# Program MT3Dinv: Forward modelling

Colin G. Farquharson and Douglas W. Oldenburg

# History

- ⋆ **Doug**, **Eldad Haber** (now at Emory University, Atlanta) & **Roman Shekhtman** developed 3D EM controlled-source forward and inversion programs.

- ⋆ I did modifications to get MT forward and inversion programs.

# Forward modelling: fundamental equations

⋆ Forward modelling for MT …

→ homogeneous equations:

$$
\begin{aligned}
\nabla \times \mathbf{E} \; - \; i\omega\mu_0 \mathbf{H} \; &= \; 0, \\
\nabla \times \mathbf{H} \; - \; \mathbf{J} \; &= \; 0, \\
\nabla \cdot \mathbf{E} \; - \; \rho/\epsilon_0 \; &= \; 0, \\
\nabla \cdot \mathbf{J} \; &= \; 0, \\
\mathbf{J} \; - \; \sigma \, \mathbf{E} \; &= \; 0
\end{aligned}
$$

($e^{-i\omega t}$ time dependence);

→ *in*homogeneous boundary conditions.

# Forward modelling: potentials

⋆ Introduce vector and scalar potentials such that

$$\mathbf{E} \;=\; \mathbf{A} \;+\; \nabla\phi.$$

Hence, **A** in active space of $\nabla\times$ operator, $\phi$ in null space of $\nabla\times$ operator.

⋆ Use the Coulomb gauge condition, namely,

$$\nabla\cdot\mathbf{A} \;=\; 0.$$

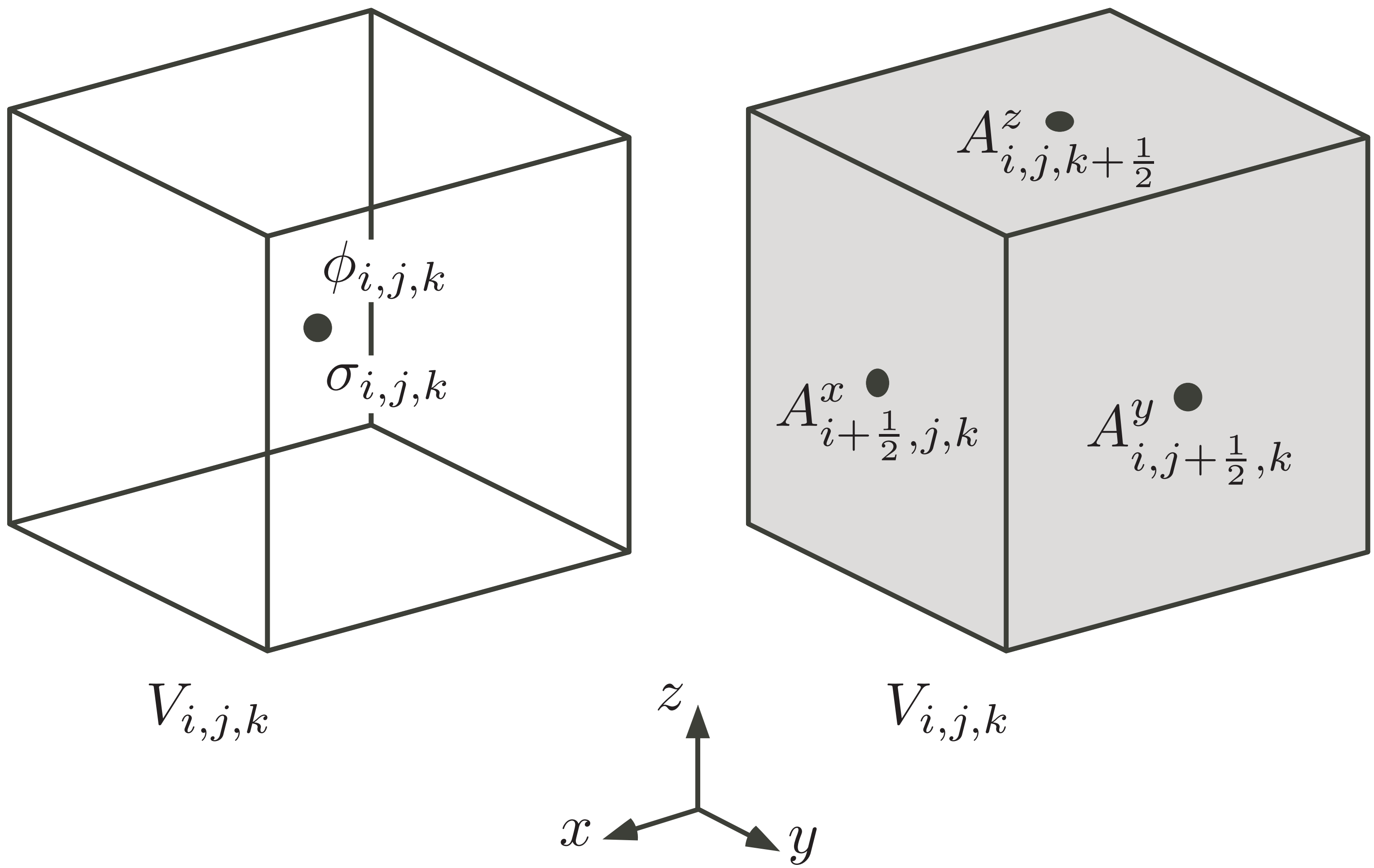
Hence, **A** is smooth:

