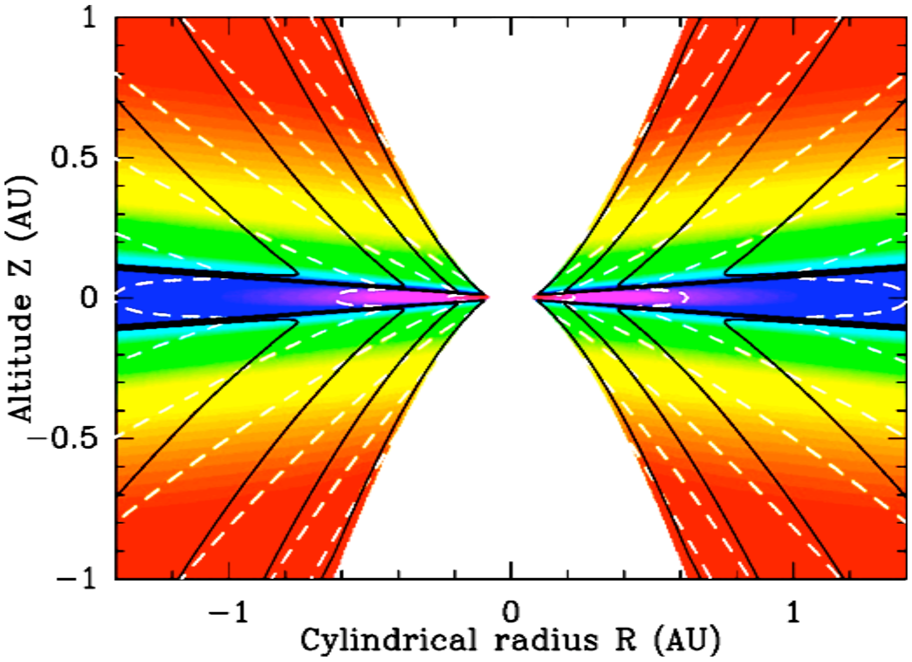
5. Efficient acceleration (up to $\gamma \sim \gamma_{max}$) is possible only in causally connected flows ($\gamma\Theta \le 1$).

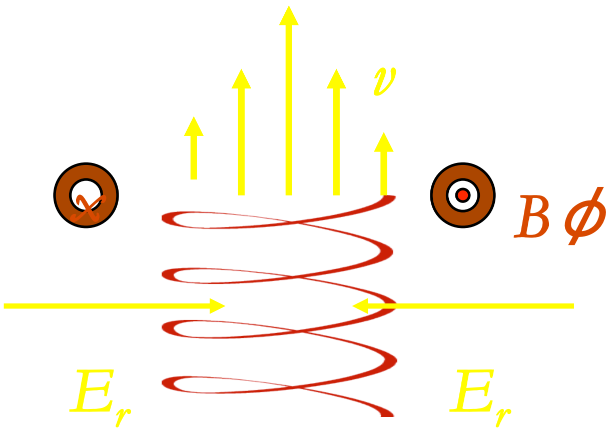
Conclusions II

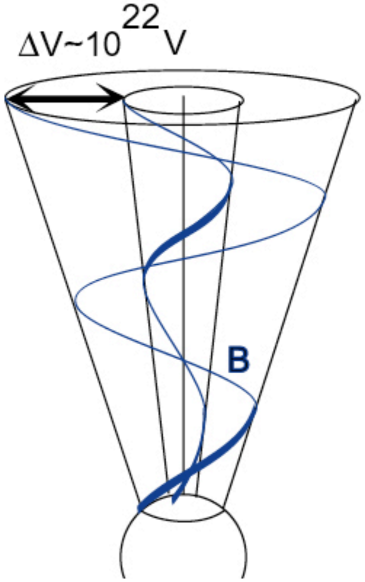
1. Even though efficient transformation of the Poynting into the kinetic energy is possible in principle, the conditions are rather restrictive.
2. Dissipation (reconnection) is necessary in order to utilize the EM energy of the outflow.
3. If alternating field preexisted in the flow, they are efficiently dissipated via the Rayleigh-Taylor instability. The necessary effective gravity is self-consistently maintained because magnetic dissipation results in the acceleration of the flow.
4. Magnetic dissipation naturally provides high radiative efficiency and strong variability necessary to fit observations.

Slow acceleration and collimation of these jets is probably the norm

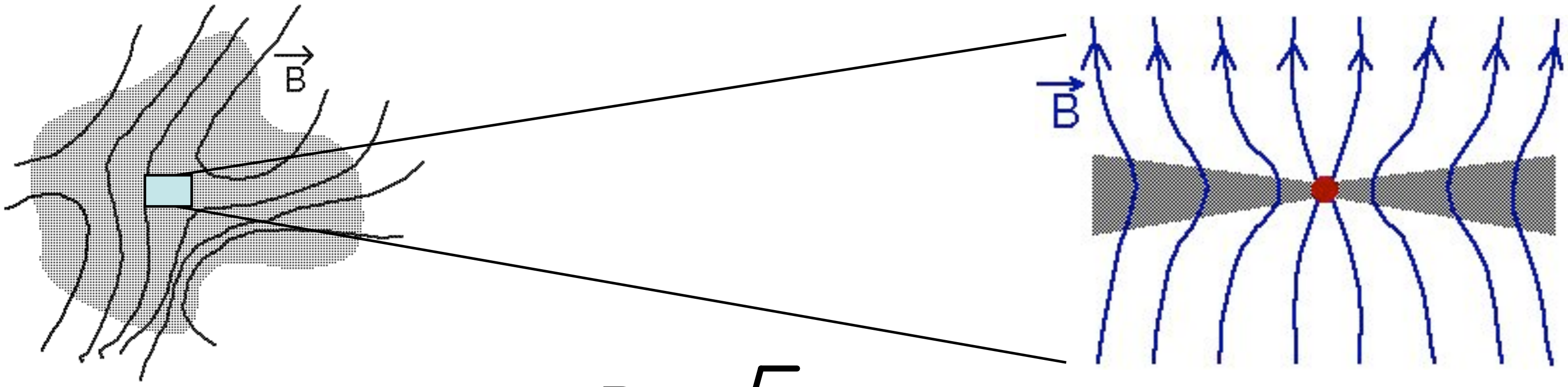
A growing body of data indicates that AGN jets undergo a bulk of their acceleration on parsec scales (>> size of the central black hole)




