Cosmic Ray Acceleration: A Plasma Physics Perspective

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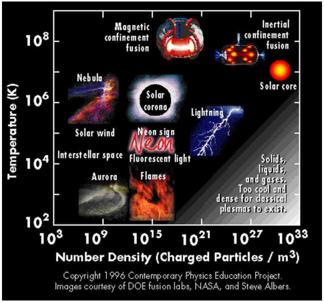
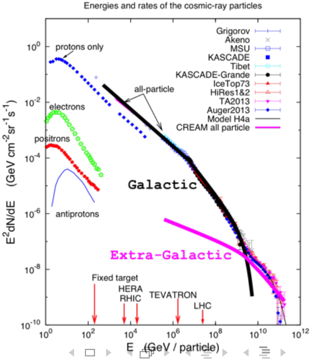
CRs and Plasma Physics: A Difficult Relationship



- Plasma comprises charged particles that do not frequently collide with one another
- Interaction between them occurs through long-range forces resulting from the plasma collective motion

- CRs are also a gas of charged particles
- Much lower density, its collective, plasma-like properties were not obvious



Contents: Collisionless shocks

Contents: CR shock Phenomena

3. State of the CR field and current issues
   - Overview of Observations
   - CR Theory: Objectives and Issues
   - Concepts of CR origin/acceleration
     - Energy sources and extraction mechanisms
   - Collisionless Shocks and Cosmic Rays

4. Diffusive Shock Acceleration (DSA, or I-Fermi)
   - Injection of particles from the thermal pool
   - Acceleration: linear vs nonlinear
   - Bifurcation to "efficient" acceleration regime
   - Particle release/escape into ISM
   - NL waves in CR shock precursor

5. Concluding Remarks

Simple Approach to Shock Phenomenon

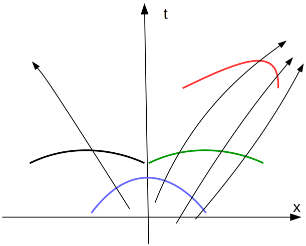
- Wave equation

$$\frac{\partial^2 \rho}{\partial t^2} - C^2 \frac{\partial^2 \rho}{\partial x^2} = Dissip \to 0$$

characteristics: $x = \pm Ct$,

$x \mapsto x - Ct$

$d\xi = dx - C(\rho)\, dt$



- Sound speed

$$C^2 = \frac{\partial \mathcal{P}}{\partial \rho} = C_0^2 + C_1^2(\rho)$$

- Pressure

$$\mathcal{P} = P_{gas} + \frac{B^2}{8\pi} + P_{en-part} + \ldots$$

may include magnetic, energetic particle (e.g. Cosmic Ray) pressure, etc.

Burgers Model and its Generalizations

- Transform to one of the characteristic frames
- convert from density to velocity
- Obtain Burgers Equation

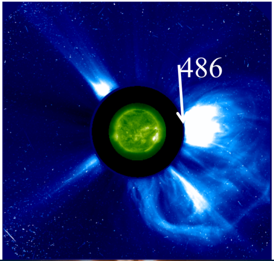
$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} = \nu\frac{\partial^2 V}{\partial x^2} + \ldots$$

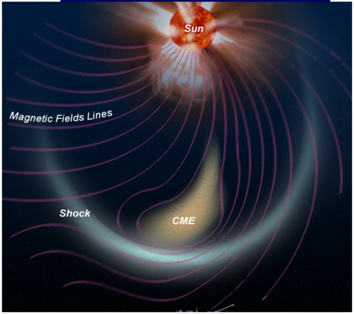
- more versatile versions available
- include driving forces in form of instabilities, wave dispersion, MHD generalizations, etc.
- however small ν, it's crucial for obtaining a single-valued solution
- what plays its role if no binary collisions?

Ordinary Shock Waves

- Naturally come from wave motion with $\nu \ll 1$
- result from wave steepening (intersection of characteristics)
- self-organize in thin sheets where nonlinearity balances even vanishing dissipation (viscosity)
- mass, momentum, and energy fluxes pass through the shock interface unchanged
- Why need then collisionless shocks? Why not just let $\nu \to 0$ in the ordinary ones?
  - $\nu \to 0+$ (but $\nu \neq 0$) - important difference from collisionless shock phenomenon where $\nu = 0$
  - media where other scales (e.g., dispersive, anomalous diffusivity of energetic particles, etc.) determine the shock structure
  - particle reflection (shock-surfing) upstream/downstream suprathermal particle interpenetration

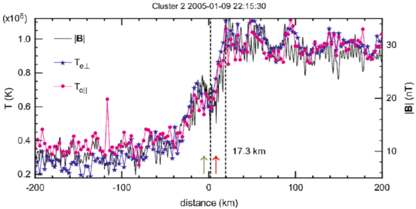
Solar Flares, CME, Interplanetary Shocks





Courtesy Mihir Desai

- Solar coronal mass ejection
  - Drives shocks into interplanetary space
  - particle mean free path is comparable to 1 AU
  - Coulomb collisions cannot account for thin shocks
- Need different mechanism for shock formation



from Krasnoselskikh, Balikhin et al. 2013 Cluster review

Thomas Gold Envision (slide borrowed from R.Z. Sagdeev)

Thomas Gold

1955

Suggested that an explanation of a Sudden Commencement Of Magnetic Storms caused by Solar Flares requires:

1) ***Presence of media in the interplanetary space***
2) ***Shock wave in that media***

A Quote from another Visionary (slide from R.Z. Sagdeev)

Harry Petschek

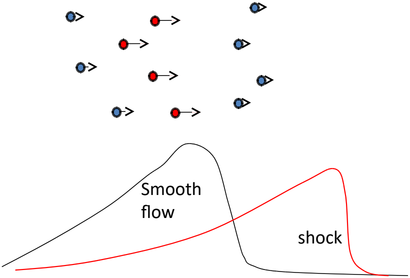
“I am working on a theory of collisionless shocks

(whatever those were!); we have tried to make them in the laboratory but have failed

because we couldn’t eliminate particle Collisions”

-Oblique whistler waves’ multiple crossing of Shock Front (like Fermi mechanism for waves;

Shock Dissipation via "Effective" collisions with waves

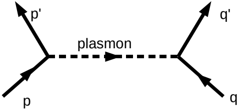


flow velocity

-Fast flow envelope overtakes the slow one
-Is the multi-stream state unavoidable?

Problem:

- How to transfer momentum and energy from fast to slow gas envelopes if there are no binary collisions?
- waves...
- driven by particles whose distribution is almost certainly unstable...



Laminar shocks: Nonlinearity + Dispersion

- Korteweg-deVries (Burgers) equation

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + \frac{\partial^3 V}{\partial x^3} = \nu\frac{\partial^2 V}{\partial x^2} + \ldots$$

- Although derived for weakly-nonlinear regimes
  - provides paradigmatic tool for studying shock phenomena in dispersive and viscose media
  - allows straightforward generalizations to shocks driven by unstable particle populations
  - elucidates quasi-periodic and chaotic dynamics of ensembles of shocks
  - accommodates shock-trains (shocklets, SLAMS, etc.)
- Serves as an accessible limit of more general shock models such as based on
  - Derivative Nonlinear Schrödinger equation
- By contrast to Burgers model, admits meaningful (soliton) solution in the strict case $\nu = 0$, not just $\nu \to 0+$.

