3D Radiative Transfer

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### Outline Part I

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   Connecting heterogeneous resources
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   - compute jobs
   - queries
   - telescopes
   - work flows
\[ \int_0^\infty \frac{d\lambda}{\pi} \kappa_{abs}(\lambda, \vec{x}) \int d\vec{n}' I(\lambda, \vec{x}) \]
# Introduction: Radiation Transfer in the Initial Star Formation Phase

<table>
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<tr>
<th>Time [yrs]</th>
<th>10^3</th>
<th>10^4</th>
<th>10^5</th>
<th>10^6</th>
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- **Magnetic fields?**
- **Turbulence?**
- **1D/2D/3D?**
- **Fragmentation**
- **External Initialization?**
- **Dust**
- **Cooling**
- **Chemistry**
- **Molecular Cloud Cores**
- **Radiation Transfer**

- **Solar-type**

Globules IC2944, HST
Introduction: Radiative Transfer in the Circumstellar Disk Phase

- Self-Gravity
- Circumstellar Disks
- Magnetic Fields
- Jets
- Multiple Systems
- Radiation Transfer
- Angular Momentum Transport
- Dust Coagulation
- Chemistry

Time [yrs]: $10^3$, $10^4$, $10^5$, $10^6$, $10^7$

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HH30, HST WFPC2
Introduction: Radiative Transfer in the Planet Formation Phase

Radiation Transfer

Chemistry

Dust Growth

Formation of Planetesimal Bodies

Disappearance of the Disk

Solar-type

| Time [yrs] | $10^3$ | $10^4$ | $10^5$ | $10^6$ | $10^7$ |
(Magneto-) Hydro Dynamics

velocity field
\[ \vec{v} (x, y, z, t) \]

Gravity: coupling all particles
Radiative Transfer

Interaction of all cells

Intensity I(\lambda,x,y,z,\theta,\phi,t)
Radiative processes

- Absorption, $\tau > 1$ opt. thick
- Emission
- Scattering

Wavelength range:
- 0.1$\mu$m
- 1$\mu$m
- 10$\mu$m
- 100$\mu$m
- 1mm

Stokes vector

Intensity

Anisotropic
3D Transport Equation for Continuum Radiation

\[ \vec{n} \nabla_x I(\lambda, \vec{x}, \vec{n}) = -\kappa_{ext}(\lambda, \vec{x}) I(\lambda, \vec{x}, \vec{n}) \]

change of intensity \( I \)
along a direction \( \vec{n} \)

extinction by
dust particles

1.-order partial differential equation, solution vector \( \sim 10 \text{ GB} \)
3D Transport Equation for Continuum Radiation

\[
\overrightarrow{n} \nabla_x I(\lambda, \overrightarrow{x}, \overrightarrow{n}) = -\kappa_{ext}(\lambda, \overrightarrow{x}) I(\lambda, \overrightarrow{x}, \overrightarrow{n}) \\
+ \kappa_{abs}(\lambda, \overrightarrow{x}) B[\lambda, T(\overrightarrow{x})]
\]

re-emission
by dust particles
3D Transport Equation for Continuum Radiation

\[ \vec{n} \nabla_x I(\lambda, \vec{x}, \vec{n}) = -\kappa_{ext}(\lambda, \vec{x}) \ I(\lambda, \vec{x}, \vec{n}) \]

\[ + \kappa_{abs}(\lambda, \vec{x}) \ B[\lambda, T(\vec{x})] \]

\[ \int_0^\infty d\lambda \ \kappa_{abs}(\lambda, \vec{x}) \ B[\lambda, T(\vec{x})] = \]

\[ \frac{1}{4\pi} \int_0^\infty d\lambda \ \kappa_{abs}(\lambda, \vec{x}) \int d\vec{n}' \ I(\lambda, \vec{x}, \vec{n}') \]

all absorbed energy density

local energy density equilibrium
3D Transport Equation for Continuum Radiation

\[ \mathbf{n} \nabla_x I(\lambda, \vec{x}, \vec{n}) = -\kappa_{ext}(\lambda, \vec{x}) \ I(\lambda, \vec{x}, \vec{n}) \]
\[ + \kappa_{abs}(\lambda, \vec{x}) \ B[\lambda, T(\vec{x})] \]