v
Bϕ
E_r
E_r

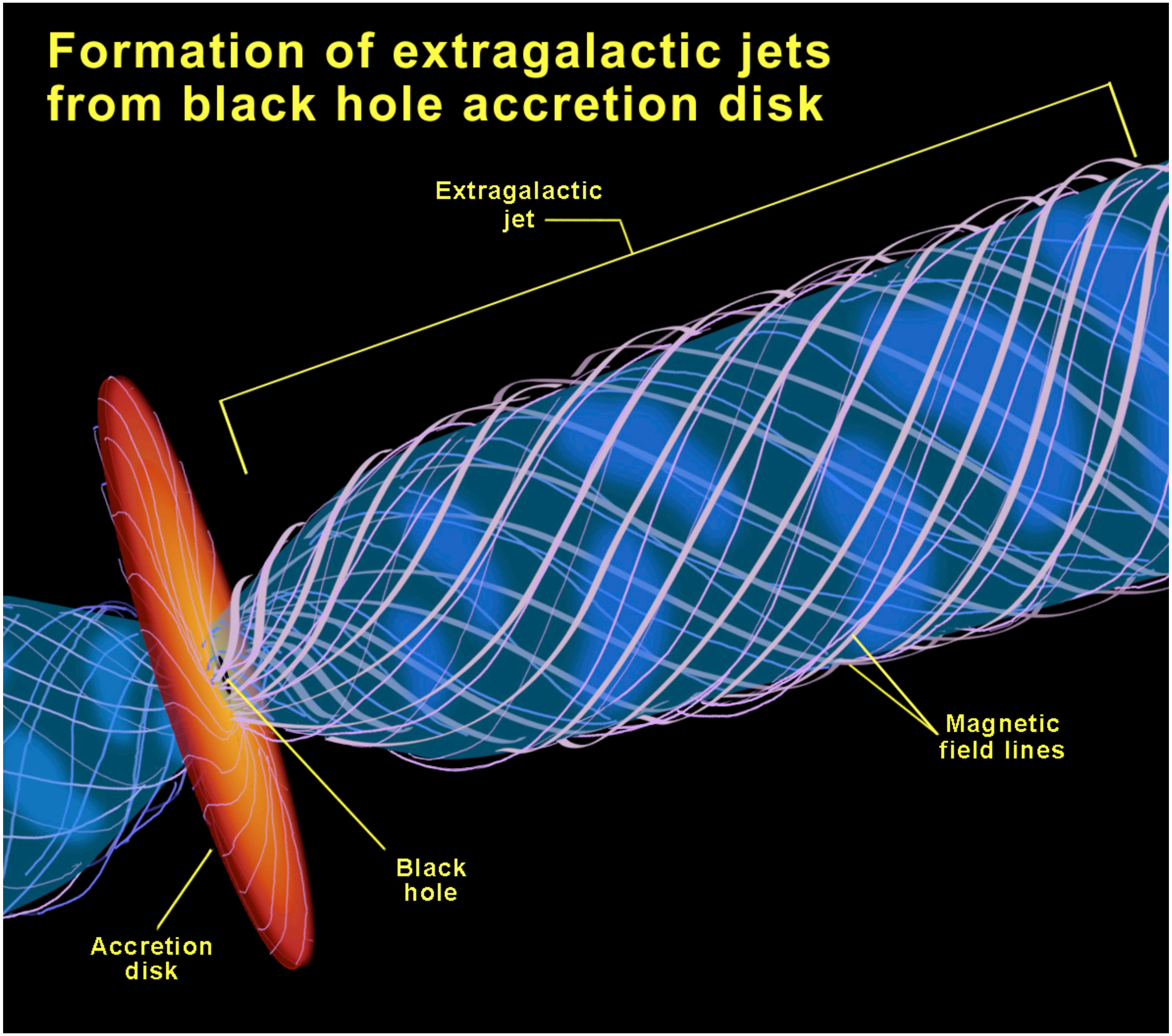

ΔV~10^22 V
B

Where is the magnetic field from?