Reconciling Reversible with Irreversible

COOPERATIVE PHENOMENA AND SHOCK WAVES
IN COLLISIONLESS PLASMAS

R. Z. Sagdeev

Rev. In Plasma Phys, 1964

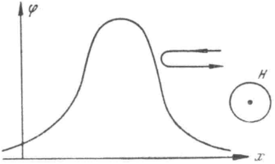


Fig. 18

Sagdeev potential

The shape of the potential well is given by

$$V(H) = \frac{1}{2}(H - H_0)^2\left[\frac{(H+H_0)^2}{16\pi n_0 M u^2} - 1\right]. \qquad (63)$$

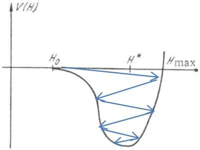


Fig. 14

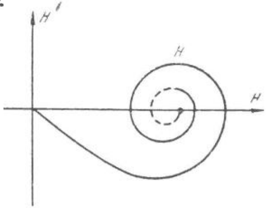


Fig. 16

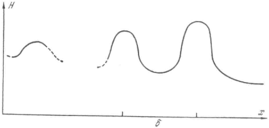


Fig. 15

A Quarter Century of Collisionless Shock Research

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This review highlights conceptual issues that have both governed and reflected the direction of collisionless shock research in the past quarter century. These include MHD waves and their steepening, the MHD Rankine-Hugoniot relations, the supercritical shock transition, nonlinear oscillatory wave trains, ion sound anomalous resistivity and the resistive-dispersive transition for subcriticial shocks, ion reflection and the structure of supercritical quasi-perpendicular shocks, the earth's foreshock, quasi-parallel shocks, and, finally, shock acceleration processes.

1985

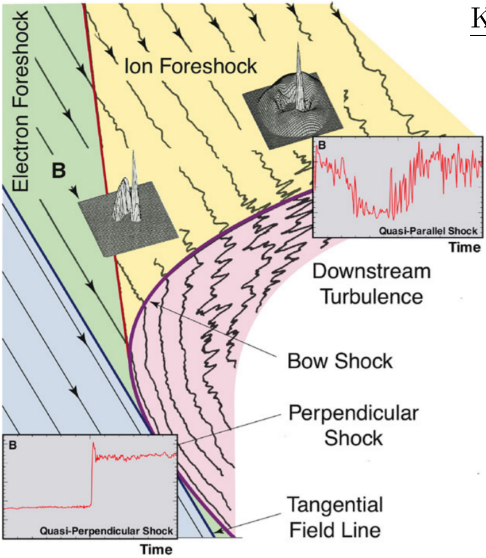
Microinstabilities and Anomalous Transport

K. Papadopoulos

Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742

The role of microinstabilities in producing dissipation and anomalous transport in collisionless shock waves is reviewed. Particular emphasis is placed on quasi-turbulent magnetosonic shocks. The review follows the historical development of anomalous transport and the incorporation of the coefficients into multifluid and hybrid models. A general formalism is presented which describes in a self-consistent manner, the macroscopic transport produced by short wavelength microinstabilities. Similarities and differences with models incorporating classical transport are emphasized. The important instabilities and their transport properties are summarized. It is shown that multifluid simulations with anomalous transport explain many features of the experimental observations. The relevance of ion reflection and the necessity for kinetic ion description for supercritical shocks along with state of the art numerical studies are also discussed. The review concludes with a brief discussion of the piston shock problem and of quasi-parallel turbulent shocks.

All in one picture of difficult problem of shock geometry



from Burgess and Scholer 2012

Key Distinction:

- Quasi-Parallel vs Quasi-Perpendicular shock geometry
- Quasi-Parallel harder to understand, as hot downstream plasma and shock-reflected penetrate far upstream
  - partially smear out the shock transition
  - drive instabilities, thus making shock front accessible to energetic particles
    → can accelerate particles to very high energies

Surfatron acceleration

Shock-drift acceleration, surfatron acceleration etc.

We wish to point out a curious acceleration mechanism that operates on certain ion bunches in such a shock wave. Ions whose velocities are very close to the velocity of the shock wave will have small Larmor radii. Upon being reflected from the potential barrier they are immediately "turned" by the magnetic field and reflected again; this process occurs several times. After several reflections (Fig. 19) these ions acquire a very high velocity in the y-direction (in the plane of the front and transverse to H). However, this velocity cannot become arbitrarily large because as v_y increases the Lorentz force $(e/c)\, v_y H$ becomes important in the region of the barrier; ultimately this force becomes greater than the "reflection" force $-e\nabla\varphi$, and the ion passes through the barrier. The maximum energy of such an ion is of order $(M/m) \cdot Mu^2/2$, where $Mu^2/2$ is the mean energy of the ordered motion executed by an ion in these oscillations.

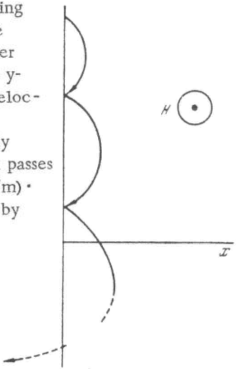


Fig. 19

PHYSICAL REVIEW VOLUME 75, NUMBER 8 APRIL

On the Origin of the Cosmic Radiation

1949...

ENRICO FERMI
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
(Received January 3, 1949)

A theory of the origin of cosmic radiation is proposed according to which cosmic rays are originated and accelerated primarily in the interstellar space of the galaxy by collisions against moving magmetic fields. One of the features of the theory is that it yields naturally an inverse power law for the spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.

ON THE ACCELERATION OF PARTICLES IN SHOCK FRONTS (*)

1963...

E. SCHATZMAN

Mount Wilson and Palomar Observatories,
Carnegie Institution of Washington,
California Institute of Technology
and Institut d'Astrophysique, Paris.

ABSTRACT. — *Charged particles can be accelerated in a perpendicular magnetohydrodynamic shock wave. The scattering of particles by the clumpiness of the magnetic field provides the statistical mechanism which is at the origin of the energy spectrum.*

The energies which can be obtained are quite high, even for weak shocks, but the stronger is the shock, the larger is the number of accelerated particles. ?

It is possible to derive the energy spectrum, which obeys to a power law, deeply related to the nature of the clumps, which can be those found in collision-free shocks.

The conditions for acceleration (injection in the accelerating process) are the same as those for the existence of the clumps in collision-free shocks.

