High Energy Radiation Processes: I. Electromagnetic Processes

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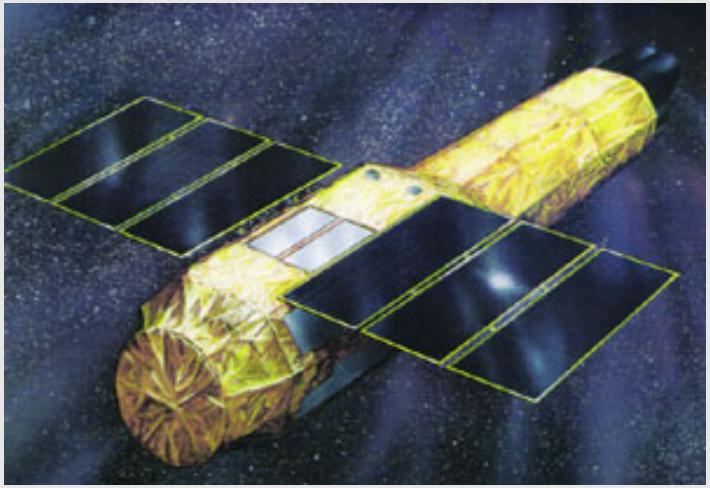
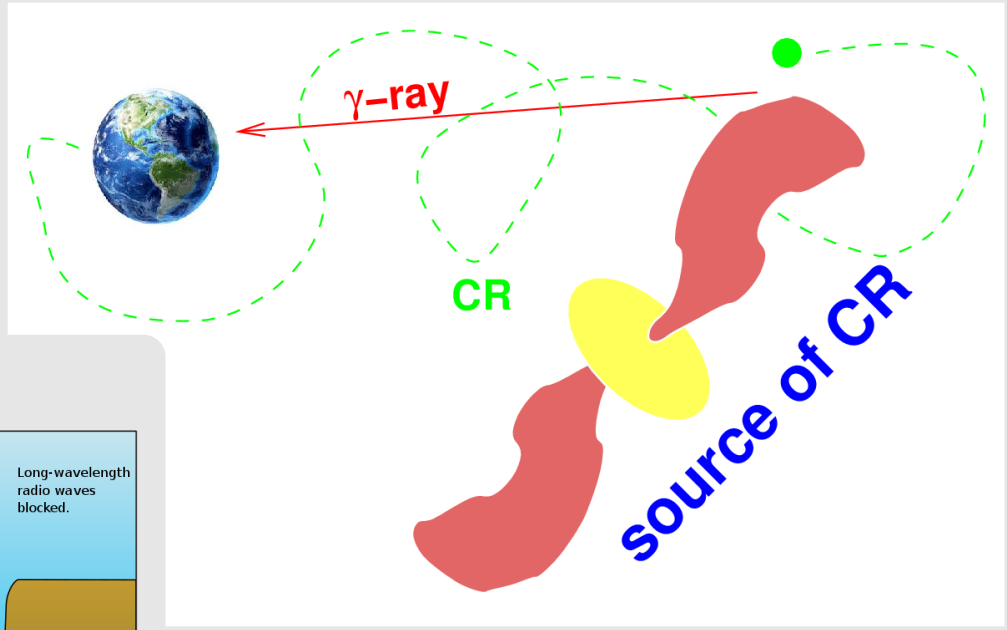
DIAS Summer School in High-Energy Astrophysics 2018

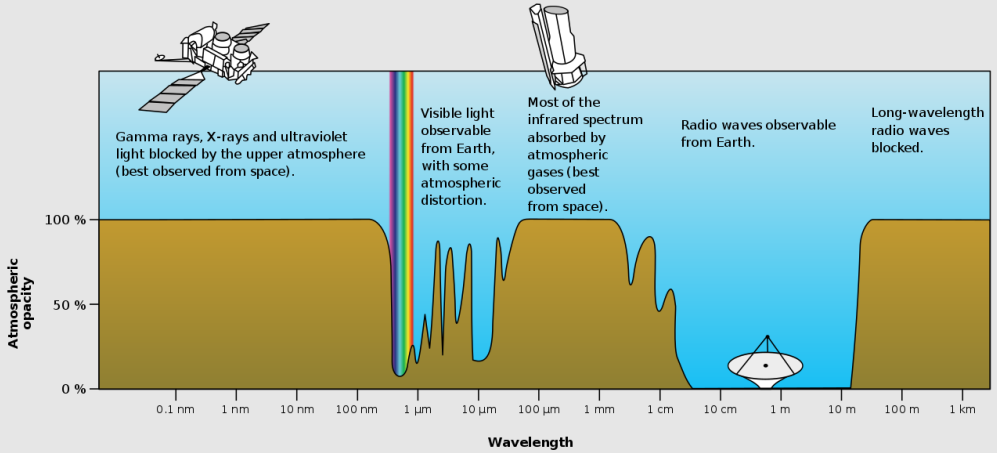
“High Energy Electromagnetic Radiation Processes”

"High Energy Electromagnetic Radiation Processes"



The talk is about production of *electromagnetic radiation*





"High Energy Electromagnetic Radiation Processes"

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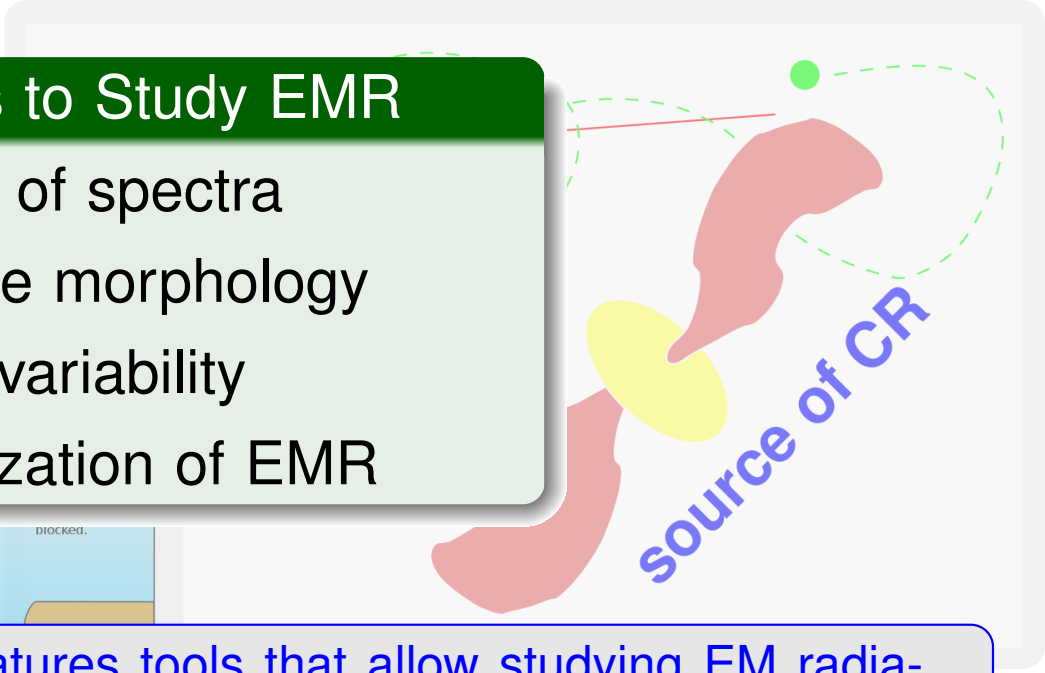
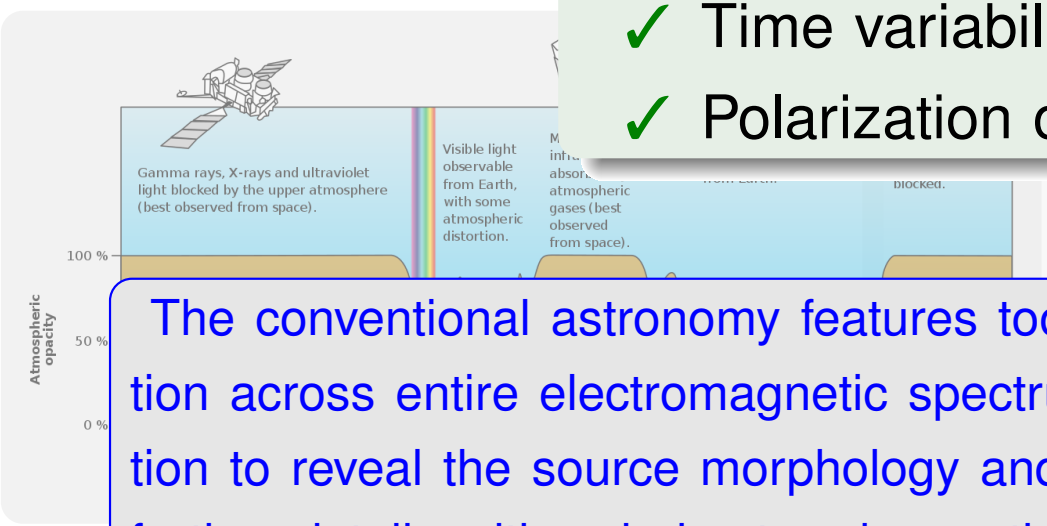


Four Tools to Study EMR

- ✓ Study of spectra
- ✓ Source morphology
- ✓ Time variability
- ✓ Polarization of EMR



The conventional astronomy features tools that allow studying EM radiation across entire electromagnetic spectrum, fine angular and time resolution to reveal the source morphology and time-dependent processes and further details with polarimetry observations.

"High Energy **Electromagnetic** Radiation Processes"

The talk is limited to the processes enabled by particle's *electric charge*:

Lorentz Transformation for EM field

$$E_x = E'_x$$

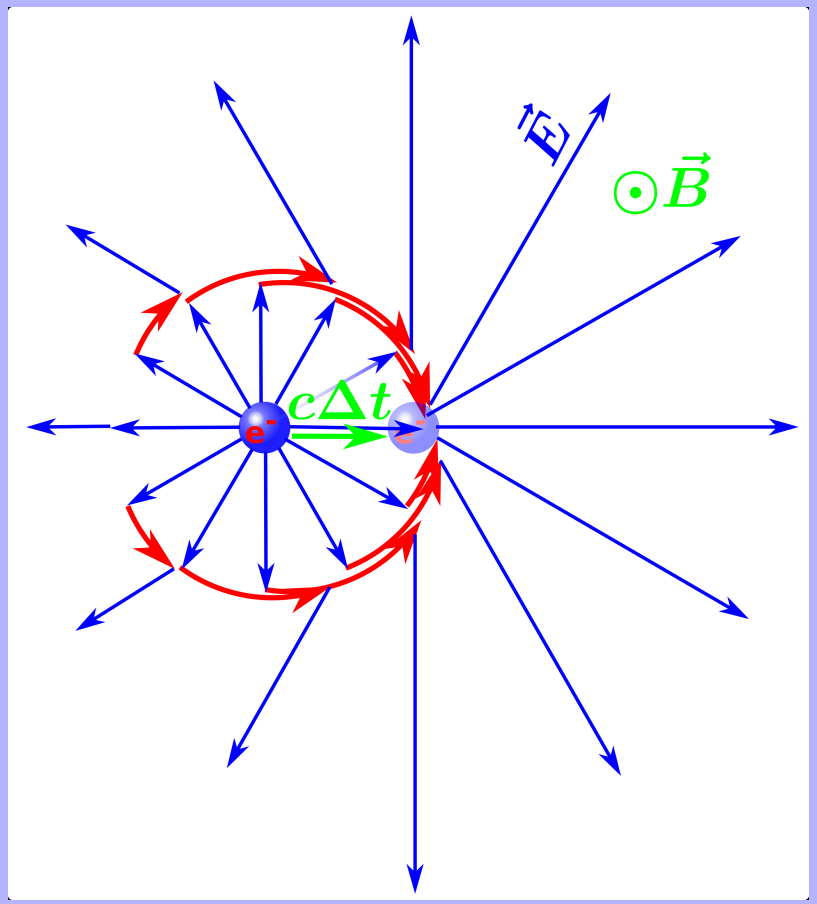
$$E_y = \Gamma E'_y$$

$$E_z = \Gamma E'_z$$

$$B_x = 0$$

$$B_y = -\beta\Gamma E'_z$$

$$B_z = \beta\Gamma E'_y$$

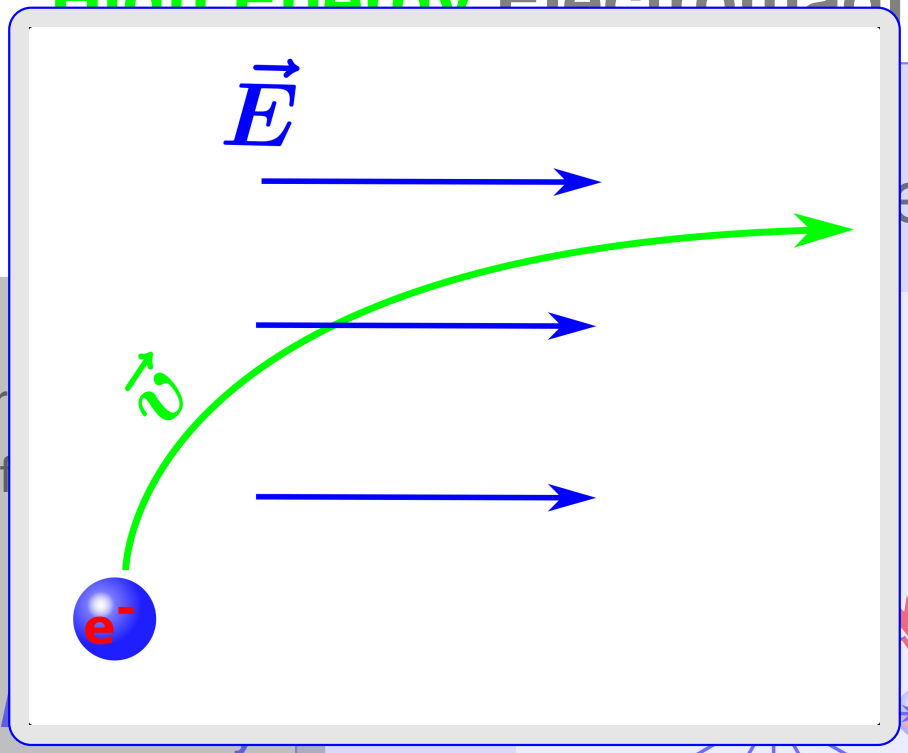


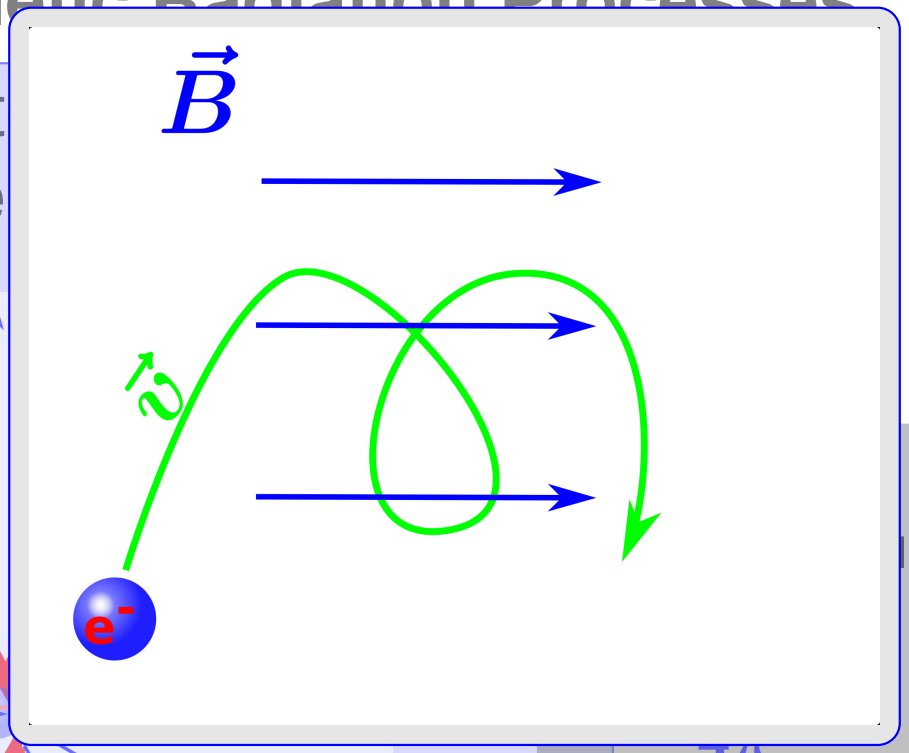
Poynting Flux

$$S = \frac{c}{4\pi}\vec{E}\times\vec{B}$$

Introduction

"High Energy Electromagnetic Radiation Processes"



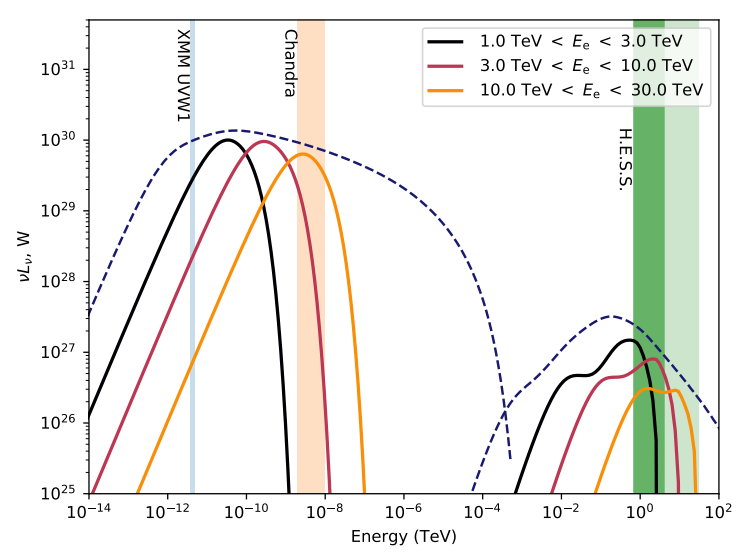


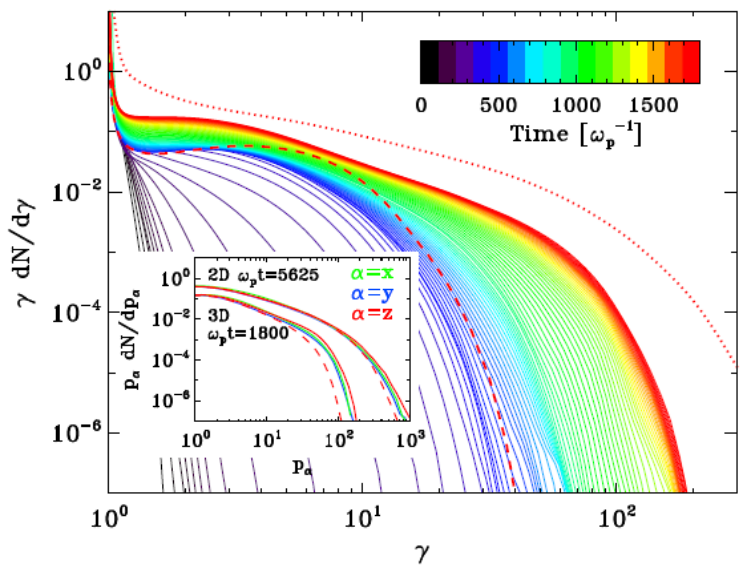
If a charged particle gets accelerated, then the configuration of the electromagnetic field is such that the Poynting flux is non-zero. Magnetic field provides ideal conditions for radiation. Acceleration in electric field saturates quickly, so one needs time variable E-field (i.e., a wave) or strong spatial dependence (e.g., collisions of charged particles).