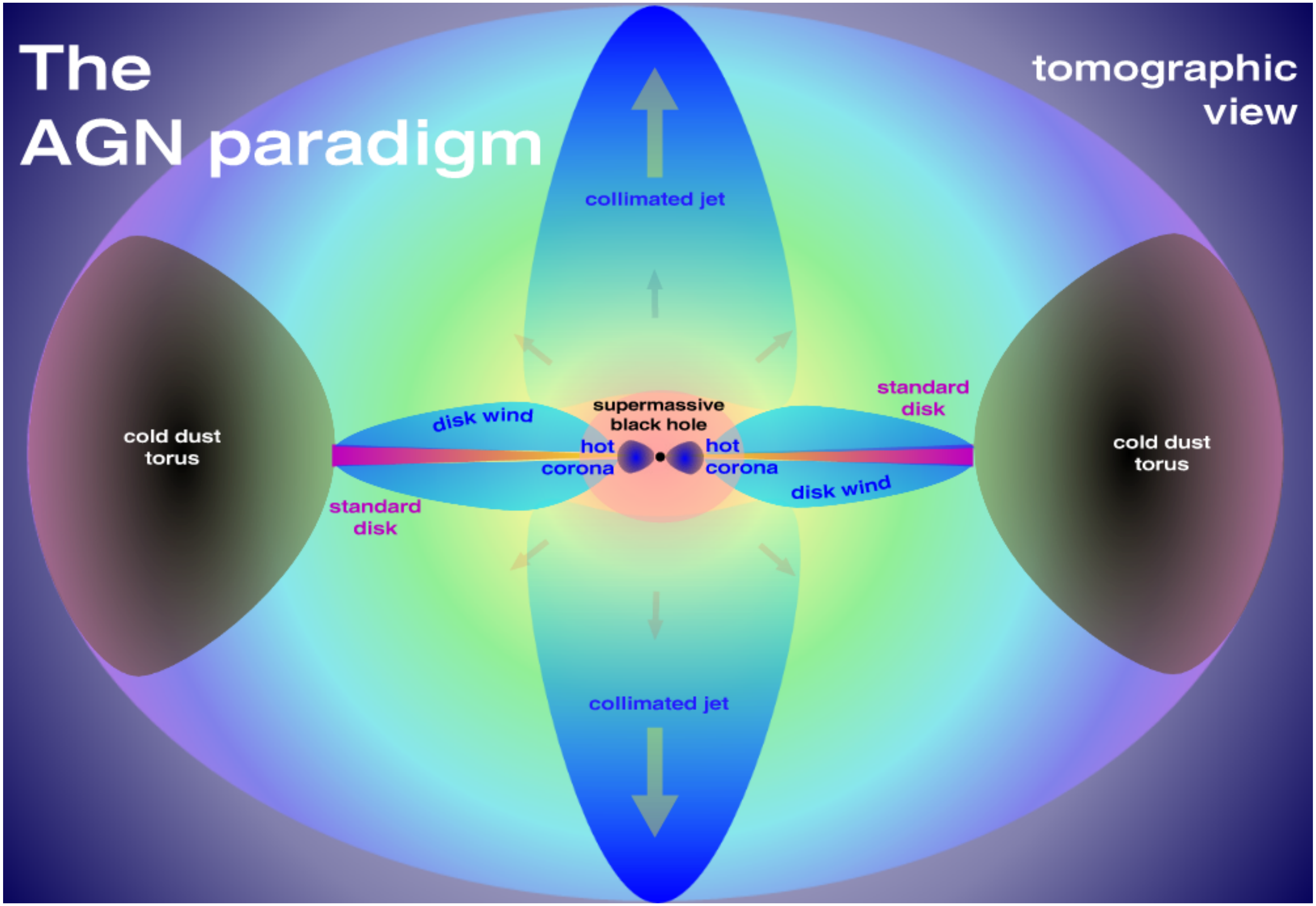
$$B \propto \sqrt{n}$$

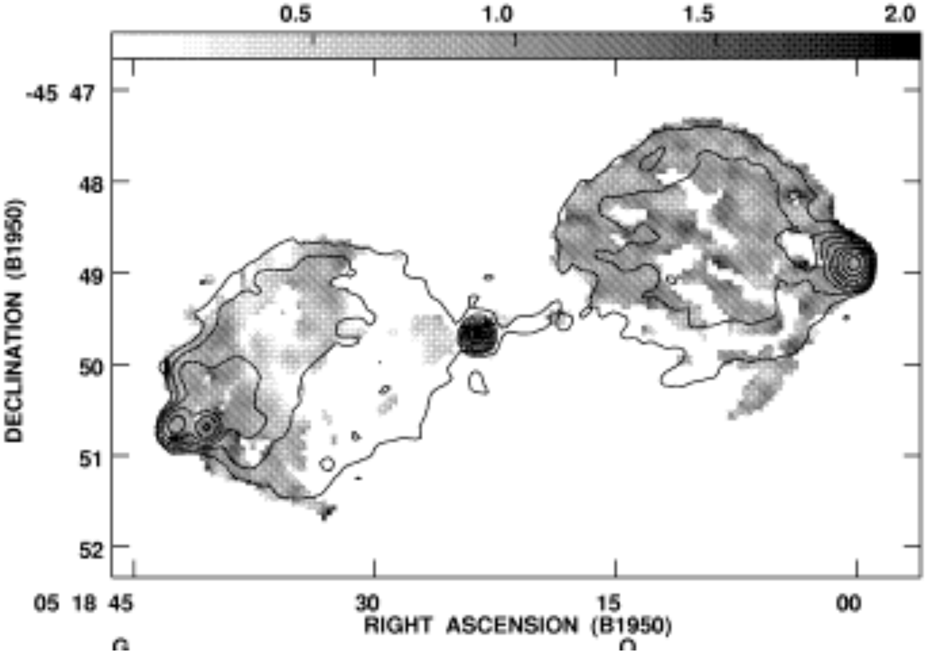
Formation of extragalactic jets from black hole accretion disk