Diffusive shock acceleration

Mon. Not. R. astr. Soc. (1978) **182**, 147–156

The acceleration of cosmic rays in shock fronts – I

+ Krymsky, 1977
Axford et al, 1977

A. R. Bell *Mullard Radio Astronomy Observatory, Cavendish Laboratory, Madingley Road, Cambridge CB3 0HE*★

Received 1977 June 23

Summary. It is shown that charged particle
energies in astrophysical shock fronts. Fast
streaming away upstream of a shock front b
which they themselves generate. This scatterin
region around the shock and results in first-o
the particles crossing the shock many times. Th
is a power law with an index close to that ob
The discussion relates to particles which are
initial acceleration from thermal energies is not

PARTICLE ACCELERATION BY ASTROPHYSICAL SHOCKS

R. D. Blandford
California Institute of Technology
AND
J. P. Ostriker
Princeton University Observatory
Received 1977 December 12; accepted 1978 January 6

ABSTRACT

A new mechanism is proposed for acceleration of a power-law distribution of cosmic rays with approximately the observed slope. High-energy particles in the vicinity of a shock are scattered by Alfvén waves carried by the converging fluid flow leading to a first-order acceleration process in which the escape time is automatically comparable to the acceleration time. Shocks from supernova explosions propagating through the interstellar medium can account for the acceleration of galactic cosmic rays. Similar processes occurring in extragalactic radio sources can lead to efficient in situ acceleration of relativistic electrons.

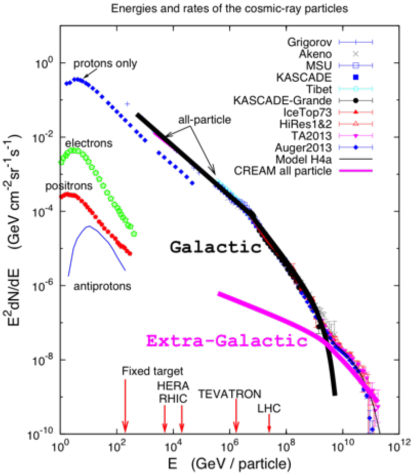
Subject headings: cosmic rays: general — shock waves

CS: Concluding Remarks

- Over almost 60 years of research work, collisionless shock discipline has grown beyond its initial, primarily laboratory and space plasma applications
- Left out of this CS discussion
  - collisionless shocks in laser plasma
  - planetary and cometary bow shocks, interplanetary shocks
  - other shock phenomena in Heliosphere, including the termination shock
- not covered are many astrophysical shocks, such as
  - pulsar-wind terminations shocks
  - galactic wind termination shocks
  - termination and internal shocks of AGN jets
  - large structure formation shocks
  - .....
- The focus on SNR shocks was dictated by their closest relation to aspects of CS mechanism associated with wave-particle interaction
  - In this regard, recent high-fidelity observations of galactic cosmic rays, presumably generated in SNR shocks, is an acid test of our understanding of how the collisionless shock mechanism works

A recent CS review: Marcowith et al. 2015

State of the Art CR energy spectrum



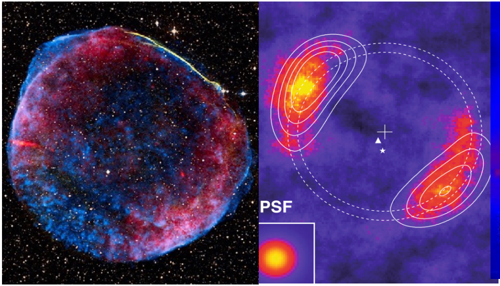
IceCube compilation of CR spectrum

- CR energy spectrum long thought to be featureless (power law):
- consistent with popular acceleration

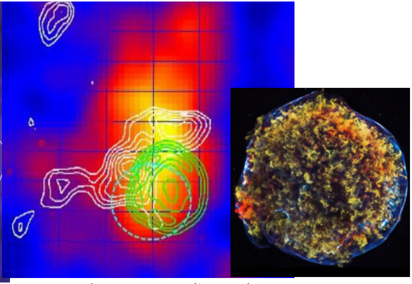
  mechanism:
  diffusive shock acceleration, DSA
- DSA rigidity (p/Z) spectra should be the same for all species
- propagation through the ISM may only change the PL-index
  - steepening by propagation losses (0.3-0.6 [!] in PL index)
- some predictions proved incorrect
  - difference in elemental rigidity spectra
  - breaks in individual spectra
- however, conclusion about PL holds up!

Goals and Issues

- Goal: where and how are CR accelerated?
- long-standing hypothesis for galactic CRs: Supernova Remnant (SNR) shocks
- proof "beyond a *reasonable* doubt", only by indirect reasoning. Why?
  - impossible to depropagate CR from Earth back to their putative sources (e.g., SNR)
  - difficult to disentangle hadronic and leptonic emission



SNR 1006: X, radio, optical, gamma

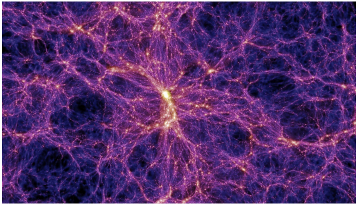


Tycho (1572): radio, mol. gas, gamma

Macroscopic Energy Sources for Cosmic Rays

Generic source: gravitational energy of

- stars, black holes
- clouds of dense molecular gases
- dark matter filaments and nodes of the "cosmic web" (galaxy clusters)
- exotic sources: strings (topological defects from BB), DM decay and annihilation

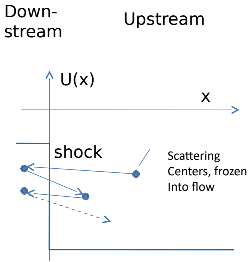


Energy extraction mechanisms:

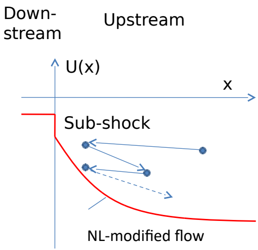
- inhomogeneous flows of conducting gases (plasmas) usually terminated by **SHOCKS**
- accretion flows on galactic clusters, BHs, jets, ..
- stellar winds, colliding winds, galactic winds, SN explosions →SNR shocks

Essential DSA (aka Fermi-I process, E. Fermi, ~1950s)

Linear (TP) phase of acceleration



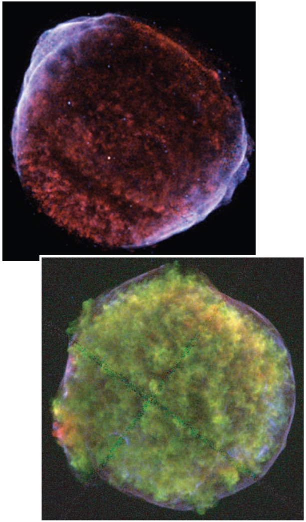
NL, with CR back-reaction



- CR trapped between converging mirrors: $p\Delta x \approx const$
- CR spectrum depends on shock compression, r: $f \sim p^{-q}$, $q = 3r/(r-1)$, $r = q = 4$, Mach $M \to \infty$

- Ind $q \to q(p)$: soft at low p:
  - $q = 3r_s/(r_s - 1) \sim 5$
- hard at high p : $q \to 3.5$
- for $M > 10$, $E_{max} \gtrsim 1$ TeV (MM'97) acceleration must go nonlinear (confirmed by other analyses and numerics in 2000s)

CR acceleration in SNRs



SN 1006 and SN 1572 (Tycho), Reynolds 2008 and Warren et al 2005

- At least some of the galactic SNR are expected to produce CR up to $10^{15} eV$ (knee energy)
- "Direct" detection is possible only as secondary emission
  - observed from radio to gamma
  - electron acceleration up to $\sim 10^{14} eV$ is considered well established, synchrotron emission in x-ray band (Koyama et al 1995)
  - tentative evidence of proton acceleration from nearby molecular clouds:

$$pp \to \gamma$$

Fermi-LAT, HESS, Agile,..