$$\hat{\mathbf{n}}\times\left(\mathbf{A}_1-\mathbf{A}_2\right) \;=\; 0, \quad \hat{\mathbf{n}}\cdot\left(\mathbf{A}_1-\mathbf{A}_2\right) \;=\; 0,$$

and $\phi$ is linked to charge density on conductivity discontinuities:

$$\hat{\mathbf{n}}\cdot\left(\nabla\phi_1-\nabla\phi_2\right) \;=\; \tilde{\rho}/\epsilon_0.$$

# Forward modelling: discretization



# Forward modelling: harmonic averaging

- ⋆ Harmonic averaging of conductivities across cell boundaries (from finite volume treatment):

$$\sigma_{i+\frac{1}{2},j,k} = h^x_{i+\frac{1}{2}} \left( \frac{h^x_i}{2\sigma_{i,j,k}} + \frac{h^x_{i+1}}{2\sigma_{i+1,j,k}} \right)^{-1}.$$

So,

$$J^x_{i+\frac{1}{2},j,k} = \sigma_{i+\frac{1}{2},j,k} \left( A^x_{i+\frac{1}{2},j,k} + \frac{\phi_{i+1,j,k} - \phi_{i,j,k}}{h^x_{i+\frac{1}{2}}} \right).$$

# Forward modelling: boundary conditions

$\star$ Boundary conditions for total-field solution:

$\rightarrow$ $\hat{\mathbf{n}} \cdot \mathbf{A}$ (i.e., $\hat{\mathbf{n}} \cdot \mathbf{J}$) and $\hat{\mathbf{n}} \times \nabla \times \mathbf{A}$ (i.e., $\hat{\mathbf{n}} \times \mathbf{H}$) specified on boundaries for two polarizations;

$\rightarrow$ requires 2D E-polarization solutions.

# Forward modelling: discrete equations

⋆ Substitution, manipulation & discretization leads to the system to be solved:

$$\begin{aligned}\nabla^2 \mathbf{A} + i\omega\mu_0\sigma\mathbf{A} \ + \ i\omega\mu_0\sigma\nabla\phi \ &= \ 0,\\ \nabla\cdot\big(\sigma\mathbf{A}\big) \ + \ \nabla\cdot\big(\sigma\nabla\phi\big) \ &= \ 0;\end{aligned}$$

i.e.,

$$\begin{pmatrix} L + \alpha M_\sigma & \alpha M_\sigma \nabla_h \\ \nabla_h \cdot M_\sigma & \nabla_h \cdot M_\sigma \nabla_h \end{pmatrix} \begin{pmatrix} \mathbf{A} \\ \phi \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ 0 \end{pmatrix}.$$

## Forward modelling: solution of system of equations

⋆ Solution via BCGSTAB.

⋆ Preconditioner: ILU decomposition of blocks of the matrix

$$\begin{pmatrix} L + \alpha M_\sigma & \alpha M_\sigma \nabla_h \\ 0 & \nabla_h \cdot M_\sigma \nabla_h \end{pmatrix}.$$

## Forward modelling: primary-secondary separation

⋆ Also, primary–secondary separation of fields and potentials:

$$\mathbf{E} = \mathbf{E}_p + \mathbf{E}_s \quad \text{and} \quad \mathbf{H} = \mathbf{H}_p + \mathbf{H}_s,$$

$$\mathbf{A} = \mathbf{A}_p + \mathbf{A}_s \quad \text{and} \quad \phi = \phi_p + \phi_s.$$