"High Energy Electromagnetic Radiation Processes"

"High Energy" stays for "gamma ray"? Yes, but what is more important: this is about the emission of *high-energy particles*. This may seem to be a trivial step: high energy emission can only be produced by high energy particles, however there are two important points:

- Emission peak energy depends on the radiation mechanism, thus a high energy particle may emit photons with very different energy
- High Energy particles are essentially non-thermal, i.e., there is no fundamental/general way to describe their distribution





Introduction

Four tools for studying non-thermal sources:

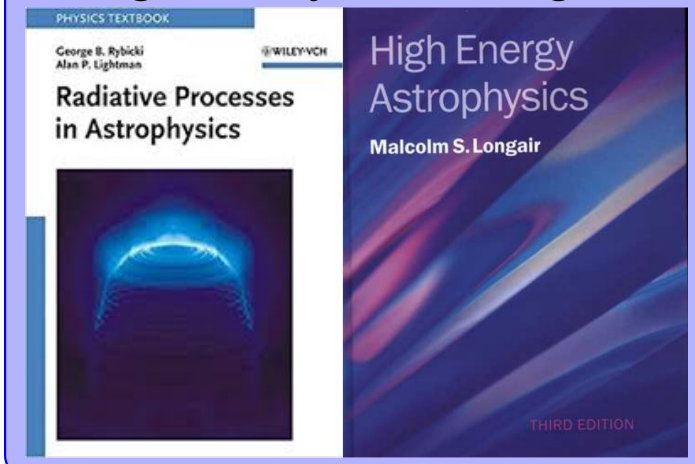
	Depends on particle distribution	Depends on radiation process
Spectrum	✓	✓
Morphology	✓	✓
Time Variability	✓	✓
Polarization	✓	✓

where ✓ means a strong dependence

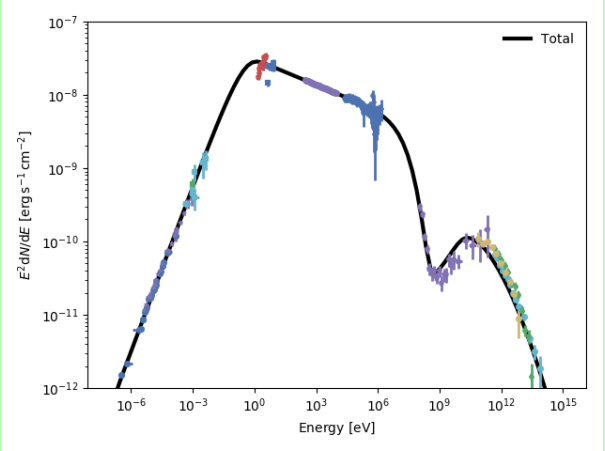
For interpretation of non-thermal emission an accurate description of the particle spectrum is as important as accurate calculation of the emission. When particles lose energy by radiating this influences the particle distribution thus the description of the particle spectrum should include the impact of radiation. However, non-radiative processes may also have a substantial influence on the particle spectrum.

"High Energy Electromagnetic Radiation Processes"

There are many excellent text books covering radiation process: Landau&Lifshitz, Longair, Rybicki&Lightman



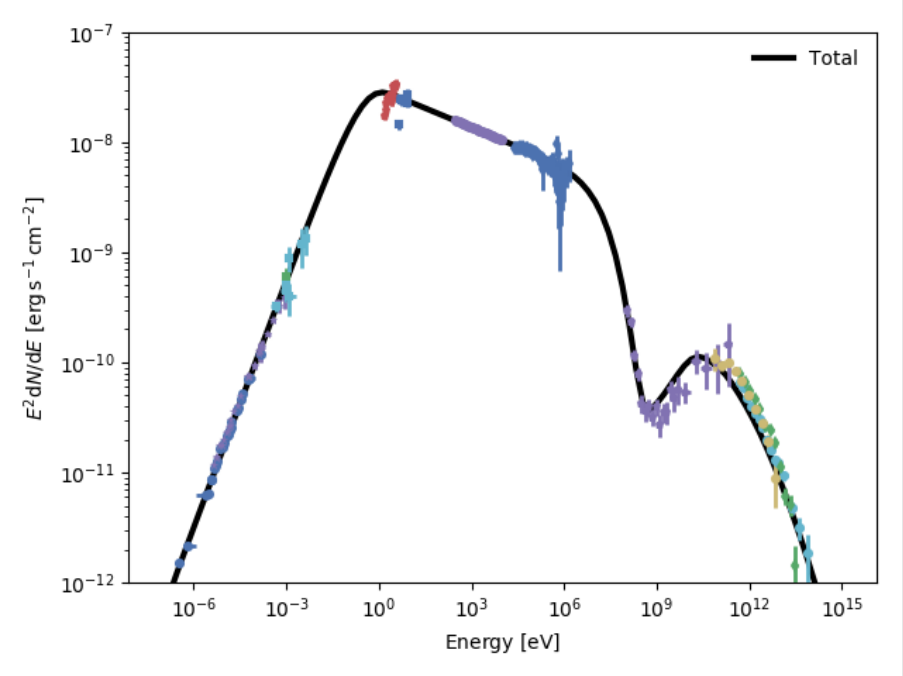
There are advance numerical tools that allow quick calculations of spectra, e.g. naima by Victor Zabalza:



"High Energy Electromagnetic Radiation Processes"

```
from astropy.constants import c
import astropy.units as u
import naima
from naima.models import (ExponentialCutoffBrokenPowerLaw,
                          Synchrotron,
                          InverseCompton)
ECBPL = ExponentialCutoffBrokenPowerLaw(
    amplitude=3.699e36 / u.eV,
    e_0=1 * u.TeV,
    e_break=0.265 * u.TeV,
    alpha_1=1.5,
    alpha_2=3.233,
    e_cutoff=1863 * u.TeV,
    beta=2.)
SYN = Synchrotron(
    ECBPL,
    B=125 * u.uG,
    Eemax=50 * u.PeV,
    Eemin=0.1 * u.GeV)
IC = InverseCompton(
    ECBPL,
    seed_photon_fields=['CMB',
                        ['FIR', 1.e2 * u.K, 1.5 * u.eV / u.cm**3],
                        ['NIR', 5.e3 * u.K, 1.0 * u.eV / u.cm**3]],
    Eemax=50 * u.PeV, Eemin=0.1 * u.GeV)
data = ascii.read('CrabNebula_spectrum.ecsv')
figure = naima.plot_data(data, e_unit=u.eV)
ax = figure.axes[0]
energy = np.logspace(-7, 15, 100) * u.eV
ax.loglog(energy,
          IC.sed(energy, 2 * u.kpc) + SYN.sed(energy, 2 * u.kpc),
          lw=3, c='k', label='Total')
```

There are advance numer-



naima allows not only computing synchrotron, IC, bremsstrahlung, and pion decay emission, but also advance MCMC fitting radiative models to observed spectra

"High Energy Electromagnetic Radiation Processes"

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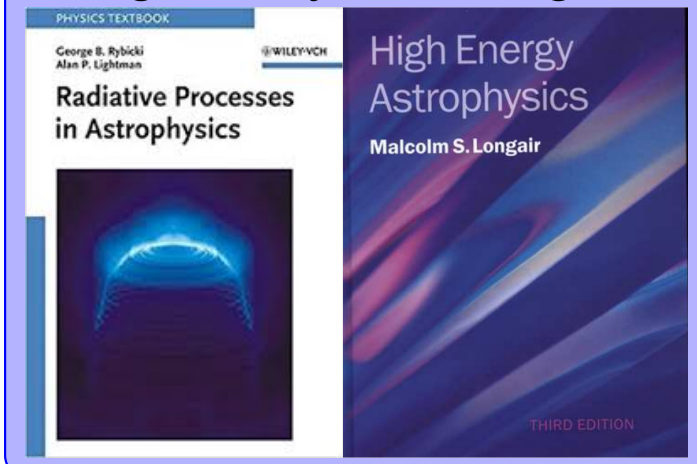
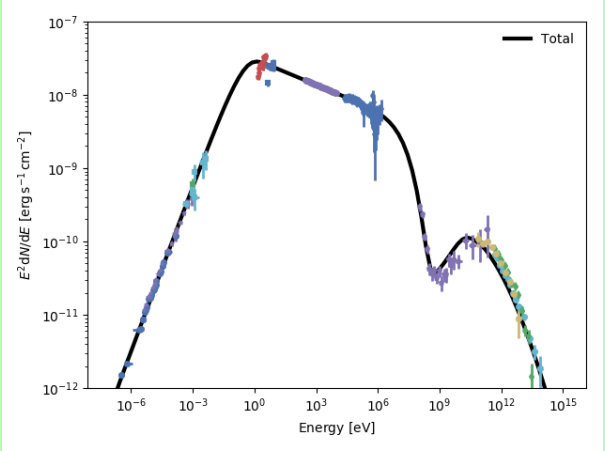


This talk

There are advance numerical tools that allow quick calculations of spectra, e.g. naima by Victor Zabalza:



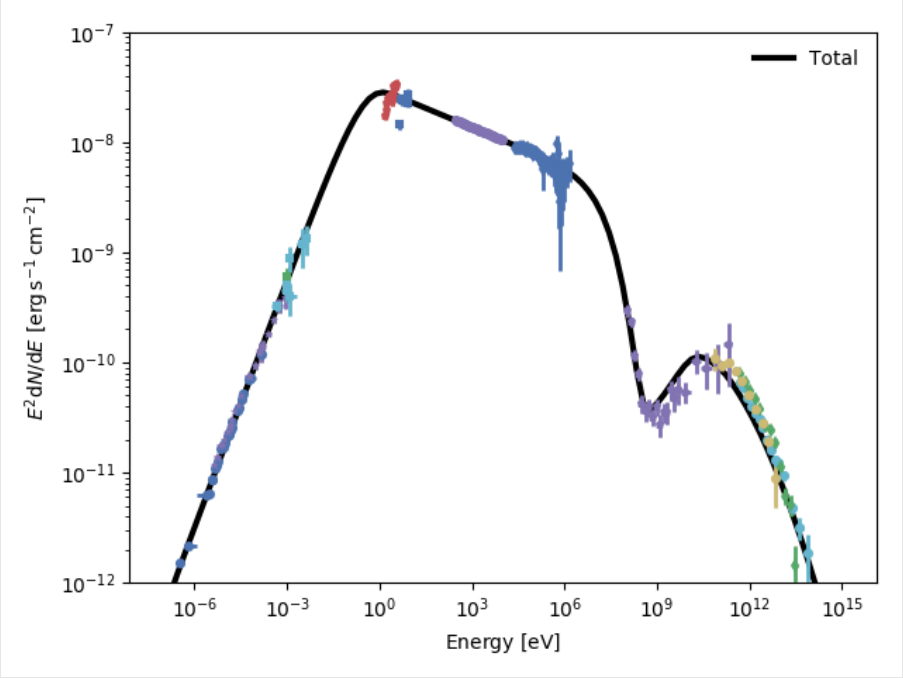
This talk aims to discuss aspects that are beyond the standard textbooks and that are still useful for modeling of high-energy sources.

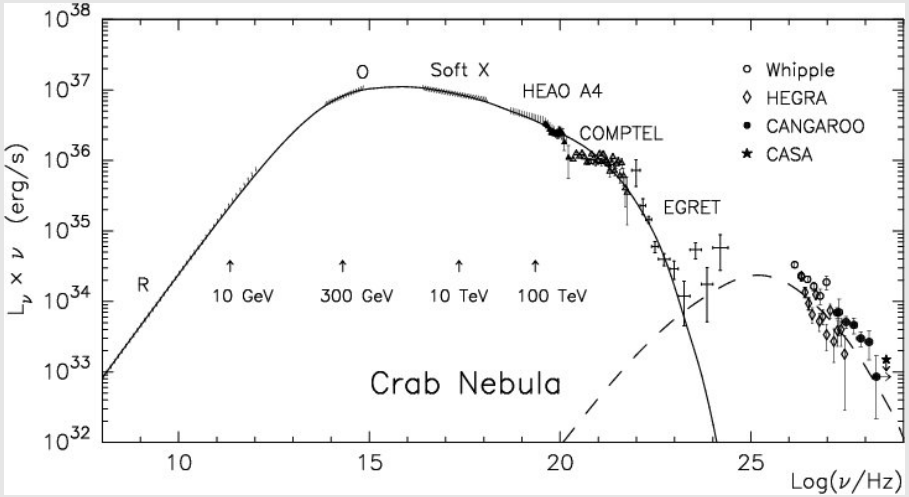
Description of Non-Thermal Particles

Distribution of high energy particles depends on some parameter(s):

- Energy: $dN = fdE$
- Momentum: $dN = fd^3p$
- Coordinate: $dN = fdEdx$
- Coordinates: $dN = fdEd^3r$
- Phase-space coordinates: $dN = fd^3rd^3p$

Here f is distribution function, and its definition may vary dependingly on the context. For each problem one needs to select an adequate distribution function that allows accounting for all relevant processes.





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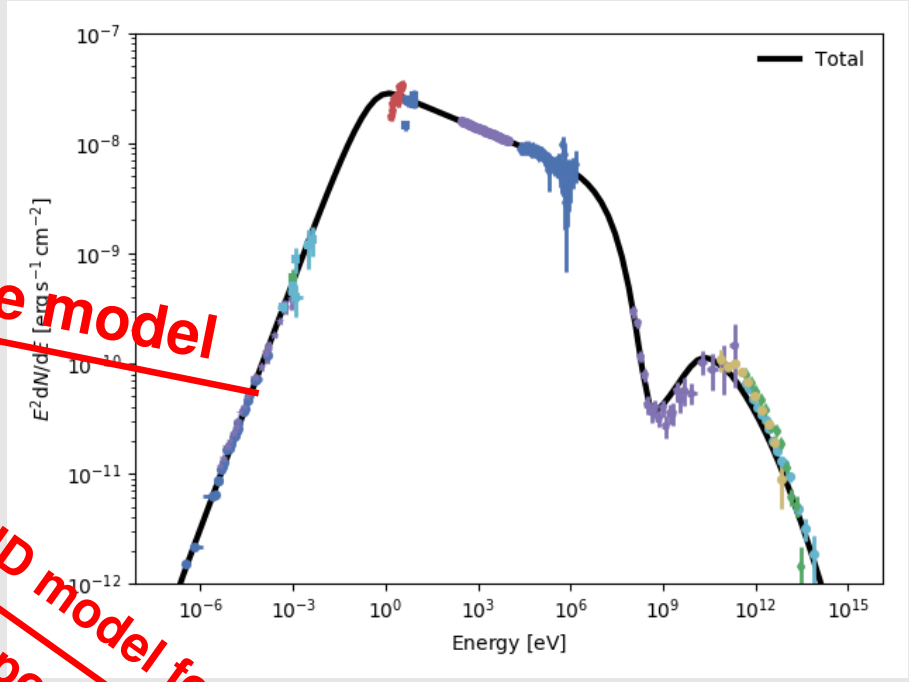
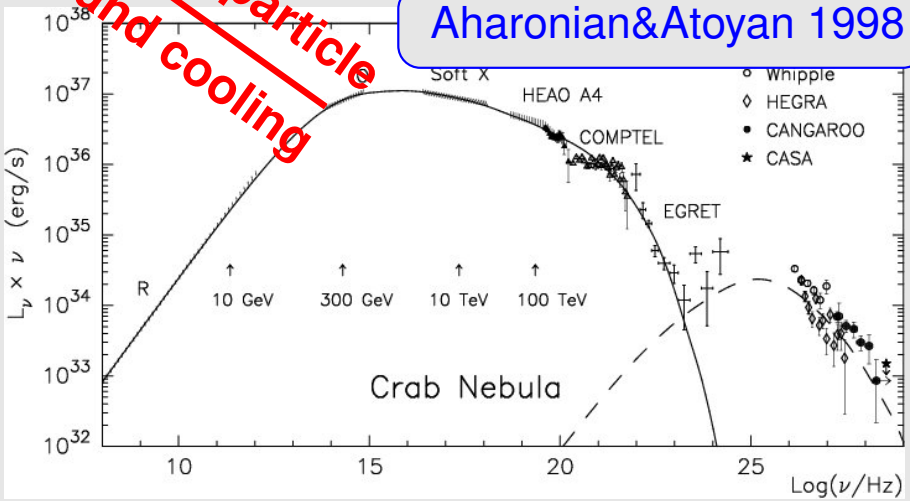
One-zone model

MHD model for particle transport and cooling





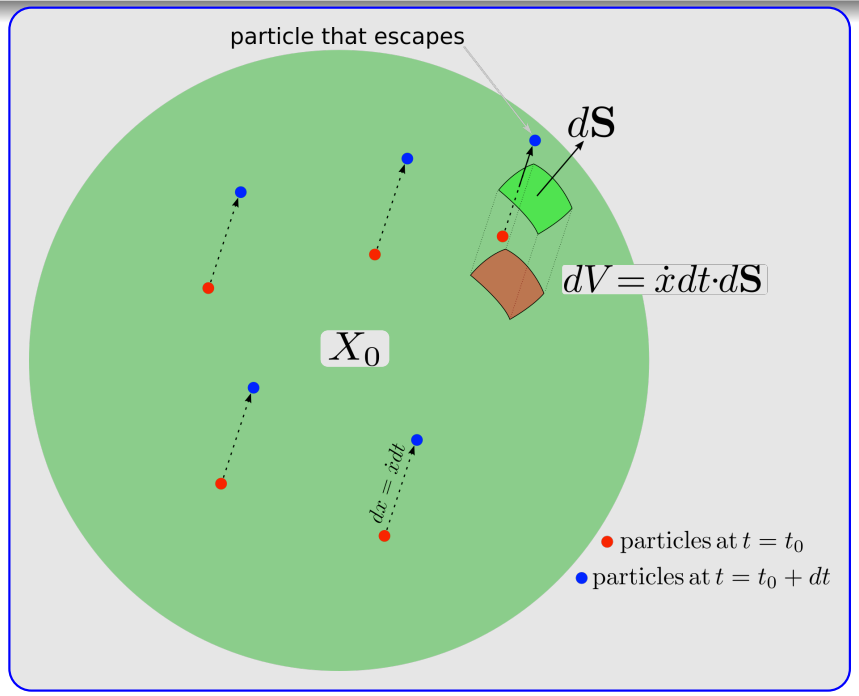
Aharonian&Atoyan 1998

Description of Non-Thermal Particles

If one ignores the particle spin – which still might be important in some astrophysical conditions, e.g., in pulsar magnetosphere– the phase-space distribution function provides the most complete description: $dN = f d^3r d^3p$.

There is a quite simple equation for the distribution function, *Boltzmann Equation*:

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + \mathbf{F}\frac{\partial f}{\partial \mathbf{p}} = 0$$



$$\frac{\partial}{\partial t}\int_{X_0} f\, dx = -\int_{\partial X_0} \mathcal{F}(f)\, d\mathbf{S}_x$$

$$\int_{X_0}\left(\frac{\partial f}{\partial t} + \mathrm{div}\mathcal{F}(f)\right) dx = 0$$

$$\frac{\partial f_a}{\partial t} + \frac{\partial(\dot{\mathbf{r}} f_a)}{\partial \mathbf{r}} + \frac{\partial(\dot{\mathbf{p}} f_a)}{\partial \mathbf{p}} = 0$$

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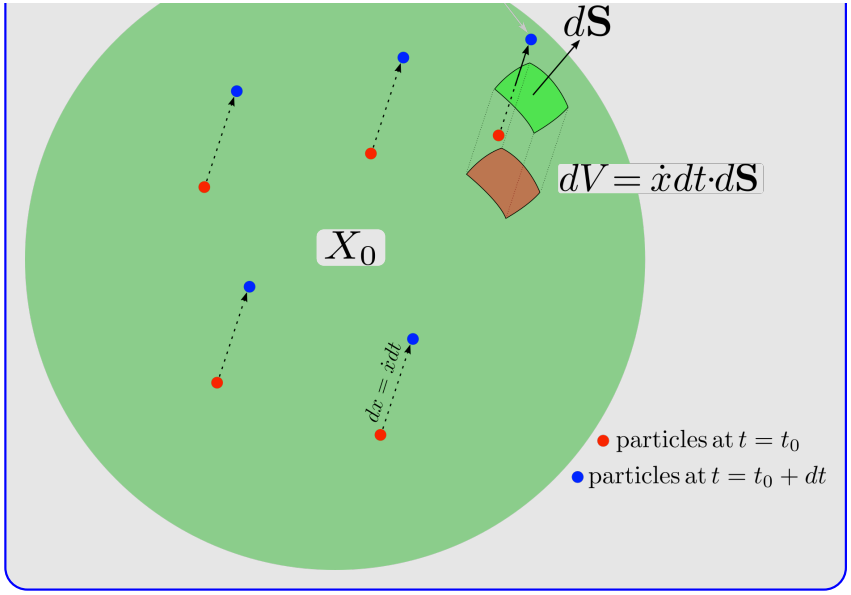
What about particle collisions?



