MT3D_INV:
A 3D MT Inversion Algorithm

MT3D INVERSION WORKSHOP
Dublin Institute of Advanced Studies
12-14 March 2008

Gregory A. Newman & Michael Commer
Earth Sciences Division
Lawrence Berkeley National Laboratory
Berkeley California

Inverse Problem Formulation

Minimize the cost functional:

$$\varphi = \sum_{j=1}^{2N} \{(Z_j^{obs} - Z_j^{p}) D_j\}^2 + \lambda\, \mathbf{m}\, \mathbf{W}^T \mathbf{W}\, \mathbf{m}.$$

$\mathbf{Z}^{obs}$ and $\mathbf{Z}^{p}$ are N observed and predicted impedances

D= data weights

m = M conductivity model parameters.

W = ∇^2 operator; constructs a smooth model

λ = tradeoff parameter

Non-Linear Conjugate Gradients

We need the gradient of the cost functional

$$\nabla_m \varphi = \nabla \varphi_d + \nabla \varphi_m$$

Ability to determine a scalar α such that

$$\varphi(\mathbf{m} + \alpha \mathbf{p})$$

is minimized along the conjugate search direction $\mathbf{p}$

Computational Efficiencies

- Gradient requires 4 applications of the forward code at each frequency
- Line search usually requires 2 forward modeling applications at each frequency
- Typically four forward modeling applications per frequency needed per inversion iteration
- Ideal method for problems with extremely large data sets and model parameterizations

Algorithm implemented on the Franklin-Cray XT4 machine at NERSC: 9660 nodes/19320 cores

An Iterative Solution

Make initial model guess

Select tradeoff parameter λ

During the iteration process λ is reduced

(this can help accelerate convergence)

Dublin Model Example

Inversion launched with 100 Ω.m half-space

9912 data points (Z_{xx},Z_{xy},Z_{yx},Z_{yy})

used all 21 frequencies; 59 detector locations

Data weighting:

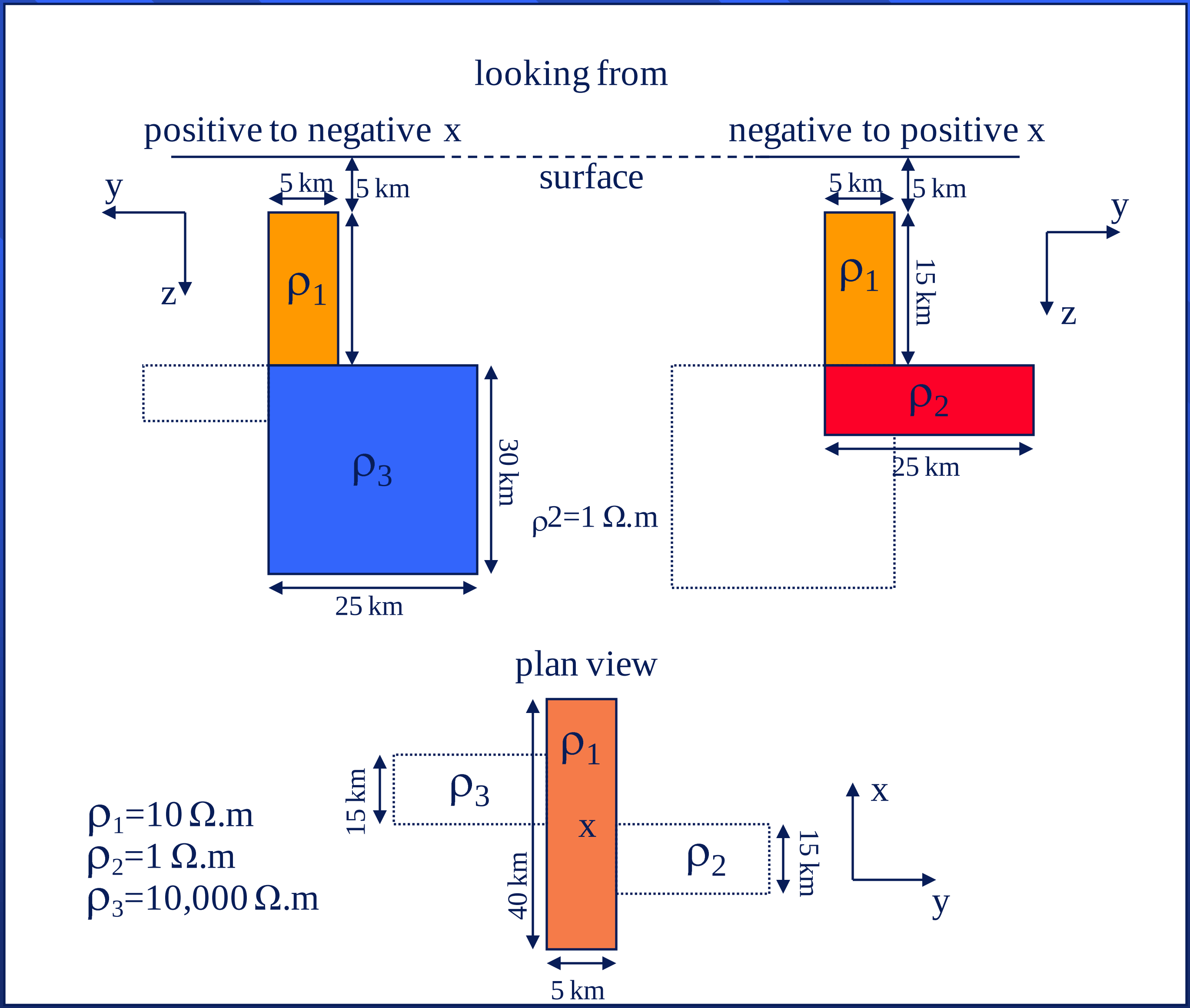
Z_{xx} weights => 5% ‖ Z_{xy} ‖ Z_{xy} weights => 5% ‖ Z_{xy} ‖

