Basics of 3D EM IE modelling

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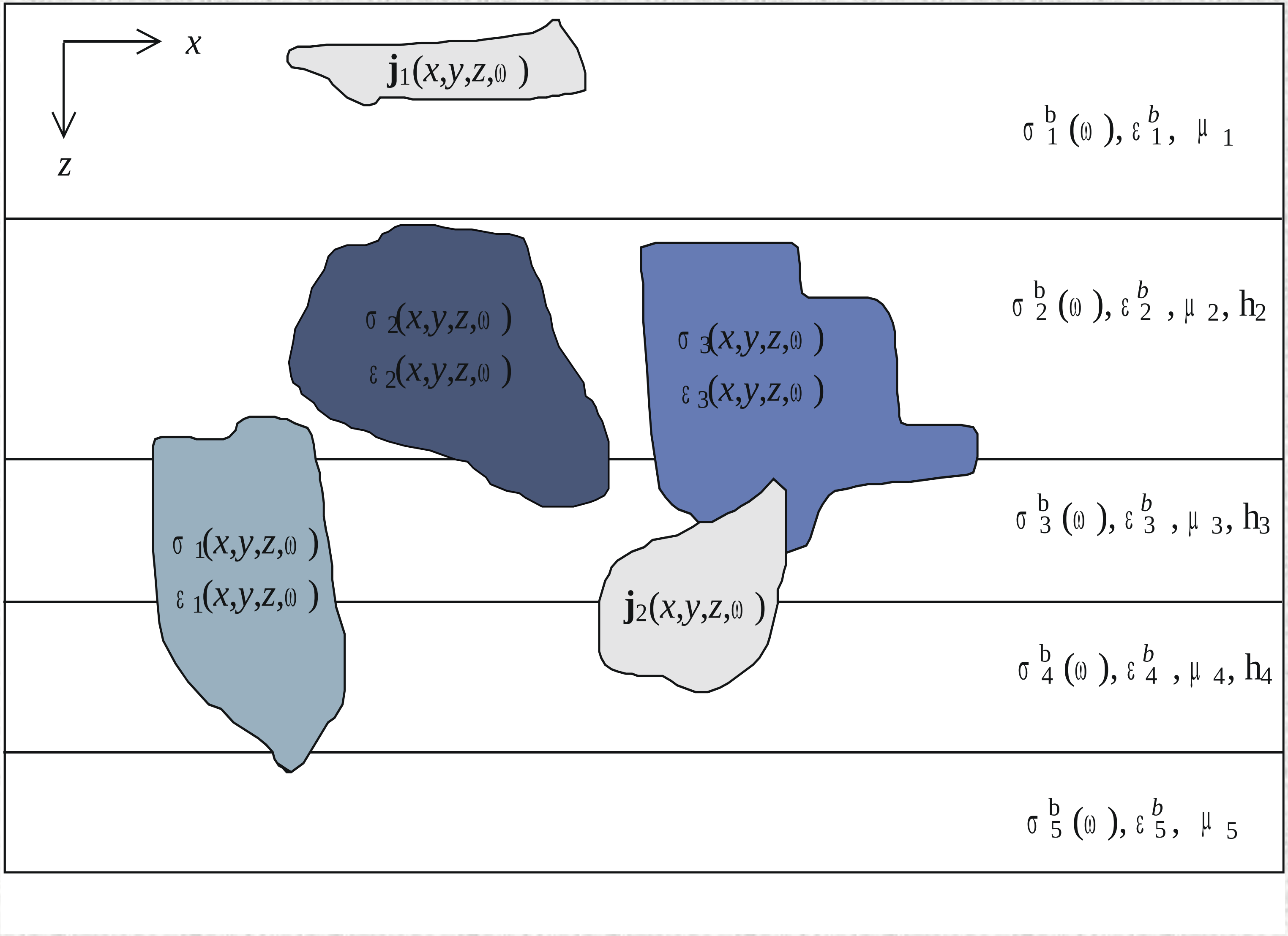


Fig. Example of a 3D problem (side view).

