MT3D:
A 3D MT Modeling Algorithm

MT3D INVERSION WORKSHOP
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Gregory A. Newman & Michael Commer
Earth Sciences Division
Lawrence Berkeley National Laboratory
Berkeley California

The Forward Problem

The electric field vector equation for MT:

$$\nabla \times \nabla \times \mathbf{e} + i\omega\mu\sigma\, \mathbf{e} = \mathbf{0}.$$

Finite Difference Approximation

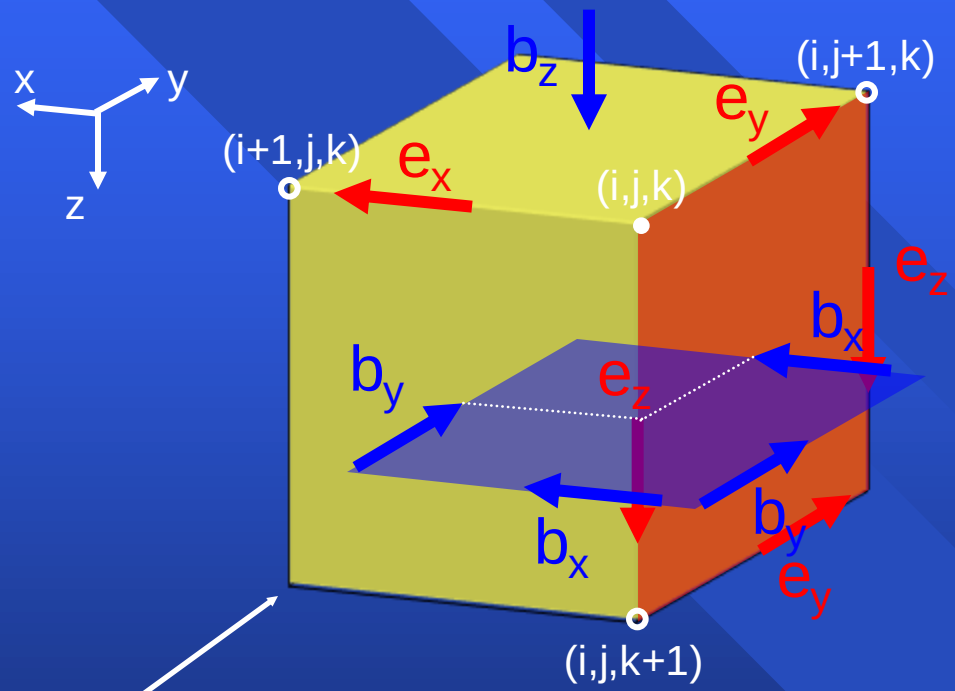
(1) The vector equation approximated with finite differences

(2) Staggered grid employed

(3) Dirichlet boundary conditions used

(4) Conductivity averaged on cell edges

„Staggered grid“ (after Yee, 1966)



$$-i\omega\mu h = \nabla \times e$$

$$\sigma\, e = \nabla \times h$$

$$j = \sigma\, e$$

$$b = \mu\, h$$

$$\Rightarrow \quad \nabla \cdot j = 0 \qquad \nabla \cdot b = 0$$

$$\sigma(i,j,k)$$
$$\mu(i,j,k)$$

Discretization of Maxwell equations

The sampled field components are continuous

The sampled electrical properties are based on volume averages cell edges and faces

GRID SEPARATION EFFICIENCIES

Advantages

- Taylor an optimal simulation grid Ω_s for each frequency-receiver set
- Modeling grid Ω_m covers basin-scale modeling volumes at fine resolution
- Simulations grids based upon skin depth considerations, can be much coarser than the modeling grid
- Faster solution times follow for coarser simulation grids

What's Required

- A mapping of conductivity from Ω_m to Ω_s and Ω_m to Ω_s
  - » Conductivity on Ω_s edged based
  - » Conductivity on Ω_m cell based
- An appropriate mixing law for the conductivity mappings