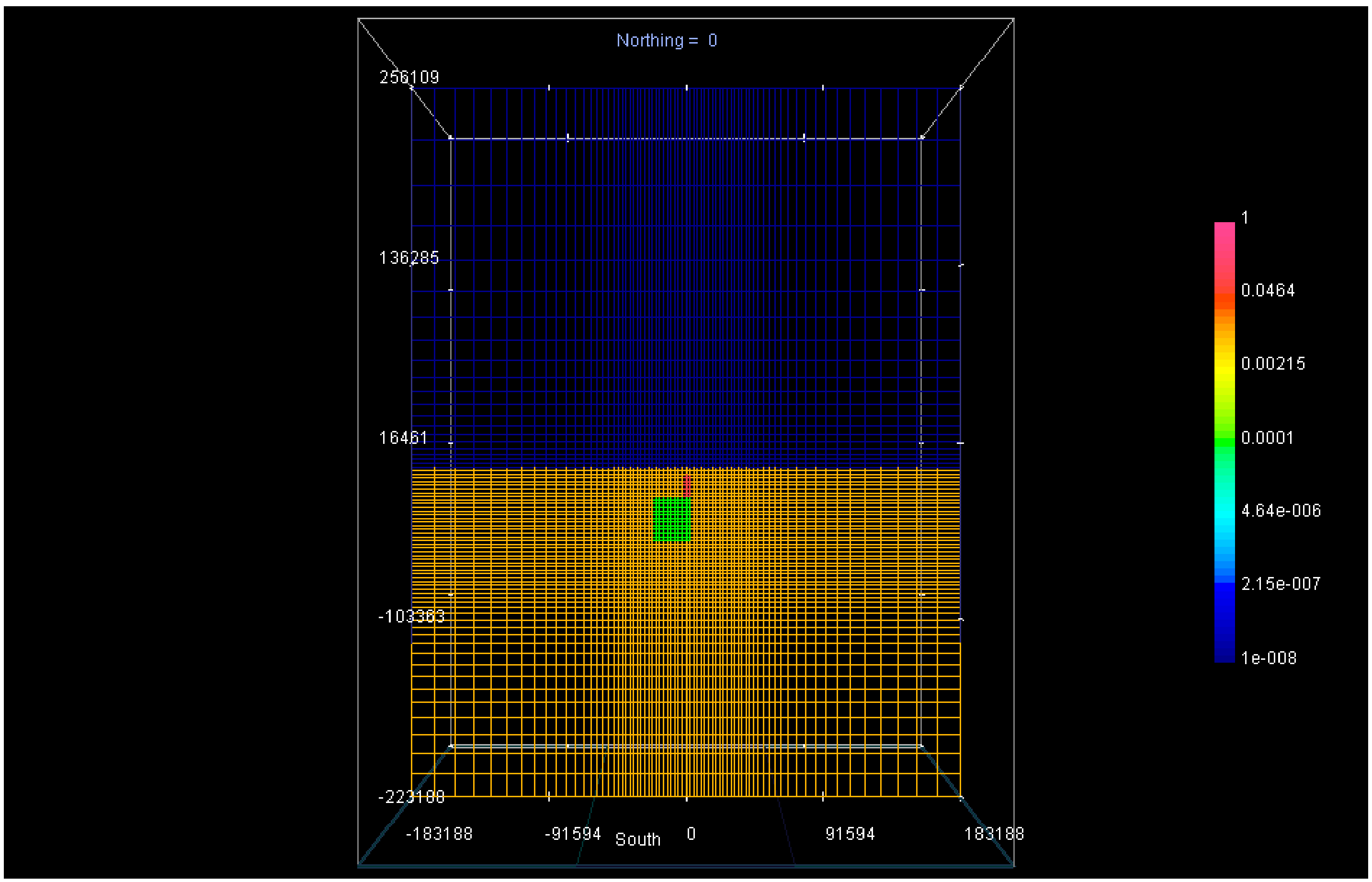
Hence . . .

$$\begin{aligned} \nabla^2 \mathbf{A}_s + i\omega\mu_0\sigma\mathbf{A}_s \; + \; i\omega\mu_0\sigma\nabla\phi_s \; &= \; -i\omega\mu_0\Delta\sigma\mathbf{E}_p, \\ \nabla\cdot\big(\sigma\mathbf{A}_s\big) \; + \; \nabla\cdot\big(\sigma\nabla\phi_s\big) \; &= \; -\nabla\cdot\big(\Delta\sigma\mathbf{E}_p\big), \end{aligned}$$

with homogeneous boundary conditions on $\mathbf{A}_s$ & $\phi_s$.