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Boltzmann Equation

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + \mathbf{F}\frac{\partial f}{\partial \mathbf{p}} = \left[\frac{\partial f}{\partial t}\right]_{\mathrm{col}}$$

- The collision integral $\left[\frac{\partial f}{\partial t}\right]_{\mathrm{col}}$ accounts for many processes: *particle injection, acceleration, scattering, energy losses, etc* – i.e., ALL plasma physics (lecture by M.Malkov illustrated the complexity of the topic)
- In the simplest case, Boltzmann collision integral is

$$\left[\frac{\partial f_a}{\partial t}\right]_{\mathrm{st}} = \sum_b \int dp_1 v_{\mathrm{rel}} d\sigma \left(f_a(x')f_b(x_1') - f_a(x)f_b(x_1)\right)$$

Boltzmann Collision Integral in Astrophysics

$$\left[\frac{\partial f_a}{\partial t}\right]_{\rm st} = \sum_b \int dp_1 v_{\rm rel} d\sigma \left(f_a(x')f_b(x_1') - f_a(x)f_b(x_1)\right)$$

- Boltzmann collision integral is widely used in kinetics of neutral gases, e.g., to describe an admixture propagation
- In astrophysics the collision integral in this form is used to describe, e.g., the electromagnetic cascading:

$$\frac{\partial f_e}{\partial t} + \mathbf{v}\frac{\partial f_e}{\partial \mathbf{r}} + \mathbf{F}\frac{\partial f_e}{\partial \mathbf{p}} = \int dp_\gamma c\, d\bar{\sigma}_{\gamma\gamma} f_\gamma(p_\gamma) - c\bar{\sigma}_{ic} f_e$$

$$\frac{\partial f_\gamma}{\partial t} + \mathbf{c}\frac{\partial f_\gamma}{\partial \mathbf{r}} = \int dp_e c\, d\bar{\sigma}_{ic} f_e(p_e) - c\bar{\sigma}_{\gamma\gamma} f_\gamma$$

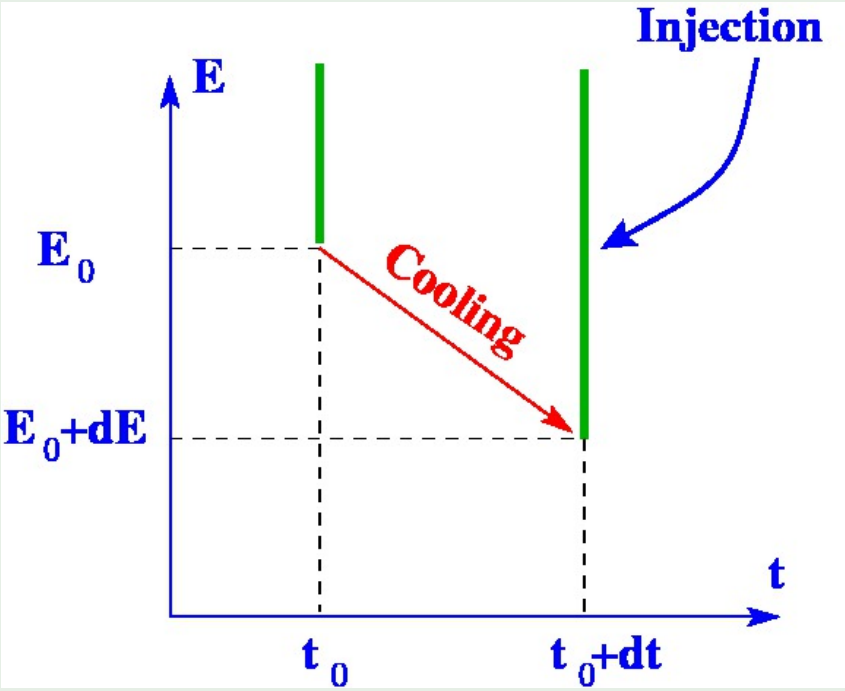
Boltzmann Equation

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + \mathbf{F}\frac{\partial f}{\partial \mathbf{p}} = \left[\frac{\partial f}{\partial t}\right]_{\mathrm{col}}$$

- Equation with the collision integral $\left[\frac{\partial f}{\partial t}\right]_{\mathrm{col}}$ cannot be solved for astrophysical applications
- It is possible to divide the physics included in the collision integral in two parts: complex (acceleration – talks by Bykov, Malkov, Blasi, Lemoine) and simple (cooling, which can be treated under the *continuous-loss approximation*)
- Also in the most cases particles are isotropic in some system, thus particle energy is a good parameter

Significant simplification in the case of energy losses

Cont. Loss Approx.



Distribution function & Injection

$$\mathrm{d}N = f(E,t)\mathrm{d}E \quad \mathrm{d}N = q(E,t)\mathrm{d}E\mathrm{d}t$$

$$F(E,t) = \int_E^\infty f(E',t)\mathrm{d}E'$$

$$F(E+\dot{E}\mathrm{d}t, t+\mathrm{d}t) =$$

$$F(E,t) + \mathrm{d}t \int_E^\infty q(E',t)\mathrm{d}E'$$

Fokker-Planck Equation

$$\frac{\partial f}{\partial t} + \frac{\partial(\dot{E}f)}{\partial E} = q(E,t)$$

Significant simplification in the case of energy losses

$$F(E+\dot{E}\mathrm{d}t, t+\mathrm{d}t) = F(E,t) + \mathrm{d}t \int_E^\infty q(E',t)\mathrm{d}E'$$

$$F(E,t) + \frac{\partial F}{\partial E}\dot{E}\mathrm{d}t + \frac{\partial F}{\partial t}\mathrm{d}t = F(E,t) + \mathrm{d}t \int_E^\infty q(E',t)\mathrm{d}E'$$

$$\frac{\partial}{\partial E} \Longrightarrow$$

accounting for $\frac{\partial F}{\partial E} = -f$ $\qquad \frac{\partial}{\partial E}\int_E^\infty q(E',t)\mathrm{d}E' = -q$

$$\frac{\partial f}{\partial t} + \frac{\partial(\dot{E}f)}{\partial E} = q(E,t)$$

Particle Energy Distribution

Fokker-Planck Equation Solution

$$f(E,t) = \frac{1}{\dot{E}} \int_{E}^{E_{\rm eff}} \mathrm{d}E'\, q(E'), \quad \textbf{where} \quad t = \int_{E}^{E_{\rm eff}} \frac{\mathrm{d}E'}{\dot{E}'}$$

$$\dot{E} = \dot{E}_{\rm syn} + \dot{E}_{\rm ic} + \dot{E}_{\rm ad} + etc / \dot{E}_{\rm syn} + \dot{E}_{\rm pp} + \dot{E}_{\rm p\gamma} + etc$$

Fast Cooling *(Saturation)*

$$E_{\rm eff} \to \infty$$

$$f(E) = \frac{1}{\dot{E}} \int_{E}^{\infty} \mathrm{d}E'\, q(E')$$

Slow Cooling

$$t \ll E/\dot{E}$$

$$f(E,t) = q(E) \cdot t$$

Spectral Breaks: Particle Distribution

Solution of the Fokker-Planck Equation:

$$f(E,t) = \frac{1}{\dot{E}} \int_{E}^{E_{\rm eff}} \mathrm{d}E'\, q(E'), \quad \text{where} \quad t = \int_{E}^{E_{\rm eff}} \frac{\mathrm{d}E'}{\dot{E}'}$$

Let us consider the simplest case:

$$q(E,t) = \theta(E - E_{\rm MIN})\theta(E_{\rm MAX} - E)E^{-\alpha}, \quad \text{where} \quad \dot{E} \propto E^{\beta}$$

Cooling energy is $E_c = \dot{E}(E_c)t$, where t is the source age

$E_c > E_{\rm MIN}$ then break at E_c and range of energy is $E_{\rm MIN} < E_{\rm MAX}$:

$$f(E) \propto \begin{cases} (E^{-\alpha+1} - E_{\rm MAX}^{-\alpha+1})E^{-\beta} \\ E^{-\alpha} \end{cases}$$

$E_c < E_{\rm MIN}$ then break at $E_{\rm MIN}$ and range of energy is $E_c < E_{\rm MAX}$

$$f(E) \propto \begin{cases} (E^{-\alpha+1} - E_{\rm MAX}^{-\alpha+1})E^{-\beta} \\ E^{-\beta} \end{cases}$$

Spectral Breaks: Particle Distribution

Solution of the Fokker-Planck Equation:

$f(E, t)$

Let us cons

$q(E, t) = \theta$ E^{β}

Cooling en ge

$E_c > E_{\mathrm{MIN}}$ ther at E_{MIN} and

of energy is E $< E_{\mathrm{MAX}}$

$f(E) \propto \begin{cases} (E \quad {}_{\mathrm{MAX}}) \\ E^{-\alpha} \end{cases}$ $f(E) \propto \begin{cases} (\quad E_{\mathrm{MAX}}^{-\alpha+1})E^{-\beta} \\ E^{-\beta} \end{cases}$

In case with pure power-law injection and cooling the particle distribution may have one break and three different slops $\alpha+\beta+1$, α, and β are allowed (here α and β are the power-law indexes of the acceleration spectrum and the cooling rate). For example, the particle distribution cannot have two breaks. More complicated particle distributions are allowed if (i) the injection spectrum is not a power-law (e.g., each acceleration spectrum has a high-energy cutoff); (ii) the loss rate has a non-power-law dependence on energy.

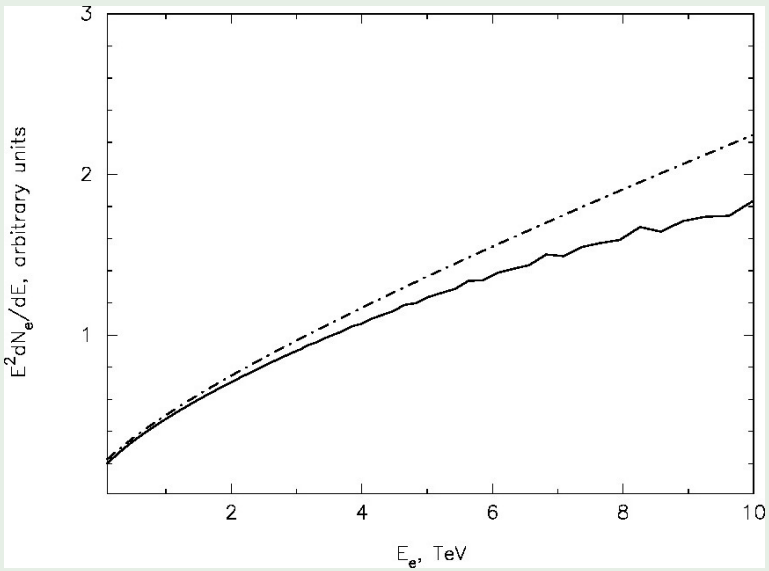
Used simplifications

- Phenomenological treatment of the acceleration process
  - High energy cut-off may depend on the loss rate
  - Some acceleration processes cannot be treated as a power-law injection, e.g. *converter mechanism by Derishev*
- Particles lose energy by small fractions, which is not true for some processes, e.g. IC in the Klein-Nishina regime
- The Fokker-Planck equation describes a one-zone model

Continuous Loss Approximation for Klein-Nishina regime

Klein-Nishina and Continues Loss approximation

- Particles lose energy by small fractions, which is not true for some processes, e.g. IC in the Klein-Nishina regime



Solid line: $c\sigma_{\rm ic} f(\gamma) = q(\gamma) + c\int\limits_{\gamma}^{\infty} {\rm d}\gamma' f(\gamma') \frac{{\rm d}\sigma}{{\rm d}E_\gamma}(\gamma', \gamma' - \gamma)$

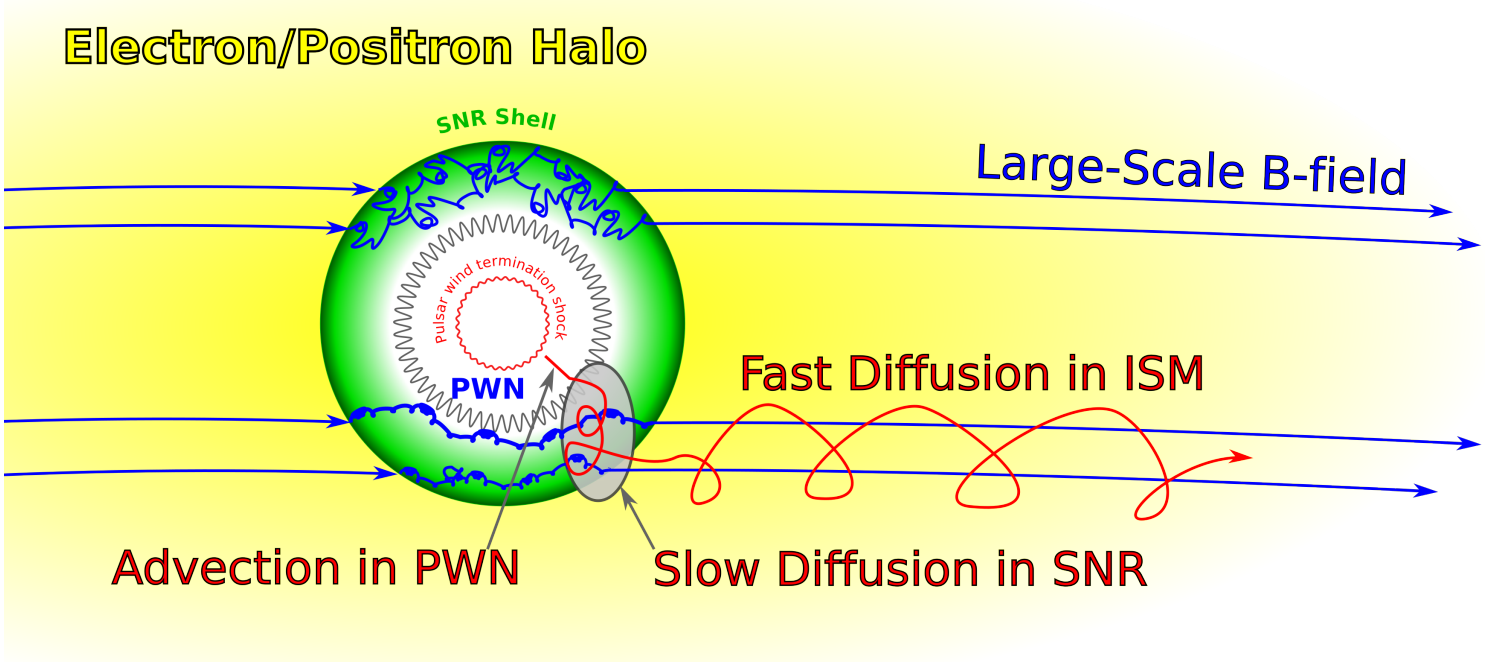
Dash-dotted line: $f(\gamma) = \frac{1}{\dot{E}_{\rm ic}} \int\limits_{\gamma}^{\infty} {\rm d}\gamma' q(\gamma')$

Transport Equation

Transport Equation with Diffusion and Escape

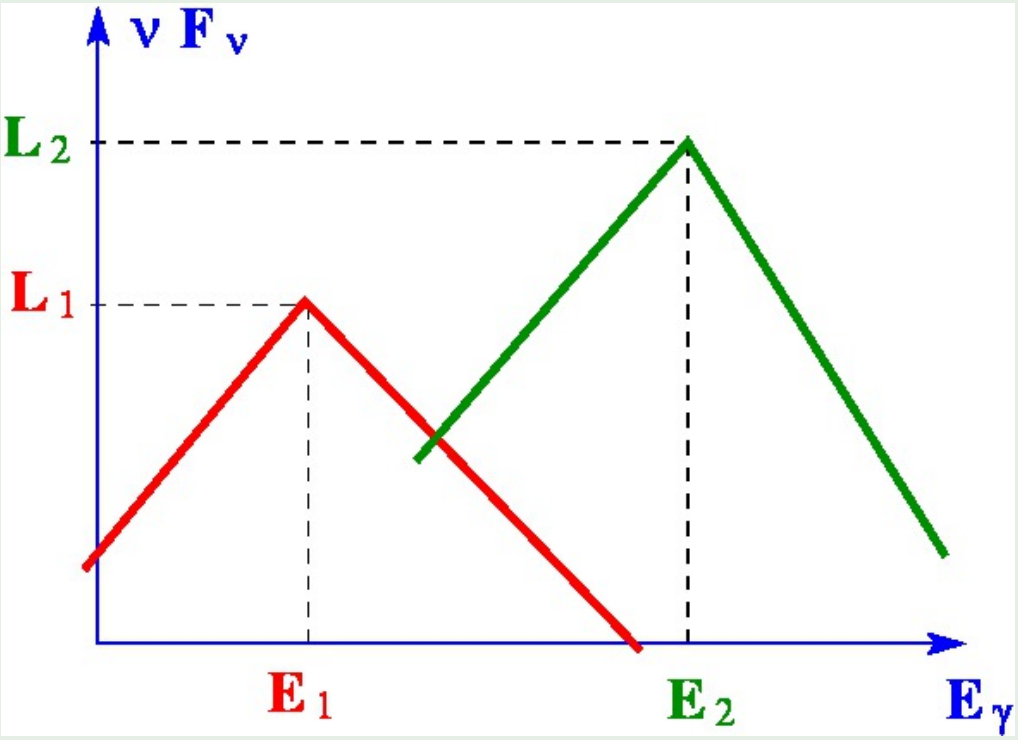
$$\frac{\partial f}{\partial t} + \frac{\partial(\dot{E} f)}{\partial E} + \nabla\left(D \nabla f\right) + \frac{f}{\tau} = q(E, t)$$

also can be solved analytically, see e.g. Ginzburg's "Astrophysics of Cosmic Rays"



Radiation Production

Emission of a Particle (two channels)