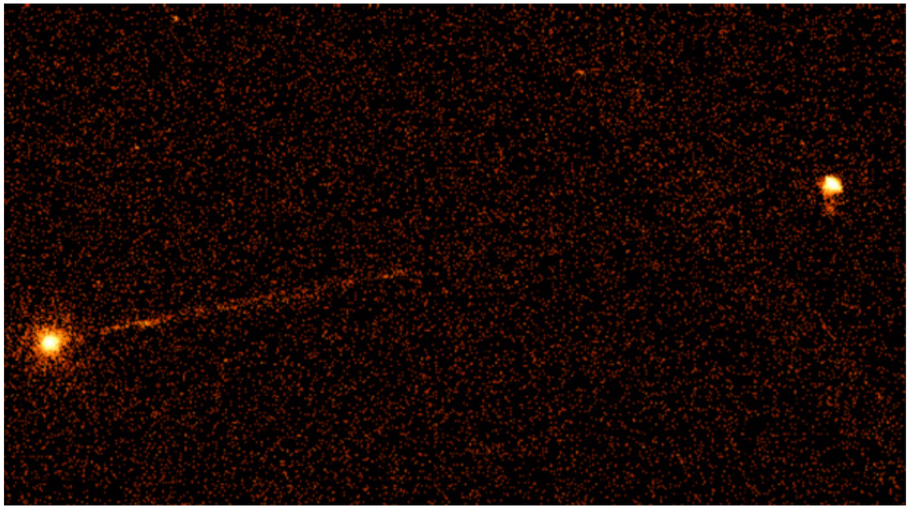
The AGN paradigm

tomographic view





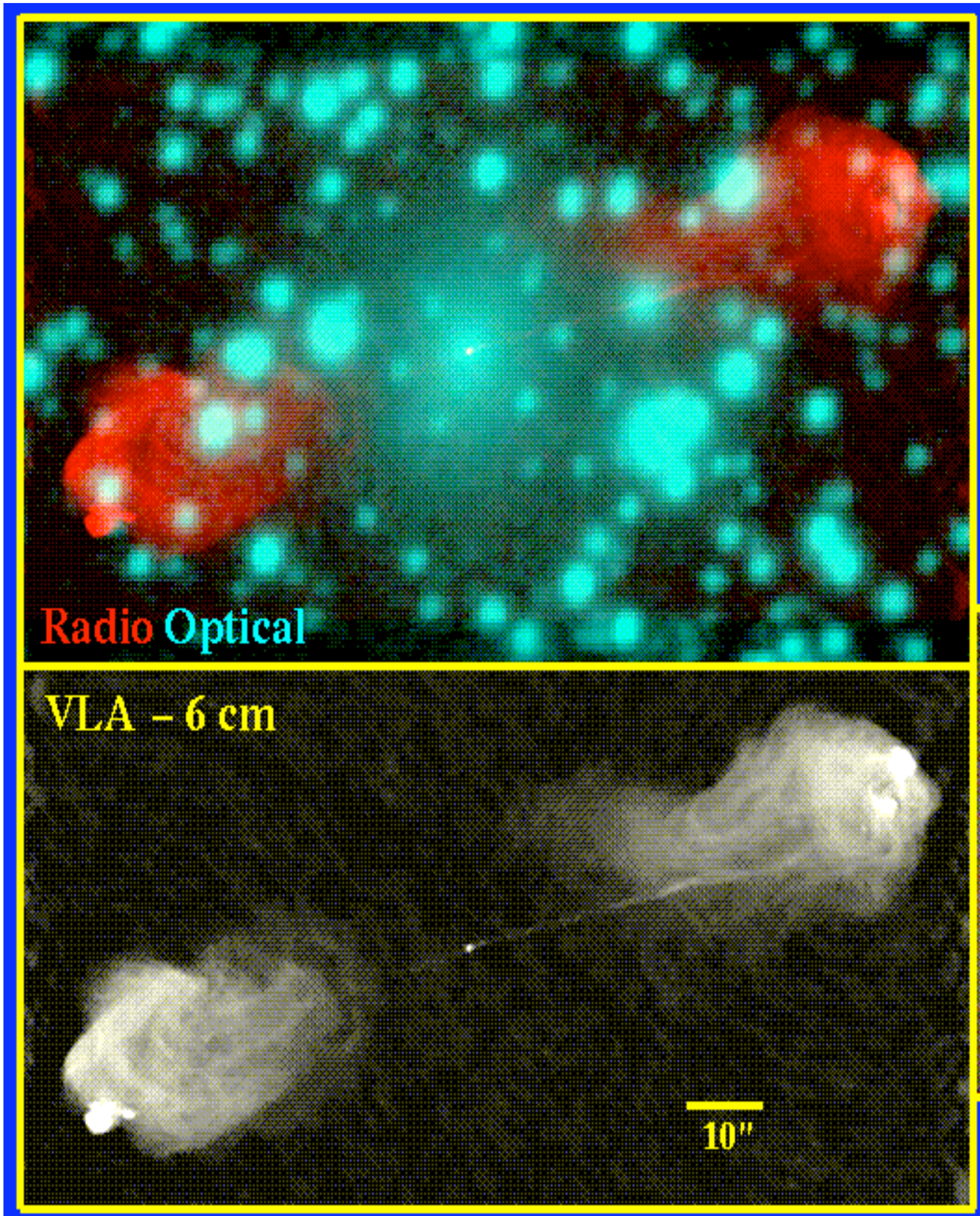
Radio



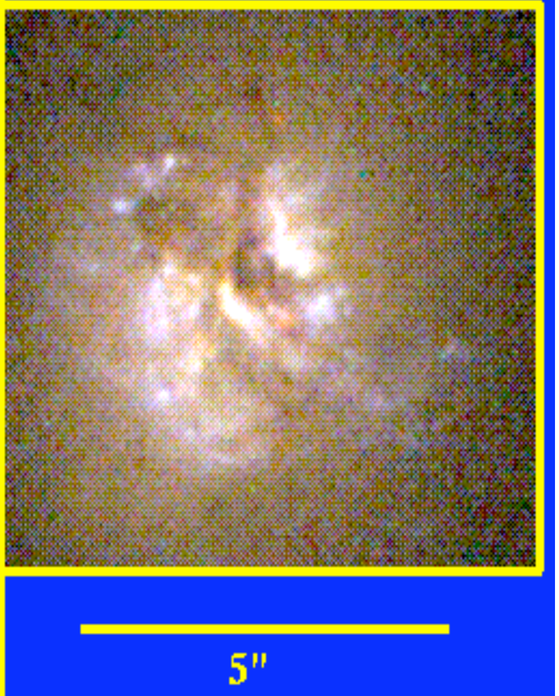
X-rays

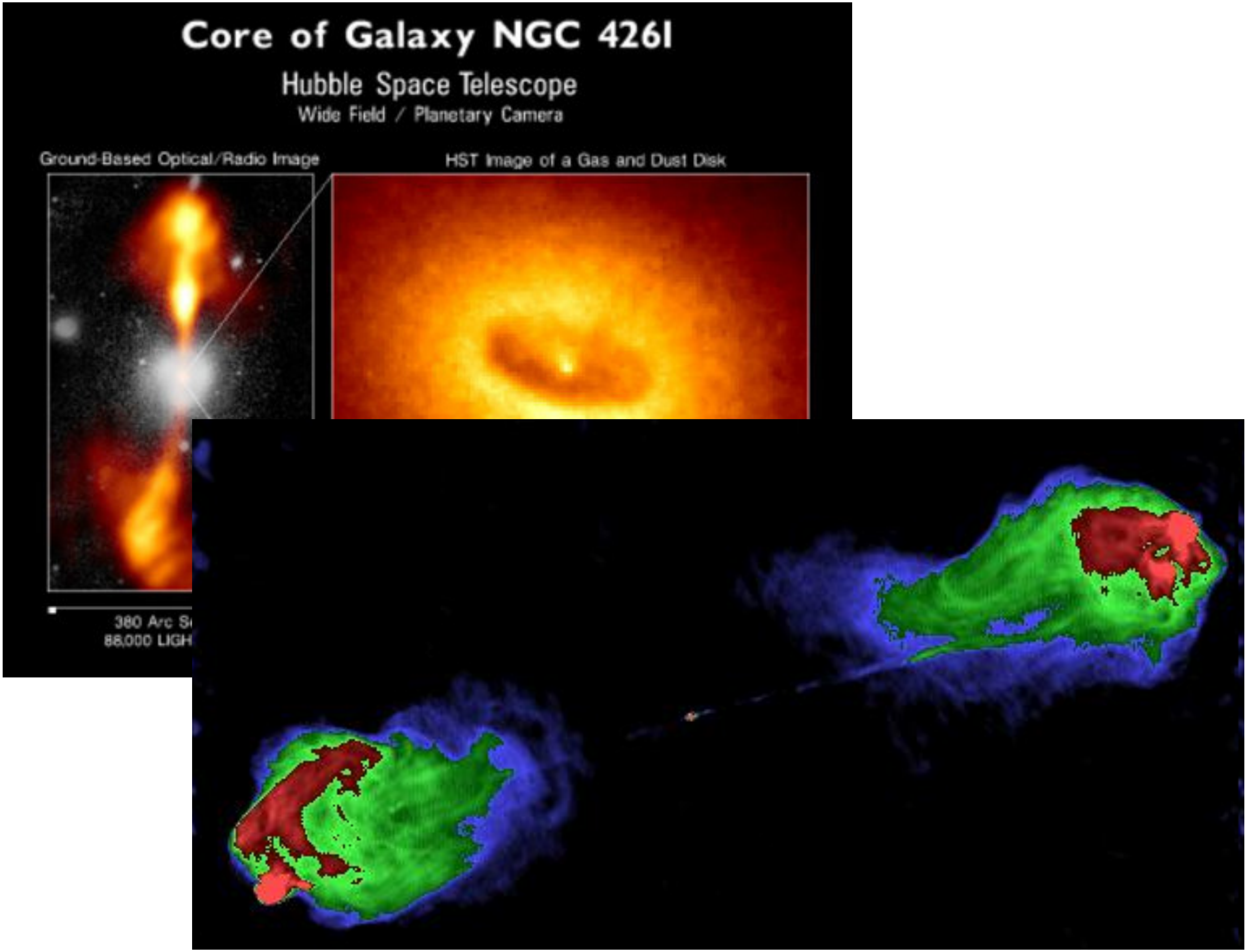
Pictor A

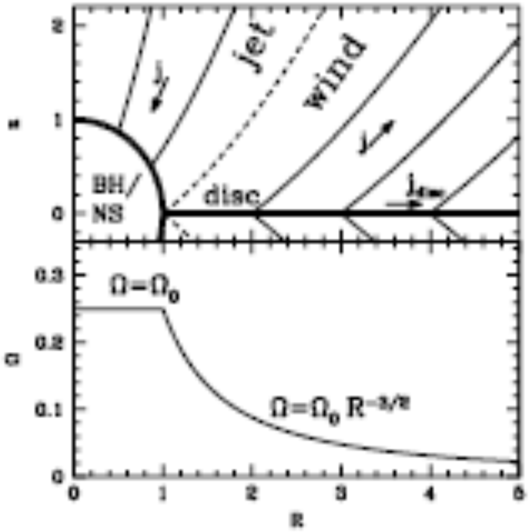
Cygnus A
(3C 405)

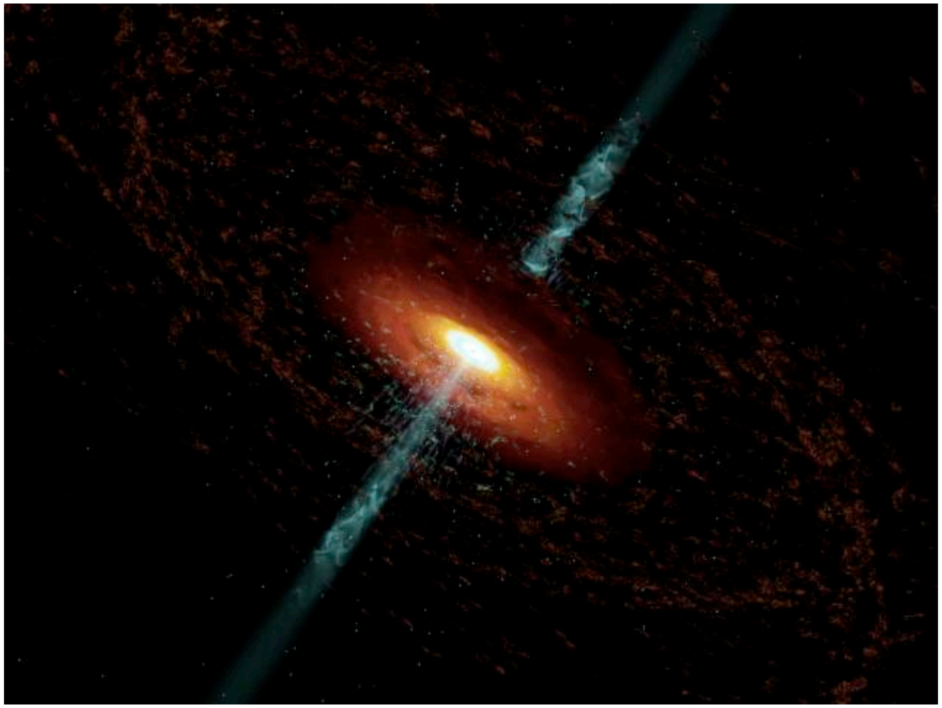


HST closeup