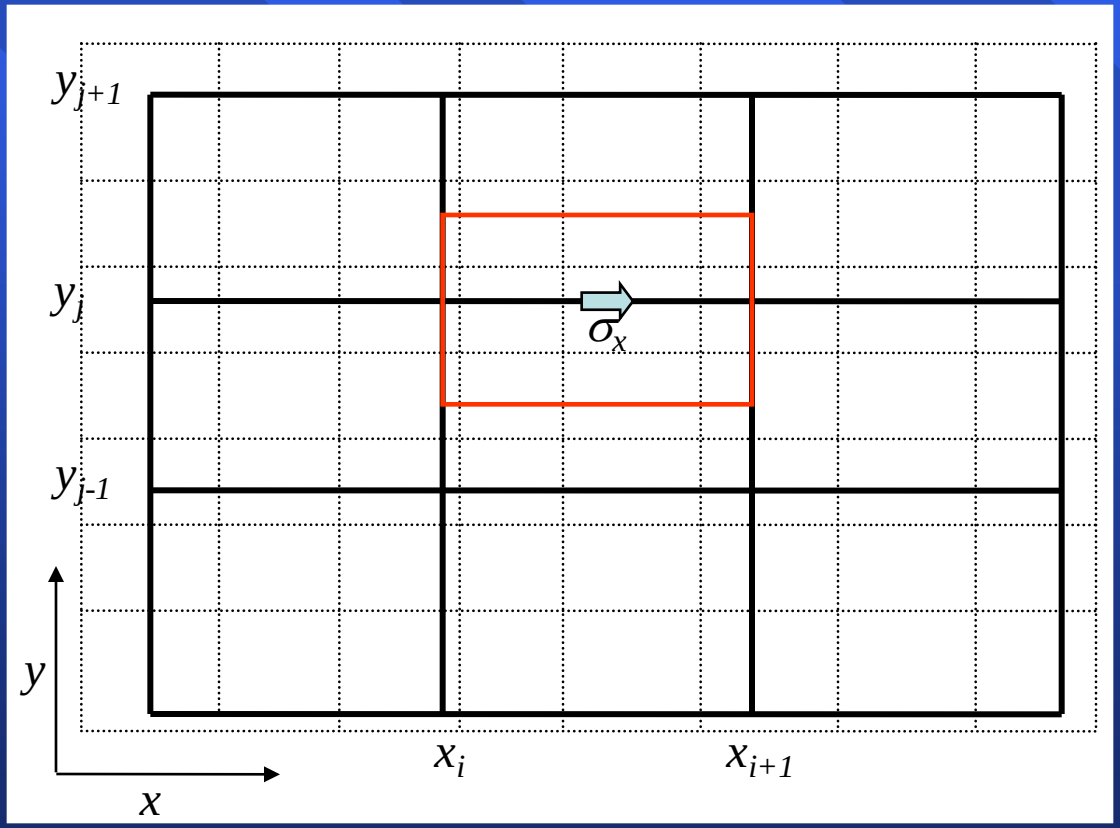
CONDUCTIVITY MAPPING (2D CASE)

$$\sigma_x = \left[\int_{x_i}^{x_{i+1}} \left(\int_{y_{j-1/2}}^{y_{j+1/2}} \sigma(x,y)\,dy \right)^{-1} dx \right]^{-1}$$

parallel-series circuit mixing law

Dashed grid model/inversion mesh

Solid grid simulation mesh



Red cell: conductivity region required to compute σ_x on the simulation mesh

The Krylov Solver

assemble complex-symmetric sparse linear system

$$\mathbf{Ke = s}$$

s depends on MT source polarization

(two for each frequency)

solve system with iterative Krylov methods

(also employ static divergence correction to improve convergence)