Z_{yx} weights => 5% ‖ Z_{yx} ‖ Z_{yy} weights => 5% ‖ Z_{yy} ‖

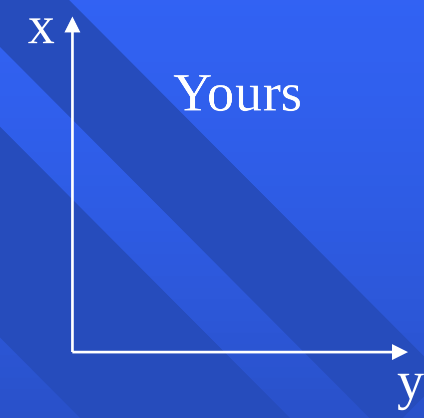
~ 10^7 resistivity parameters imaged

Problem solved on 64 cores

Dublin Test Model



Coordinate Systems

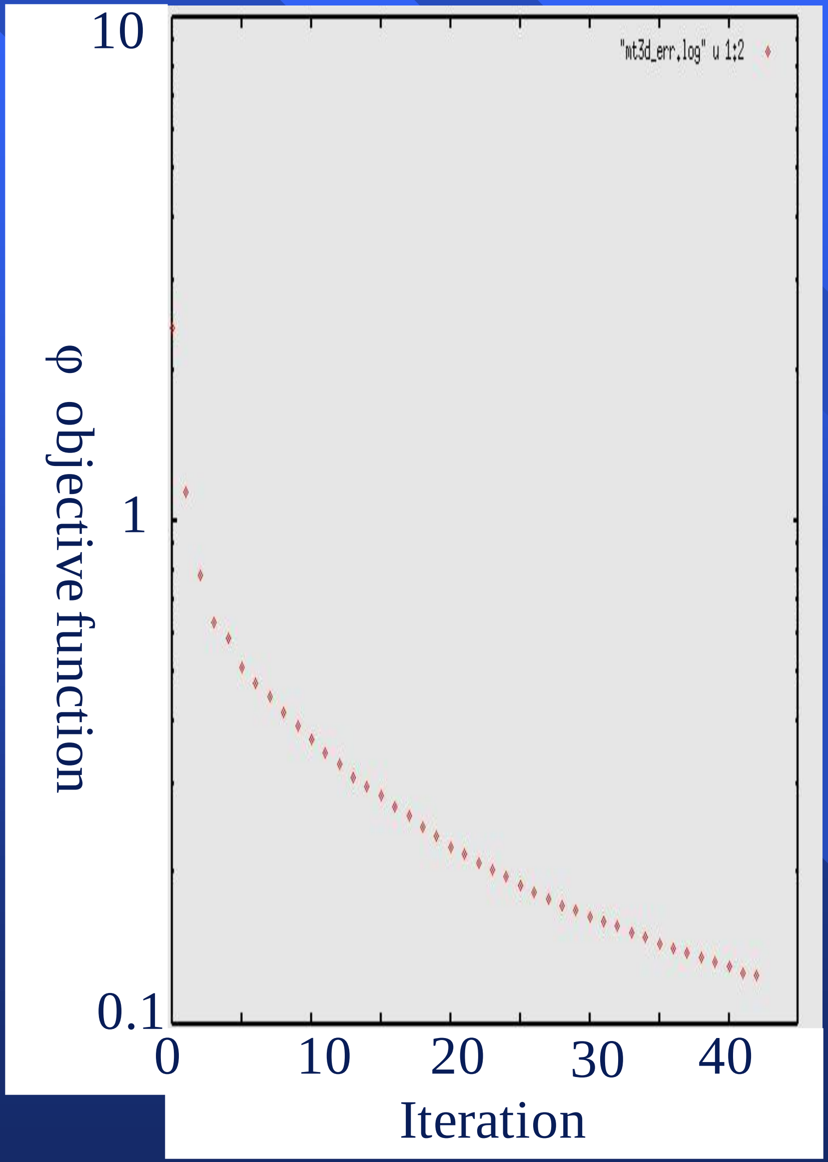


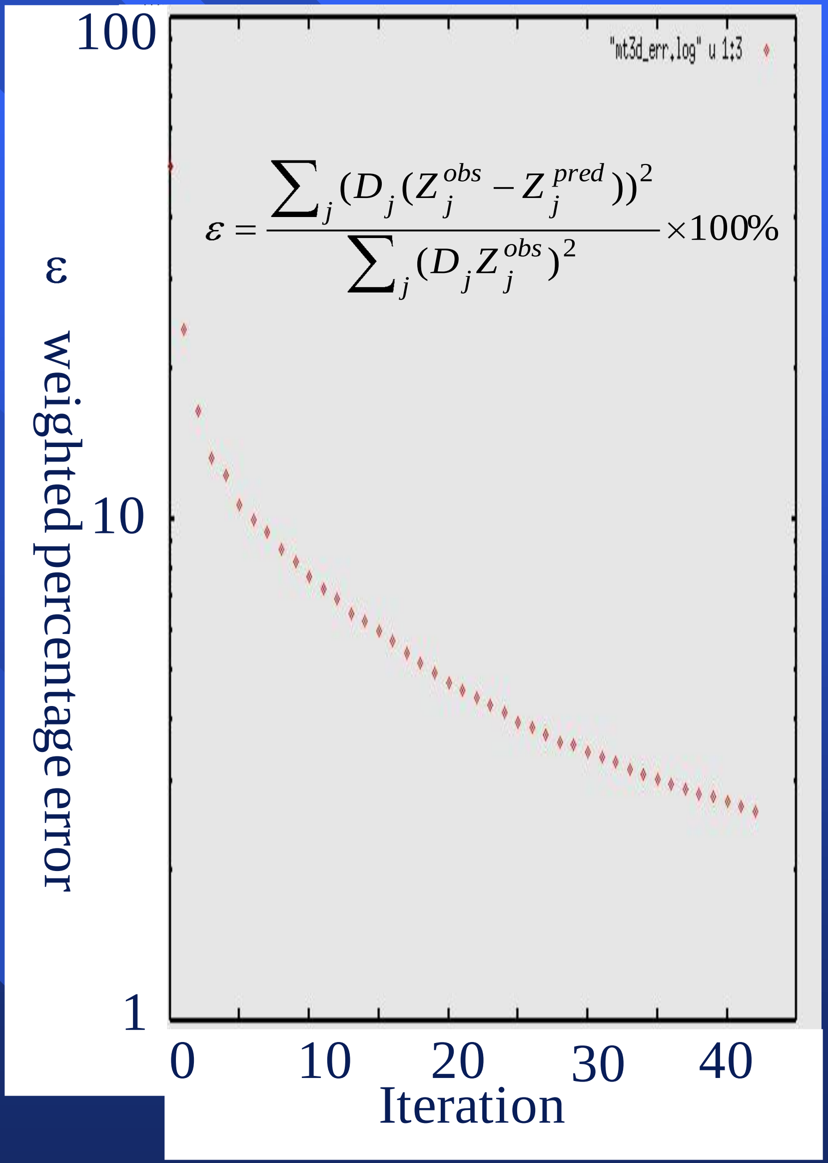
Coordinate systems related to each other by a -90 degree rotation