local energy density equilibrium

all re-emitted energy density

\[ \int_0^\infty d\lambda \ \kappa_{abs}(\lambda, \vec{x}) \ B[\lambda, T(\vec{x})] = \]
\[ \frac{1}{4\pi} \int_0^\infty d\lambda \ \kappa_{abs}(\lambda, \vec{x}) \int d\vec{n}' \ I(\lambda, \vec{x}, \vec{n}') \]
3D Transport Equation for Continuum Radiation

\[ \vec{n} \nabla_x I(\lambda, \vec{x}, \vec{n}) = -\kappa_{\text{ext}}(\lambda, \vec{x}) \ I(\lambda, \vec{x}, \vec{n}) \]

\[ + \kappa_{\text{abs}}(\lambda, \vec{x}) \ B[\lambda, T(\vec{x})] \]

\[ + \frac{\kappa_{\text{sca}}(\lambda, \vec{x})}{4\pi} \int d\vec{n}' \ p(\lambda, \vec{n}, \vec{n}') \ I(\lambda, \vec{x}, \vec{n}') \]

scattering in the considered direction \( \vec{n} \)

phase function

\[ \int_0^\infty d\lambda \ \kappa_{\text{abs}}(\lambda, \vec{x}) \ B[\lambda, T(\vec{x})] = \]

\[ \frac{1}{4\pi} \int_0^\infty d\lambda \ \kappa_{\text{abs}}(\lambda, \vec{x}) \int d\vec{n}' \ I(\lambda, \vec{x}, \vec{n}') \]
Isotropic or anisotropic scattering?

\[
\frac{\kappa_{\text{Scal}}(\lambda, \vec{x})}{4\pi} + \frac{1}{4\pi} \int d\vec{n}' p(\lambda, \vec{n}, \vec{n}') I(\lambda, \vec{x}, \vec{n}')
\]

Phase function (polar coordinates)

- Dust particle size: 0.12 \(\mu\)m
The most-used radiative transfer approximation in MHD: 
Flux-limited Diffusion

Radiative transfer equation into moment equations

\[ J_\nu = \frac{1}{4\pi} \int I_\nu \, d\Omega \quad H_\nu = \frac{1}{4\pi} \int I_\nu \hat{s} \, d\Omega. \quad \text{etc.} \]

Diffusion (optically thick):
\[ H \sim D \text{ grad } J \]

"Vacuum" (optically thin):
\[ H \sim J \]

closure equation:

Levermore & Pomraning 1981:
Create a closure relation fitting both limiting cases
How to solve the radiative transfer equation?

- Ray-Tracing
- Monte-Carlo
- Grid

Others:
- Moment
- Spectral
- Multi-Grid
- Finite Volume
- ...

Various approaches to improve the solution methods:
- Unstructured Grids
- Beam splitting
- Photon splitting
- ...

Solver overview
How to solve the radiative transfer equation?

Monte-Carlo

Not a solution but a simulation technique:
Follow a photon interaction at $\ell$

• Calculates interaction precisely (polarization)
• Flexible technique for complicated structures
• Code development within one PhD lifetime
• No explicit error control
• Problem with low and high optical depth
Task: Resolution in Monte Carlo Simulations

3a: How many photons have to be send to this location to cover all possible directions (isotropc scattering) with a mean resolution of about 10 degrees?

$$18 \times 36 = 648 \ (500)$$
Task: Resolution in Monte Carlo Simulations

3b: How many photons are needed to cover the direction space after 6 scattering events with a mean resolution of about 10 degrees?
Task: Resolution in Monte Carlo Simulations

3b: How many photons are needed to cover the direction space after 6 scattering events with a mean resolution of about 10 degrees?

$1.7 \times 10^{15}$
3c: During each interaction, $\frac{1}{2}$ of the radiation is absorbed and distributed over the unit sphere. What is left of the intensity after 6 interactions?
Task: Resolution in Monte Carlo Simulations

\[ \lambda \]

mean free path \( \lambda \)

scattering event

3c: During each interaction, \( \frac{1}{2} \) of the radiation is absorbed and distributed over the unit sphere. What is left of the intensity after 6 interactions?

\[ 4 \times 10^{-9} \]
How to solve the radiative transfer equation?

Grid

- Technique: Solve equation on grid recursively

- Good portion can be precalculated (including errors)
- Flexible technique for complicated structures
- Numerical diffusion
- Interpolation errors
- Coarse direction resolution
Adaptive grids for radiative transfer

Adapt to what?

\[ \tau = \kappa \Delta x \ll 1 \]

Multi-wavelengths grids
Adaptive grids for radiative transfer

3. Solution strategies: grid solver

Steinacker, Bacmann, Henning 2002
Steinacker, Hackert, Steinacker, Bacmann 2002
How to solve the radiative transfer equation?