Maxwell's equations in frequency-domain

$$\nabla\times\mathbf{H}=\underline{\underline{\zeta}}(x,y,z,\omega)\mathbf{E}+\mathbf{j}^{ext},\quad \nabla\times\mathbf{E}=i\omega\underline{\underline{\mu}}(z)\mathbf{H}.$$

$$\underline{\underline{\zeta}}(x,y,z,\omega)=\underline{\underline{\sigma}}-i\omega\underline{\underline{\varepsilon}}=\begin{pmatrix}\zeta_{xx} & 0 & 0\\ 0 & \zeta_{yy} & 0\\ 0 & 0 & \zeta_{zz}\end{pmatrix},\qquad \underline{\underline{\mu}}(z,\omega)=\begin{pmatrix}\mu_{\tau} & 0 & 0\\ 0 & \mu_{\tau} & 0\\ 0 & 0 & \mu_{z}\end{pmatrix}.$$

Maxwell's equations for reference EM field

$$\nabla\times\mathbf{H}^{o}=\underline{\underline{\zeta}}_{o}(z,\omega)\mathbf{E}^{o}+\mathbf{j}^{ext},\quad \nabla\times\mathbf{E}^{o}=i\omega\underline{\underline{\mu}}(z)\mathbf{H}^{o}.$$

$$\underline{\underline{\zeta}}_{0}(z,\omega)=\begin{pmatrix}\zeta_{0\tau} & 0 & 0\\ 0 & \zeta_{0\tau} & 0\\ 0 & 0 & \zeta_{0z}\end{pmatrix}.$$

Maxwell's equations for scattered EM field

$$\nabla\times\mathbf{H}^{s}=\underline{\underline{\zeta}}_{o}(z,\omega)\mathbf{E}^{s}+\mathbf{j}^{q},\quad \nabla\times\mathbf{E}^{s}=i\omega\underline{\underline{\mu}}(z)\mathbf{H}^{s}.$$

$$\mathbf{j}^{q}=(\underline{\underline{\zeta}}-\underline{\underline{\zeta}}_{o})(\mathbf{E}^{s}+\mathbf{E}^{o}).$$

$$\nabla\times\frac{1}{i\omega}\underline{\underline{\mu}}^{-1}(z)\nabla\times\mathbf{E}^{s}-\underline{\underline{\zeta}}_{o}(z,\omega)\mathbf{E}^{s}=\mathbf{j}^{q}.\quad\longleftrightarrow\quad \mathbf{E}^{s}(\mathbf{r})=\int_{V^{S}}\underline{\underline{G}}_{o}^{ee}(\mathbf{r},\mathbf{r}')\mathbf{j}^{q}(\mathbf{r}')dv'.$$

$$\mathbf{r}=(x,y,z),\quad dv'=dx'dy'dz'.$$

Conventional scattering equation (Dmitriev, 1969; Weidelt, 1975)

$$\mathbf{E}^{s}(\mathbf{r})=\mathbf{E}_{o}+Q\mathbf{E}^{s}=\mathbf{E}_{o}(\mathbf{r})+\int_{V^{S}}\underline{\underline{G}}_{o}^{ee}(\mathbf{r},\mathbf{r}')\left(\underline{\underline{\zeta}}(\mathbf{r}')-\underline{\underline{\zeta}}_{o}(z')\right)\mathbf{E}^{s}(\mathbf{r}')dv'. \tag{1}$$

3x3 dyadic for the electric-to-electric Green's function of the 1D reference formation:

$$\underline{\underline{G}}_{o}^{ee}=\begin{pmatrix}G_{xx} & G_{xy} & G_{xz}\\ G_{yx} & G_{yy} & G_{yz}\\ G_{zx} & G_{zy} & G_{zz}\end{pmatrix}$$

Formal solution of eq. (1) can be expressed as an infinite Neumann series

$$\mathbf{E}^{s}(\mathbf{r})=(I-Q)^{-1}\mathbf{E}_{o}=\mathbf{E}_{o}+Q\mathbf{E}_{o}+Q^{2}\mathbf{E}_{o}+\ldots$$

As a rule, the series doesn't converge at all.

Conventional scattering equation

$$\mathbf{E}^s(\mathbf{r}) = \mathbf{E}_o + Q\mathbf{E}^s = \mathbf{E}_o(\mathbf{r}) + \int_{V^S} \underline{\underline{G}}_o^{ee}(\mathbf{r},\mathbf{r}')\left(\underline{\underline{\zeta}}(\mathbf{r}') - \underline{\underline{\zeta}}_o(z')\right)\mathbf{E}^s(\mathbf{r}')dv'.$$

$$\underline{\underline{\Lambda}}(z,\omega) = \begin{pmatrix} \sqrt{\mathrm{Re}\,\zeta_{0\tau}} & 0 & 0 \\ 0 & \sqrt{\mathrm{Re}\,\zeta_{0\tau}} & 0 \\ 0 & 0 & \sqrt{\mathrm{Re}\,\zeta_{0z}} \end{pmatrix}$$