magnetic field then determined from Faraday's law

Computational Efficiencies

(1) forward code parallelized over model/simulation grids & frequency

3D Dublin Test Model: 21 frequencies

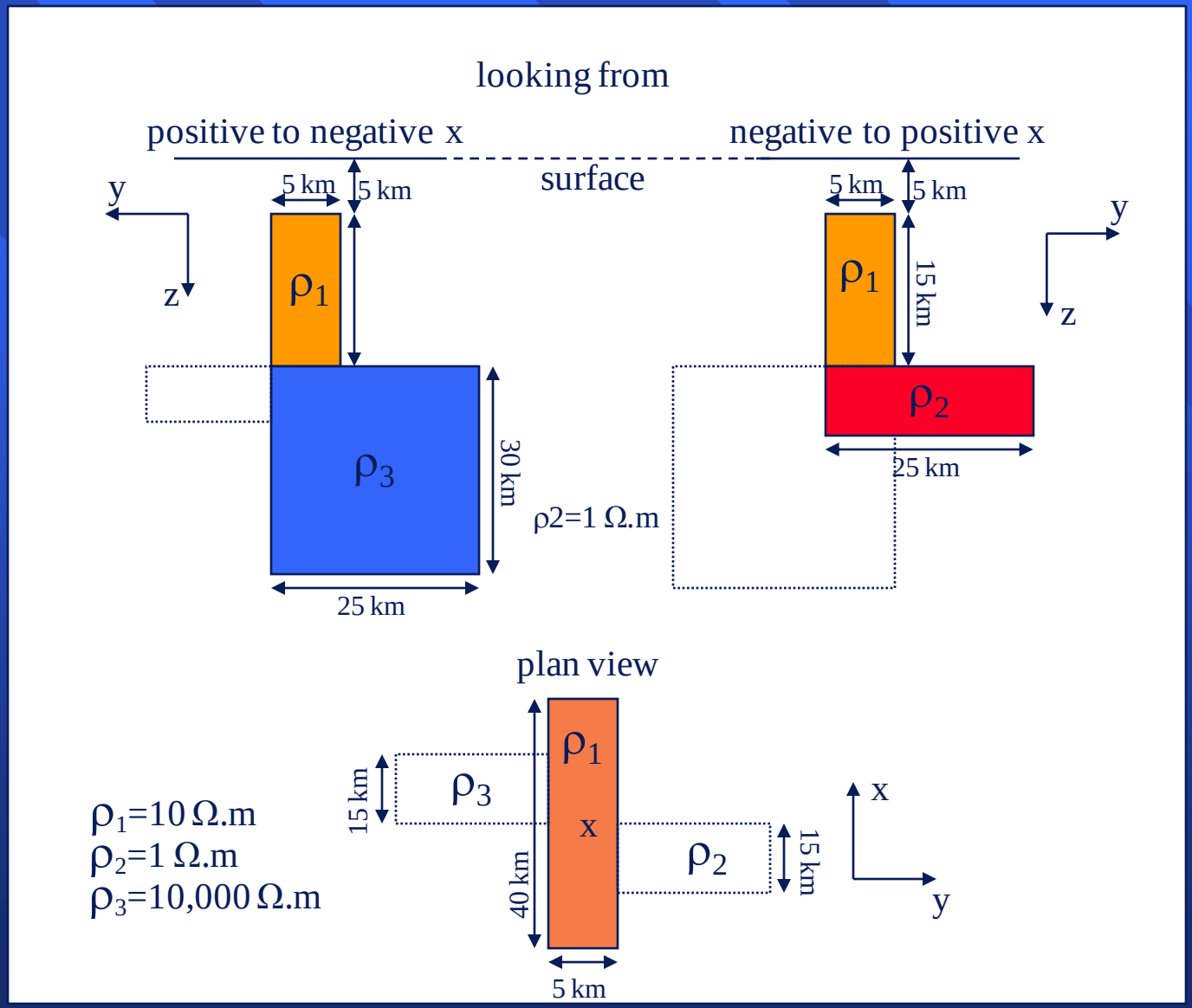
consider model domain decomposed over 180 cores

total number of cores that can be used 3720

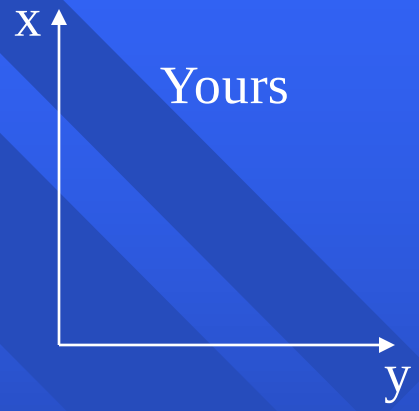
(2) ideal method for problems with extremely large data sets and model parameterizations

(3) scheme is implemented on the lab Linux clusters & Franklin-Cray XT4 machine at NERSC: 9660 nodes/19320 cores

Dublin Test Model



Coordinate Systems



Coordinate systems related to each other by a -90 degree rotation

$$\begin{pmatrix} Z_{yy} & -Z_{yx} \\ -Z_{xy} & Z_{xx} \end{pmatrix}_{\text{yours}} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix}_{\text{mine}} \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}_{\varphi=-\frac{\pi}{2}}$$

z positive downward for both

Meshing Considerations

Earth Radius 6350 km

Freq (Hz)	Skin Depth (km)	Cell Size (km)	Mesh Size (km)
10	1.59	0.318	±8
5.5	2.14	0.427	± 11
3.1	2.84	0.569	± 14
1.8	3.76	0.753	± 19
1	5.03	1.000*	± 25
5.5e-1	6.78	1.356	± 34
3.1e-1	9	1.800	± 45
1.8e-1	11.9	2.380	± 60
1e-1	15.9	3.180	± 80*
5.5e-2	21.45	4.290	± 107
3.1e-2	28.47	5.694	± 142
1.8e-2	37.64	7.528	± 188
1e-2	50.3	10.000	± 252
5.5e-3	67.82	13.564	± 339
3.12e-3	90.05	18.010	± 450
1.78e-3	119.	23.800	± 595.
1e-3	159	31.800	± 795
5.5e-4	214	42.800	± 1070
3.12e-4	285	57.000	± 1425
1.8e-2	376	75.200	± 1880
1e-4	503	100.600	± 2515

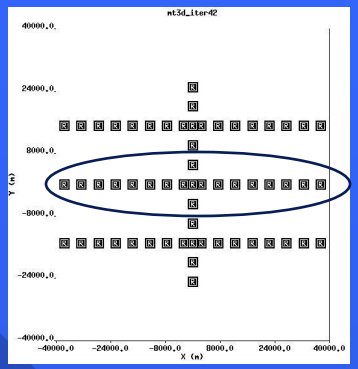
skin depth (δ) based on 100 Ω.m half-space; cell szie δ /5; mesh size based on detector at the origin