Convection-Diffusion Equation: shock solution

- energetic particles, pitch-angle scattered by MHD waves frozen into a plasma flow of the speed $u(x)$

$$\frac{\partial f}{\partial t} + u(x)\frac{\partial f}{\partial x} - \frac{\partial}{\partial x}\kappa(p)\frac{\partial f}{\partial x} = \frac{1}{3}\frac{du}{dx}\frac{\partial f}{\partial p}$$

- at a simple shock, u is a step function $u = u_1, u_2$ for $x > 0$, $x < 0$

$$f(x,p) = f_0(p)\exp\left[-\frac{u_1}{\kappa}x\right], \quad x > 0; \qquad f(x,p) = f_0(p), \quad x \le 0$$

- matching at $x = 0$ (shock position) leads to the particle spectrum

$$f_0(p) \propto p^{-q}, \qquad q = 3r/(r-1), \qquad r = u_1/u_2$$

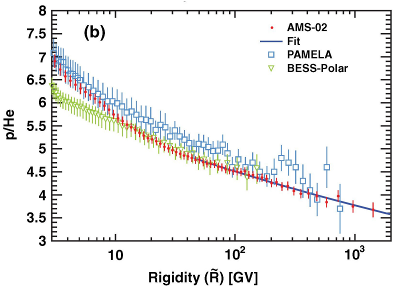
Krymsky 77, Blandford & Ostriker 78, Bell 78, Axford et al 78

Problems with simple test particle solution

1. Does not determine the normalization
   1. number of accelerated particles N_{CR} remains unknown
      1. So-called "injection problem": how and in what number are particles extracted from the thermal pool
   2. the level of turbulence driven by them remains unknown
      1. since DSA is a bootstrap process, acceleration rate, i.e. $p_{max}(t)$ depends on the scattering rate, that is on turbulence level
   3. particle backreaction on the shock structure is unknown
2. for high Mach numbers, typical for young SNRs ($M \sim 100$), $r = 4$, $q = 4$ →CR pressure diverges with p_{max}
3. high pressure of CR may totally change the shock structure, drive instabilities near the shock, change the CR confinement condition and the shock compression rate
   - in fact, it does!

- Bottom line: even the PL index is no longer determinate

New instruments make injection models testable



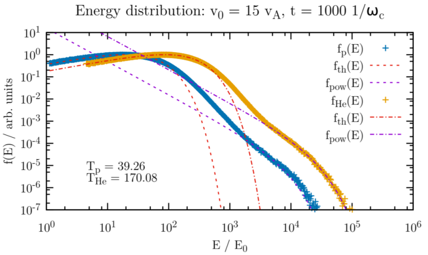
AMS-02 (2015) results along with earlier data

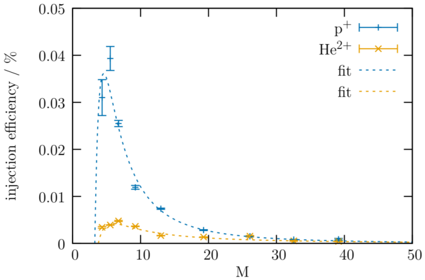
- strange result, at the first glimpse
- all elements with the same rigidity must have the same spectra under a steady state acceleration conditions

Key Observations and Disagreements with theory:

- Several instruments revealed deviation ≈ 0.1 in spectral index between He and p's (claimed inconsistent with DSA (e.g., Adriani et al 2011)
- DSA predicts a flat spectrum for the He/p ratio
- points to initial phase of acceleration where elemental similarity (rigidity dependence only) does not apply
- A/Z is the same for He and C

Validating Physical ideas by hybrid Simulations

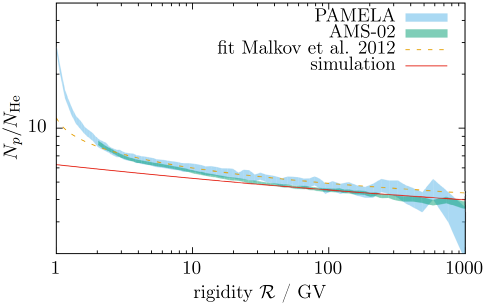




- 1D in configuration space, full velocity space simulations
  - shock propagates into ionized homogeneous plasma
- p and He are thermalized downstream according to Rankine-Hugoniot relations
- preferential injection of He into DSA for higher Mach numbers is evident
- injection dependence on Mach is close to theoretically predicted $\eta \sim M^{-1} \ln M$ (MM'98)

plots from A. Hanusch, T. Liseykina, MM, 2017

p/He ratio integrated over SNR life



p/He from A. Hanusch, T. Liseykina, MM, 2017

- p/He result automatically predicts the p/C ratio since the rest rigidity (A/Z) is the same for C and He

Some Conclusions

- the p/He ratio at $\mathcal{R} \gg 1$, is not affected by CR propagation, regardless the individual spectra
- telltale signs, intrinsic to the particle acceleration mechanism
- reproducible theoretically with no free parameters
- PIC and hybrid simulations confirm p and He injection scalings with Mach number Hanusch et al, ICRC 2017

Backreaction of accelerated CRs on shock

- DC equation with $u(x)$ to be determined self-consistently with $f(x,p)$:
$$u\frac{\partial f}{\partial x}+\kappa(p)\frac{\partial^2 f}{\partial x^2}=\frac{1}{3}\frac{du}{dx}p\frac{\partial f}{\partial p}, \tag{1}$$

- $f(x,p)=\langle f(x,\boldsymbol{p})\rangle$
- BC: $f\to 0,\ x\to\infty;\ f<\infty,\ f<\infty,\ x\to-\infty$
- CR diffusivity $\kappa(p)$ is of the Bohm type, $\kappa(p)=Kp^2/\sqrt{1+p^2}$ (p is normalized to mc, $\kappa\sim r_{\rm g}(p)$
- K depends on $\delta B/B$ of MHD turbulence that scatters the particles in pitch angle
- $K\sim mc^3/eB$ if $\delta B\sim B$.
- $x<0$ $\quad f(x,p)=f_0(p)\equiv f(0,p),\ \ u\equiv u_2$
- $x>0$: $\quad$ need to solve eq.(1) coupled with eq. for $u(x)$