$$\begin{pmatrix} Z_{yy} & -Z_{yx} \\ -Z_{xy} & Z_{xx} \end{pmatrix}_{\text{yours}} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix}_{\text{mine}} \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}_{\varphi=-\frac{\pi}{2}}$$

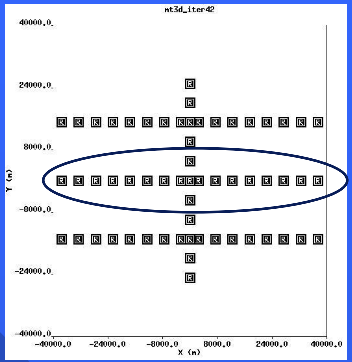
z positive downward for both

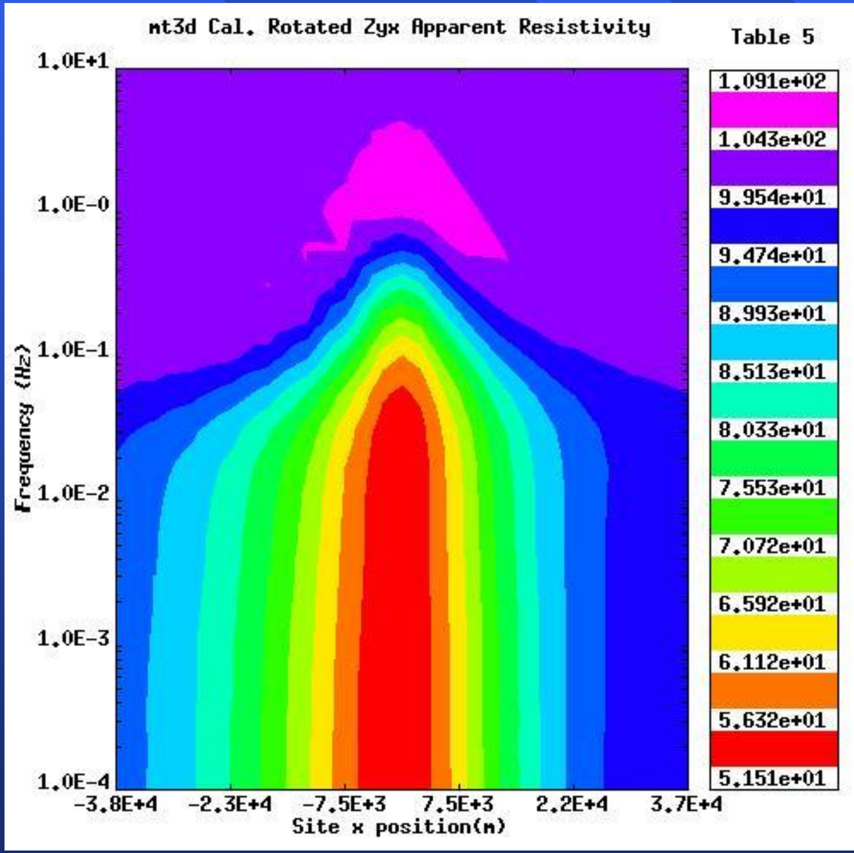
Inversion Metrics

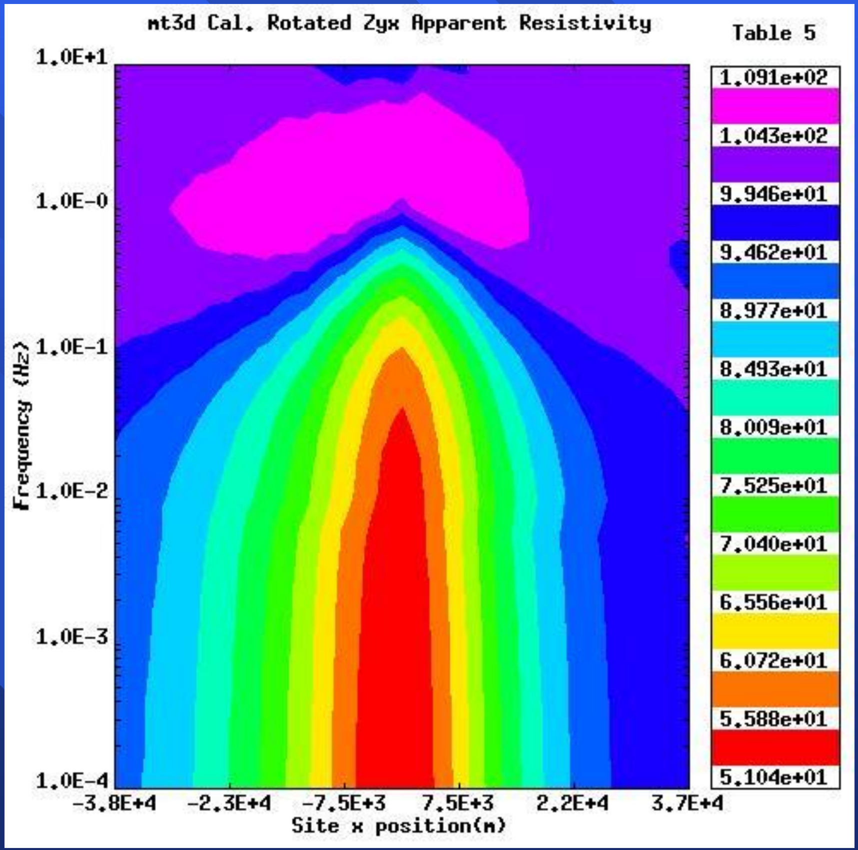




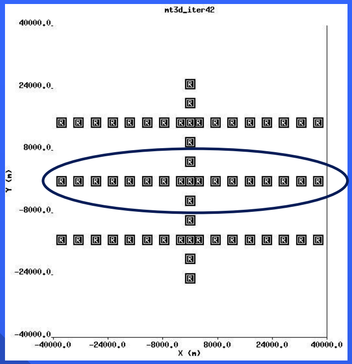
Z_{yx} Apparent Resistivity Fits