Simulation Grids Employed

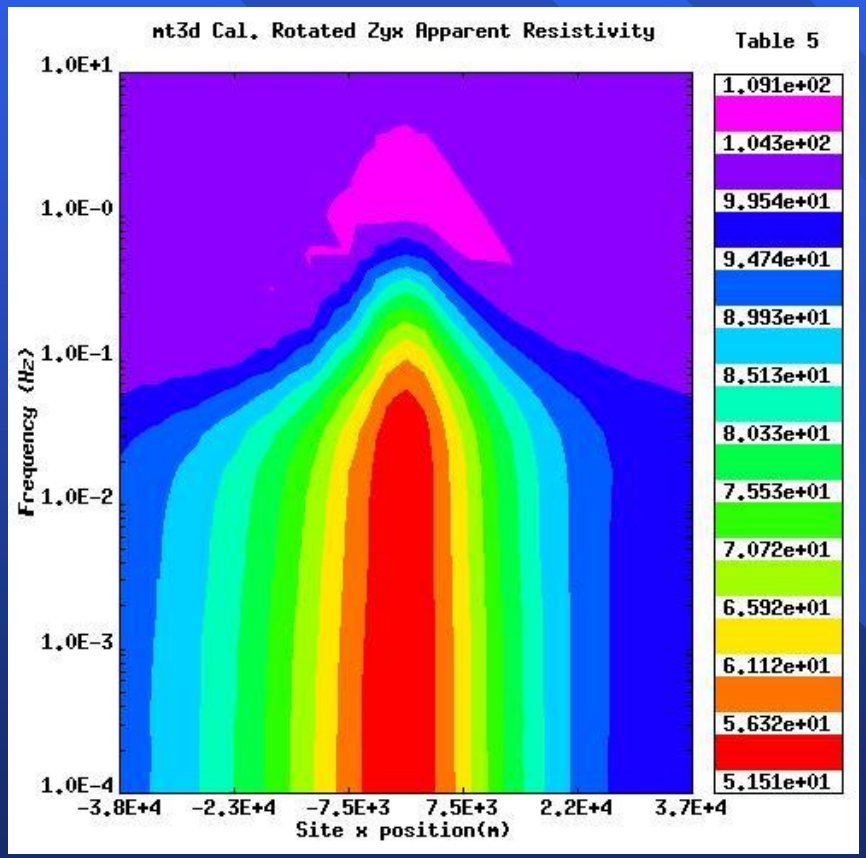
Freq (Hz)	Nodes: (nx	ny	nz)	Δmin(x(m)	y(m)	z(m))	Δmax (x(m)	y(m)	z(m))	time(min:sec)	Krylov Iter.
10	285	207	125	318.1	318.1	71.4	318.1	318.1	285.4	29:17 / 8:50	10000 / 2228
5.5	225	167	125	427.0	427.0	98.0	427.0	427.0	392.0	4:35 / 4:20	1808 / 2002
3.1	181	137	125	569.1	569.1	134.4	569.1	569.1	537.5	2:55 / 3:23	1648 / 2451
1.8	149	117	119	752.8	752.8	184.0	752.8	752.8	735.9	1:52 / 2:19	1605 / 2143
1	125	101	105	1000	1000	250.0	1000	1000	1000	0:52 / 0:40	1108 / 801
5.5e-1	143	117	119	1000	1000	250.0	1000	1000	1000	1:33 / 1:13	1336 / 1064
3.1e-1	125	101	105	1000	1000	250.0	1000	1000	1000	2;45 / 2:33	1637 / 1569
1.8e-1	143	117	113	1000	1000	250.0	1000	1000	1000	3;33 / 4:24	1601 / 2264
1e-1	161	139	135	1000	1000	250.0	1000	1000	1000	5:44 / 5:03	2721 / 2338
5.5e-2	161	165	145	1000	1000	250.0	1000	1000	1000	5:30 / 4:37	2665 / 2261
3.1e-2	161	171	145	1000	1000	250.0	1000	1000	1000	4:55 / 4:02	2482 / 1715
1.8e-2	161	171	145	1000	1000	250.0	1000	1000	1000	3:48 / 4:13	1667 / 1835
1e-2	161	171	145	1000	1000	250.0	1000	1000	1000	3:49 / 4:19	1682 / 1952
5.5e-3	161	171	145	1000	1000	1000	1000	1000	1000	3:58 / 3:23	1766 / 1448
3.12e-3	161	171	145	1000	1000	1000	1000	1000	1000	4:24 / 3:57	1842 / 1756
1.78e-3	161	171	145	1000	1000	1000	1000	1000	1000	3:01 / 3;37	1502 / 1291
1e-3	161	171	145	1000	1000	1000	1000	1000	1000	2:43 / 2:44	1190 / 1196
5.5e-4	161	171	145	1000	1000	1000	1000	1000	1000	2:28 / 2:26	1000 / 1093
3.12e-4	161	171	145	1000	1000	1000	1000	1000	1000	1:48 / 2:11	742 / 909
1.8e-2	161	171	145	1000	1000	1000	1000	1000	1000	3:20 / 1:46	1173 / 704
1e-4	161	171	145	1000	1000	1000	1000	1000	1000	2:47 / 1:45	986 / 697

Solution tolerance 1e-12; 10000 maximum Krylov iterations; run times based on Intel Xeon CPU (3.6 GHz); infinband interconnect
Total run time 2 hr 49 min; model dcomposed over 64 cores, data parallelization not used.