Solving DC self-consistently with backreaction of CR

Introduce : $$P_{\rm CR}(x) = \frac{4\pi}{3} mc^2 \int_{p_0}^{p_1} \frac{p^4\, dp}{\sqrt{p^2+1}} f(p,x)$$

$$P_{\rm CR} + \rho u^2 = \rho_1 u_1^2, \qquad \rho u = \rho_1 u_1, \quad x > 0$$

$$u\frac{\partial f}{\partial x} + \kappa(p)\frac{\partial^2 f}{\partial x^2} = \frac{1}{3}\frac{du}{dx} p \frac{\partial f}{\partial p}, \qquad u(x) = u_1 - P_{CR}/\rho_1 u_1$$

$$r_{\rm s} \equiv \frac{u_0}{u_2} = \frac{\gamma+1}{\gamma - 1 + 2R^{\gamma+1} M^{-2}}, \qquad u_2 < u_0 \equiv u(x+0) < u_1 \tag{2}$$

precursor compression $R \equiv u_1/u_0$ and γ -the adiabatic index

- key substitution

$$f(x,p) = f_0(p) \exp\left[-\frac{q}{3\kappa}\Psi\right]$$

$$q(p) = -d\ln f_0/d\ln p, \qquad \Psi = \int_0^x u(x')dx'$$

Self-consistent solution of DC equation

- one-parameter (λ) family (MM '99)

$$f_0 = f_0\left(p_0\right)\left[1 + \frac{q\left(p_0\right)}{\lambda\kappa\left(p_0\right)} p_0^{-3/\lambda} \int_{p_0}^{p} \kappa\left(p'\right) p'^{3/\lambda-1} dp'\right]^{-\lambda}$$

- flow potential

$$\Psi\left(x\right) = \Psi_0^{-\lambda/(\lambda-1)} \left[\left(1-\lambda\right) u_0 x + \Psi_0\right]^{1/(1-\lambda)}$$

- λ=1/2 comes from the condition of pressure balance in the shock precursor $P_{CR} + \rho u^2 = const.$
- solution implicitly (trough Ψ_0) depends on p_1- maximum momentum (cut-off)
- tends to exact solution in the limit $p_1 \to \infty$ ($M = \infty$, as no thermal pressure), zeroth order term in $1/p_1$
- for this solution to exist $p_1 > 10^3$ (SNRs $p_1 > 10^6$)
- current hybrid simulations $p_1 \sim 1$ (e.g., Caprioli & Spitkovsky, 2017)

Integral Transform of DC: $u\frac{\partial f}{\partial x}+\kappa(p)\frac{\partial^2 f}{\partial x^2}=\frac{1}{3}\frac{du}{dx}p\frac{\partial f}{\partial p}$

- use self-similar solution to build a kernel of integral transform of DC

$$f(x,p)=f_0(p)\exp\left[-\frac{q}{3\kappa}\Psi\right],\quad \Psi=\int_0^x u(x')dx',\quad q(p)=-d\ln f_0/d\ln p$$

The integral transform is as follows (MM '97)

$$U(p)=\frac{1}{u_1}\int_{0-}^{\infty}\exp\left[-\frac{q(p)}{3\kappa(p)}\Psi\right]du(\Psi) \tag{3}$$

and it is related to $q(p)$ through

$$q(p)=\frac{d\ln U}{d\ln p}+\frac{3}{r_s RU(p)}+3 \tag{4}$$

$U(p)$ yields both the flow profile and particle distribution. Using the linearity of equation $P_{\rm CR}(x)+\rho u^2=\rho_1 u_1^2$ ($\rho u=const$),

- obtain an integral equation for U by applying transform (3)

$$U(t) = \frac{r_s-1}{Rr_s}+\frac{\nu}{Kp_0}\int_{t_0}^{t_1}dt'\left[\frac{1}{\kappa(t')}+\frac{q(t')}{\kappa(t)q(t)}\right]^{-1}\frac{U(t_0)}{U(t')}\exp\left[-\frac{3}{Rr_s}\int_{t_0}^{t'}\frac{dt''}{U(t'')}\right] \tag{5}$$

where $t=\ln p$, $t_{0,1}=\ln p_{0,1}$. Injection parameter ν

$$\nu = \frac{4\pi}{3}\frac{mc^2}{\rho_1 u_1^2}p_0^4 f_0(p_0)= Kp_0\left(1-R^{-1}\right)\left\{\int_{t_0}^{t_1}\kappa(t)dt\frac{U(t_0)}{U(t)}\exp\left[-\frac{3}{Rr_s}\int_{t_0}^{t}\frac{dt'}{U(t')}\right]\right\}^{-1} \tag{6}$$

Approximate solution of Integral Equation

- re-scaling, remapping, simplifications..., obtain $U \to F, \quad p \to \tau,$ $p_0 \to \epsilon \ll 1,\ p_1 \to \epsilon^{-1}$

$$F(\tau) = \int_{\epsilon}^{1/\epsilon} \frac{d\tau'}{\tau+\tau'} \frac{1}{\tau' F(\tau')} + A(R, r_s, \nu) \tag{7}$$

- expand in $\sqrt{\epsilon} \ll 1$: $F = F_0(\tau) + \ldots, \quad \tau + \epsilon \equiv y$:

$$F_0(y) = \int_0^{\infty} \frac{dy'}{y+y'} \frac{1}{y' F_0(y')}$$

- solution

$$F_0 = \sqrt{\pi/y}$$

- using the symmetry of eq.(7) $\tau \mapsto 1/\tau,\ F \mapsto \tau F\ (A=0)$
- using the branch points $\tau = \epsilon, \epsilon^{-1}$, restore the full solution

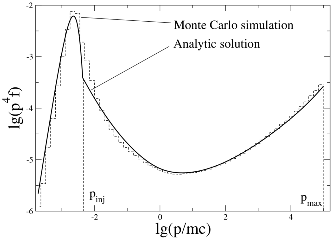
$$F = \sqrt{\frac{\pi}{(\tau+\epsilon)(1+\epsilon\tau)}}$$

Nonlinear Spectrum

- returning to physical variables, $p \gg p_0$ (simplified spectrum)

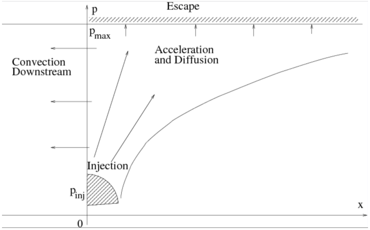
$$f_0(p) = \frac{C}{p^{7/2}} \sqrt{3q(p) + p/p_1}$$

- more accurately, from int. eq.:

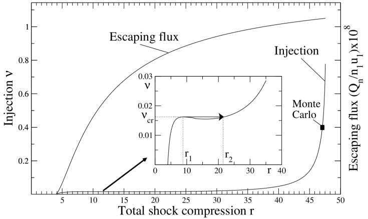


MC: Ellison & Berezhko '99
Anal. Sol.: MM '97

Phase space of acceleration

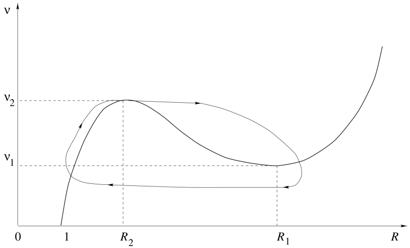


Escape flux and bifurcation of acceleration regime

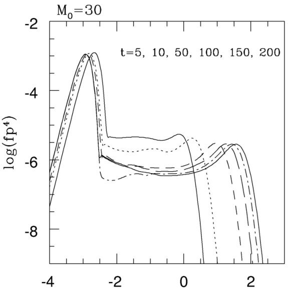


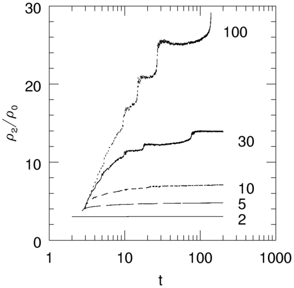
Limit Cycle Oscillations in CR Acc'n

- calculated bifurcation diagram
  (schematically from MM&L.Drury 2001)