Ray-Tracing

- Follow a ray through the domain calculating the change in I from A to B

- Very accurate (e.g. high-order Runge Kutta)
- Allows for adaptive step size control (error control)
- Often calls the density function
- Problems with high optical depth
0. Create AMR grid
1. Propagate the source energy

Boundary Conditions:
- Stars
- IS Radiation field

Store per cell per $\lambda$:
- absorbed energy
- scattered $I$
- scattering direct.
- $\lambda$
2. Propagate the scattered radiation

Per cell per $\lambda$:
- follow rays

Store per cell per $\lambda$:
- absorbed energy
- scattered $I$
- scattering direct.
- $\lambda$

$I$ drops with each scattering iteration:
- energy distributed over directions
- absorption takes away energy

Solvers: Ray-tracing on AMR grids
3. Determine start temperature

Re-emitted hot dust radiation

$\lambda > 3 \, \mu m$

→ neglect scattering
4. Switch to MIR grid

No big error: Interpolation from fine to coarse grid
5. Determine temperature

Cell-cell illumination

for N cells: $N^2N/2$ steps
Ray-tracing at high optical depth

optically thin

\[ \tau \sim 10^6 \]

optically thick

Ray-tracer is using diffusion limit

speed-up by factor 800

Jürgen Steinacker

Steinacker, Bacmann, Henning 2006
2D SED Benchmark

calculated with five state-of-the-art European continuum radiative transfer codes (Monte-Carlo & grid codes)

Spectral Energy Density: Star and Circumstellar Disk

Power Density [Wm$^{-2}$] vs. wavelength $\lambda$ [µm] for phase-on and edge-on orientations.
Line transfer compared to continuum transfer

- Derivative with respect to $\lambda$
- Solve for the level population of the considered atom/molecule

The step to line radiative transfer
Fits incorporating many parameter

How to find the minimum of the $\chi^2$?

How to cover the entire parameter space?

Simulated Annealing

Cubature of the sphere

$\text{p} = e^{-1/T}$
Do you believe in a universal fluid force?

Large group of fanatics:
The sect of fluid believers

- hydro stars
- hydro planet cores
- hydro molecular clouds
- hydro winds
- hydro air
- hydro accretion disks
- hydro beams/jets
Beyond the fluid world ...
Beyond the fluid world ...
Particle Distribution Function

Kinetic theory

Where is each particle in phase space?
particle distribution

\[ f(x_1, v_1, x_2, v_2, \ldots, x_N, v_N, t) \]

large number of particles N

How many particles are in a certain phase space element?

\[ f(\mathbf{x}, \mathbf{v}, t) \]
We need an equation. 

like: particle number constant over time 

\[
\frac{df}{dt} = 0
\]

\[
df = \frac{\partial f}{\partial t} \, dt + \frac{\partial f}{\partial x_1} \, dx_1 + \ldots + \frac{\partial f}{\partial v_3} \, dv_3
\]

Particle distribution function: 

\[
f(\vec{x}, \vec{v}, t)
\]

\[
d\vec{v} = \frac{q}{m} \left( \vec{E} + \frac{1}{c} \vec{v} \times \vec{B} \right) \, dt
\]

Lorentz force

\[
\vec{E} = \vec{E}_0 + \Delta \vec{E}
\]

"Boltzmann equation"
Moments of the Boltzmann equation

integrate over $d^3v$: \text{continuity equation}

\[ *mv, \text{ integrate over } d^3v: \text{ momentum equation} \]

\[ *mv^2, \text{ integrate over } d^3v: \text{ energy equation} \]

sum over all particles

because collisions are keeping them together

charge density = 0

equation of state: incompressible/isothermal/adiabatic
Electrons
Protons
Ions
thermal to
highly relativistic

\[ \omega = k v \cos \theta + n \Omega / \gamma \]

"Co-moving
gyro-resonance"

When \( f(\theta) \) gets anisotropic?