Z_{yx} Apparent Resistivity



100 Ω.m Half Space BC



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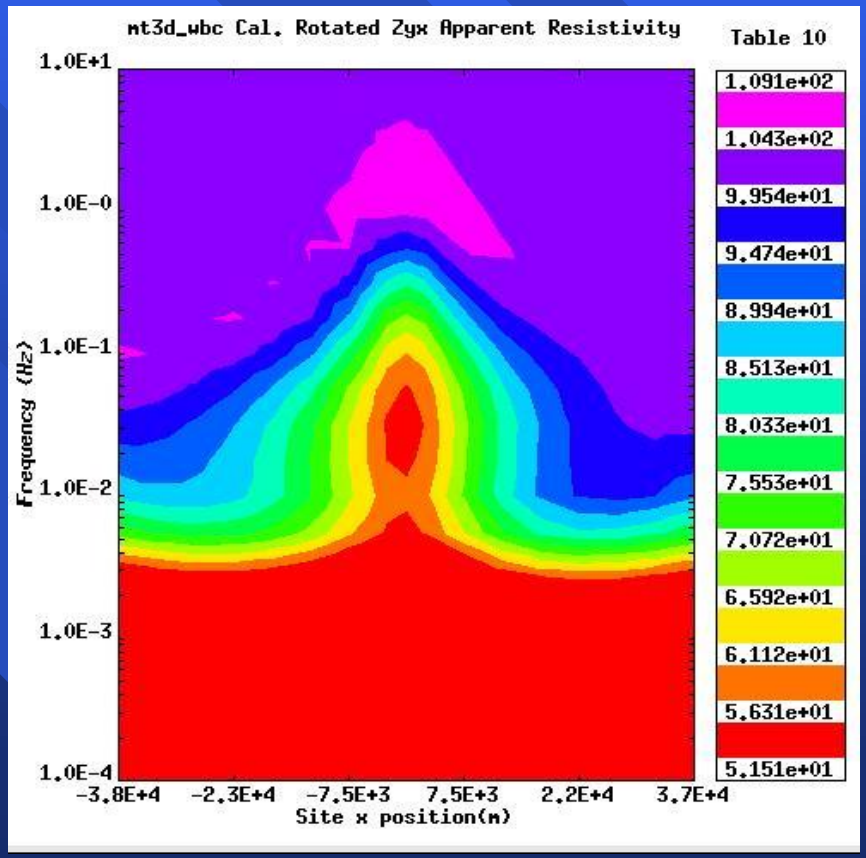

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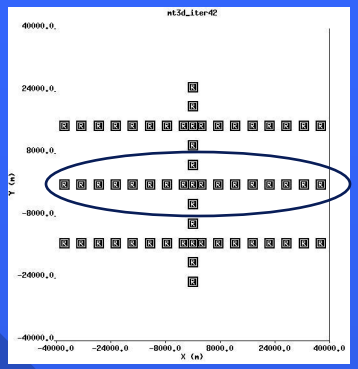