Single particle spectra:

$$\frac{\mathrm{d}N_i}{\mathrm{d}E_\gamma} = K_i\left(E_\gamma, E_0\right)$$

Total luminosity:

$$L = \dot{E}_1 + \dot{E}_2$$

Luminosity per channel:

$$L_i = \frac{\dot{E}_i}{\dot{E}_1 + \dot{E}_2} L$$

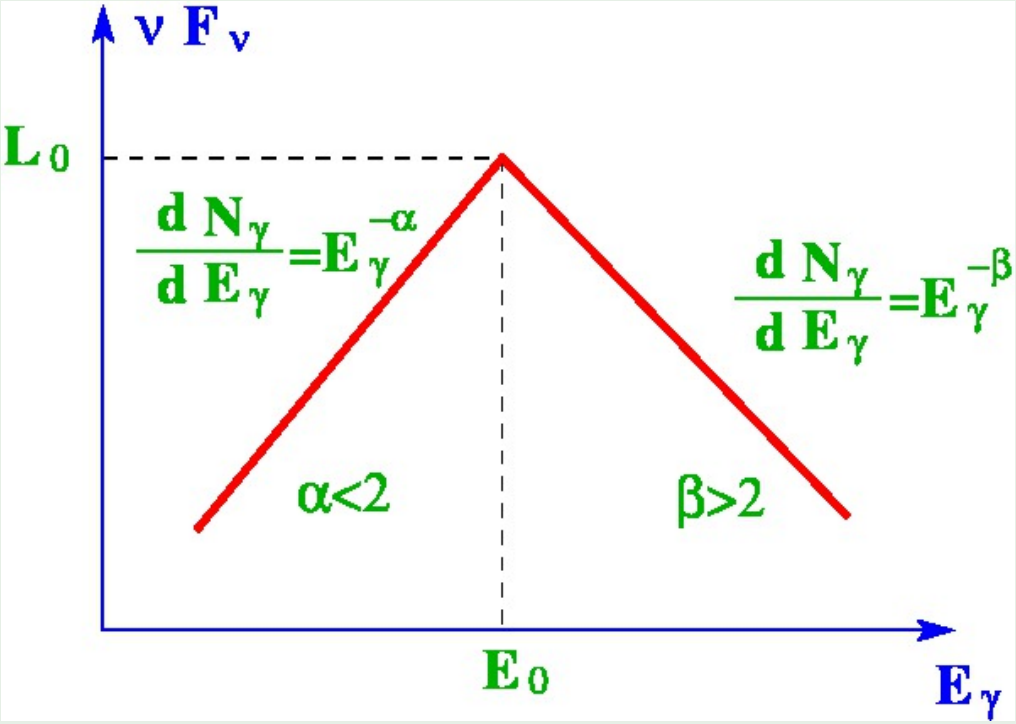
Ratio of the humps:

$$\frac{L_1}{L_2} = \frac{\dot{E}_1}{\dot{E}_2}$$

νF_ν peak gives the luminosity

νF_ν peaking distribution

$$\nu F_\nu = E_\gamma^2 \frac{\mathrm{d}N_\gamma}{\mathrm{d}E_\gamma}$$



$$L_\gamma = \int \mathrm{d}E_\gamma \, E_\gamma \frac{\mathrm{d}N_\gamma}{\mathrm{d}E_\gamma} =$$

$$\int_{E_{\min}}^{E_0} \mathrm{d}E_\gamma \, E_\gamma^{1-\alpha} + \int_{E_0}^{E_{\max}} \mathrm{d}E_\gamma \, E_\gamma^{1-\beta}$$

$$E_{\min} \ll E_0 \ll E_{\max}$$

$$L_\gamma = L_0 \left(\frac{1}{2-\alpha} + \frac{1}{\beta - 2} \right)$$

For $\alpha = 1.5$ and $\beta = 2.5$

$$L_\gamma = 4 L_0$$

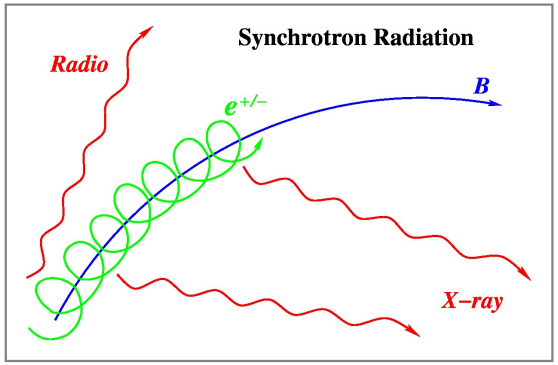
Synchrotron Radiation

- Single Particle Spectrum:

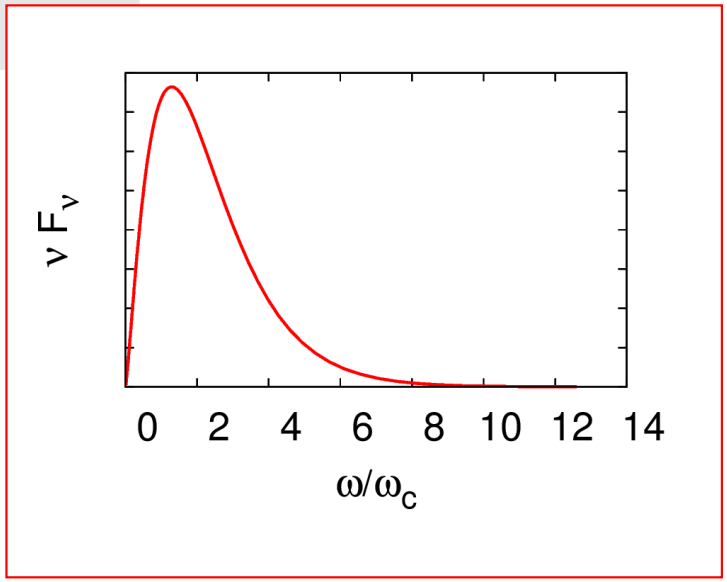
$$\frac{\mathrm{d}I_{\mathrm{syn}}}{\mathrm{d}\omega} = \frac{\sqrt{3}}{2\pi}\frac{e^3 B}{mc^2} F\left(\frac{\omega}{\omega_{\mathrm{c}}}\right)$$

where $\omega_{\mathrm{c}} = \frac{3eB\gamma^2}{2mc}$ and $F(x) = x\int_x^{\infty} K_{5/3}(x')\mathrm{d}x'$

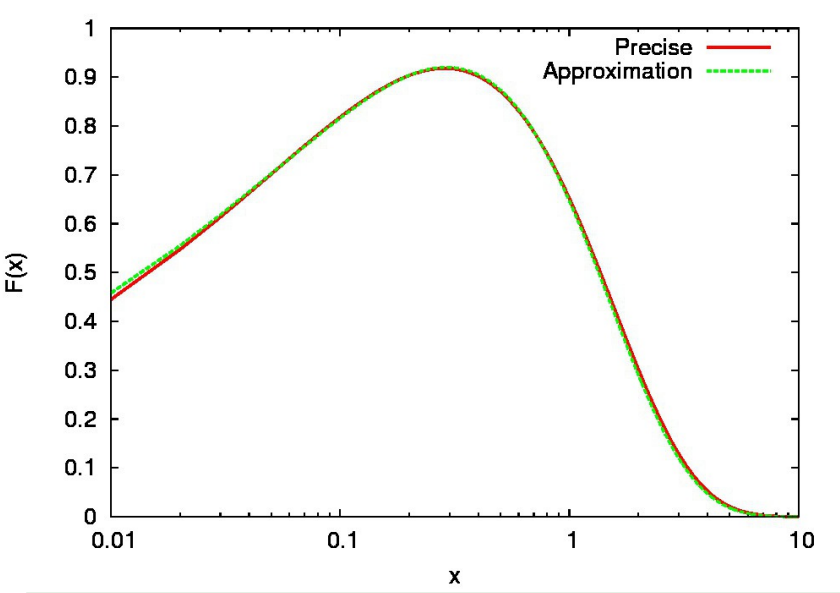
- Energy Losses: $\dot{E}_{\mathrm{syn}} = -\frac{4}{3}U_{\mathrm{B}}c\gamma^2$
- Spectrum transformation: $\alpha \Rightarrow \Gamma = \frac{\alpha+1}{2}$



Acceleration of non-thermal particle proceeds in magnetized media therefore accelerated particles unavoidable interact with magnetic field generating non-therm emission – synchrotron radiation



Approximation for the peak



- $x \gg 1$

$$F(x) = \left(\frac{\pi}{2}\right)^{1/2} x^{1/2} \mathrm{e}^{-x}$$

- $x \ll 1$

$$F(x) = \frac{4\pi}{\sqrt{3}\Gamma(1/3)} x^{1/3}$$

- Useful approximation (but not an asymptotic):

$$F(x) = x \int_x^\infty K_{5/3}(x')\mathrm{d}x' = 1.76\, x^{0.29} \mathrm{e}^{-x}$$

Synchrotron Test for MHD Regime

MHD regime implies that

$$\mathcal{E} = B/\eta \qquad \eta > 1$$

Acceleration Time: $E/e\mathcal{E}$

$$t_{\rm acc} = \eta \frac{r_{\rm g}}{c} = 0.1\eta E_{\rm TeV} B_{\rm G}^{-1}{\rm s}$$

Synchrotron Cooling Time

$$t_{\rm syn} \approx 400 E_{\rm TeV}^{-1} B_{\rm G}^{-2}{\rm s}$$

Synchrotron Self Cutoff

$$\epsilon = 1.15\hbar\omega_{\rm c} \approx 300\eta^{-1}{\rm MeV}$$

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MHD regime implies that

Shock acceleration: $\eta = 2\pi \left(\frac{c}{v_{sh}}\right)^2$

Acceleration Time: $E/e\mathcal{E}$

$$t_{acc} = \eta \frac{r_g}{c} = 0.1\eta E_{TeV} B_G^{-1} s$$

$E_{br} \sim 1\,\mathrm{keV}$, i.e. $\eta \sim 10^5$ ($v_{sh} \sim 10^3\ km/s$)

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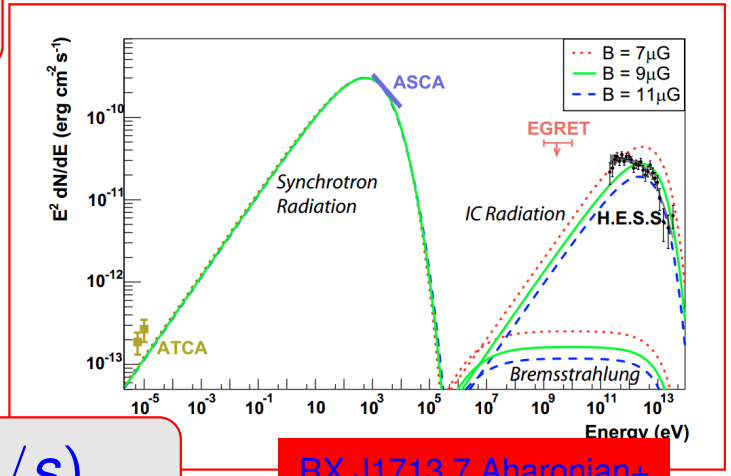

RX J1713.7 Aharonian+

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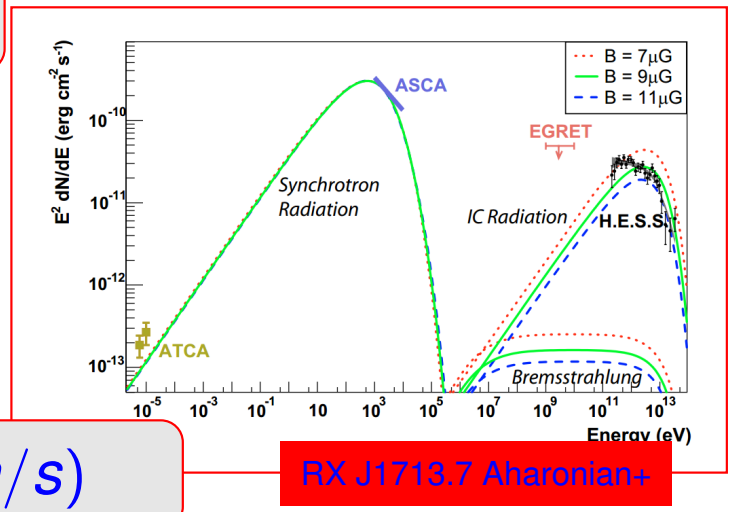

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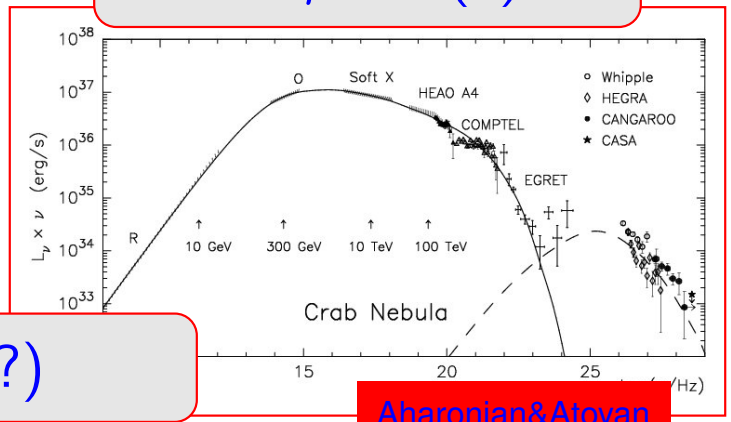
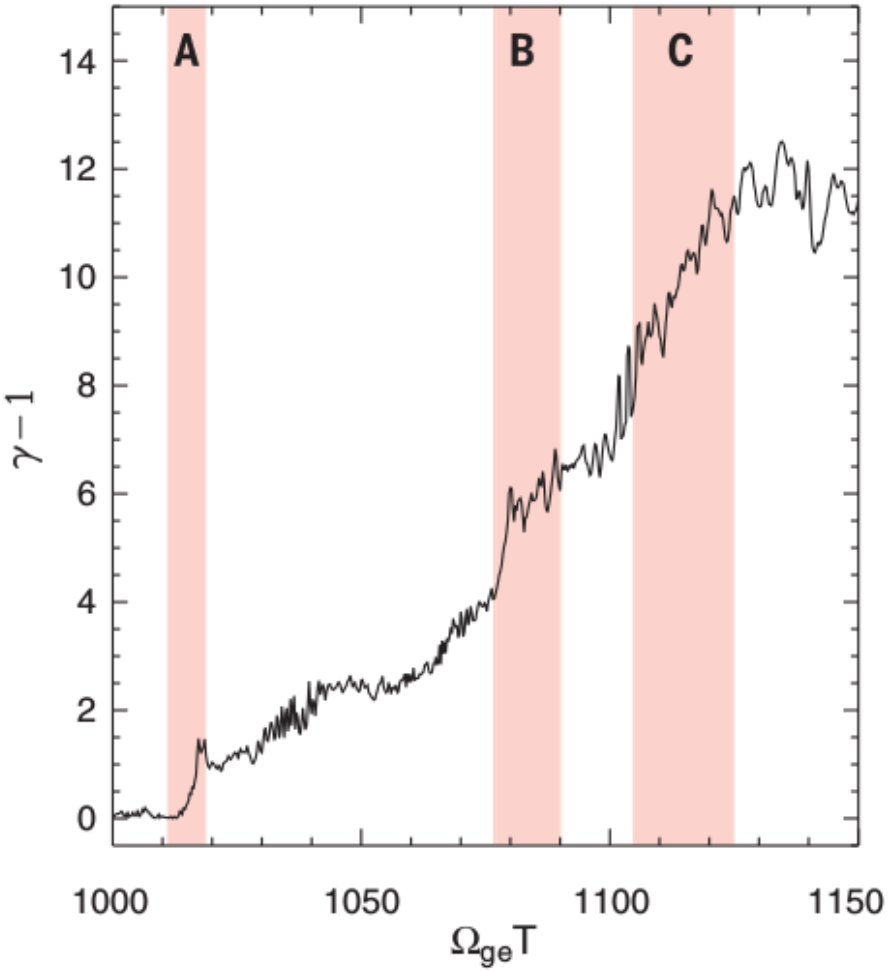
PWN: $\eta \to 1(?)$



$E_{br} \sim 10^{21-22}\mathrm{Hz}$, i.e. $\eta \sim 2\pi$ ($v_{sh} \to c$?)

Aharonian&Atoyan

Particle Acceleration in Astrophysics



Matsumoto+ 2015

Gyro freq. is $\Omega_{ge} = \frac{eB}{mc}$, so

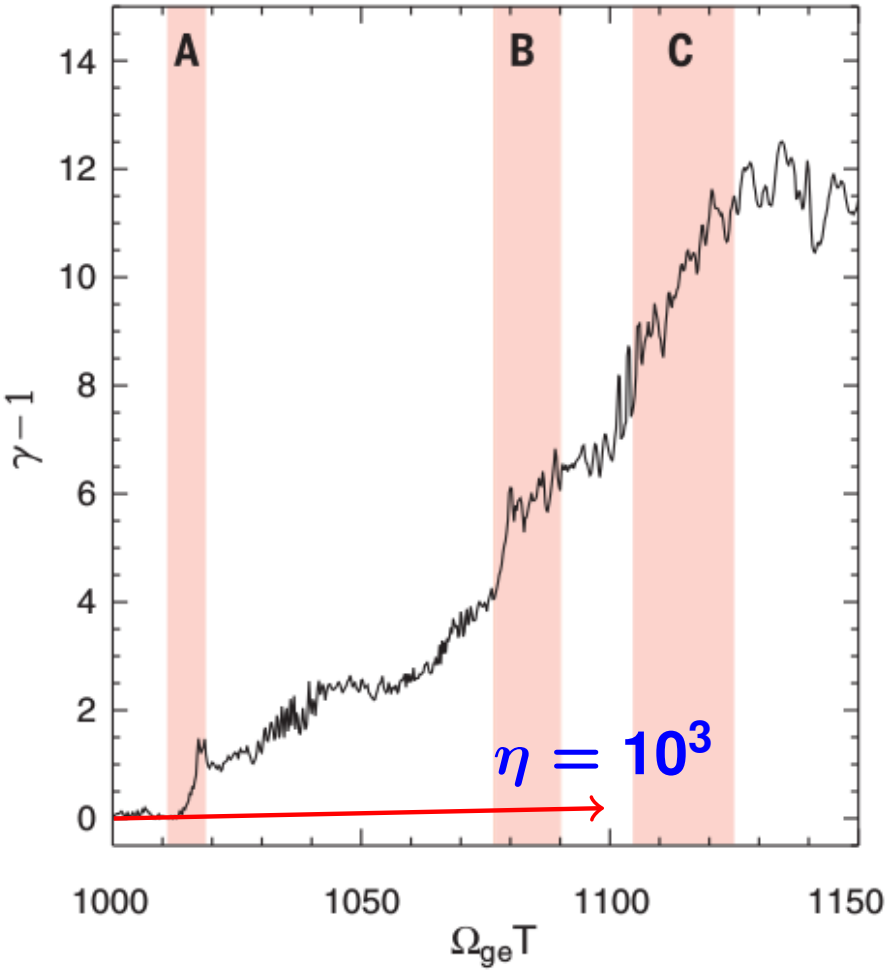
$$\Omega_{ge} T = \gamma T / t_{\rm acc}$$

$$\frac{d\gamma}{d(\Omega_{\rm ge} T)} = \eta^{-1}$$

Different acceleration regimes

- DSA ($v = 0.1c$)$\rightarrow \eta = 10^3$
- Crab Nebula $\rightarrow \eta = 10$
- Can it be even more efficient? $\eta \rightarrow 1$

Particle Acceleration in Astrophysics



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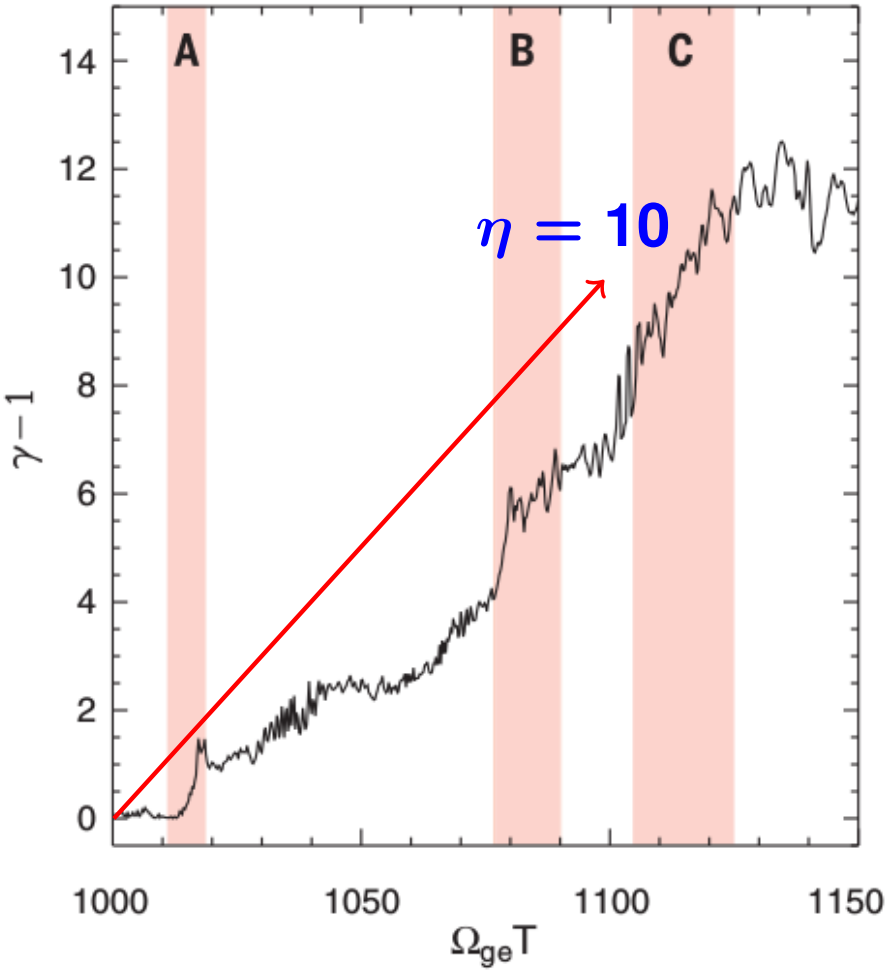