Core of Galaxy NGC 4261
Hubble Space Telescope
Wide Field / Planetary Camera
Ground-Based Optical/Radio Image
HST Image of a Gas and Dust Disk
380 Arc S
88,000 LIGH

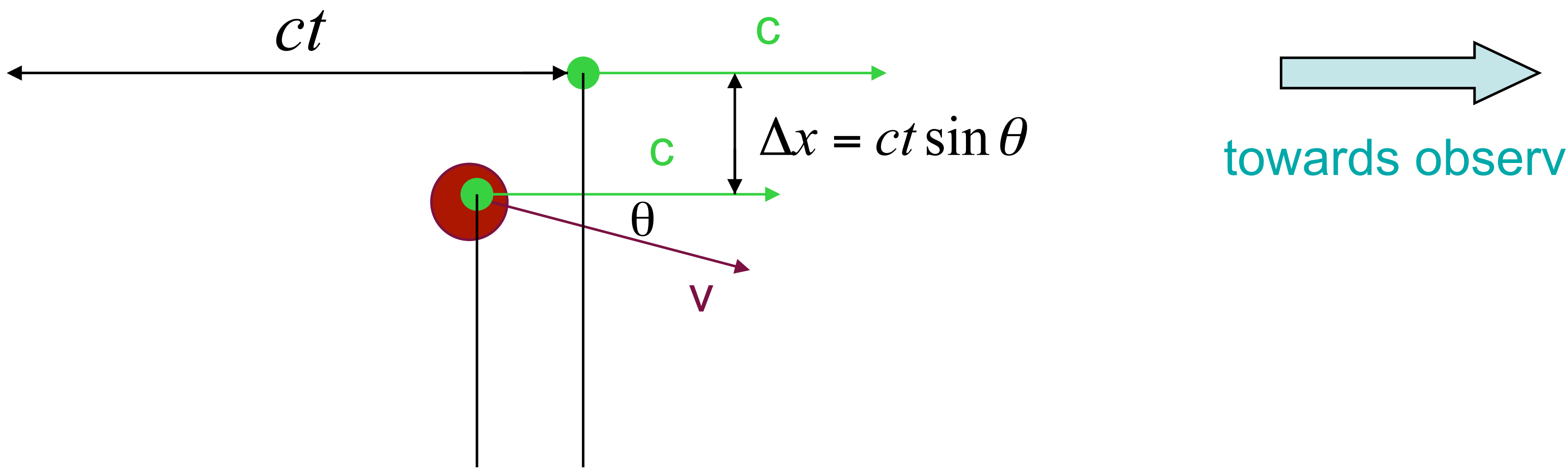




MHD outflow from rotating accretion disks Blandford (1976); Lovelace(1976)
MHD outflow from rotating black holes (Blandford & Znajek 1977)