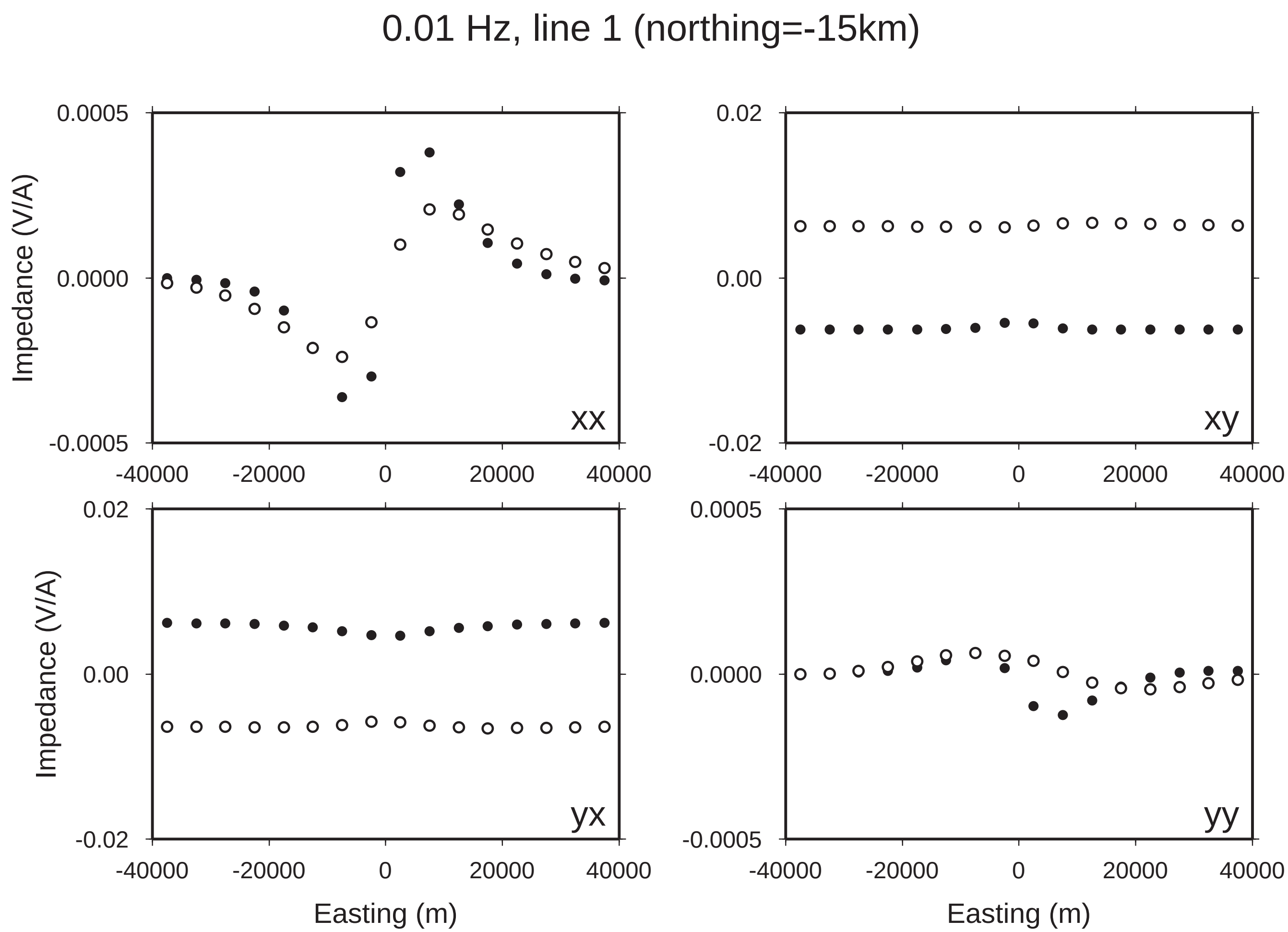
# Forward modelling: example 1

- ⋆ Mesh: $72 \times 72 \times 72$ cells;
  centre: $80 \times 80 \times 80$ km, $32 \times 32 \times 32$ cells, 2.5 km cubes;
  padding: additional $\sim 140$ km, $\sim 250$ km in air, 20 cells each.