- undergoes adiabatic deformation when $p_{max}(t)$ grows
- suggests hysteresis and limit-cycle oscillations in course of acceleration
- such LCO's have indeed been observed in numerical modeling of acceleration by Kang and Jones 2002

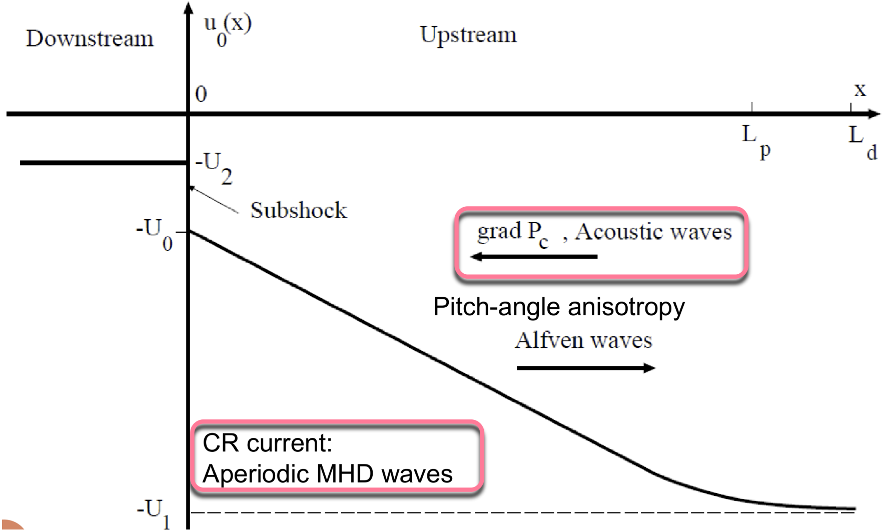




- long, $\kappa\left(p_{max}\right)/U_{shock}$ CR precursor with velocity and CR pressure gradients
- CR modified shock constitutes an NL front propagating into weakly turbulent ISM
- the background ISM turbulence does not provide enough CR scattering to accelerate them to appreciable energy
- particles need to create waves by themselves
  - Bootstrap acceleration
- most relevant instabilities
  - resonant ion-cyclotron instability of CRs in shock precursor
  - nonresonant aperiodic (Bell's) instability driven by the return current of CRs
  - acoustic (Drury's) instability driven by the CR pressure gradient
- large scale magnetic field generation
- plasma heating in CR precursor

(last two items poorly understood)

Instabilities, important for particle transport in CRP



MHD Equations

$$\frac{d\rho}{dt} + \rho \frac{\partial}{\partial x} U_x = 0$$

$$\frac{d\mathbf{B}_\perp}{dt} = B_0 \frac{\partial \mathbf{U}_\perp}{\partial x} - \mathbf{B}_\perp \frac{\partial U_x}{\partial x}$$

$$\frac{dU_x}{dt} = -\frac{1}{\rho}\frac{\partial}{\partial x}\left(P_c + P_g + \frac{B_\perp^2}{8\pi}\right)$$

$$\frac{d\mathbf{U}_\perp}{dt} = \frac{B_0}{4\pi\rho}\frac{\partial \mathbf{B}_\perp}{\partial x} + \frac{J_c}{c\rho}\mathbf{n} \times \mathbf{B}_\perp$$

$$d/dt \equiv \partial/\partial t + U_x \partial/\partial x$$

Lagrangian variable

$$d\xi = \frac{\rho}{\rho_0}(dx - U_x dt)$$

MHD reduction

$$\frac{\partial^2 b}{\partial t^2} - C_A^2 \frac{\partial^2}{\partial \xi^2} \frac{\rho}{\rho_0} b = \frac{i}{c\rho_0} B_0 J_c \frac{\partial b}{\partial \xi}$$

$$\frac{\partial^2}{\partial t^2} \frac{\rho_0^2}{\rho} + \frac{\partial^2}{\partial \xi^2} \left(P_g + \frac{|b|^2}{8\pi} \rho^2 \right) = -\frac{\partial^2}{\partial \xi^2} P_c$$

Instability

$$b = \frac{1}{\rho} (B_y + iB_z) \qquad C_A^2 = \frac{B_0^2}{4\pi\rho_0}$$

$$P_c = \frac{4\pi}{3} mc^2 \int \frac{p^4}{\sqrt{1+p^2}} f dp \qquad J_c = eun_c = 4\pi eu \int p^2 f dp$$

Linear vs. nonlinear regime (sh. precursor)

Two-way balance $\longrightarrow$ $f = f_0(p)\exp\left[\frac{1}{\kappa(p)}\phi(x)\right], \quad x \geq 0$

$$\boxed{\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} - \kappa(p)\frac{\partial^2 f}{\partial x^2} = \frac{p}{3}\frac{\partial u}{\partial x}\frac{\partial f}{\partial p}}$$

Three-way balance $\longrightarrow$ $f = f_0(p)\exp\left[\frac{q(p)}{3\kappa(p)}\phi(x)\right], \quad x \geq 0$

$u = \partial\phi/\partial x \qquad q(p) = -\partial \ln f_0/\partial \ln p$

9

q is well constrained: 3.5 < q < 4 (< q_sub, close to the sub-shock)

Instabilities (summary)

❑ Resonant, anisotropy-driven Alfven waves $k \sim r_g^{-1}(p)$
(use as a seed for the next two)

→Bell '78

✓ Non-resonant, CR-current driven (Bell's) instability

→Achterberg '83, Shapiro and Quest '98,
Bell and Lucek '01, 04, Reville et al 08, Bykov et al 08

→ Bell '04 particularly complete characterization

(short-scale, needs an inverse cascade or extreme amplification to confine particles)

✓ Hydrodynamic, CR-pressure-gradient-driven (Drury's) instability

→Drury 84, Drury and Falle 86, Zank et al 90, Kang et al 92…

Advantages:

- drive all wave numbers, $\gamma(k) \approx const$
- insensitive to CR distribution function
- staibilizes only nonlinearly (not quasi-linearly)
- long scales, much needed for particle confinement are naturally produced → Diamond and MM 07