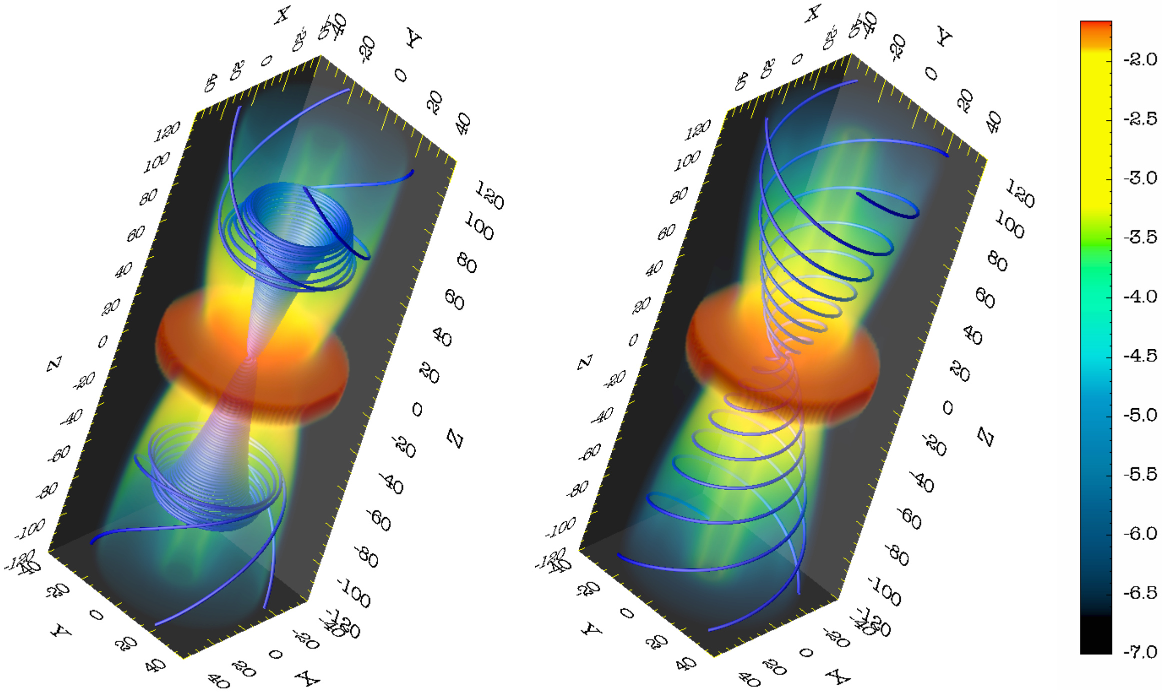
Apparent superluminal motion (Rees 1967)

Apparent superluminal motion (Rees 1967)

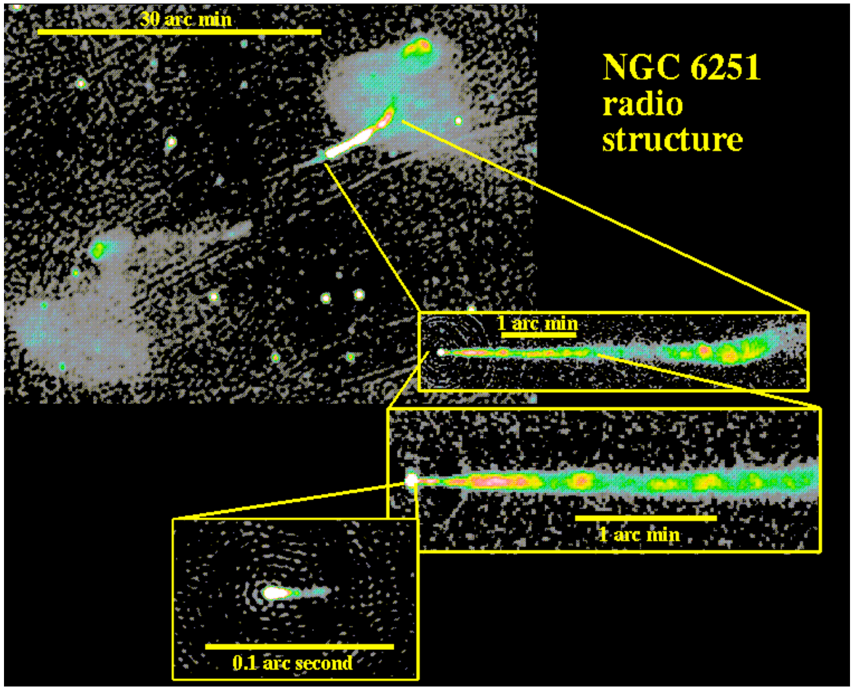


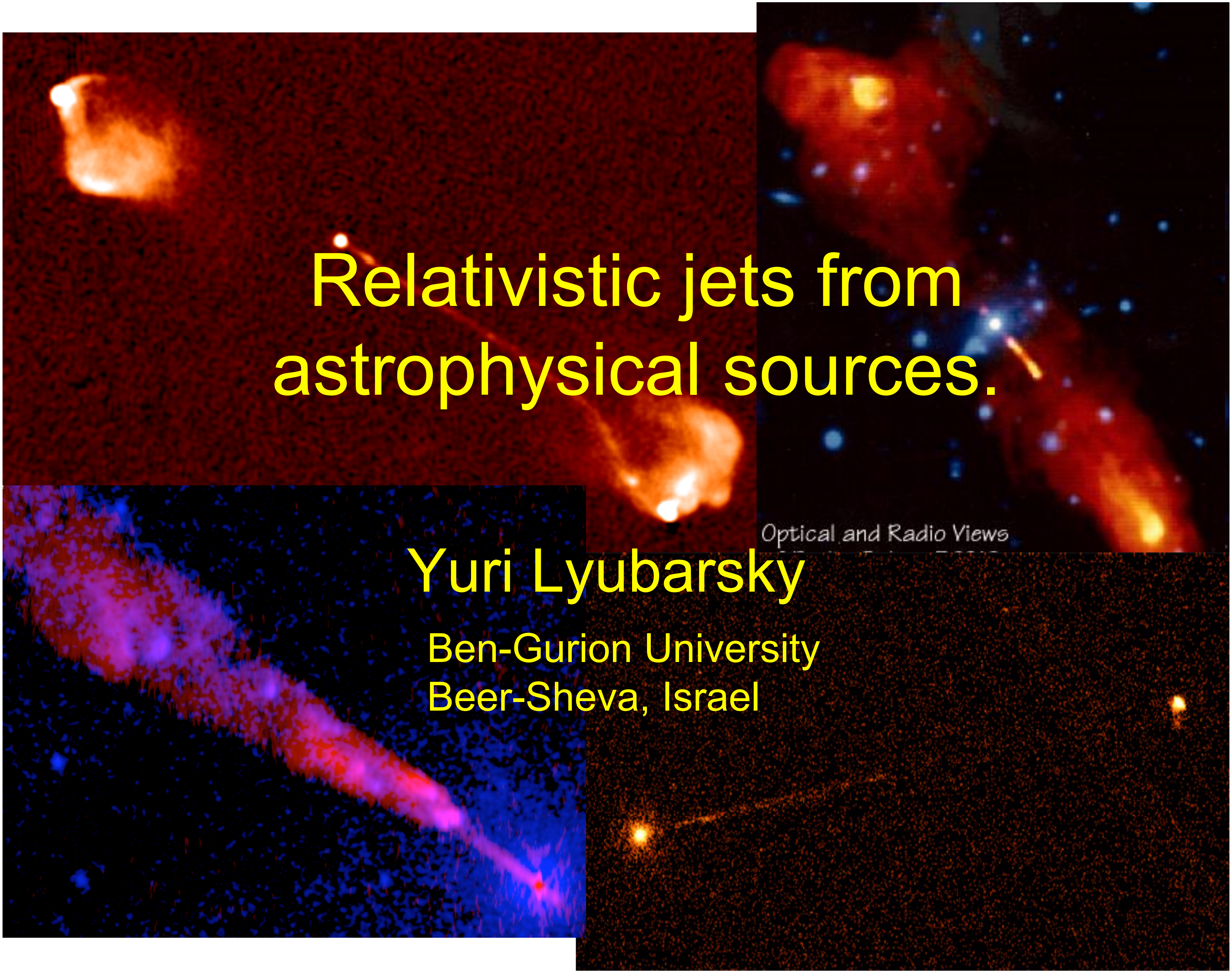
$$\Delta t = t\left[1 - (\mathrm{v}/c)\cos\theta\right]$$