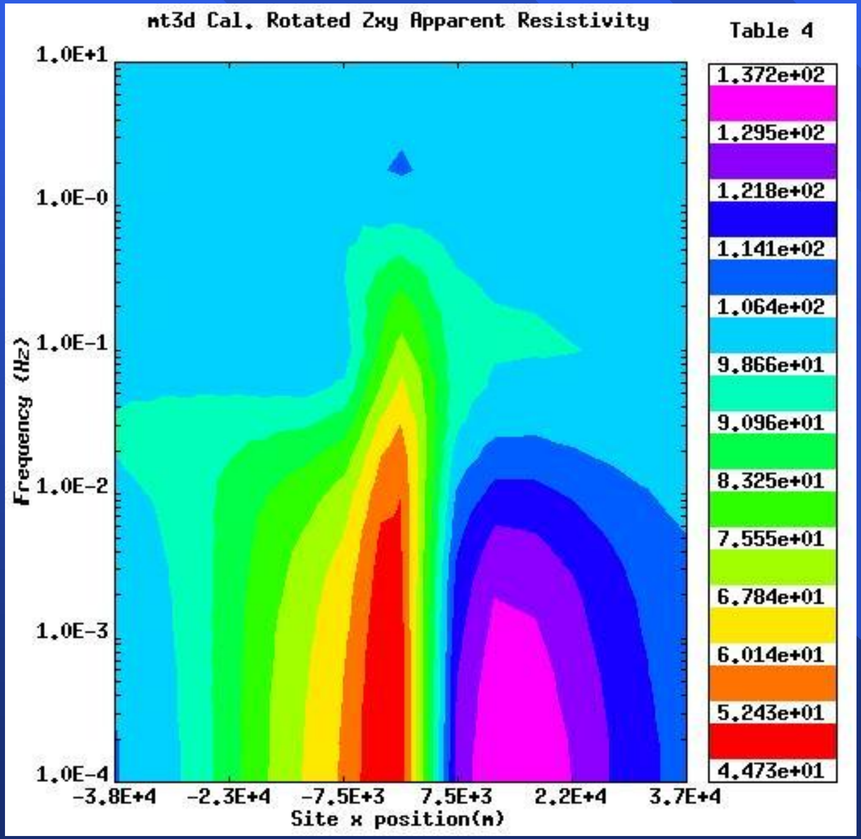


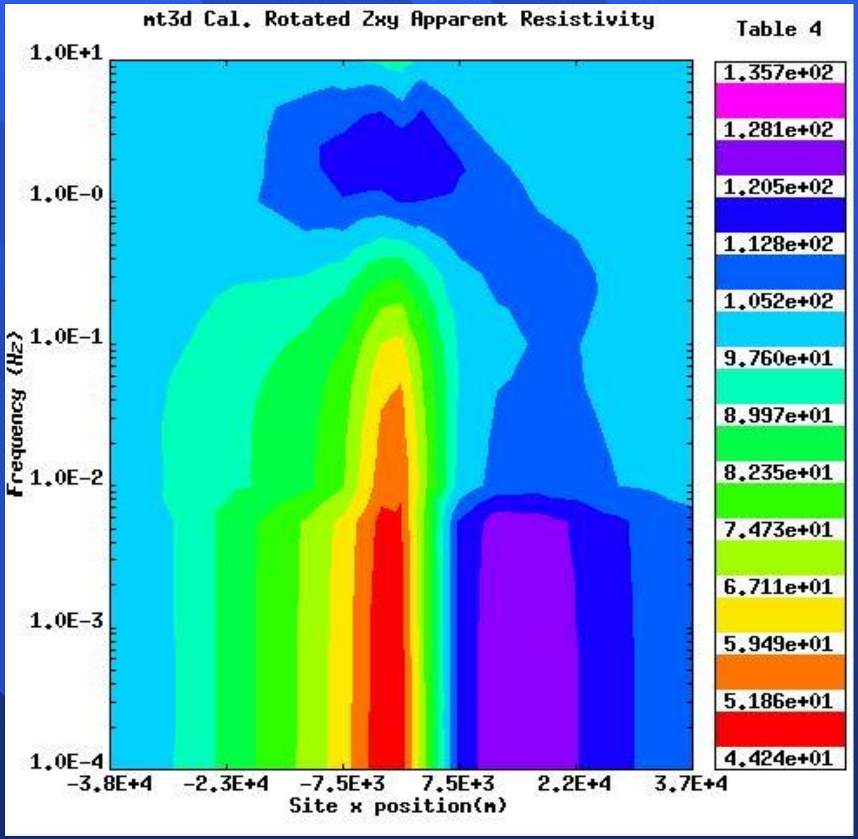




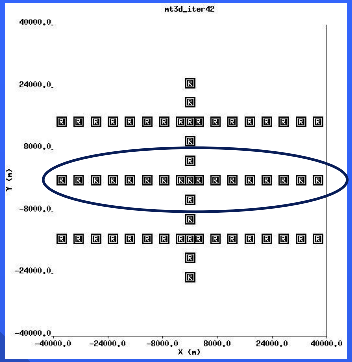
Z_{xy} Apparent Resistivity Fits

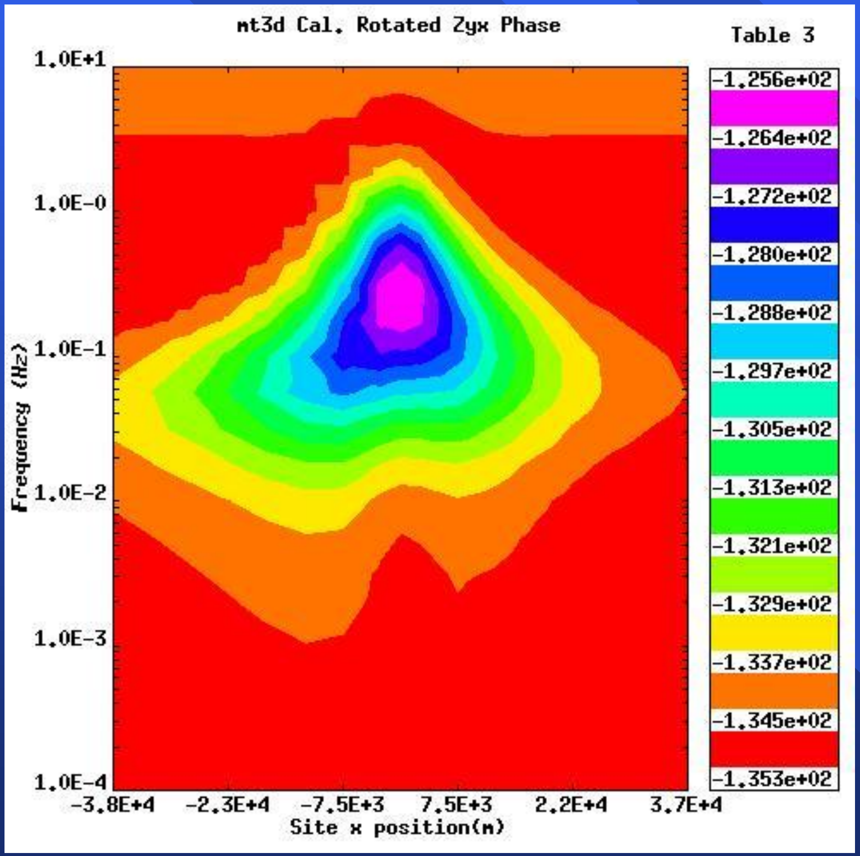


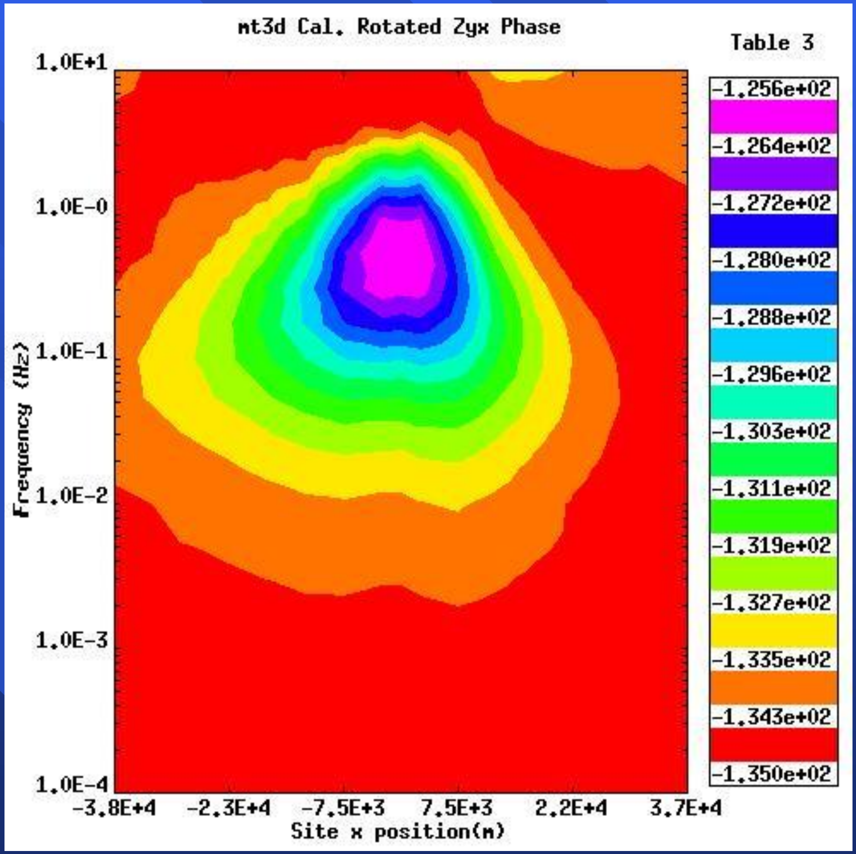




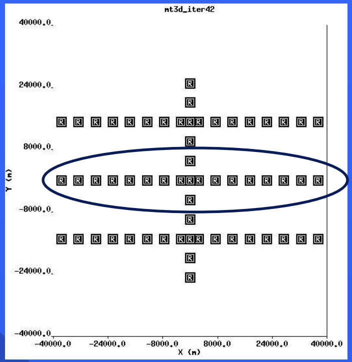