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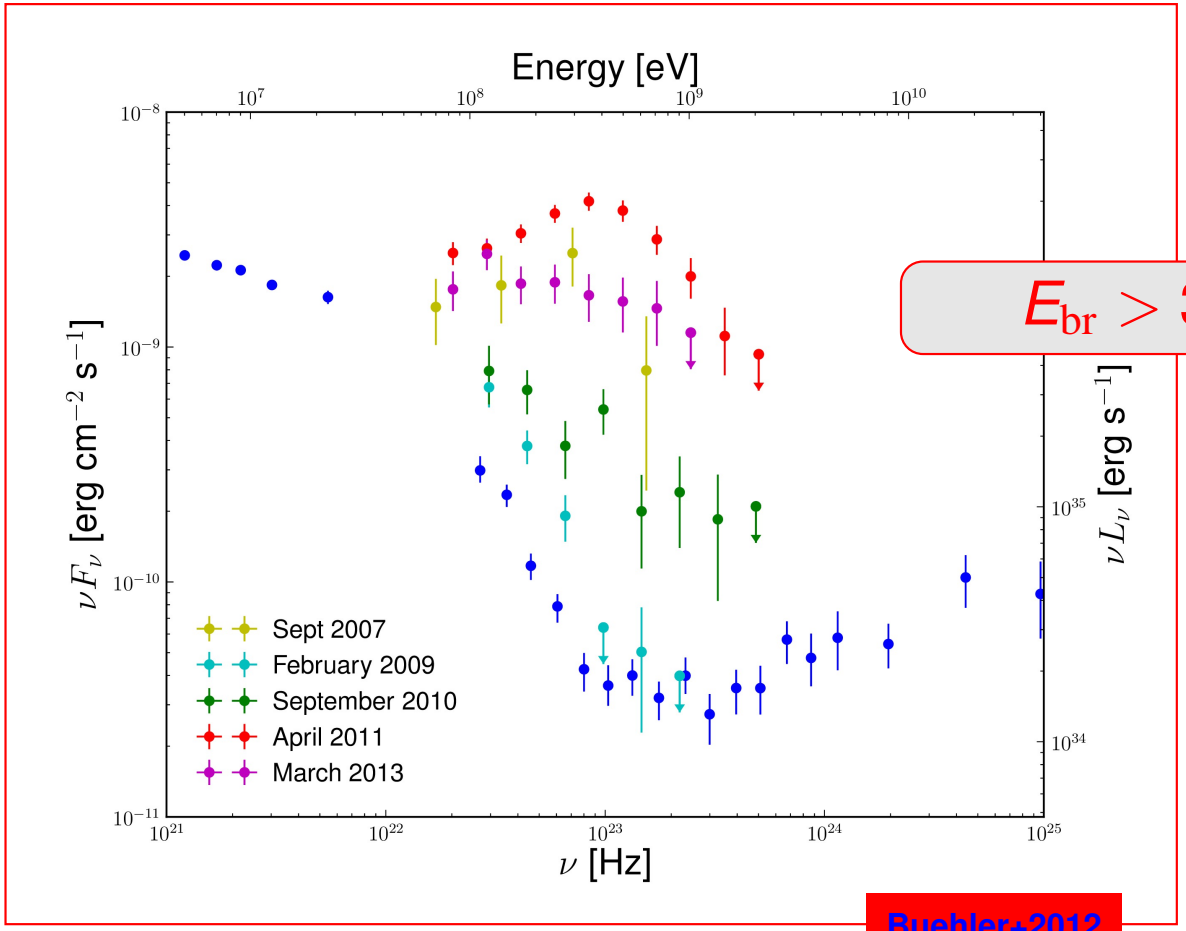
Crab Flares: Even More Efficient Acceleration?



$E_{\rm br} > 300\,{\rm MeV}$

Buehler+2012

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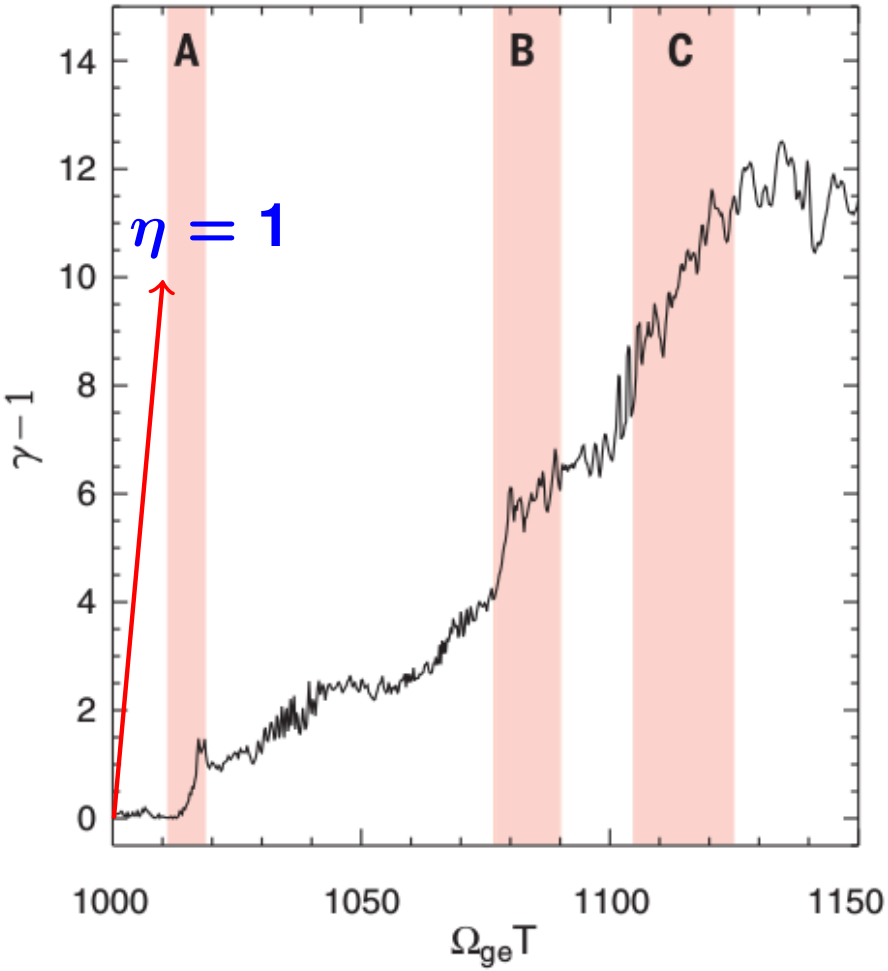

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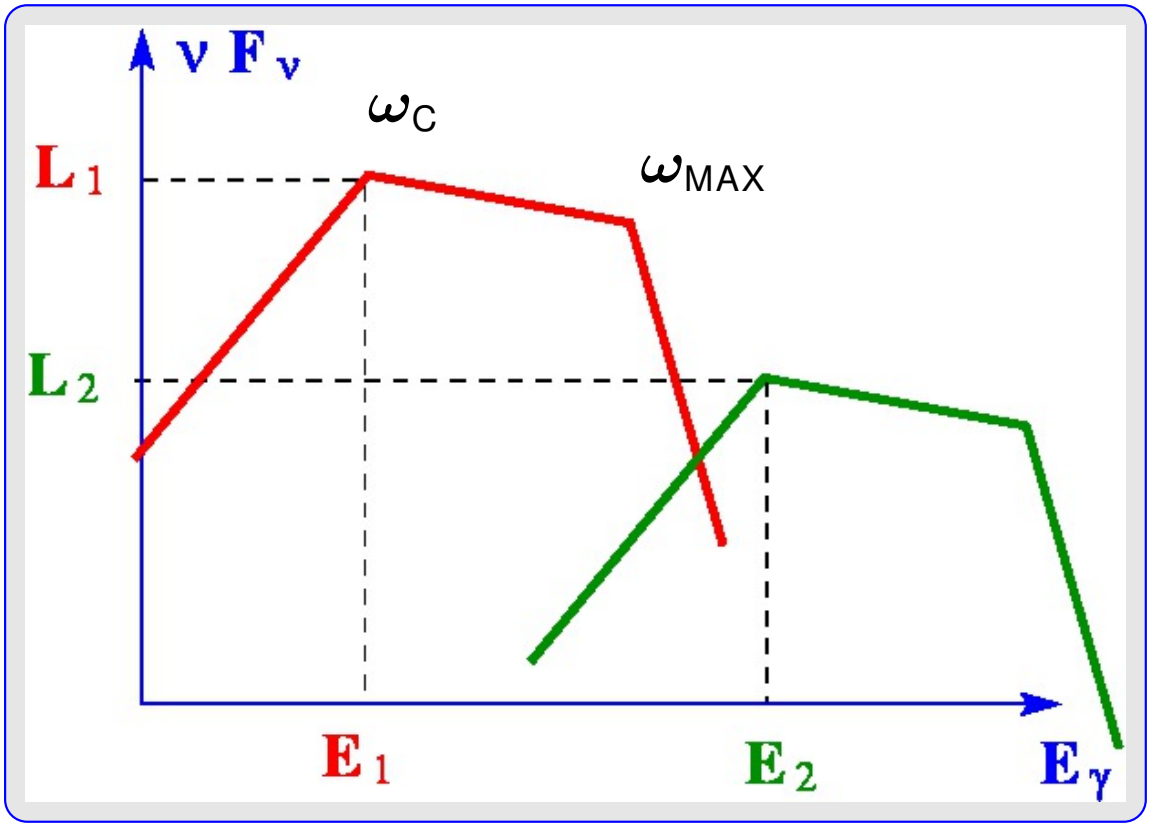
Synchrotron Model



Synchrotron Emission component

- If synchrotron losses are the dominant

$$\omega_{MAX} = 300\eta^{-1} MeV$$

- If the break is cooling

$$\omega_c = 60 \left(\frac{t}{400s}\right)^{-2} B_G^{-3}\,\mathrm{keV}$$

- If the break is cooling

$$\Delta\Gamma = \frac{1}{2}$$

- Hardest cooled spectrum

$$\Gamma = 1.5$$

Proton Synchrotron

Electrons

- Cooling: $t_{\rm cool} = 400 E_{\rm TeV}^{-1} B_{\rm G}^{-2}\ {\rm s}$
- Photon Energy: $\hbar\omega = 20 E_{\rm TeV}^{2} B_{\rm G}\ {\rm keV}$
- Highest Energy Cooling: $t_{\rm cool} = 6\eta^{1/2} B_{\rm G}^{-3/2}\ {\rm s}$
- Highest Energy Photons: $\hbar\omega = 100\eta^{-1}\ {\rm MeV}$

Protons

- Cooling: $t_{\rm cool} = 400 E_{\rm TeV}^{-1} B_{\rm G}^{-2} \left(\frac{m_{\rm p}}{m_{\rm e}}\right)^{4}\ {\rm s}$
- Photon Energy: $\hbar\omega = 20 E_{\rm TeV}^{2} B_{\rm G} \left(\frac{m_{\rm e}}{m_{\rm p}}\right)^{3}\ {\rm keV}$
- Highest Energy Cooling: $t_{\rm cool} = 6\eta^{1/2} B_{\rm G}^{-3/2} \left(\frac{m_{\rm p}}{m_{\rm e}}\right)^{2}\ {\rm s}$
- Highest Energy Photons: $\hbar\omega = 100\eta^{-1} \left(\frac{m_{\rm p}}{m_{\rm e}}\right)\ {\rm MeV}$

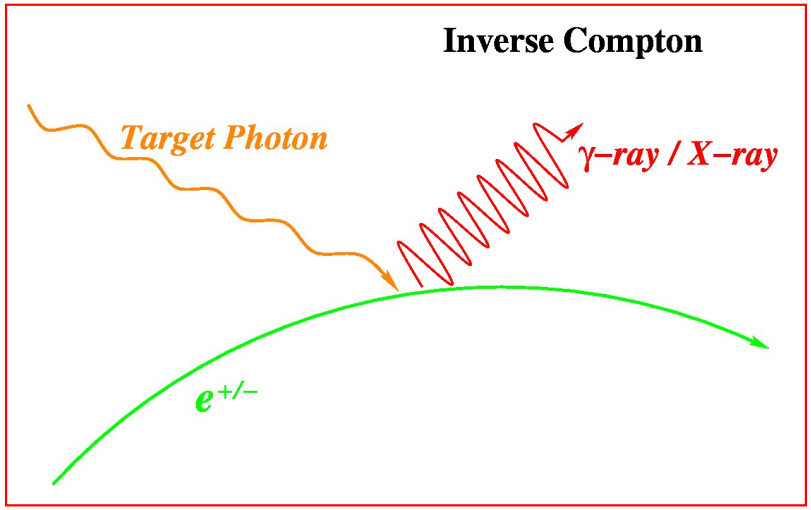
Inverse Compton Scattering

- Single Particle Spectrum:

$$\frac{\mathrm{d}I_{\mathrm{ic}}}{\mathrm{d}\omega} = \frac{r_o^2 \pi m_{\mathrm{e}}^3 c^4 \kappa T^2}{3\hbar^3 E} \times \left[\frac{(\omega/E)^2}{2(1-\omega/E)} + 1 \right]$$

- Energy Losses: $\dot{E}_{\mathrm{syn}} = -\frac{4}{3} U_{\mathrm{ph}} c \gamma^2$
- Spectrum transformation: $\alpha \Rightarrow \Gamma = \frac{\alpha+1}{2}$

Background photons should present in any source, in many cases IC scattering appears to be comparable to the synchrotron radiation.



Synchrotron-IC Model

Very similar processes

- Energy Losses

$$\dot{E}_{\mathrm{IC}} = -\frac{4}{3} U_{\mathrm{ph}} c \gamma^2 \qquad \dot{E}_{\mathrm{syn}} = -\frac{4}{3} U_{\mathrm{B}} c \gamma^2$$

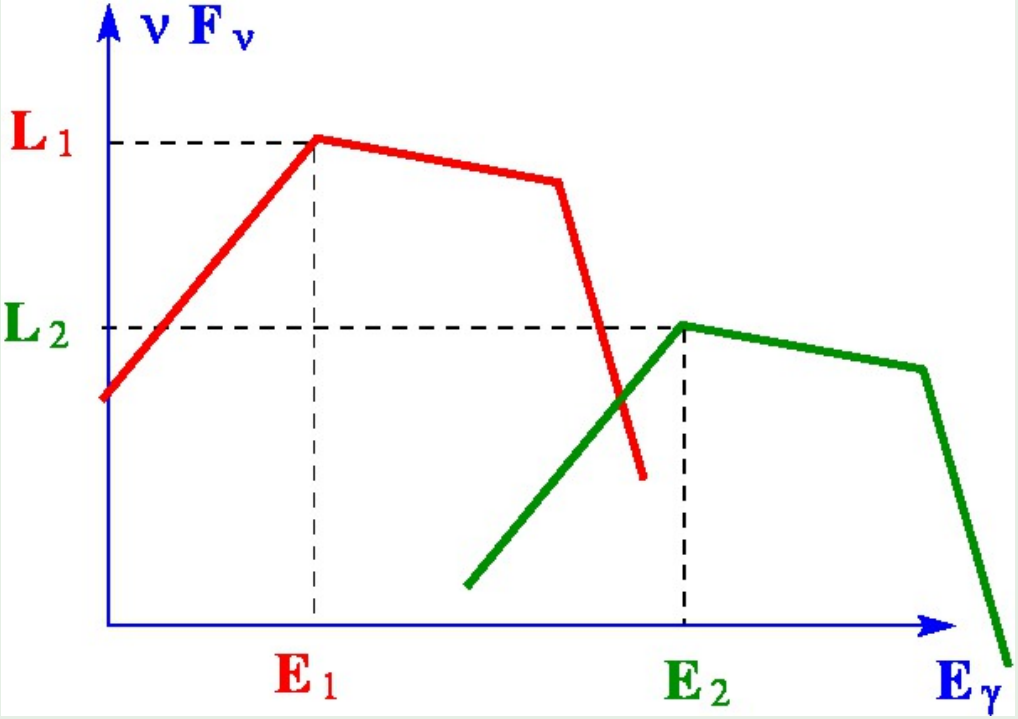
- Mean Energy

$$\hbar\omega = \left(\frac{4}{3}\hbar\omega_0\right)\gamma^2 \qquad \hbar\omega = \left(\frac{eB\hbar}{2\pi m_{\mathrm{e}} c}\right)\gamma^2$$

$$\frac{L_{\mathrm{syn}}}{L_{\mathrm{ic}}} = \frac{U_{\mathrm{B}}}{U_{\mathrm{ph}}} \qquad \frac{\hbar\omega_{\mathrm{syn}}}{\hbar\omega_{\mathrm{ic}}} = \frac{\left(\frac{eB\hbar}{2\pi m_{\mathrm{e}} c}\right)}{\left(\frac{4}{3}\hbar\omega_0\right)}$$

Synchrotron-IC Model

Two identical humps?