$$\mathrm{V}_{\mathrm{apparent}} = \frac{\Delta x}{\Delta t} = \frac{\sin\theta}{1-(\mathrm{v}/c)\cos\theta}\,c$$



Zanni et al 2007


NGC 6251
radio
structure
30 arc min
1 arc min
1 arc min
0.1 arc second



Relativistic jets from astrophysical sources.

Yuri Lyubarsky

Ben-Gurion University
Beer-Sheva, Israel

Longitudinal dynamics of the flow

$$\gamma - \frac{r\Omega B_\phi}{\eta} = \mu(\Psi)$$ energy

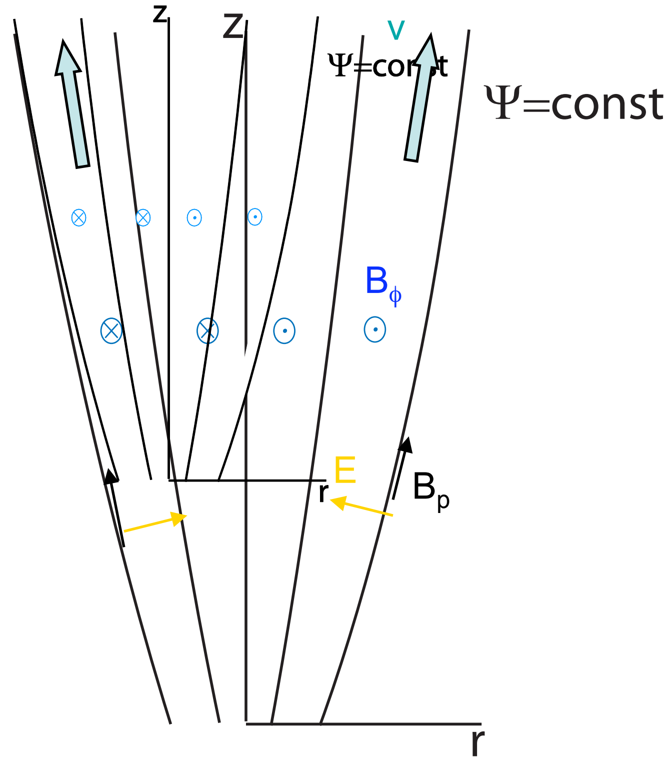
$$\gamma r \mathrm{v}_\phi - \frac{rB_\phi}{\eta} = l(\Psi)$$ angular momentum

$$4\pi\rho \mathrm{v}_p \gamma = \eta(\Psi) B_p$$ continuity

$$B_p \mathrm{v}_\phi - B_\phi \mathrm{v}_p = r\Omega(\Psi) B_\mathrm{p}$$ “bead on wire”

$$Y(\Omega r, \gamma, \Psi, \nabla\Psi) = 0$$ Bernoulli equation

$$B_p = \tfrac{1}{r} |\nabla\Psi|$$



At $r \to \infty$, $\quad Y = 0$ is reduced to $\quad E^2 = B_\phi^2$

Drift $\quad \mathrm{v}_p = \frac{\mathrm{E}}{\mathrm{B}_\phi} \mathrm{c} \to \mathrm{c}$

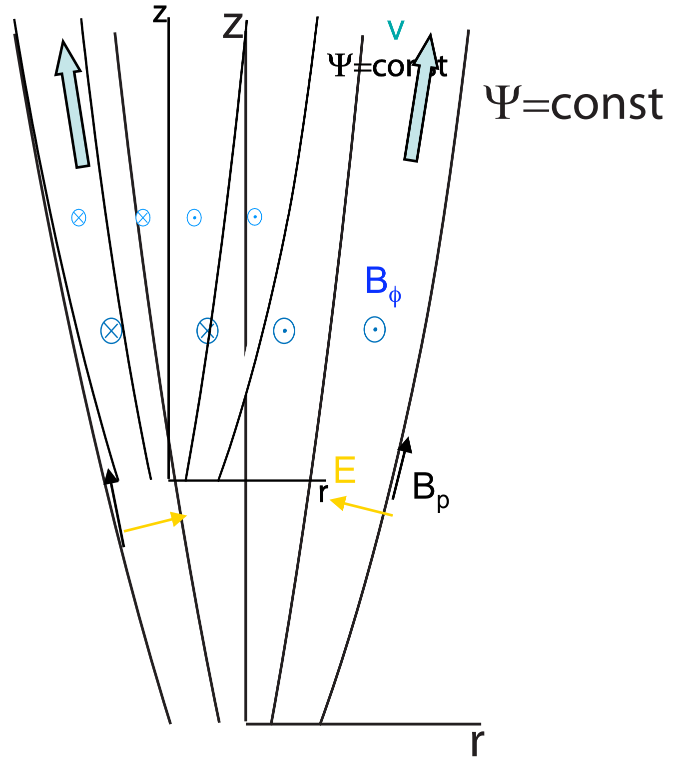
Transverse force balance

$$\left[4\pi\rho\gamma(\mathbf{v}\cdot\nabla)\mathbf{v}-\mathbf{E}\left(\nabla\cdot\mathbf{E}\right)+\mathbf{B}\times\left(\nabla\times\mathbf{B}\right)\right]\cdot\nabla\Psi=0$$

$$G\left(\Omega r,\gamma,\Psi,\nabla\Psi,\nabla^{2}\Psi\right)=0$$

At $r\to\infty$, G = 0 is reduced to

$$\nabla\Psi\cdot\nabla\left[r^{2}\left(B_{\phi}^{2}-E^{2}\right)\right]=0$$



The hoop stress is balanced by the electric force

$$\text{At } \Omega r >> c, \quad E, B_\phi >> B_p$$

$$\text{Transverse force balance, } G\left(\Omega r, \gamma, \Psi, \nabla\Psi, \nabla^2\Psi\right) = 0, \text{ implies } E^2 \approx B_\phi^2$$