Z_{yx} Impedance Phase Fits

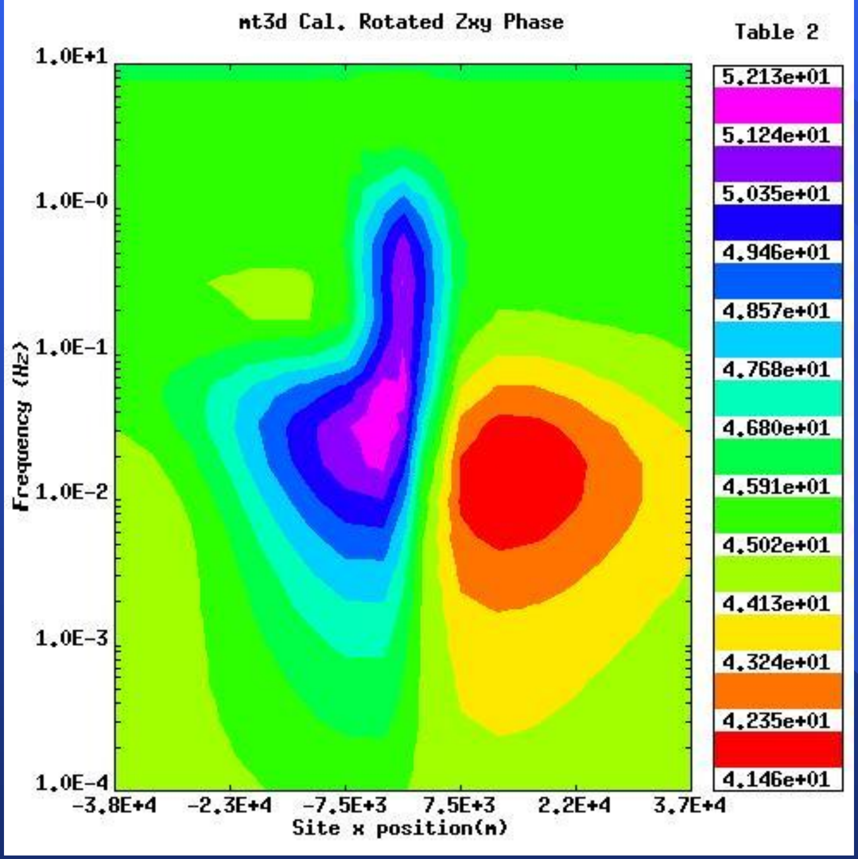


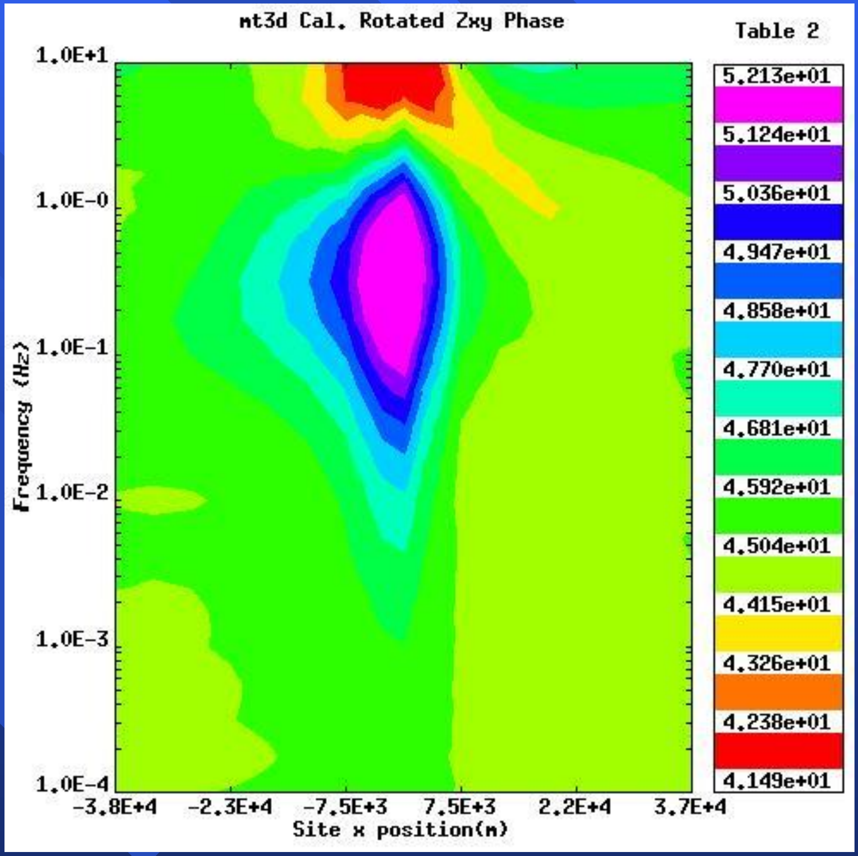




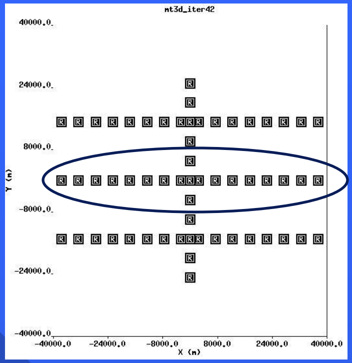
Z_{xy} Impedance Phase Fits

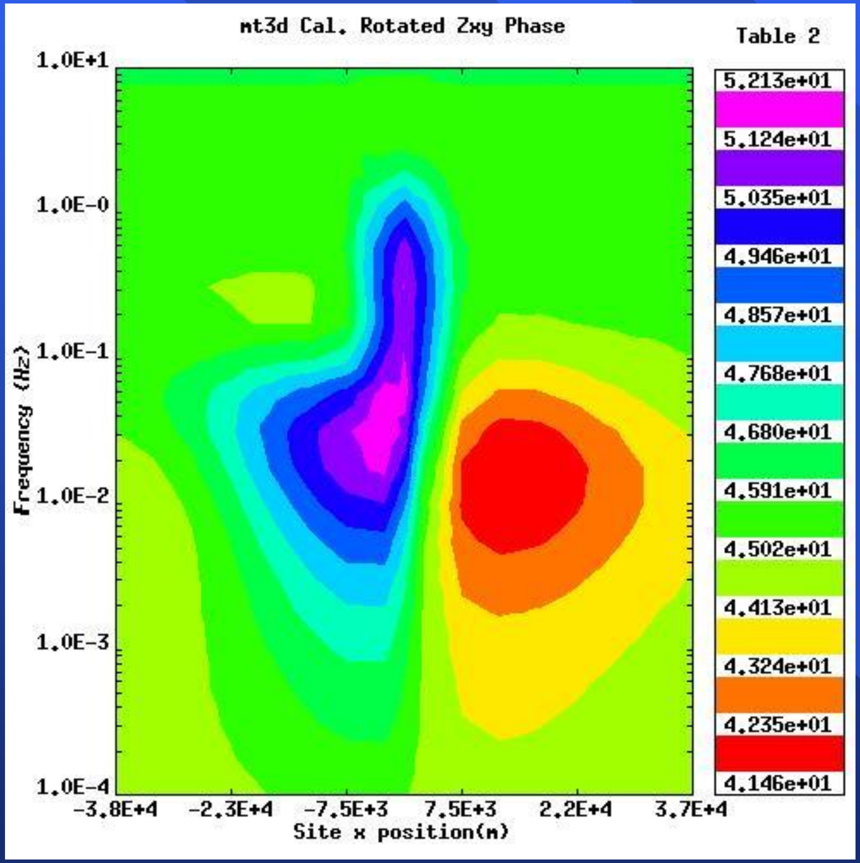