Ratio of the peaks

$$\frac{L_{\rm syn}}{L_{\rm ic}} = \frac{U_{\rm B}}{U_{\rm ph}}$$

Ratio of the energies

$$\frac{\hbar\omega_{\rm syn}}{\hbar\omega_{\rm ic}} = \frac{\left(\frac{eB\hbar}{2\pi m_e c}\right)}{\left(\frac{4}{3}\hbar\omega_0\right)}$$

Kein-Nishina Cut-off

Photon Energy < Electron Energy

$$\hbar\omega = \left(\frac{4}{3}\hbar\omega_0\right)\gamma^2 < \gamma m_{\mathrm{e}} c^2$$

$$\gamma < \left(\frac{m_{\mathrm{e}} c^2}{\frac{4}{3}\hbar\omega_0}\right)$$

$$\gamma < 5 \times 10^5 \omega_{0,\,\mathrm{eV}}^{-1}$$

$$\hbar\omega = \left(\frac{eB\hbar}{2\pi m_{\mathrm{e}} c}\right)\gamma^2 < \gamma m_{\mathrm{e}} c^2$$

$$\gamma < \left(\frac{2\pi m_{\mathrm{e}}^2 c^3}{eB\hbar}\right)$$

$$\gamma < 3 \times 10^{14} B_{\mathrm{G}}^{-1}$$

Inverse Compton Radiation

Interact with photon field

- Maximum Energy of Gamma Rays $E_\gamma < \frac{\gamma\left(1-\frac{1}{4\gamma^2}\right)}{1+\frac{1}{4\gamma\epsilon}}$
- Single Particle Spectrum:

$$\frac{\mathrm{d}\sigma_\gamma}{\mathrm{d}E_\gamma} = \frac{\pi r_e^2 \epsilon^2 E_\gamma}{\gamma^3(\gamma - E_\gamma)} \left[\ln \frac{E_\gamma}{\gamma\epsilon(\gamma - E_\gamma)} + \left(4\epsilon\gamma^2 - 4\gamma\epsilon E_\gamma - E_\gamma\right) \right.$$

$$\left. \frac{\left(2\epsilon\gamma^2 - 2\gamma\epsilon E_\gamma + \epsilon E_\gamma^2 + E_\gamma\right)}{4E_\gamma\gamma\epsilon(\gamma - E_\gamma)} \right]$$

Treatment of IC losses and scattering

- IC scattering on BB:

$$\frac{dN_{\mathrm{ani/iso}}}{d\omega\, dt} = \frac{T^3 m_{\mathrm{e}}^3 c^3 \kappa}{\pi^2 \hbar^3} \int\limits_{\epsilon_{\mathrm{ani/iso}}/T}^{\infty} \frac{d\nu_{\mathrm{ani/iso}}}{d\omega\, dN_{\mathrm{ph}}\, dt} \frac{x^2 dx}{\mathrm{e}^x - 1}\,.$$

- Approximate treatment as in Khangulyan+2014

$$\frac{dN_{\mathrm{ani/iso}}}{d\omega\, dt} = \frac{2 r_o^2 m_{\mathrm{e}}^3 c^4 \kappa T^2}{\pi \hbar^3 E^2} \times \left[\frac{z^2}{2(1-z)} F_1(x_0) + F_2(x_0) \right]$$

where $x_0 = \frac{z}{2(1-z)\, E_{\mathrm{e}} T (1-\cos\theta) t_\theta}$, $z = E_\gamma / E_{\mathrm{e}}$, and $F_{1,2}$ are simple functions of 1 argument

Treatment of IC losses and scattering

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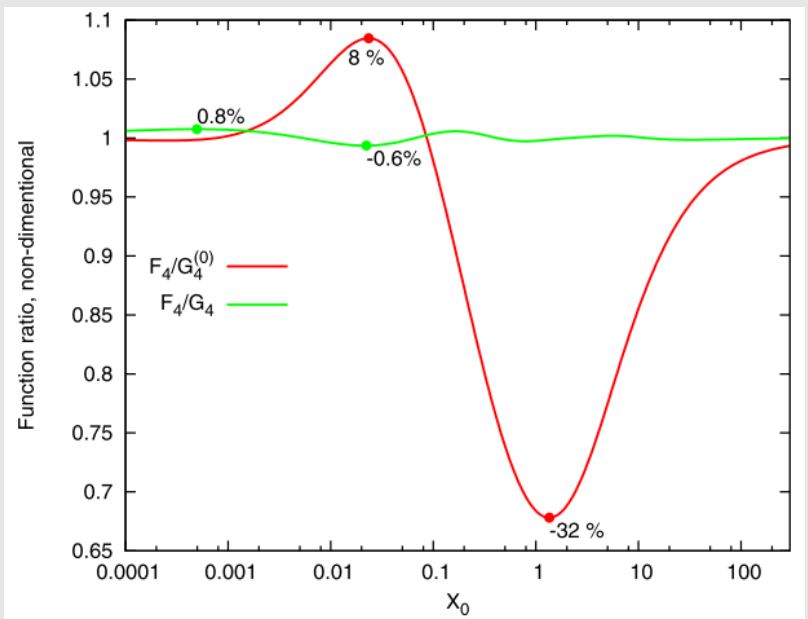
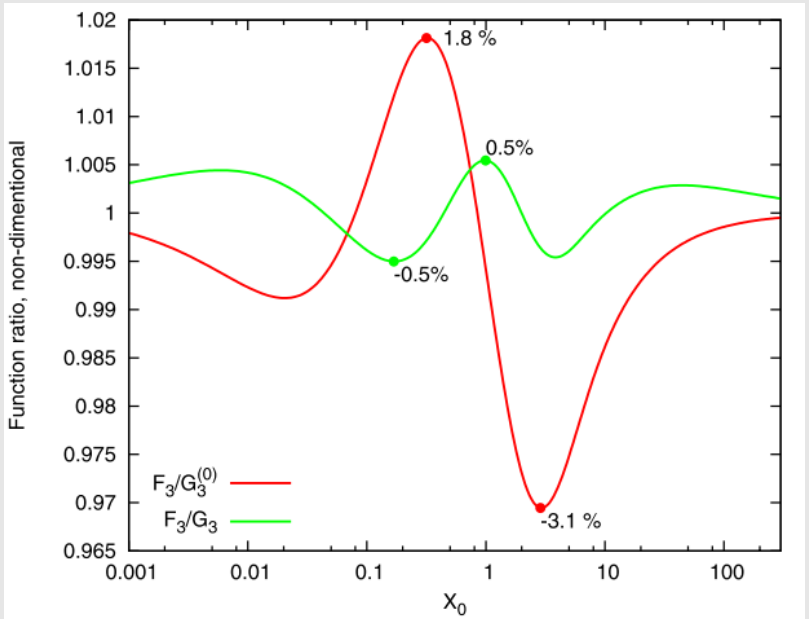
$$\frac{dN_{\mathrm{ani/iso}}}{d\omega\, dt} = \frac{T^3 m_e^3 c^3 \kappa}{\pi^2 \hbar^3} \int_{\epsilon_{\mathrm{ani/i}}}^{\infty} \frac{d\nu_{\mathrm{ani/iso}}}{d\omega\, dN_{\mathrm{ph}}\, dt} \frac{x^2 dx}{e^x - 1} \cdot$$

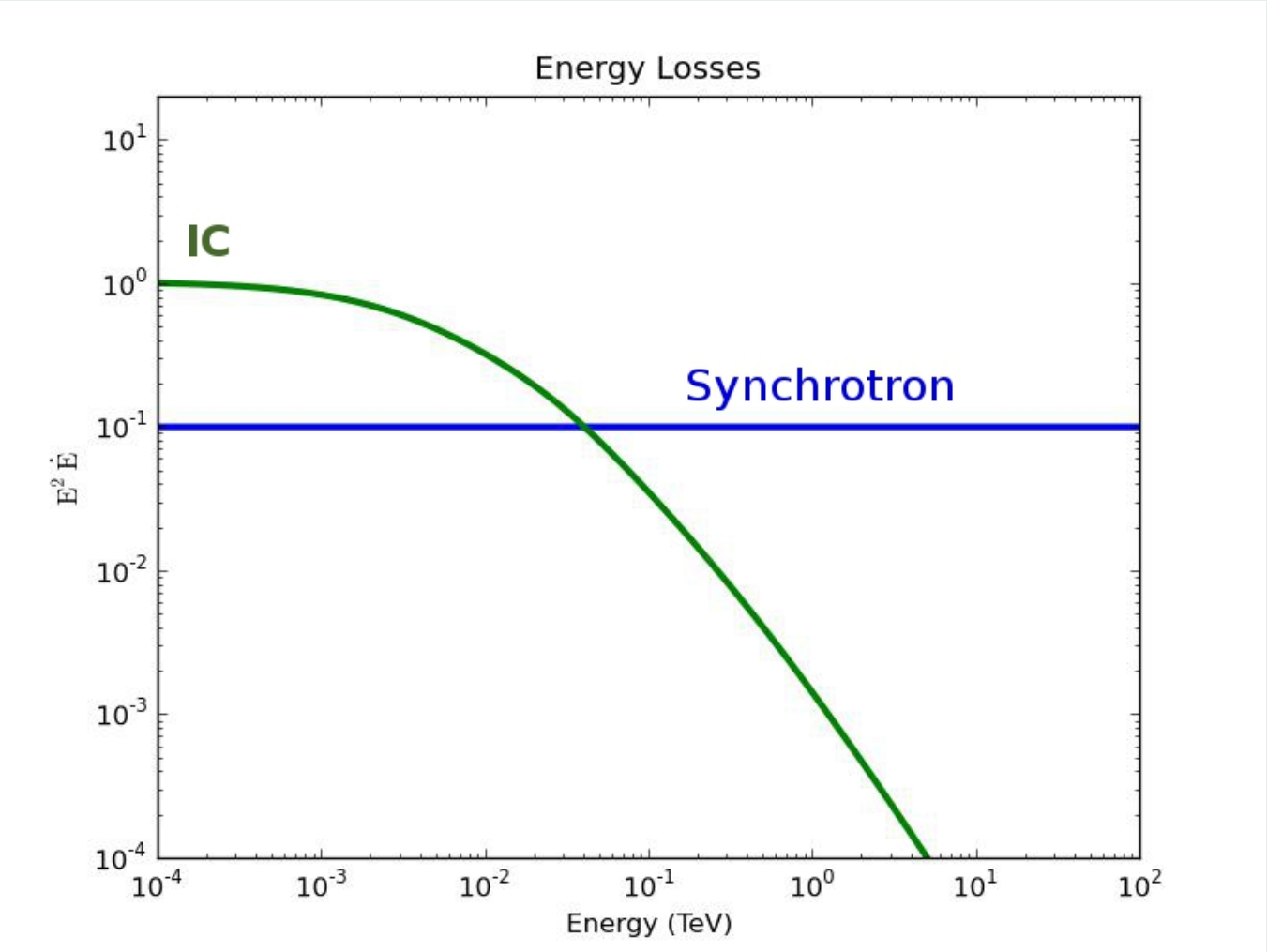




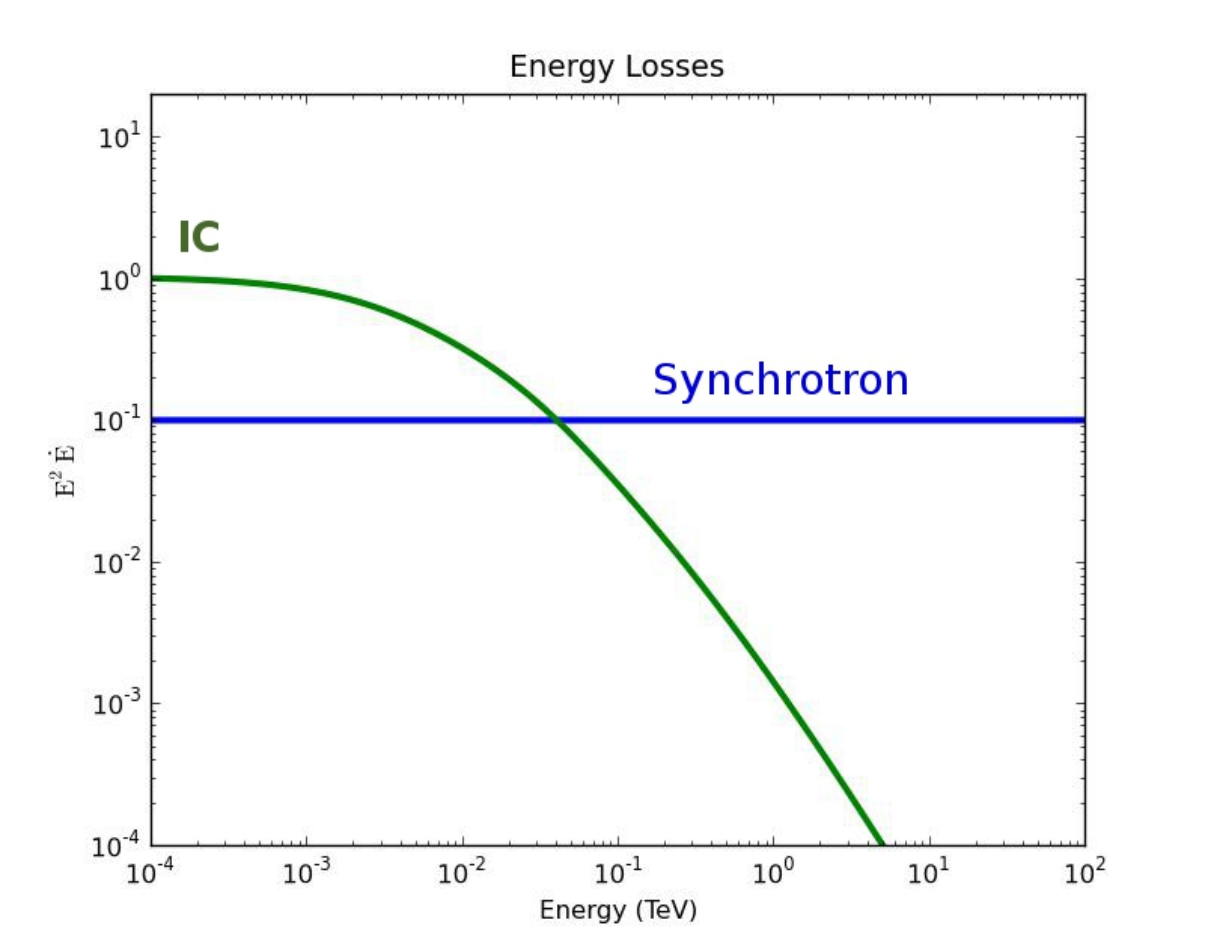
Treatment of IC losses and scattering

			Representation of spectra							
F_1	Equation (11)	$G_1^{(0)} = \left(\frac{\pi^2}{6} + x_0\right) e^{-x_0}$	$\frac{z}{1-z}\frac{1}{2ET(1-\cos\theta)}$	...	...	...	...	...	~10%	Figures 1 and 2
F_1	Equation (11)	$G_1^{(0)}(x_0) \times g(x_0)$	$\frac{z}{1-z}\frac{1}{2ET(1-\cos\theta)}$	0.153	0.857	0.254	1.84	...	<1%	Figures 1 and 2
F_2	Equation (11)	$G_2^{(0)} = \left(\frac{\pi^2}{6} + x_0\right) e^{-x_0}$	$\frac{z}{1-z}\frac{1}{2ET(1-\cos\theta)}$	...	...	...	...	...	~50%	Figures 1 and 3
F_2	Equation (11)	$G_2^{(0)}(x_0) \times g(x_0)$	$\frac{z}{1-z}\frac{1}{2ET(1-\cos\theta)}$	1.33	0.691	0.534	1.668	...	<1%	Figures 1 and 3
F_3	Equation (14)	$G_3^{(0)} = \frac{\pi^2}{6}\frac{1+c\,x_0}{1+\frac{\pi^2 c}{6}x_0} e^{-x_0}$	$\frac{z}{1-z}\frac{1}{4ET}$	...	...	...	...	2.73	~3%	Figures 4 and 5
F_3	Equation (14)	$G_3^{(0)}(x_0) \times g(x_0)$	$\frac{z}{1-z}\frac{1}{4ET}$	0.443	0.606	0.54	1.481	0.319	<1%	Figures 4 and 5
F_4	Equation (14)	$G_4^{(0)} = \frac{\pi^2}{6}\frac{1+c\,x_0}{1+\frac{\pi^2 c}{6}x_0} e^{-x_0}$	$\frac{z}{1-z}\frac{1}{4ET}$	...	...	...	...	47.1	~30%	Figures 4 and 6
F_4	Equation (14)	$G_4^{(0)}(x_0) \times g(x_0)$	$\frac{z}{1-z}\frac{1}{4ET}$	0.726	0.461	0.382	1.457	6.62	<1%	Figures 4 and 6
			Energy losses							
$F_{\rm ani}$	Equations (33) and (55)	$G_{\rm ani}^{(0)} = \frac{cu\log(1+2.16u/c)}{1+cu/0.822}$	$2ET(1-\cos\theta)$	...	...	...	—	6.13	~1%	Figure 7
$F_{\rm iso}$	Equations (36) and (57)	$G_{\rm iso}^{(0)} = \frac{cu\log(1+0.722u/c)}{1+cu/0.822}$	$4ET$	...	...	...	—	4.62	~5%	Figure 7
$F_{\rm iso}$	Equation (36)	$G_{\rm iso}^{(0)}(u) \times g(u)$	$4ET$	−0.362	0.682	0.826	1.281	5.68	~1%	Figure 7
			Interaction rate							
$F_{\rm n,ani}$	Equations (46) and (55)	$G_{\rm n,ani}^{(0)} = 0.822\log(1+1.949u)$	$2ET(1-\cos\theta)$	...	...	...	...	...	~25%	Figure 8
$F_{\rm n,ani}$	Equation (46)	$G_{\rm n,ani}^{(0)}(u) \times g(u)$	$2ET(1-\cos\theta)$	1.05	0.885	2.46	1.213	...	~1%	Figure 8
$F_{\rm n,iso}$	Equations (50) and (57)	$G_{\rm n,iso}^{(0)} = 0.822\log(1+0.97u)$	$4ET$	...	...	...	...	...	~30%	Figure 8
$F_{\rm n,iso}$	Equation (50)	$G_{\rm n,iso}^{(0)}(u) \times g(u)$	$4ET$	0.829	0.88	1.27	1.135	...	~1%	Figure 8
			Mean energy of emitted photons							
$\bar{z}_{\rm ani}$	Equation (55)	$G_{\rm ani}^{(0)}/G_{\rm n,ani}^{(0)}$	$2ET(1-\cos\theta)$	...	...	...	—	4.26	~3%	Figure 9
$\bar{z}_{\rm ani}$	Equation (56)	$\frac{u}{u+0.2}\frac{\log(1+u/2)}{\log(1+2u)}$	$2ET(1-\cos\theta)$	...	...	...	—	...	~8%	Figure 9
$\bar{z}_{\rm iso}$	Equation (57)	$G_{\rm iso}^{(0)}/G_{\rm n,iso}^{(0)}$	$4ET$	...	...	...	—	2.9	~5%	Figure 9
$\bar{z}_{\rm iso}$	Equation (58)	$\frac{u}{u+0.3}\frac{\log(1+u/4)}{\log(1+u)}$	$4ET$	...	...	...	—	...	~8%	Figure 9
			Energy of emitting particle							
t_θ	Equation (59)	$\frac{v^{1/2}(1+2v^{1/2})}{2}\sqrt{\frac{\log(1+v^{1/2})}{\log(1+v^{1/2}/3)}}$	$2\bar{\omega}T(1-\cos\theta)$	...	...	...	—	...	~10%	...
t	Equation (59)	$\frac{v^{1/2}\left(1+2v^{1/2}\right)}{2}\sqrt{\frac{\log(1+v^{1/2})}{\log(1+v^{1/2}/4)}}$	$4\bar{\omega}T$	...	...	...	—	...	~10%	...