$\mathbf{E}^S \rightarrow \chi:$

$$\chi = \tfrac{1}{2}\underline{\underline{\Lambda}}^{-1}\left((\underline{\underline{\zeta}} + \underline{\underline{\zeta}}_o^*)\mathbf{E}^s + (\underline{\underline{\zeta}} - \underline{\underline{\zeta}}_o)\mathbf{E}^o\right)$$

Scattering equation MIDM (Singer, 1995; Pankratov, Avdeev,Kuvshinov, 1995; Pankratov, Kuvshinov, Avdeev, 1997; Singer & Fainberg, 1995, 1997)

$$\chi(\mathbf{r}) = \chi_o + M\chi = \chi_o(\mathbf{r}) + \int_{V^S} \underline{\underline{K}}(\mathbf{r},\mathbf{r}')\underline{\underline{R}}(\mathbf{r}')\chi(\mathbf{r}')dv'$$

$$\underline{\underline{K}}(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}')\underline{\underline{I}} + 2\underline{\underline{\Lambda}}(z)\underline{\underline{G}}_o^{ee}(\mathbf{r},\mathbf{r}')\underline{\underline{\Lambda}}(z')$$

$$\underline{\underline{R}}(\mathbf{r}) = \left(\underline{\underline{\zeta}}(\mathbf{r}) - \underline{\underline{\zeta}}_o(z)\right)\left(\underline{\underline{\zeta}}(\mathbf{r}) + \underline{\underline{\zeta}}_o^*(z)\right)^{-1}$$

$$\chi = (1-M)^{-1}\chi_o = \chi_o + M\chi_o + M^2\chi_o + \ldots$$

The Neumann's series converges for any frequency and any electrical resistivity contract $\|\underline{\underline{M}}\chi\| < \|\chi\|, \ \forall\chi$

Simple iteration (MIDM; Avdeev et al., 2000)

$$\chi^{(n+1)} = \chi_o + M\chi^{(n)}, \quad n = 1,2,\ldots$$

Krylov iteration (Avdeev et al., 2002)

$$A\chi = \chi_o, \qquad A = 1 - M.$$

$$\kappa(A) = \|A\|\|A^{-1}\| \leq \sqrt{C_l} \qquad (C_l = 10^4 \Rightarrow \kappa(A) \leq 10^2)$$

EM field determination

$\chi \rightarrow \mathbf{j}^q$

$$\mathbf{j}^q(\mathbf{r}) = 2\underline{\underline{\lambda}}(\underline{\underline{\zeta}} + \underline{\underline{\zeta}}_o^*)^{-1}(\underline{\underline{\zeta}} - \underline{\underline{\zeta}}_o)(\chi + \underline{\underline{\lambda}}\mathbf{E}^o), \quad \mathbf{r} \in V^S$$

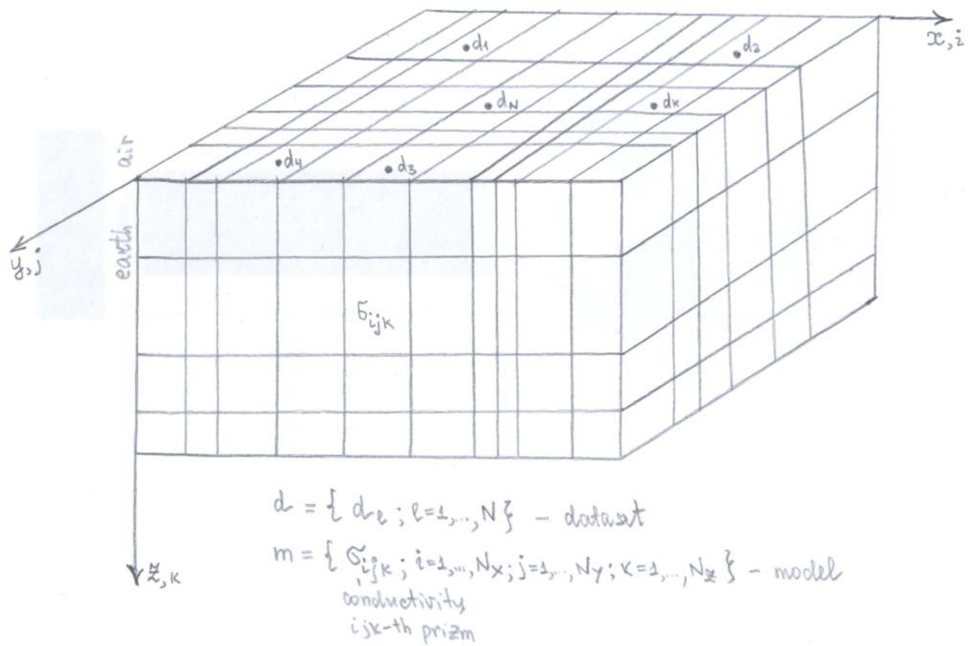
$\mathbf{j}^q \rightarrow \mathbf{E}^S, \mathbf{H}^S$