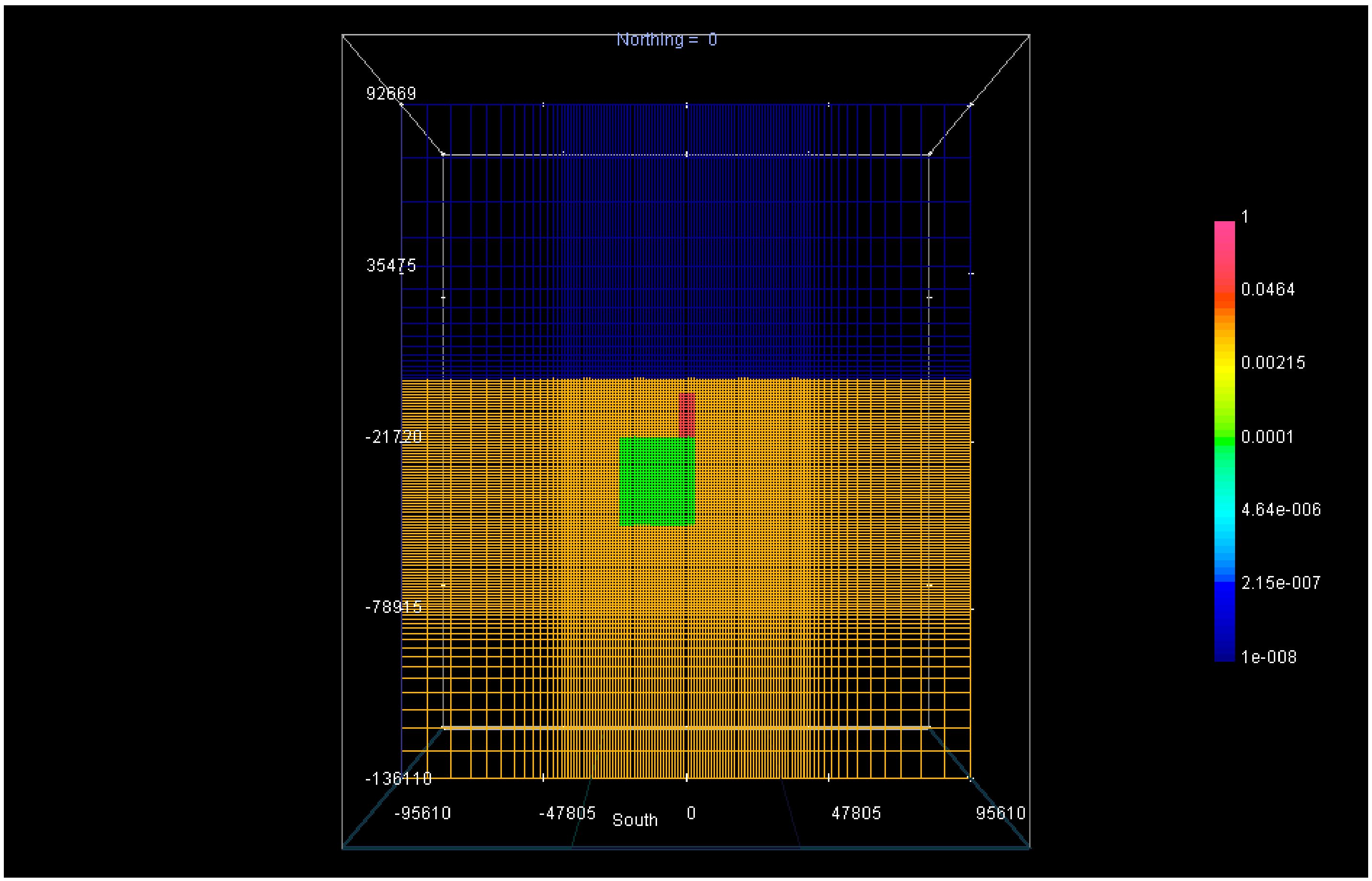
# Forward modelling: example 1

- ⋆ Memory required: $\sim 2$ Gbytes;
  Computation time for one frequency: $\sim 8$ hours (Apple Mac G5 2.2 GHz).



# Forward modelling: example 2

- ⋆ Mesh: $109 \times 110 \times 110$ cells;
  centre: $79 \times 80 \times 80$ km, $79 \times 80 \times 80$ cells, 1.0 km cubes;
  padding: additional $\sim 55$ km, $\sim 92$ km in air, 15 cells each.



# Forward modelling: example 2

⋆ Memory required: $\sim 7$ Gbytes;
Computation time for one frequency: $\sim 20$ hours (IBM A-Pro 2.2 GHz).