Testing of the 3D inversion routine engine - the 3D forward algorithm - by comparison with 2D forward modelling results

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Summary

The importance of 3D forward and inversion codes in electromagnetic interpretation is continuously increasing. As all inversion algorithms are based on a forward calculation, the forward codes are the important basis for all kind of 3D interpretation - either for forward modelling itself or integrated in the inversion. Therefore a user must acquaint himself with the strengths and weaknesses of the 3D forward code. There are many ways of accomplishing this; the one described here is the comparison of 2D and 3D forward modelling responses. The idea behind this kind of testing is that 3D model responses become more and more identical to the 2D responses if the extension of the body becomes large, in an inductive scale-length sense, in one direction. The dimensionless ratio of the body length to skin depth at a specific frequency in the host medium (Jones, 1983) gives a limit for the period range where the agreement of 2D and 3D responses is supposed to be good - if it is not that can be a hint that there is an issue either with the forward code or its implementation.

Introduction

Independent of the size of a survey area the subject of interest for electromagnetic studies is, very rarely, validly interpreted as one-dimensional. Even two-dimensional interpretations are often not valid for all frequencies and all sites - which is why three-dimensional modelling tools are required. For magnetotelluric studies there are already 3D forward modelling codes (e.g., Avdeev, Kuvshinov, Pankratov, & Newman, 1997; Mackie, Madden, & Wannamaker, 1993; Mackie, Smith, & Madden, 1994; Siripunvaraporn, Egbert, & Lenbury, 2002; Xiong, Luo, Wang, & Wu, 1986; Xiong, 1992) and more recent also 3D inversion codes (e.g. Farquharson, Oldenburg, Haber, & Shekhtman, 2002; Siripunvaraporn, Egbert, Lenbury, & Uyeshima, 2005) are available.

Using 3D inversion codes should never be done without being aware of the fact that the engines of all 3D inversion programs are the 3D forward algorithms which are running in the background, and which, in fact, dominate the time of the 3D inversion run. Therefore, the reliability of the used 3D forward routine should be tested thoroughly before undertaking extensive 3D inversion. There are different ways of testing, the one which we describe here is the comparison of 3D responses with 2D forward modelling results.

Why are 2D and 3D response comparable and what are the limits?

Using a one-dimensional subsurface model to calculate the 1D, 2D and 3D forward response it is obvious that all responses should be identical - meaning that the impedance elements at all frequencies and sites should be $Z_{xx} = Z_{yy} = 0$ and $Z_{xy} = -Z_{yx}$, whereas the magnitude of the off-diagonal elements of the 3D forward response should be identical (within acceptable bounds) to the one of the 1D and the 2D forward modelling. Also for a two-dimensional Earth model the results of the 2D forward and 3D forward algorithms must match each other at specific locations and for specific frequencies. Jones (1983) showed that the difference between data from a profile over a 3D structure (e.g. a dyke of finite length l) and a real 2D response is dependent on the dimensionless ratio $\frac{l}{\delta}$, where δ is the skin depth at a specific frequency in the host medium. If this ratio is far greater than 1, then the 2D and 3D results should approximately give the same resistivity structure, whereas if $\frac{l}{\delta}$ is smaller than 1 (i.e. either the length l is too short or the frequency is too low) this assumption will not be valid.

This means for the 2D vs. 3D comparison that the high frequencies (or short periods) will give a good correlation for a certain range, whereas at lower frequencies (longer periods) this conformity cannot be expected.

	$1\,\Omega m$	$10\,\Omega m$	$100\,\Omega m$
$0.1\,s$	253	80	25.3
$1\,s$	80	25.3	8
$10\,s$	25.3	8	2.53
$100\,s$	8	2.53	0.8
$1000\,s$	2.53	0.8	0.25
$10000\,s$	0.8	0.25	0.008

Table 1: This table gives the $\frac{l}{\delta}$ ratio for different periods ($0.1\,s$ to $10000\,s$) and different resistivities of the host medium ($1\,\Omega m$, $10\,\Omega m$ and $100\,\Omega m$). The length l of the body is assumed to be 40 km.

In Tabel 1 are $\frac{l}{\delta}$ ratios for a period range from $0.1\,s$ to $10000\,s$ listed. The values are related to a body with a length of $l = 40$ km for 3 different host medium resistivities ($1\,\Omega m$, $10\,\Omega m$ and $100\,\Omega m$).