Linear Theory: density vs. magnetic NR perturbations

$$\boxed{\begin{aligned} \omega^2 &= k^2C_A^2 && -2\gamma_B k C_A \\ \omega^2 &= k^2C_s^2 && -2i\gamma_D k C_S \end{aligned}}$$

$$\gamma_B = \sqrt{\frac{\pi}{\rho_0}} J_c/c$$

$$\gamma_D = -\frac{1}{2\rho_0 C_S}\frac{\partial \overline{P}_c}{\partial x}$$

Compare the growthrates:

$$\frac{\gamma_D}{\gamma_B} = \frac{C_A}{C_s}\frac{c^2}{3\omega_{ci}}\left\langle \frac{p^2}{\sqrt{1+p^2}}\frac{q}{3\kappa}\right\rangle$$

$$q/3 \to 1$$

In TP acceleration regime

$$\langle\cdot\rangle \equiv \frac{\int(\cdot)\, f_0(p)\, p^2 \exp(q\phi/3\kappa)\, dp}{\int f_0(p)\, p^2 \exp(q\phi/3\kappa)\, dp}$$

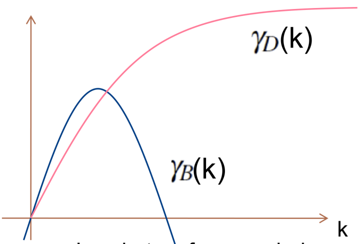
Comparing density (Drury) unstable mode with B-field (Bell) mode

Bohm diffusion $\kappa = r_g(p)\, v/3$

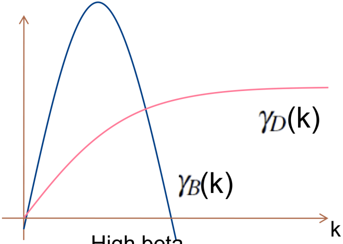
$$\boxed{\frac{\gamma_D}{\gamma_B} = \left\langle \frac{q}{3} \right\rangle \frac{C_A}{C_s}}$$

$q/3 \to 1$

In TP acceleration regime



Low beta: focus on below



High beta

Weakly nonlinear theory (acoustic)

Instability driver

CR linear response

$$\left(\frac{\partial}{\partial t} - c_s\frac{\partial}{\partial \xi}\right)\left(\frac{\partial}{\partial t} + c_s\frac{\partial}{\partial \xi}\right)\tilde{\rho} = -\frac{1}{\rho_0}\frac{\partial \bar{P}_c}{\partial \xi}\frac{\partial \tilde{\rho}}{\partial \xi} + \frac{\partial^2 \tilde{P}_c}{\partial \xi^2} + c_s^2\frac{\gamma_g - 2}{2\rho_0}\frac{\partial^2 \tilde{\rho}^2}{\partial \xi^2}$$
$$- \frac{\partial}{\partial \xi}\left(2\tilde{u}\frac{\partial \tilde{\rho}}{\partial t} + \tilde{\rho}\frac{\partial \tilde{u}}{\partial t}\right) - 2\mu\rho_0\frac{\partial^3 \tilde{u}}{\partial \xi^3}$$

Use Lagrangian coordinate: $d\xi = dx - u_0(x)\, dt$

→ Burgers eq.

$$\frac{\partial \tilde{\rho}}{\partial t} - c_s\frac{\partial \tilde{\rho}}{\partial \xi} - \frac{\gamma_g + 1}{2\rho_0}c_s\tilde{\rho}\frac{\partial \tilde{\rho}}{\partial \xi} - \mu\frac{\partial^2 \tilde{\rho}}{\partial \xi^2} = \gamma\tilde{\rho}$$

where the acoustic instability growth rate is

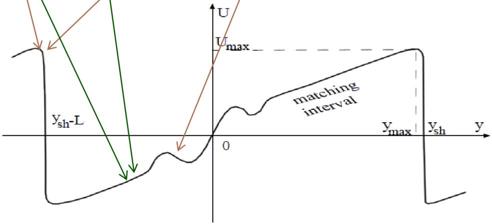
$$\gamma = -\frac{1}{2\rho_0 c_s}\frac{\partial \bar{P}_c}{\partial \xi} - \frac{\bar{q}}{18}\frac{mc^2 p_*}{\kappa_*}\frac{n_c(x)}{\rho_0}$$

Traveling wave solution driven by acoustic and cyclotron instabilities

$$\frac{\partial \hat{\rho}}{\partial t} + \hat{\rho}\frac{\partial \hat{\rho}}{\partial \zeta} - \gamma\hat{\rho} - \mu\frac{\partial^2 \hat{\rho}}{\partial \zeta^2} = Q(\zeta - vt)$$

Resonant Ion-cyclotron Instability (seed) term

balance

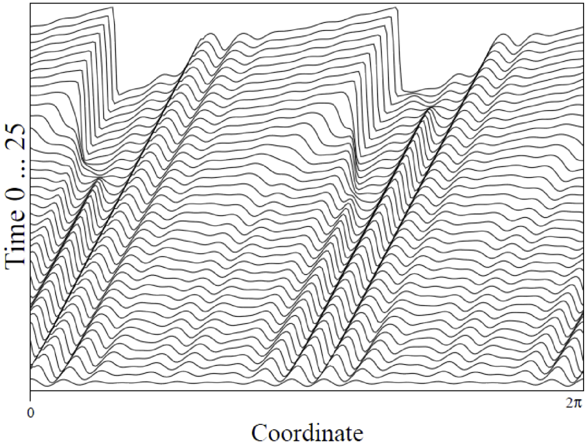


More general, 'magnetic' versions of this solution but with a cyclotron-unstable (no acoustic instability term) are also available

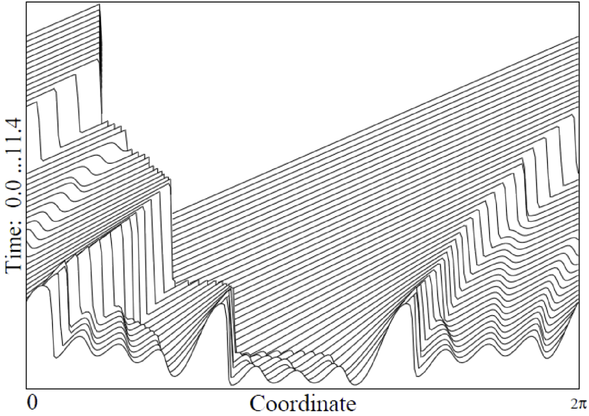
→ Kennel et al JETP Let. '88,

→ MM et al Phys. Fluids '90

Numerical verification of the traveling wave solution (acoustic instability +IC inst

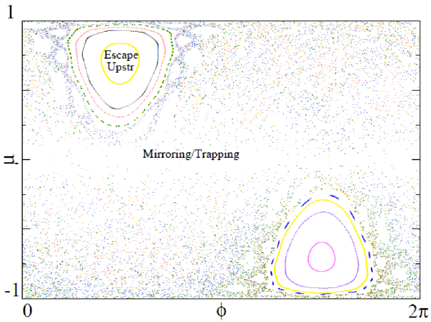


Numerical verification of the traveling wave solution (acoustic instability only)



Initial perturbation profile steepens into 3 relatively weak shocks
They merge to form one strong shock

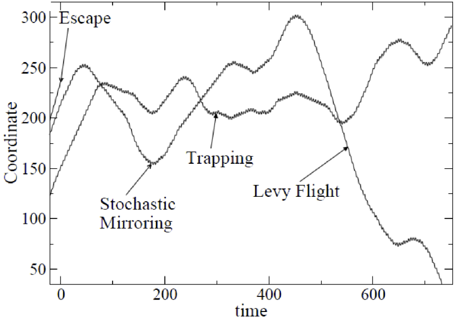
Particle dynamics in shock train



Pitch-angle/Gyro-phase
Poincare map
(Pitch-angle wrt shock normal, 45 deg here)

$r_g(p)/L \simeq 3$

Particle trajectories



Particle spectrum: change of confinement Regime leads to formation of a spectral break

For particles with momentum below the break at p_* the spectrum should be determined from nonlinear self-consistent solution of kinetic and HD equations.