Alfvén-, Whistler-, ion cyclotron waves

turbulence spectrum
decay of plasma waves

Accelerations of high-energy solar particles

Steinacker, Schlickeiser, Dröge 1989
Schlickeiser, Steinacker 1989
Steinacker, Schlickeiser 1989
Bech, Steinacker, Schlickeiser 1990
Achatz, Steinacker, Schlickeiser 1991
Steinacker, Miller 1992
Miller, Steinacker 1992
Steinacker, Jaekel, Schlickeiser 1993
Dispersion relation

Warm p-e-He-plasma:

\[
\begin{align*}
\{[k^2c^2 \cos^2 \theta - \omega_c^2 K_1][k^2c^2 - \omega_c^2(K_0 + K_1)] + \omega_c^2 K_2^2\} \omega_c^2 K_3 \\
+ k^2c^2 \sin^2 \theta \{[k^2c^2 - \omega_c^2(K_0 + K_1)]\omega_c^2 K_1 - \omega_c^4 K_2^2\} \\
+ \omega_c^2 K_4[k^2c^2 - \omega_c^2 K_5]\{[k^2c^2 \cos \theta] \omega_c^2 K_2 \omega_c^2 \sin \theta \}
\end{align*}
\]

Numerics:
Muller method with extreme accuracy:
Each number has to be stored with 1000 digits
Image construction
Forward 3D Radiative Transfer

density image
Evolution of a collapsing low-mass cloud core

Smoothed Particle Hydrodynamics

Time after start of turbulence decay [1000 years]
Produce images of the core evolution

Wavelength

7 µm

15 µm

170 µm

1.3 mm

Time

56 141 169 244 272
3D HD collapse of a massive molecular cloud core

Krumholz, Klein & McKee 2006

ORION MHD AMR code with flux-limited diffusion

size 1000 AU
Resolving the inner part of a massive core at a distance of 2 kpc

1.2 mm image of entire domain
resolution 0.6 arcsec

inner pixel:
size 1000 AU
Density re-construction
Inverse 3D Radiative Transfer

density images
ρ Ophiuchi

Molecular cloud core D

ISOCAM 7 and 15 μm

IRAM 30m 1.3 mm

MIR: in absorption

mm: in emission
Step 1: Determine the column density

Interpolate background intensity $I_b$
Step 1: Determine the column density

Use 30 Gaussian distributions to model the density

\[ I(x,y) = I_b(x,y) e^{-\sigma(x,y) N(x,y)} \]

150 parameter
1430 pixels to fit
(simulated annealing)
Step 1: Determine the column density

optical depth $\tau = \sigma N$
Step 2: Fitting the mm map

\[ I \sim \int_{-\infty}^{+\infty} dx' \ n(x') \ B[T(x')] \]
Image modeling: full 3D info

Cloud core ρ Oph D density structure

IRAM 30m 1.3 mm

ISOCAM 7 and 15 µm

MIR: in absorption

mm: in emission
Internal structure of a cold dark molecular cloud inferred from the extinction of background starlight

Alves, Lada, & Lada

*European Southern Observatory, Karl-Schwarzschild Straße 2, D-85748 Garching b. München, Germany
†Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, Massachusetts 02138, USA
‡Astronomy Department, University of Florida, Gainesville, Florida 32608, USA

Stars and planets form within dark molecular clouds, but little is understood about the internal structure of these clouds, and consequently about the initial conditions that give rise to star and planet formation. The clouds are primarily composed of molecular hydrogen, which is virtually inaccessible to direct observation. But the clouds also contain dust, which is well mixed with the gas and which has well understood effects on the transmission of light. Here we use sensitive near-infrared measurements of the light from background stars as it is absorbed and scattered by trace amounts of dust to probe the internal structure of the dark cloud Barnard 68 with unprecedented detail. We find the cloud’s density structure to be very well described by the equations for a pressure-confined, self-gravitating isothermal sphere that is critically stable according to the Bonnor–Ebert criteria. As a result we can precisely specify the physical conditions inside a dark cloud on the verge of collapse to form a star.

Molecular clouds are primarily composed of molecular hydrogen mixed with trace impurities including interstellar dust grains and rare organic and inorganic molecules. Because of its symmetric structure, the dipole moment of the molecule is zero. This implies that the molecule is homotopic, that is, it can reversibly pass through a circular shape by means of a conformational change without breaking any bonds. The molecule is therefore capable of associating into infinite complexes of any degree, and this property has important implications for the formation of macromolecules.
Low-mass star formation site
ρ Ophiuchi

Low-mass star formation site
ρ Ophiuchi

Low-mass star formation site
Dense gas cores in ρ Ophiuchi

ρ Ophiuchi
Dense gas cores in ρ Ophiuchi

Dense gas cores
Dense gas cores in ρ Ophiuchi
Bok globule Barnard 68
The standard core: B68