$$\nabla \times \frac{1}{i\omega}\underline{\mu}^{-1}(z)\nabla \times \mathbf{E}^s - \underline{\zeta}_o(z,\omega)\mathbf{E}^s = \mathbf{j}^q.$$

$$\mathbf{E}^s(\mathbf{r}) = \int_{V^S} \underline{\underline{G}}_o^{ee}(\mathbf{r},\mathbf{r}')\mathbf{j}^q(\mathbf{r}')dv', \quad \mathbf{H}^s(\mathbf{r}) = \int_{V^S} \underline{\underline{G}}_o^{me}(\mathbf{r},\mathbf{r}')\mathbf{j}^q(\mathbf{r}')dv',$$

$$\mathbf{E} = \mathbf{E}^o + \mathbf{E}^s, \quad \mathbf{H} = \mathbf{H}^o + \mathbf{H}^s(\mathbf{r}).$$

Fig. A 3-D MT model discretized with rectangular prisms.



(1) Integral equation (IE) approach

$$\chi(\mathbf{r}) = \chi_0(\mathbf{r}) + \int_{V^s} \underline{\underline{K}}(\mathbf{r},\mathbf{r}')\underline{R}(\mathbf{r}')\chi(\mathbf{r}')dv'$$

$A_{IE} \cdot x = b$ - system of linear equations (← on a rectangular 3-D grid);

A_{IE} - complex, dense with all entries filled, non-Hermitian matrix; but much more compact than FD and FE matrices

(a) Main attraction: only the scattering volume V^s is subject to discretization;

(this reduce dramatically the size of matrix A_{IE})

(b) Drawback: most EM software developers refrain from implementation of the IE approach, since accurate computation of the matrix A_{IE} is indeed an extremely tedious and nontrivial problem itself.

$$\nabla \times \underline{\underline{\mu}}^{-1}\mathbf{E} - i\omega \underline{\underline{\zeta}}(x,y,z,\omega)\mathbf{E} = \mathbf{j}^{ext}$$

(2) Finite-difference (FD) approach

$A_{FD} \cdot x = b$ - system of linear equations (← on a rectangular 3D grid);

A_{FD} - large, sparse 3Mx3M symmetric, non-Hermitian matrix;

x represents the grid nodal values of electric field;

b represents the source and boundary conditions;

$M = n_x \cdot n_y \cdot n_z$ - number of model parameters.

(a) The most commonly employed

(b) Main attraction: an apparent simplicity of its numerical implementation

(3) Finite-element (FE) approach

the EM field (or its potentials) are decomposed to some basic (usually, edge and nodal) functions. The coefficients of the decomposition, a vector *x*, are sought using the Galerkin method.

$A_{FE} \cdot x = b$

A_{FE} - large, sparse, non-symmetric, non-Hermitian matrix;

(a) Main attraction: it is commonly believed to be better able than other approaches to accurately account for geometry (shapes of ore-bodies, topography, cylindrical wells, etc.)

(b) Main drawback: construction of the finite elements themselves is another nontrivial and usually time-consuming procedure.

Based on the above approach, there have been developed 3D forward modeling solutions for various EM applications

- ✓ Induction logging in deviated boreholes
- ✓ Airborne electromagnetics
- ✓ General controlled-source EMs, MT, and CSMT
- ✓ Global induction studies

The solutions:

- give accurate results even for lateral contrast of electrical resistivity up to 100,000;
- simulate the frequency-domain responses in frequency range from DC up to 50 MHz;
- account for the induced polarization and displacement currents;
- admit an *anisotropy* of the electrical conductivity;
- allow to run large-scale models discretized by up to 1,000,000 cells;

Thank you for your attention