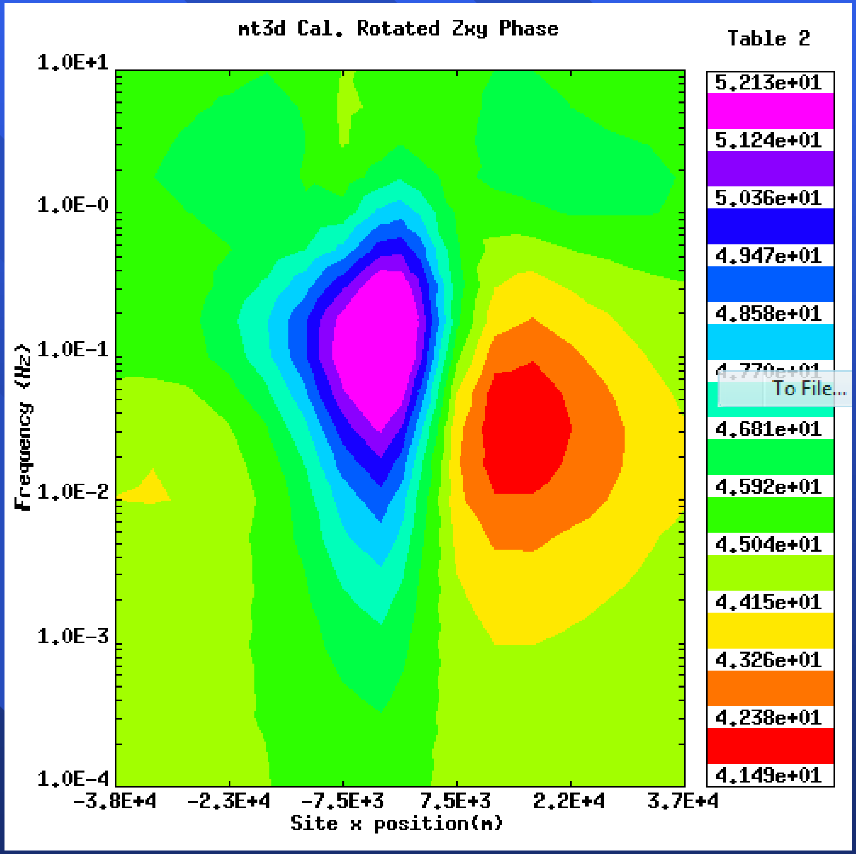




Z_{xy} Impedance Phase Fits

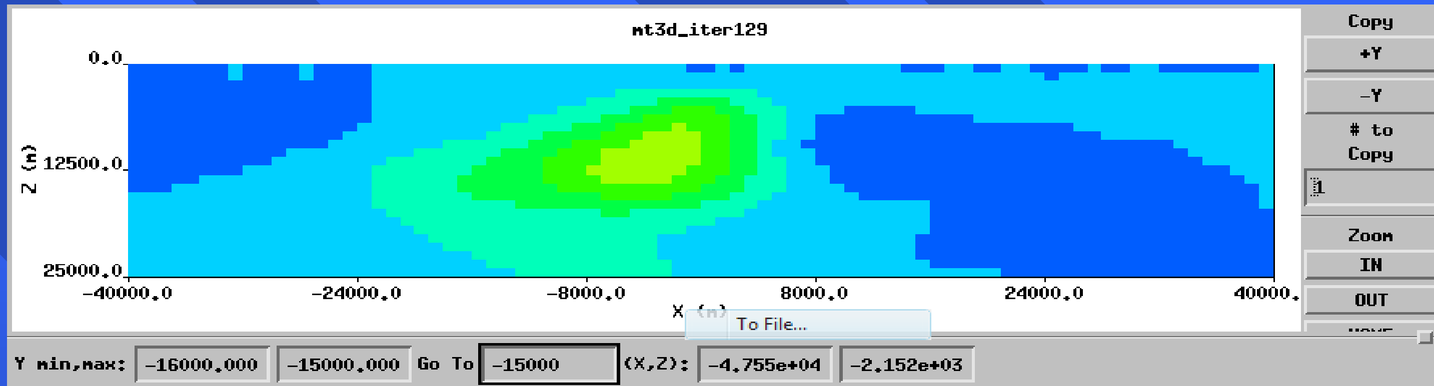




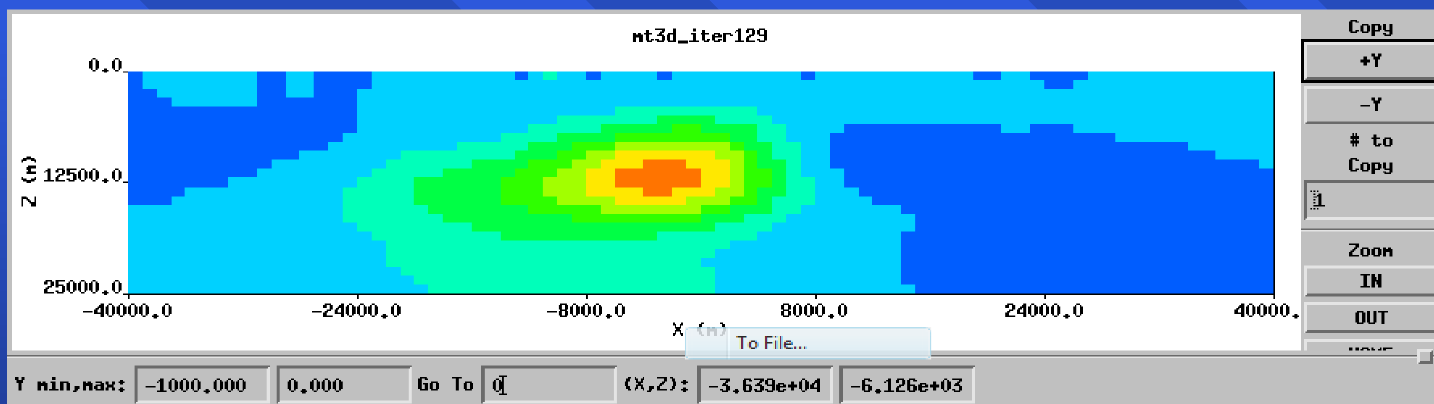


Resistivity Image in Cross Section

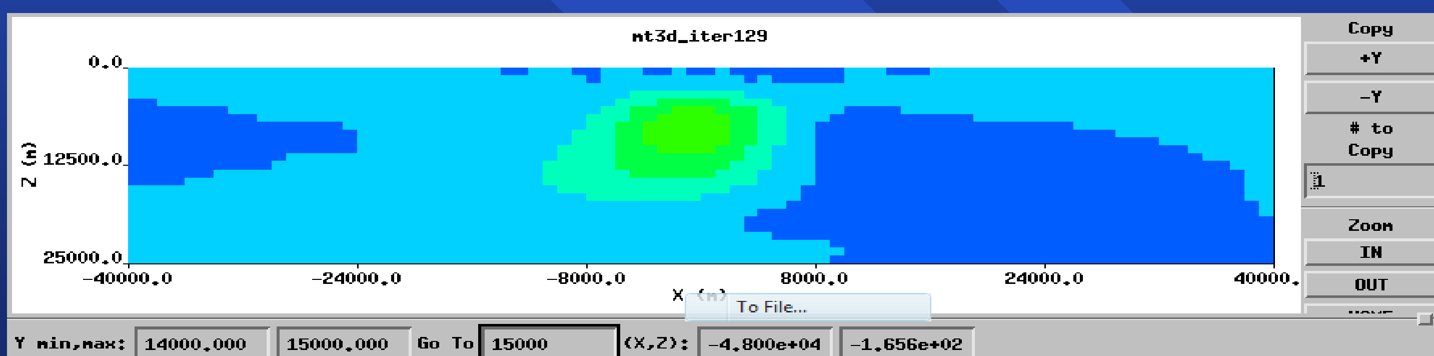
y=-15 km