Klein-Nishina Effect

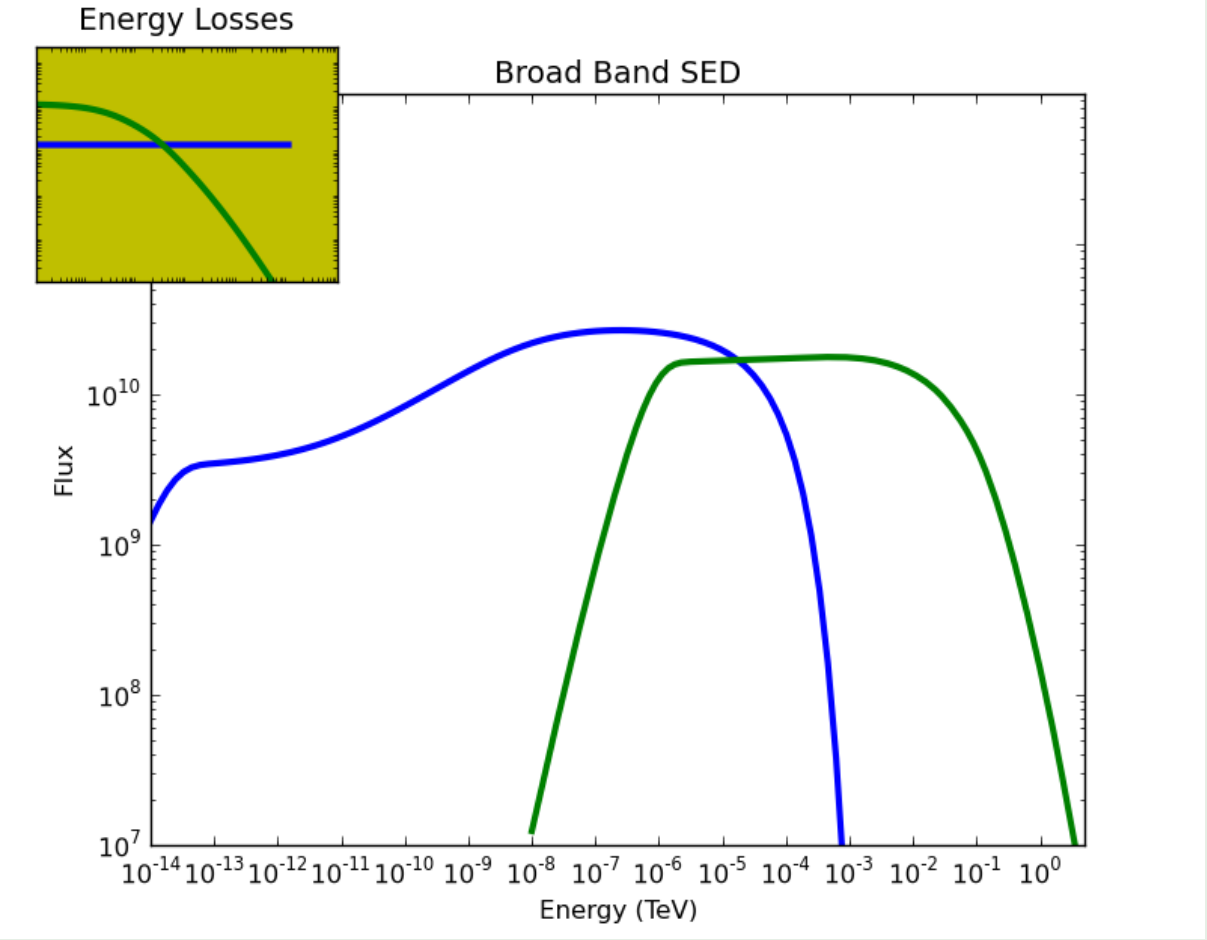


Klein-Nishina Effect



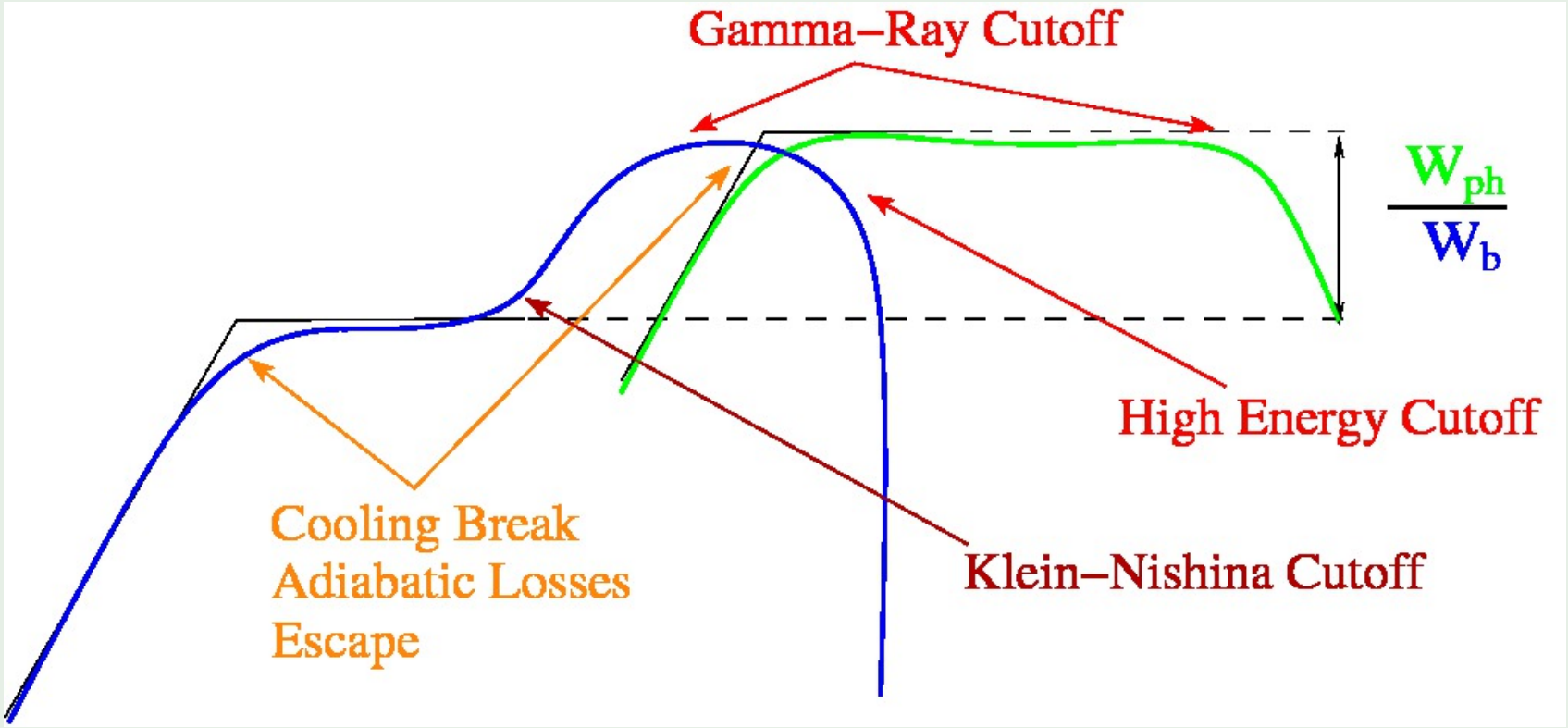
$$\frac{\mathrm{d}N_{\mathrm{e}}}{\mathrm{d}E} = \frac{1}{\dot{E}} \int_{E}^{\infty} \mathrm{d}E' \, Q(E) \qquad \dot{E} = \dot{E}_{\mathrm{syn}} + \dot{E}_{\mathrm{ic}}$$

Klein-Nishina Effect



- X-ray: hardening
- γ-rays: no Klein-Nishina cutoff

Klein-Nishina Effect



Compton Scattering Spectrum

$$\frac{\mathrm{d}N_\gamma}{\mathrm{d}E_\gamma} = \int \mathrm{d}E_\mathrm{e} c(1-\cos\theta) n_\mathrm{ph} \frac{\mathrm{d}N_\mathrm{e}}{\mathrm{d}E_\mathrm{e}} \frac{\mathrm{d}\sigma}{\mathrm{d}E_\gamma}$$

$$\frac{\mathrm{d}^2N(\theta,\omega)}{\mathrm{d}\omega\,\mathrm{d}\Omega} = \frac{r_0^2}{2\omega_0 E^2}\left[1+\frac{\omega^2}{2E(E-\omega)} - \frac{\omega}{\omega_0 E(E-\omega)(1-\cos\theta)} + \right.$$
$$\left. + \frac{\omega^2}{2\omega_0^2 E^2 (E-\omega)^2 (1-\cos\theta)^2}\right]$$
$$\equiv \frac{r_0^2}{2\omega_0 E^2}\left[1+\frac{z^2}{2(1-z)} - \frac{2z}{b_\theta(1-z)} + \frac{2z^2}{b_\theta^2(1-z)^2}\right],$$

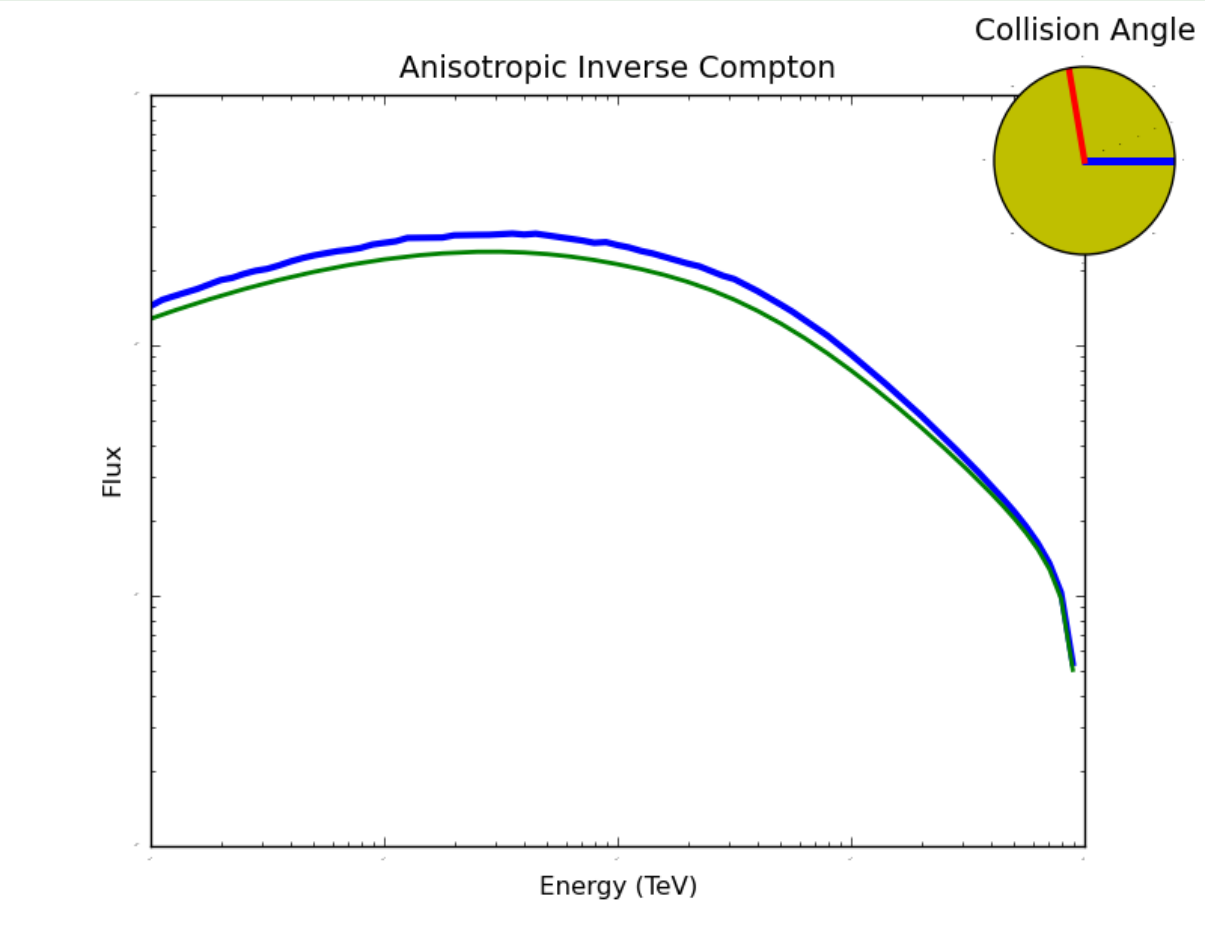
where $b_\theta \equiv 2(1-\cos\theta)\omega_0 E$, $z \equiv \omega/E$, and ω changes in the limits

$$\omega_0 \ll \omega \leqslant \frac{b_\theta}{1+b_\theta} E.$$

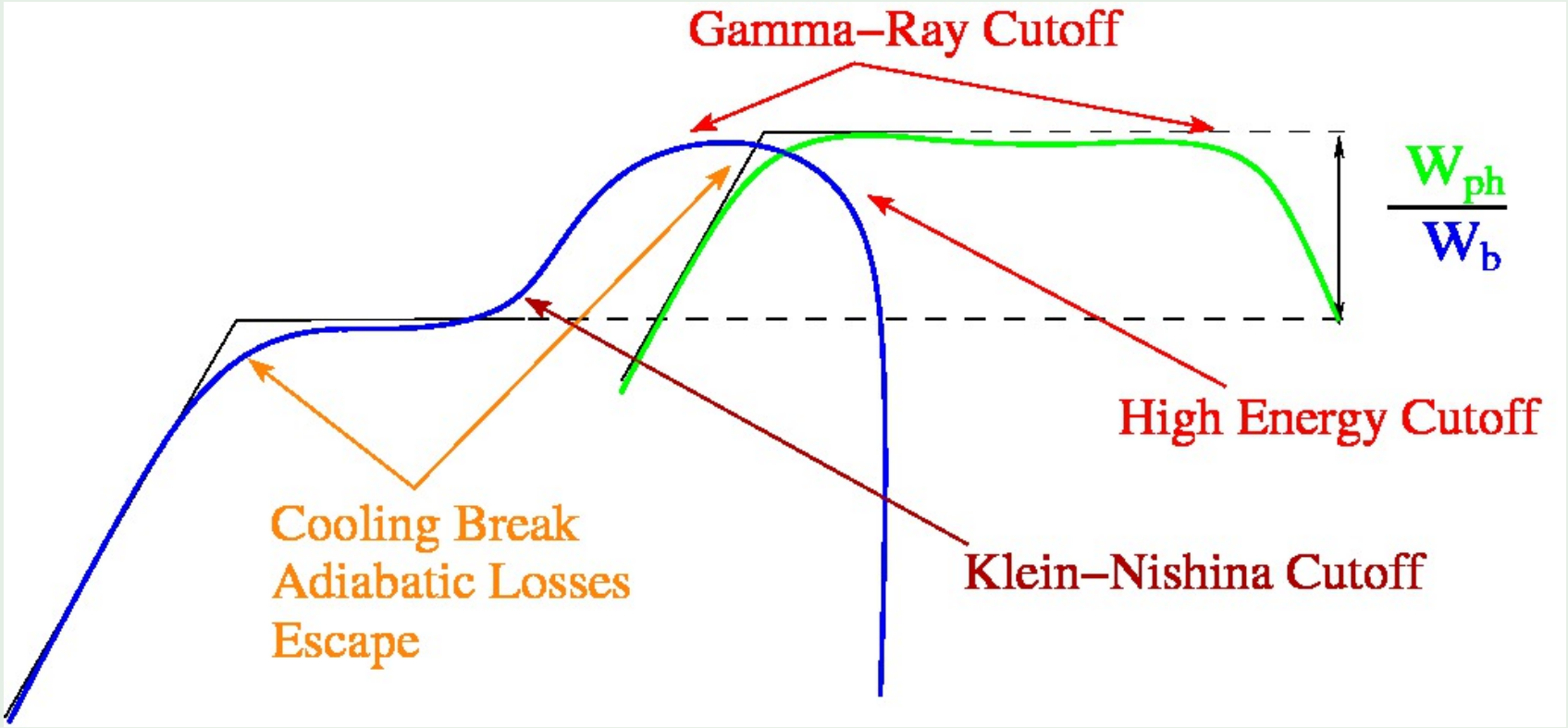
Aharonian&Atoyan, 1981

$$\frac{\mathrm{d}N_\gamma}{\mathrm{d}E_\gamma} = \int \mathrm{d}E_\mathrm{e} c(1-\cos\theta) n_\mathrm{ph} \frac{\mathrm{d}N_\mathrm{e}}{\mathrm{d}E_\mathrm{e}} \frac{\mathrm{d}\sigma}{\mathrm{d}E_\gamma}$$

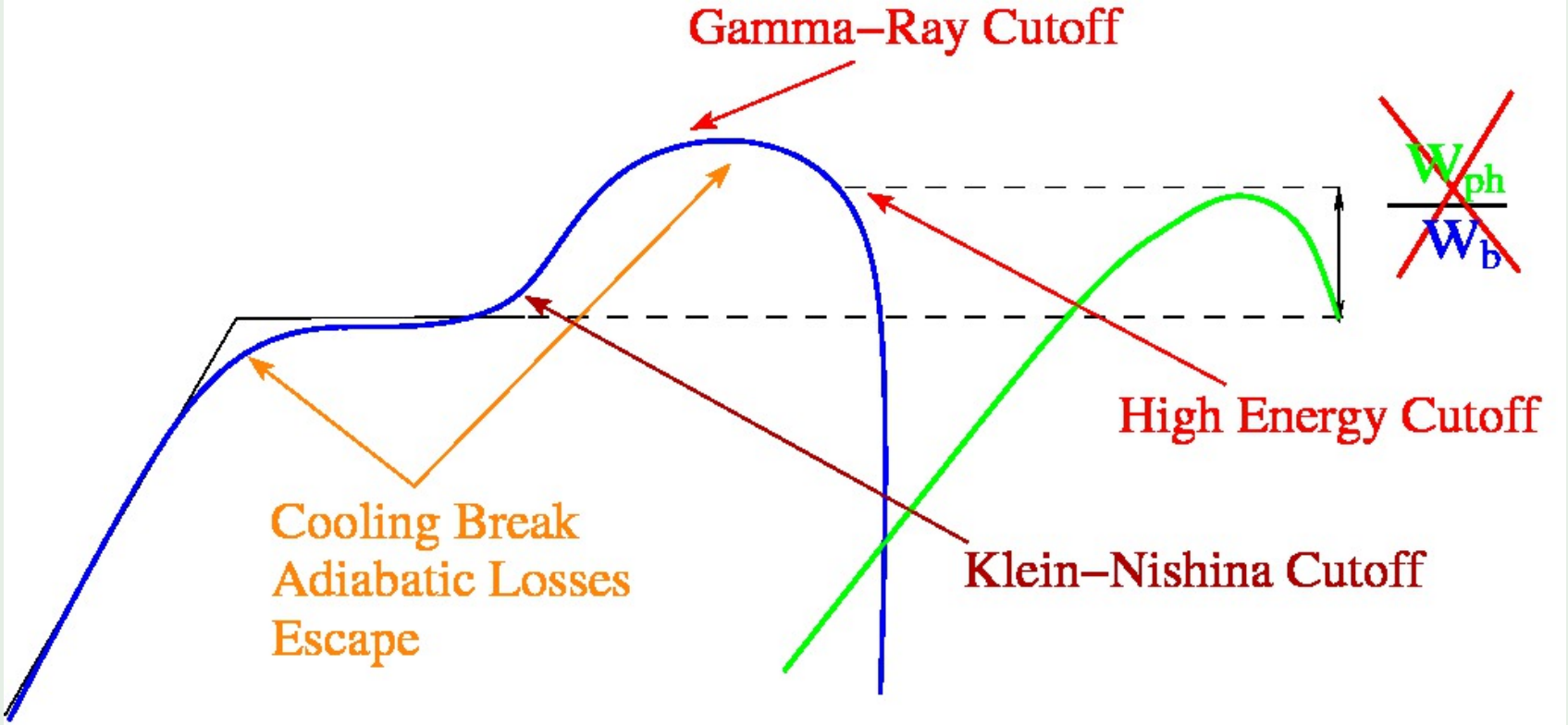
Anisotropic inverse Compton



Klein-Nishina Effect



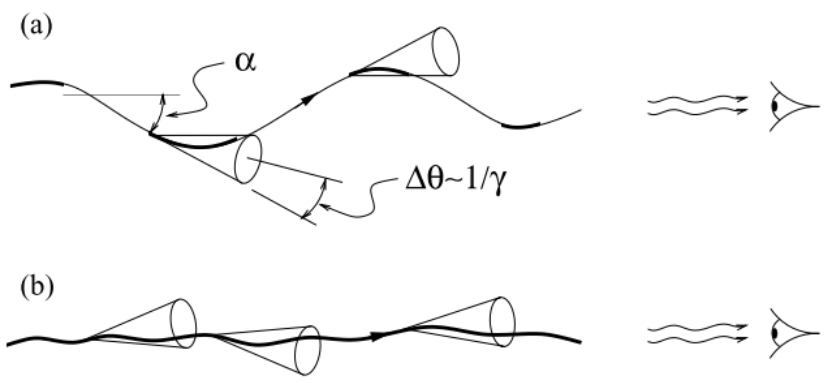
Klein-Nishina + Anisotropic IC



Radiation in Turbulent Fields

- Study of emission in turbulent media has a long history (e.g., Gatmantsev1973, Tsytovich&Kaplan1973, Chiuderi&Veltri1974)
- There are two basic approaches: perturbation theory and full description of trajectories (via kinetic equation or numerically)

credit Medvedev2000



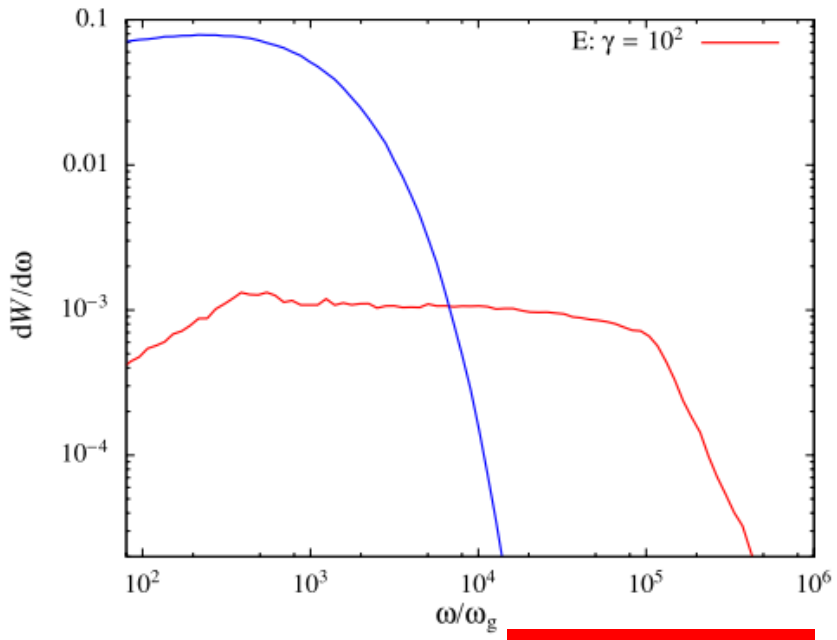
$$\frac{dW}{d\omega} = \frac{e^2\omega}{2\pi c^3}\int_{\omega/2\gamma^2}^{\infty}\frac{|\boldsymbol{w}_{\omega'}|^2}{\omega'^2}\left(1-\frac{\omega}{\omega'\gamma^2}+\frac{\omega^2}{2\omega'^2\gamma^4}\right)d\omega'$$

$$\frac{\partial W}{\partial t} + \mathbf{v}\,\frac{\partial W}{\partial \mathbf{r}} - (\boldsymbol{\Omega}\,\mathcal{O})W + e\mathbf{E}\,\frac{\partial W}{\partial \mathbf{p}} =$$

$$= \int_0^{\infty} \mathrm{d}\tau \left\{ \left(\frac{ec}{\mathscr{E}}\right)^2 \mathcal{O}_\alpha T_{\alpha\beta}(\Delta\mathbf{r}(\tau),\tau)\,\mathcal{O}_\beta + \right.$$

$$+ e^2\,\frac{\partial}{\partial p_\alpha}\,K_{\alpha\beta}(\Delta\mathbf{r}(\tau),\tau)\,\frac{\partial}{\partial p_\beta} - \frac{e^2 c}{\mathscr{E}}\left(\mathcal{O}_\beta S_{\alpha\beta}\,\frac{\partial}{\partial p_\alpha} + \right.$$

$$\left.\left. + \frac{\partial}{\partial p_\alpha}\,S_{\alpha\beta}\,\mathcal{O}_\beta\right)\right\} W(\mathbf{r}-\Delta\mathbf{r}(\tau),\,\mathbf{p}-\Delta\mathbf{p}(\tau),\,t-\tau)\,.$$