But

$$\text{Longitudinal dynamics, } \Upsilon\left(\Omega r, \gamma, \Psi, \nabla\Psi\right) = 0, \text{ also implies } E^2 \approx B_\phi^2$$

The set of equations is nearly degenerate in the far zone.
One has to retain small terms.

The correct procedure: expand Y=0 in r^{-1} and γ^{-1} to find B^2-E^2=O(r^{-1},γ^{-1}) and then to eliminate B^2-E^2 from G=0.

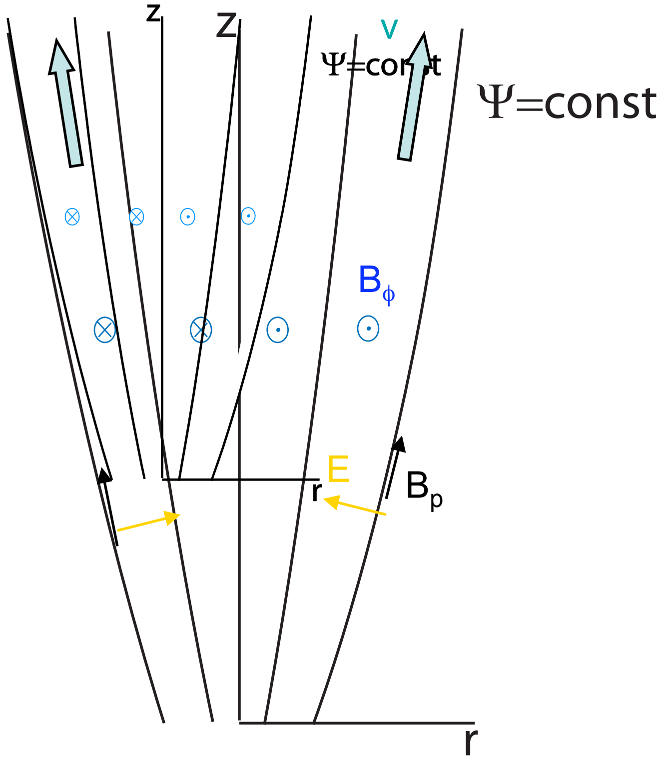
Relativistic MHD in the far zone, $\Omega r>>c$

(Vlahakis 2004; Komissarov, Vlahakis, Konigl, Barkov 2009; Lyubarsky 2009)

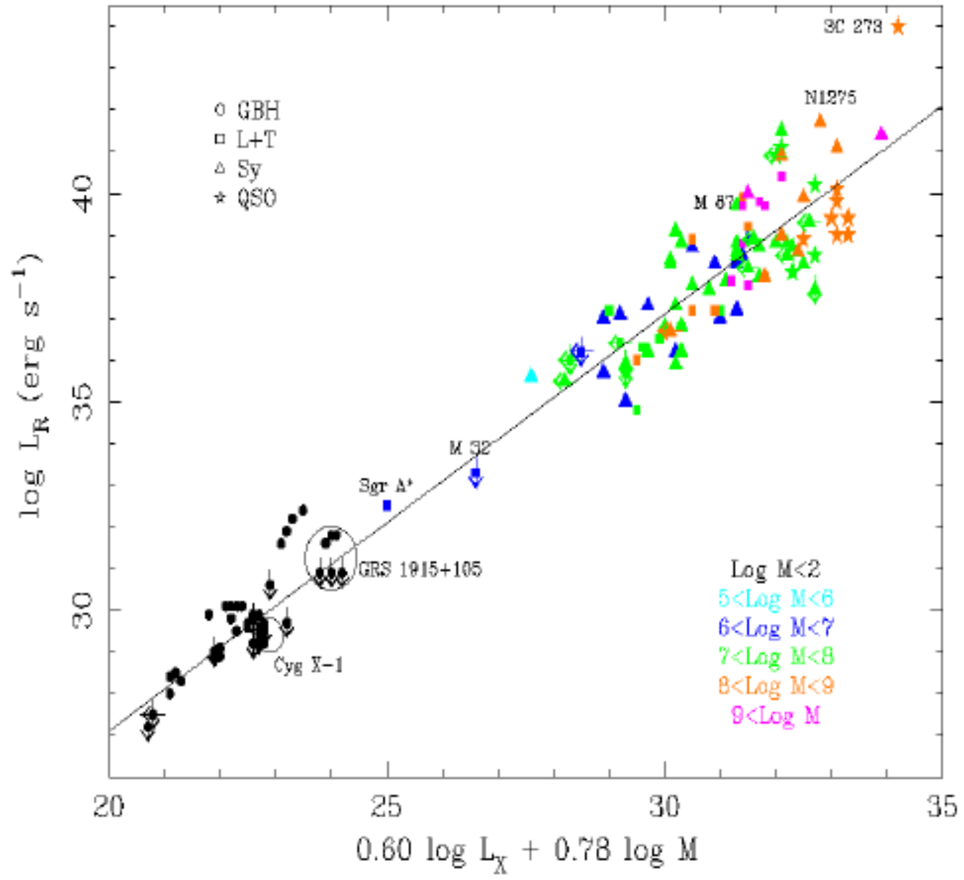
$$2\mu\eta r\left(-\frac{\partial^2 r}{\partial z^2}+\frac{c^2}{\Omega^2 r^3}\right)=\frac{\partial}{\partial\Psi}\left(\frac{\eta(\mu-\gamma)}{\Omega\gamma}\right)^2$$ transverse force balance

$$\eta(\mu-\gamma)=\Omega^2 r^2 B_p$$ longitudinal dynamics

$r(\Psi,z)$ - shape of the flux surfaces



Scaling of X-ray binaries to AGNs



Merloni et al 03; Falcke et al 04

Why radial relativistic MHD flows do not accelerate?