For the $1\,\Omega m$ host medium a 2D and a 3D response would be identical (within acceptable bounds) down to $100\,s$, may be even down to $1000\,s$, whereas for the $100\,\Omega m$ host a correlation would only be reasonable for periods smaller than $1\,s$ (may be $10\,s$).

COMPARISON OF 2D AND 3D RESULTS

The 2D forward code of Rodi and Mackie (2001) was used to calculate the responses, which were assumed to be the absolute correct and impeccable 2D responses which are used as the reference datasets. Two different subsurface models were used for the 2D vs. 3D comparison: a dyke model and a layered earth model with two blocks in the top layer.

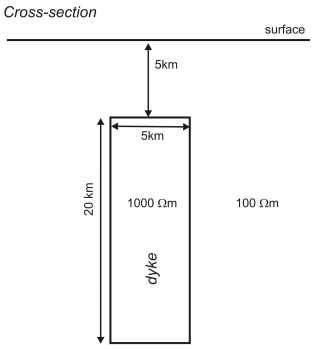

Figure 1: Cross-section of the dyke model along the x-axis.

Dyke model

A 5 km wide and 20 km thick dyke of $1000\,\Omega m$ is located 5 km below the surface and is embedded in a $100\,\Omega m$ homogeneous half-space (see Figure 1 for a cross-section of the model). The length of the dyke (in perpendicular direction to the cross-section) is varied for the 3D modelling, which is done using the forward code from Xiong et al. (1986); Xiong (1992).

Figure 2 shows the comparison of the off-diagonal elements of resistivity and phase. The solid line is the 2D reference data, the symbols represent 3D responses calculated for the different body length of 10 km, 20 km, 50 km and 100 km. All four plots show clearly that the agreement of 2D and 3D becomes better with increasing body length. This observation is in accordance with the ratio of body length to skin depth. The result of the 3D response with 100 km body length shows the same period limit of good agreement between 2D and 3D responses as it is predicted in Tabel 1.

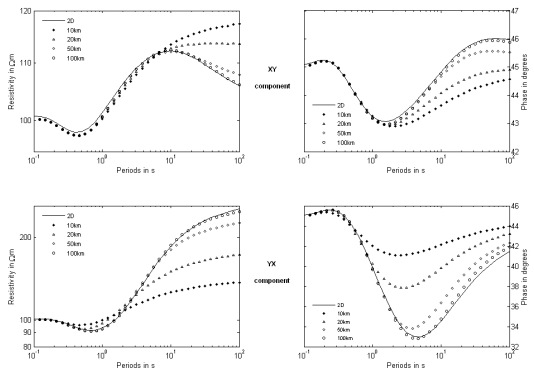

Figure 2: Resistivity and phase plots of the 2D result (solid line) and the 3D responses for different body length l of 10 km, 20 km, 50 km and 100 km.

Two blocks in layered earth

This 3D model (see Figure 3) has often been used by different authors (e.g. Mackie et al., 1993; Siripunvaraporn et al., 2005; Wannamaker, 1991) and goes back to Dr. M.S. Zhdanov. The model is based on a three-layered earth with $10\,\Omega m$ down to 10 km, $100\,\Omega m$ from 10 - 30 km and then a halfspace of $0.1\,\Omega m$. In the first layer are two 20 km x 40 km blocks (over the whole layer thickness) with resistivity values of $1\,\Omega m$ and $100\,\Omega m$ respectively embedded.

For this model responses at six different stations were calculated and for the 3D response the 3D code described by Mackie et al. (1993, 1994) was used. Setting the origin at

the boundary between the two blocks for x-direction and in the middle of the block in y-direction the site locations are -25 km, -15 km, -5 km, 5 km, 15 km and 25 km in x-direction and y = 0 for all.