Reville&Kirk2010

Radiation in Turbulent Field (Jitter Case)

$$\frac{d\mathcal{E}_{\mathbf{n}\omega}}{d\omega\, d\Omega} = \frac{e^2}{4\pi^2 c^3} \left| \int\limits_{\mathbb{R}} \frac{\mathbf{n}\times[(\mathbf{n}-\beta(t))\times\mathbf{a}(t)]}{(1-\mathbf{n}\beta(t))^2}\, e^{i\omega(t-\mathbf{n}\mathbf{r}(t)/c)}\, dt \right|^2$$

$$\frac{eB\lambda}{mc^2} \ll 1$$

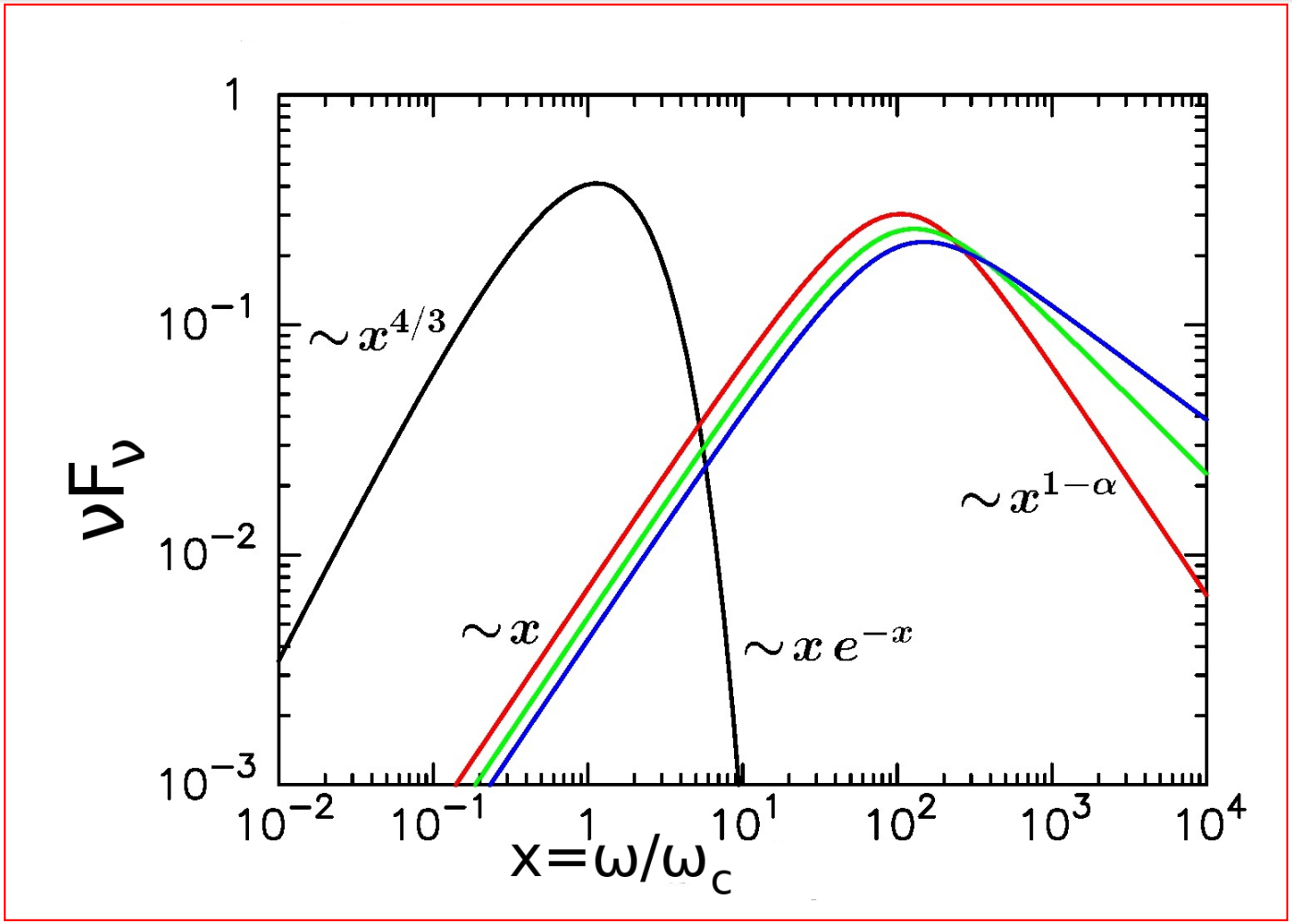
$$P_\omega(t) = \frac{e^4\langle \mathbf{B}^2\rangle}{6\pi^2 m^2 c^4} \int\limits_1^\infty u(\xi) \left(\frac{\omega\xi}{2c\gamma^2}\right)^2 \Psi\left(\frac{\omega\xi}{2c\gamma^2}\right) \frac{d\xi}{\xi}$$

$$K_{\rho\sigma} \equiv \langle B_\rho(\mathbf{r}_1, t_1) B_\sigma(\mathbf{r}_2, t_2)\rangle$$

$$\tilde{K}_{\rho\sigma}(\mathbf{q}) = \frac{1}{2}\left(\delta_{\rho\sigma} - \frac{q_\rho q_\sigma}{q^2}\right) \Psi(|\mathbf{q}|)\langle \mathbf{B}^2\rangle$$

Kelner+2014: Emission spectrum produced in small-scale turbulence is determined by the spectrum of turbulence only (both in isotropic and anisotropic regimes)

Synchrotron v.s. Jitter Emission Spectra



- low energy part
  $x^{4/3} \to x$
- maximum
  $\omega_c \to \frac{mc^2\omega_c}{eB\lambda}$
- high energy part
  $xe^{-x} \to x^{1-\alpha}$,
  where $\Psi \propto q^{-2-\alpha}$ for $q \gg 1$

Applicability of Jitter Radiation

- Jitter Regime is realized if $\frac{mc^2}{eB\lambda} \gg 1$, i.e.,

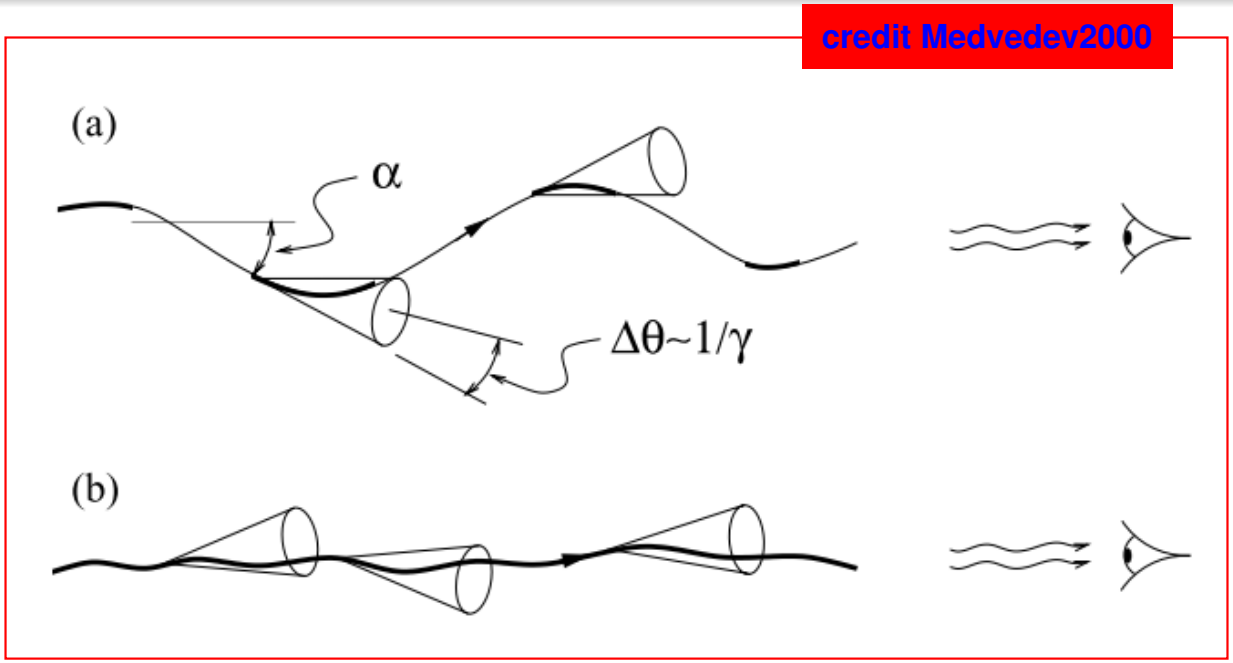
$$\lambda \ll 1.7 \times 10^2 (m/m_e)(B/1\,\mathrm{G})^{-1}\,\mathrm{cm}$$

- For conditions typical for SNR, blazar jet, and GRB that gives $10(m/m_e)\,\mathrm{km}$, $1(m/m_e)\,\mathrm{m}$, and $1(m/m_e)\,\mathrm{mm}$, respectively. That conditions might be hard to realize for electrons
- However, for protons the requirements are significantly relaxed

However, for relativistic particles there is an intermediate radiation regime (given $\gamma \gg 1$):

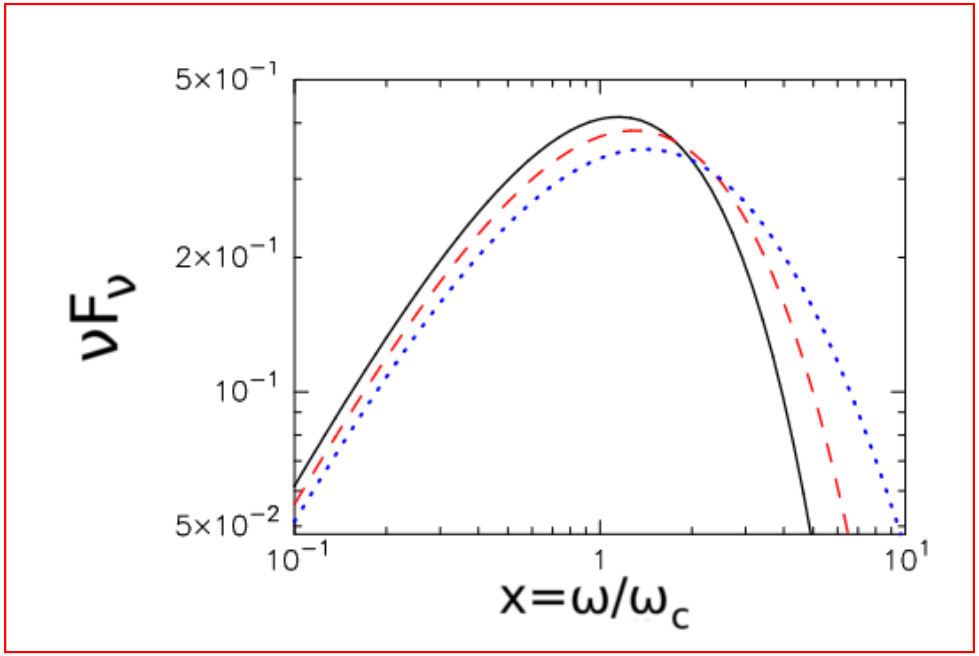
$$\frac{mc^2}{eB} \ll \lambda \ll \frac{mc^2\gamma}{eB}$$

Radiation in Turbulent Field (Large-Scale Case)


credit Medvedev2000

- the emission spectrum is determined by the trajectory curvature
- if $\lambda \gg \frac{mc^2}{eB}$, the momentary spectrum is synchrotron-type
- the time-averaged spectrum can be obtained by averaging the distribution of magnetic field: $w(B)$

Radiation in Turbulent Field (Large-Scale Case)



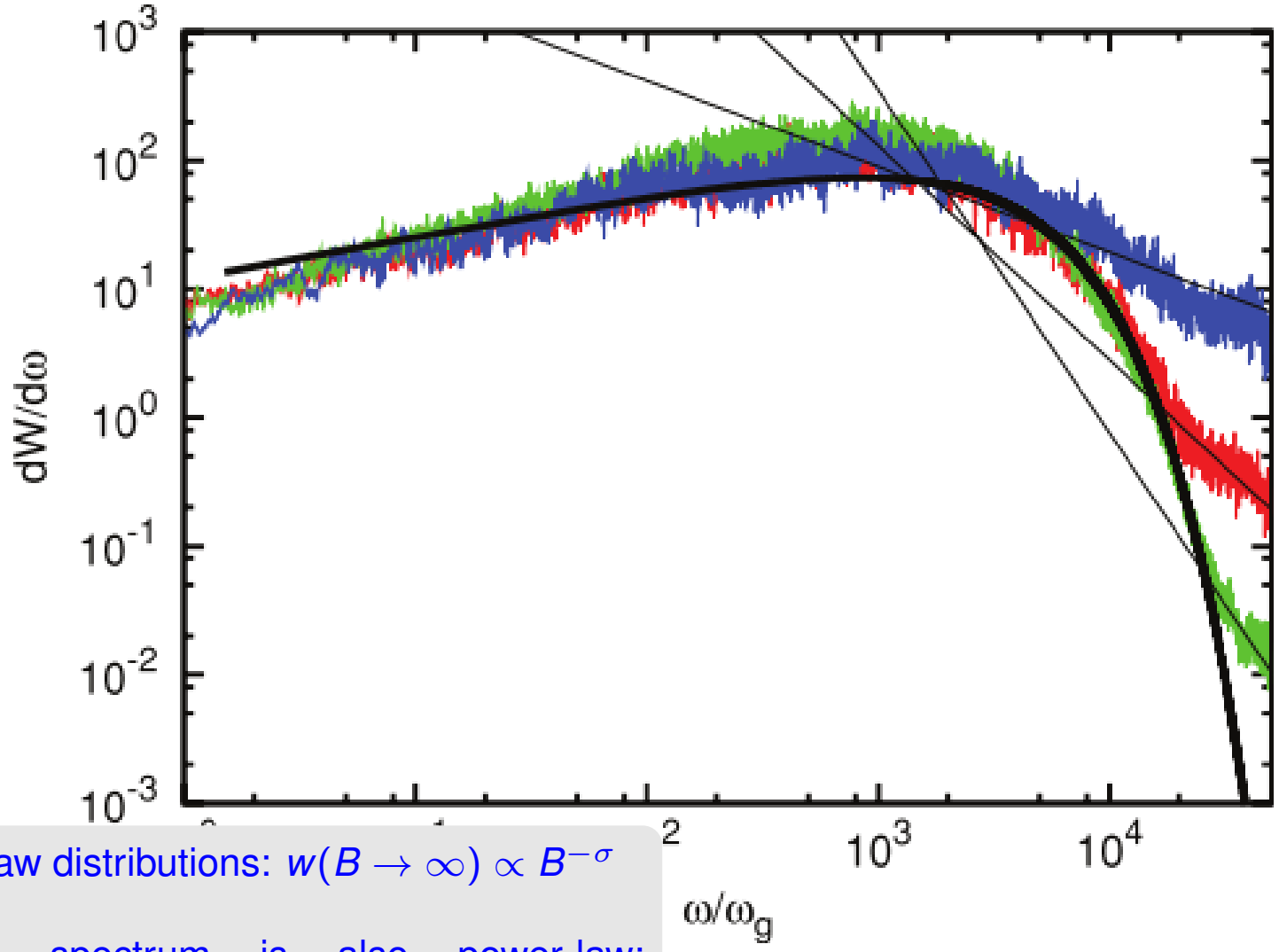
- the emission spectrum is determined b trajectory curvature
- if $\lambda \gg \frac{mc^2}{eB}$, the momentary spectru synchrotron-type
- the spectrum from an ensemble of par can be obtained by averaging the distrib of magnetic field

$w(B)dB$ is probability to interacts with field in the range $(B, B + dB)$

- $w(B)dB = \delta(B/B_0 - 1)dB/B_0$
- $w(B)dB = \frac{3\sqrt{6}}{\sqrt{\pi}}(B/B_0)^2 e^{-3(B/B_0)^2/2} dB/B_0$
- $w(B)dB = \frac{32\,(B/B_0)^2}{\pi(1+(B/B_0)^2)^4} dB/B_0$

Radiation in Turbulent Field (Large-Scale Case)

Teraki&Takahara2011



- for a power-law distributions: $w(B \to \infty) \propto B^{-\sigma}$

  the photon spectrum is also power-law: $\nu F_\nu \propto \omega^{2-\sigma}$

- High-Energy radiative models typically treat plasma physics a simplified way: phenomenological treatment of the acceleration and continues-loss approximation
- Nevertheless, obtaining the particle distribution is the most important part of the non-thermal emission modeling
- Particle spectra cannot feature arbitrary breaks, each break needs to be justified by cooling, injection or dominant radiation channel change
- One-zone models very useful and can address many physical features, but much more sophisticated models are widely used for more than 20 years
- In the high energy regime IC mechanism may proceed in the Klein-Nishina or anisotropic regime which has a strong impact on particle distribution, synchrotron, and IC spectra
- Synchrotron radiation may proceed differently in the case of a turbulent magnetic field which has a strong impact on the synchrotron spectrum, (particle distribution and IC spectra remain not affected)