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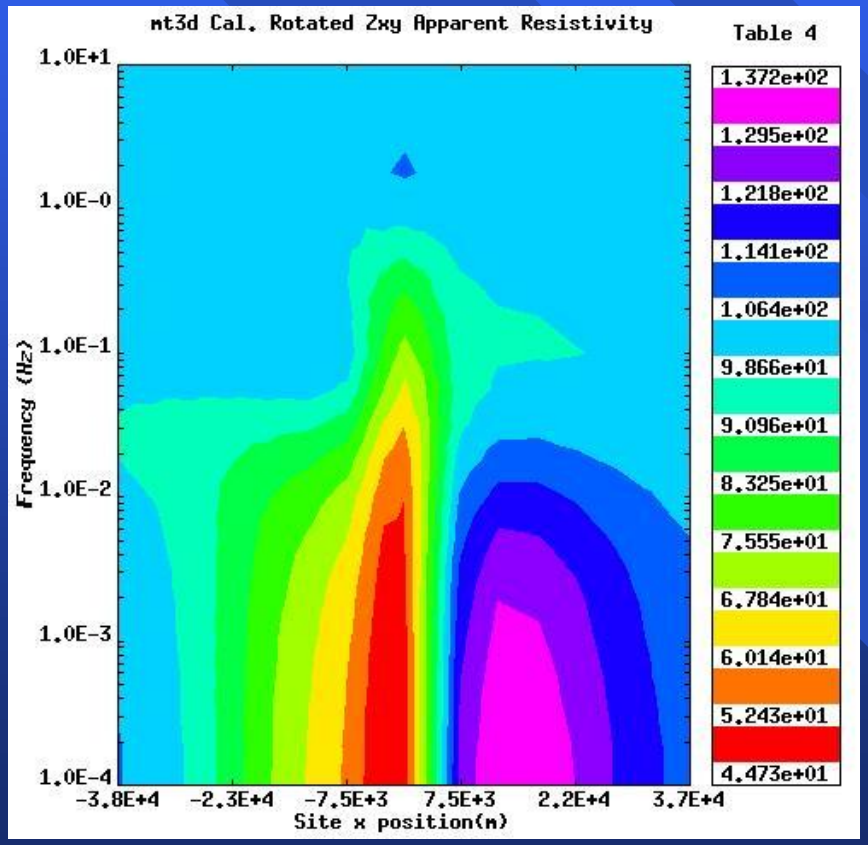

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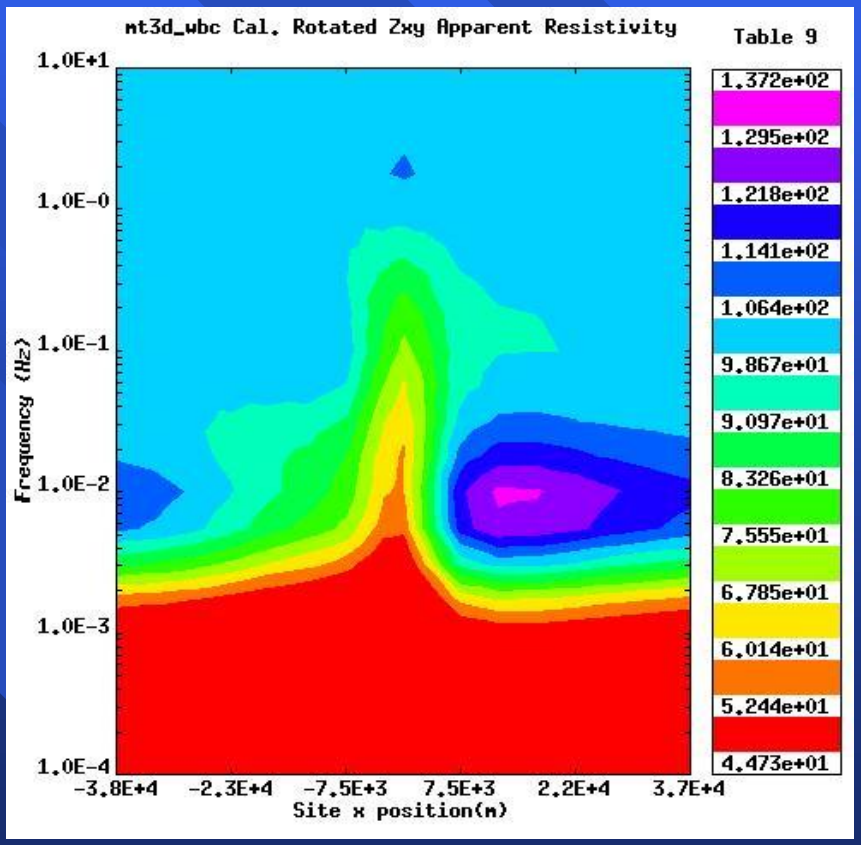
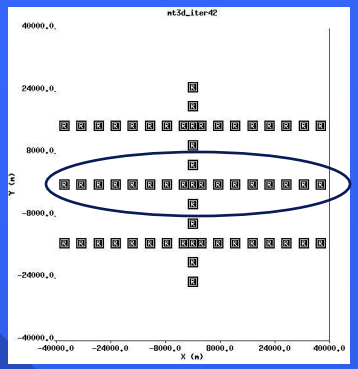