Bok globule Barnard 68

10000 AU
Very good fit by a "Bonnor-Ebert" sphere: Isothermal gas sphere balanced by thermal pressure and gravitation.

column density profile assuming 1D

\[ \xi_{\text{max}} = 6.9 \pm 0.2 \]
Fits of B68

observations

Av map Alves et al. 2001

850 µm SCUBA
Bianchi et al. 2003

1.2 mm SIMBA
Bianchi et al. 2003

model
B68 is not a sphere

R = 4500 AU

head

20

R = 9000 AU

tail

B68 is not a sphere
B68 is close to gravo-thermal balance

Lane-Emden equation

\[
\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\rho}{dr} \right) = -\frac{4\pi G m \rho}{kT}
\]

\[
V = \frac{\frac{4\pi G m \rho}{kT}}{\frac{d}{dr} \left( r^2 \frac{d\rho}{dr} \right)}
\]
Determination of kinematical properties

observers view

"dark side" view
Are stars universally formed by an accretion disk?

Final stellar mass [$M_{\text{sun}}$]

- Ultracompact HII regions
- Optically obscured
- Disks found
- Young massive stars
- Main Sequence stars
- T Tauri stars
- Herbig Ae/Be stars

$t = 0$: first hydrostatic core forms

Evolutionary time [yrs]

- 10000
- 1 million
- 10^8
- 10^10

Optically obscured

after H. Yorke
Influence of radiation pressure

How to form a 50 solar masses star?

Wolfire & Cassinelli 1987 using 1D calculations:
The should be no high-mass star above 10 solar masses!
2D models: $33 \, M_{\text{sol}}$ stars can form from accretion disks

Yorke & Sonnhalter 2002

Wavelength-dependent Radiative Transfer!
M17
Image modeling: SO-1
Inverse Radiative Transfer: A massive disk around a massive star

low mass disk

~ 8000 AU

Chini et al. Nature 2004
ISAAC/VLT 2.2 µm
Model: A disk around a young massive object

- Use the high-resolution of the VLT image (60AU/pixel)
- Use the rare configuration of a silhouette structure

**Jet**

**Hour glass-shaped**

**Reflection nebula**

**Accretion signatures**

**Symmetric pattern**

**Counter jet symmetric if edge-on disk**

**(Kepler) rotation**

Massive disk candidate model
low mass disk

~ 8000 AU

Diskmass: 0.2-13 solar masses

Steinacker et al. 2006
Given both star and disk are massive:

Is such a disk stable?
Central region resolved down to 240 AU

Extinction map from Spitzer as well as H2 emission at the disk surface indicate B4 star
UC1 – A *hyper*-compact HII region

Scattered light image modeling
A remnant disk around a massive star (B0)

ISAAC NTT 3.8 μm
TIMMI2 ESO3.6 10 μm

26 solar masses
Pa α and He I 10314 line: B0V to B1V

10000 AU
Indication for an inner cavity

Calculated radial density profile in the disk

IRS 15
Chini et al. 2006

Observed radial flux profile in the disk
Growing evidence for disk-like structures around Young Massive Stars
3D modeling of the dust distribution of the galaxy M33

Multi-λ image modeling

Input: dust model

Output: 3D dust distribution stellar population

collaboration with:
J. Braine (Bordeaux)
E. Athanassoula (Marseille)
German Astronomy Grid of compute and data resources

Submit a data query, a compute job, an observation, or a complete workflow.

The Grid will know what to do where.
Partners

Dimension 1: wealth of fields within the astro-topic

MPI für Gravitationsphysik Potsdam
MPI für Astrophysik Garching
MPI für extraterristrische Physik Garching
MPI für Radioastronomie Bonn
MPI für Astronomie Heidelberg

Computer Science
Konrad-Zuse-Zentrum für Informationstechnik Berlin
Informatik, Technische Universität München
Institut für Informatik, Universität Potsdam

Astrophysics
Astrophysikalisches Institut Potsdam
Zentrum für Astronomie der Universität Heidelberg
Universitätssternwarte der LMU München

Computer Center
Max-Planck-Rechenzentrum Garching
Leibniz-Rechenzentrum München
Forschungszentrum Karlsruhe
Dimension 2: wealth of fields within the grid-topic

D-GRID incorporates:

• Climate Research (C3)
• High Energy Physics (HEP)
• Engineering (InGrid)
• Medicine (MediGrid)
• Humanities (TextGrid)
• Astronomy (AstroGrid)
dynamo-beteigeuze.ari.uni-heidelberg.de [Done]

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a workshop devoted to radiative transfer coding

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Thank you.