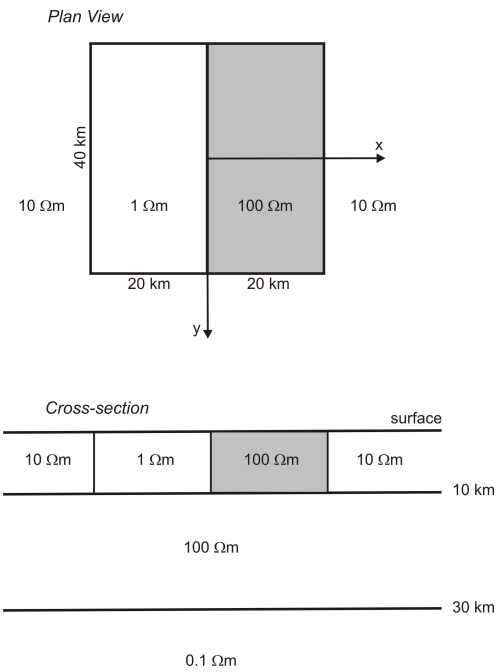

Figure 3: Plan view and cross-section of the used 3D model (redrawn after Mackie, Madden, & Wannamaker 1993).

Figure 4 shows the resistivity and phase curves for both off-diagonal elements at all six sites. The agreement of the 2D and 3D results is good down to periods of 10 s - sometimes even down to 100 s. That is in good accordance with the expected period range calculated using the ratio of length to skin depth of the host (see Table 1).

Conclusions

For the chosen models and codes the comparison for 2D and 3D response gave the expected results: good agreement for the short period range. Also the increasing of the part with reasonable agreement of the 2D and 3D responses towards longer periods with increasing body length could be seen. For this combination of codes and models the comparison can be consider as being successful.

Although the experience while testing different codes and models showed that there are a few difficulties with the 3D forwards codes (not all combinations of chosen models and codes gave the expected result). Each codes has weak and strong aspects and it is useful to get an idea of them before using a 3D inversion code where the 3D forward algorithm is used as engine.

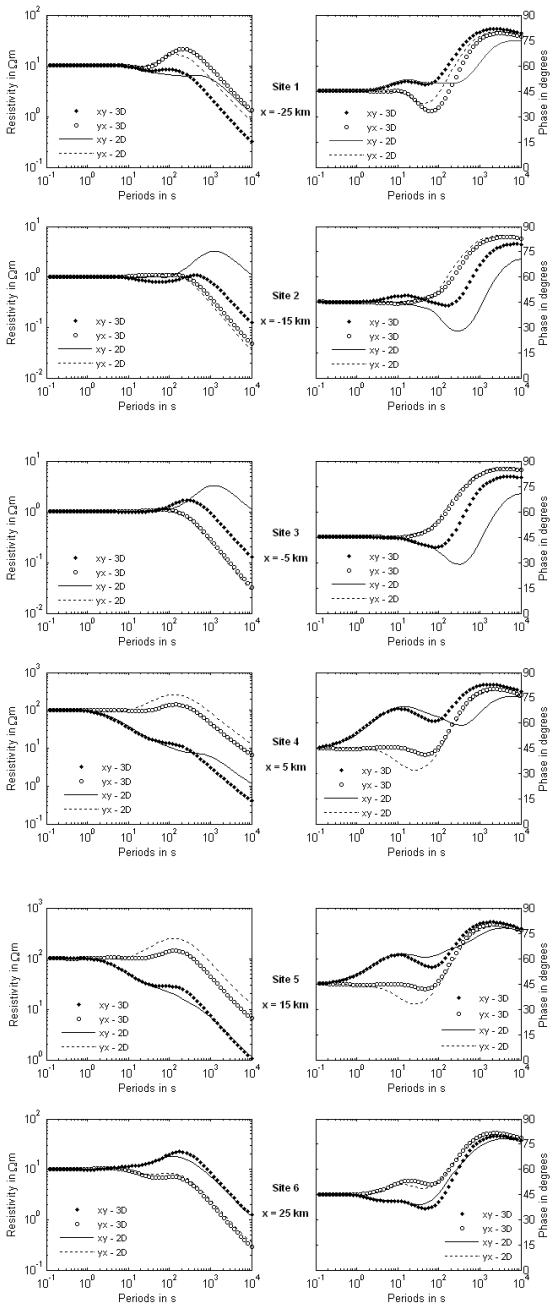

Figure 4: Resistivity and phase plots of the 2D responses (solid and dashed line) and the 3D responses (solid dots and open circles) for all 6 sites over the model shown in Figure 3.

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