Z_{yx} Impedance Phase



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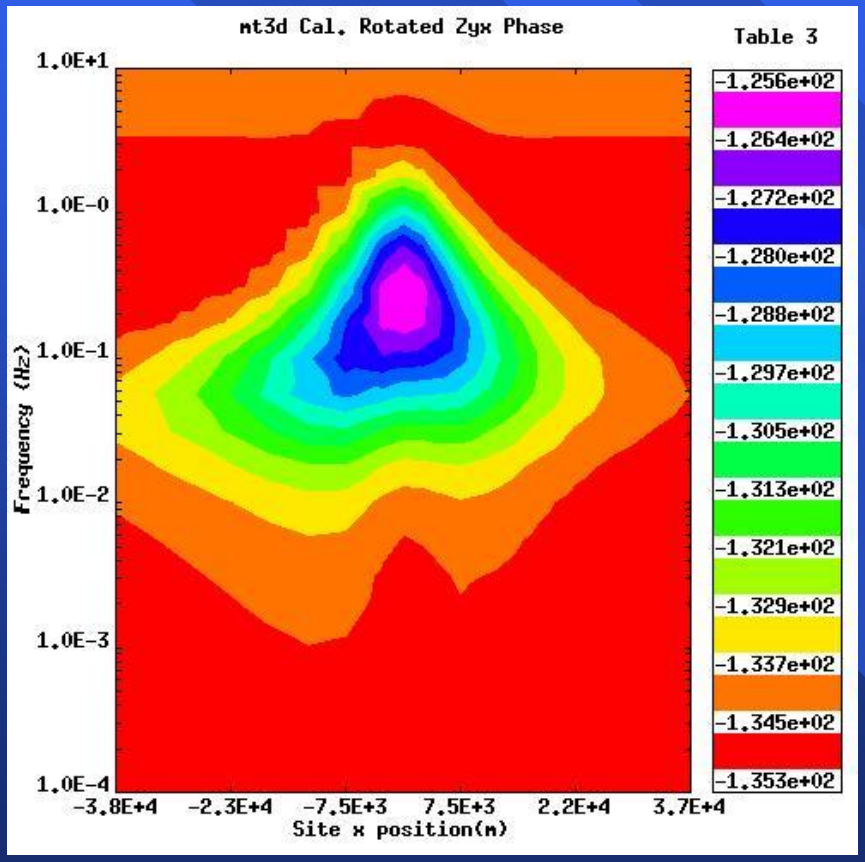

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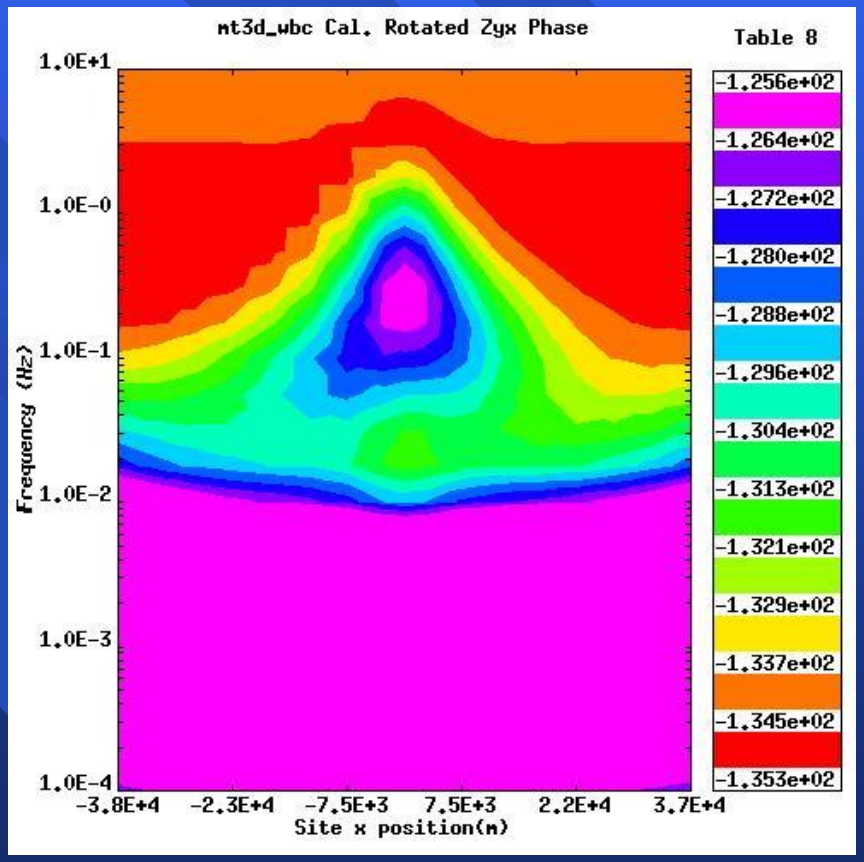

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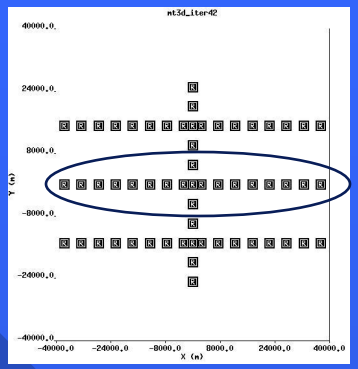