Above the break at $p=p_*$ --no significant contribution of those particles to the CR pressure

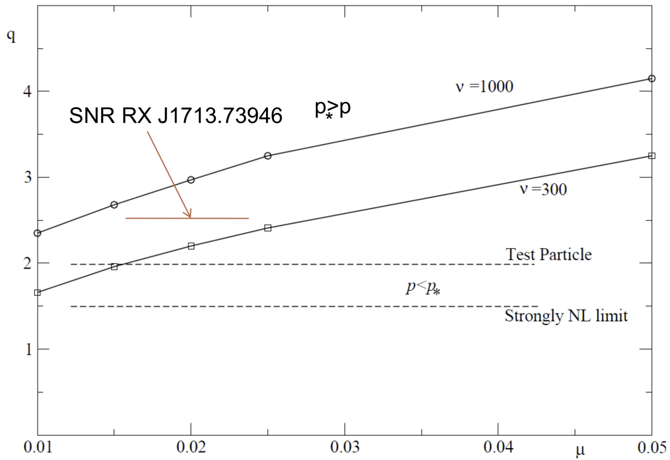
Fermi '49 general spectral index $\boxed{q = 3 + \tau_{acc}/\tau_{conf}}$

$$q_e = 3 + \frac{\ln\left[(U_+/U_0)\left(\mathscr{P}_L^2/\mathscr{P}_{tr}\right)\right]}{K(\vartheta)\ln\left(1/\mathscr{P}_L\right)}$$

$\mathscr{P}_{tr}$ Trapping probability

$\mathscr{P}_L$ Detrapping probability (Levy flight)

Softening of the spectrum



$$\nu = \frac{U_+\beta}{U_0\alpha_+} \simeq \frac{U_+}{U_0}\frac{\tau_L\tau_L^+}{\tau_{tr}^2} \sim \frac{c}{U_0} \gg 1$$

$$\mu \equiv \frac{\beta\kappa_0}{2\pi\eta^2 p_0 p_*(U_0 - U_2)^2}.$$

CR current driven Breathers

$$\frac{\partial^2}{\partial t^2}\frac{B}{\rho} - C_A^2\frac{\partial^2}{\partial \xi^2}\frac{B}{\rho_0} = \frac{i}{c\rho_0}B_0 J\frac{\partial}{\partial \xi}\frac{B}{\rho} \tag{8}$$

$$\frac{\partial^2}{\partial t^2}\frac{\rho_0^2}{\rho} + \frac{\partial^2}{\partial \xi^2}\frac{|B|^2}{8\pi} = 0, \tag{9}$$

$$B = B_y + iB_z \text{ and } C_A^2 = \frac{B_0^2}{4\pi\rho_0}.$$

- traveling wave solution

$$B = B_{\max} v(\zeta) e^{-i\omega t}, \qquad \rho = \rho(\zeta) \tag{10}$$

where $\zeta = \xi - Ct$, C is the (constant) propagation speed

$$\frac{\rho_0}{\rho} = 1 - \frac{|B|^2}{B_{\max}^2} \tag{11}$$

where $B_{\max}^2 \equiv 8\pi\rho_0 C^2$

Solution

$$\frac{\partial^2}{\partial\zeta^2}\left(a-|v|^2\right)v - iK\frac{\partial}{\partial\zeta}\left(1-|v|^2\right)v - \frac{\omega^2}{C^2}\left(1-|v|^2\right)v = 0. \tag{12}$$

with notation

$$K = \frac{B_0 J}{c\rho_0 C^2} - 2\frac{\omega}{C}, \quad a = 1 - 2\frac{B_0^2}{B_{max}^2} = 1 - \frac{C_A^2}{C^2}, \tag{13}$$

$C_A^2 = B_0^2/4\pi\rho_0$ The linear dispersion $v(\zeta) \propto e^{ik\zeta}$, $v \to 0$ in eq.(12):

$$\omega = kC \pm \sqrt{k^2 C_A^2 + B_0 Jk/c\rho_0}. \tag{14}$$

- Substituting $v(\zeta) = \sqrt{w}e^{i\Theta}$, obtain 1-st integral ($s = K\zeta/2$)

$$\left(\frac{dw}{ds}\right)^2 - \frac{w^2}{(3w-a)^2(a-w)^2}\sum_{n=0}^{4} C_n w^n = 0 \tag{15}$$

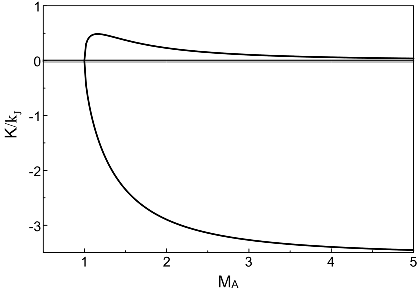
- passing through two singularities, obtain a closed-form solution (cumbersome)
- small-amplitude limit

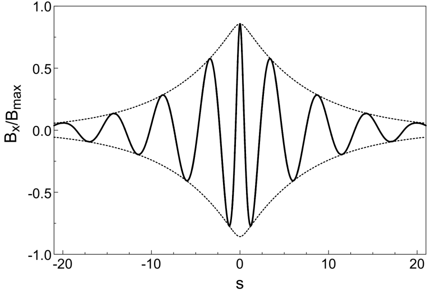
$$w(s) = w_0 \cosh^{-2}\left(\frac{\sqrt{C_0}}{2a^2}s\right)$$

NL dispersion relation

$$\omega = \frac{k_J C}{M_A^2 \left(1 \pm \sqrt{\left(1 - M_A^{-2}\right) / \left(1 + 1/8M_A^2\right)}\right)},$$

- $k_J = 2\pi J / cB_0$
- Strong solitons with $M_A \equiv C/C_A \gg 1$





Conclusions and Outlook

- observational basis of the CR research is rapidly improving
- DSA theory accounts for most observations of the main (proton) CR energy spectrum
- however, some aspects need further studies
  - no consensus as to what maximum energy achievable in SNRs
    - estimates range from $10^{14} eV$ to $10^{16} eV$ and even higher (often backed off, though)
  - reason: lack of understanding of magnetic field generation of sufficiently large scale in CR shock precursor
- chemical composition remains partly controversial
  - observations are rapidly improving on e^+, He, C, N, O, Be, ...
  - largely by the new instrument AMS-02 on board ISS
  - theoretical work is ongoing
- CR loaded shocks – excellent versatile laboratories for plasma physics and NL wave studies