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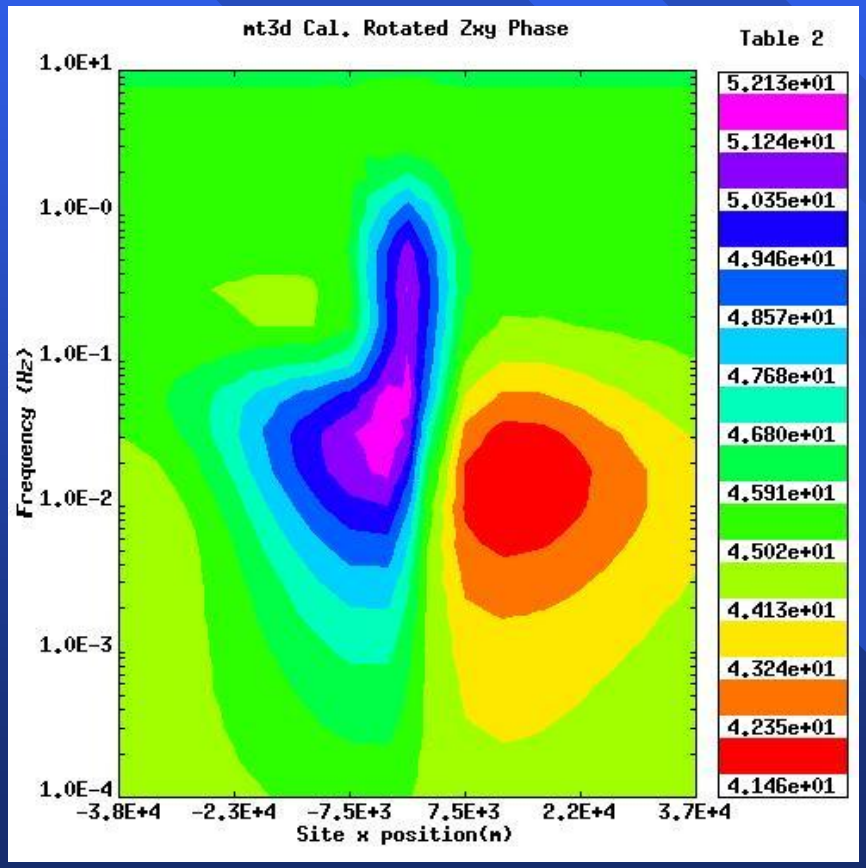

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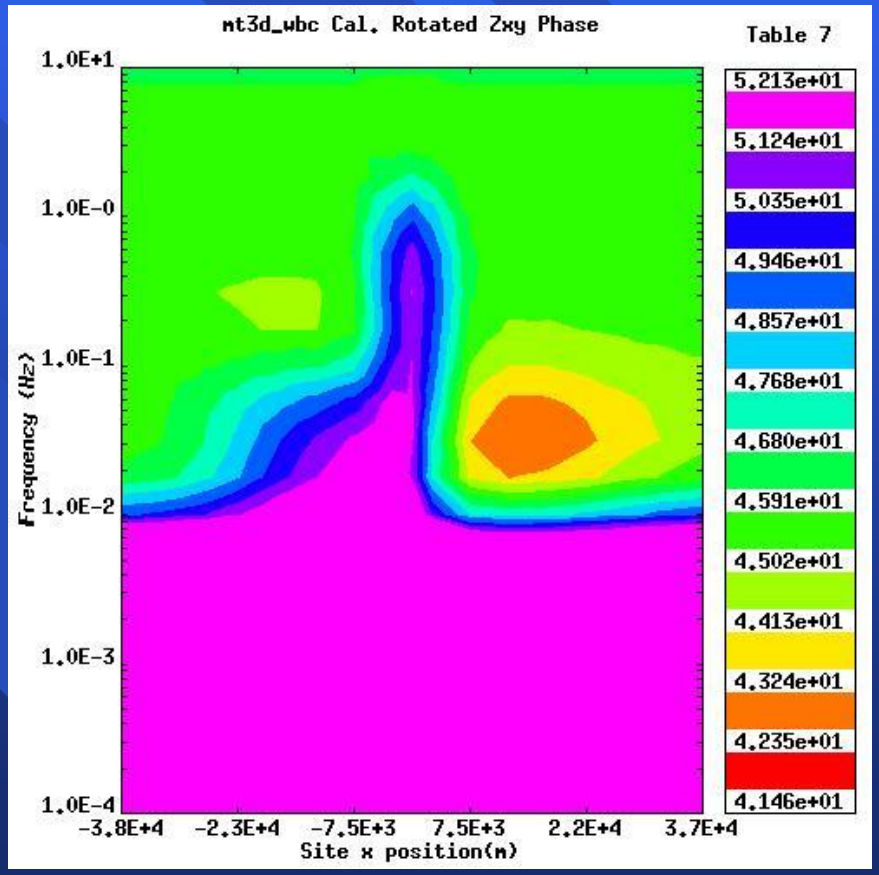

Survey Layout

in my coordinate system