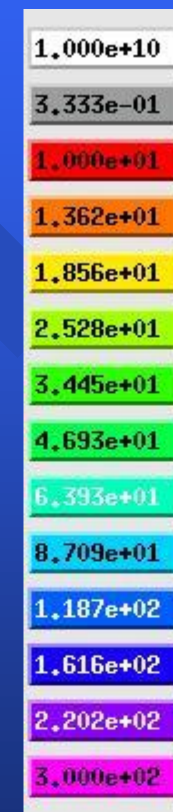
y=0 km



y=15 km



Ω.m



Outstanding Issues

Static Shifts

Survey Aperture & Station Density

Meshing

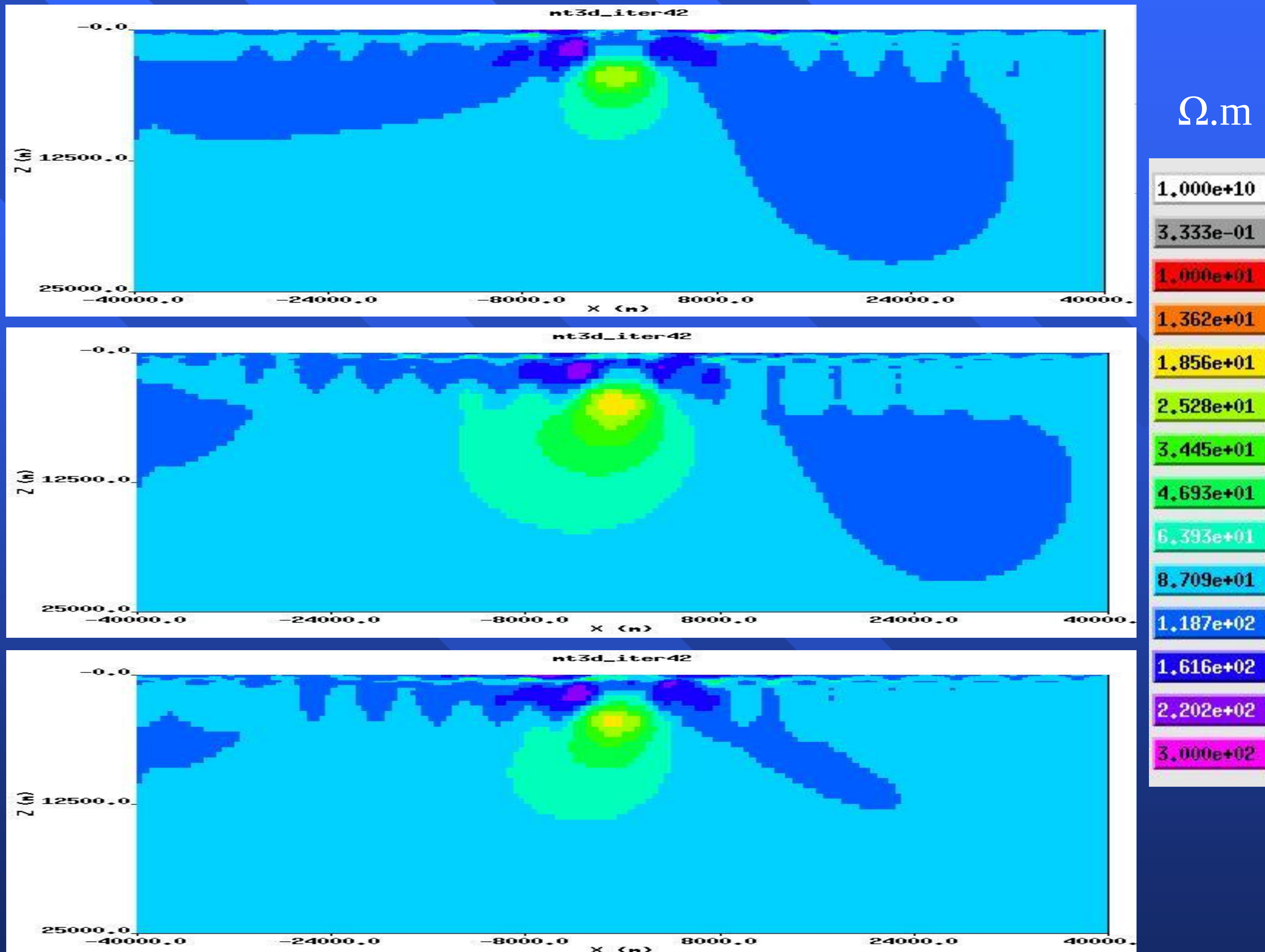
Model Stabilization

Resistivity Image in Cross Section

y=-15 km

y=0 km

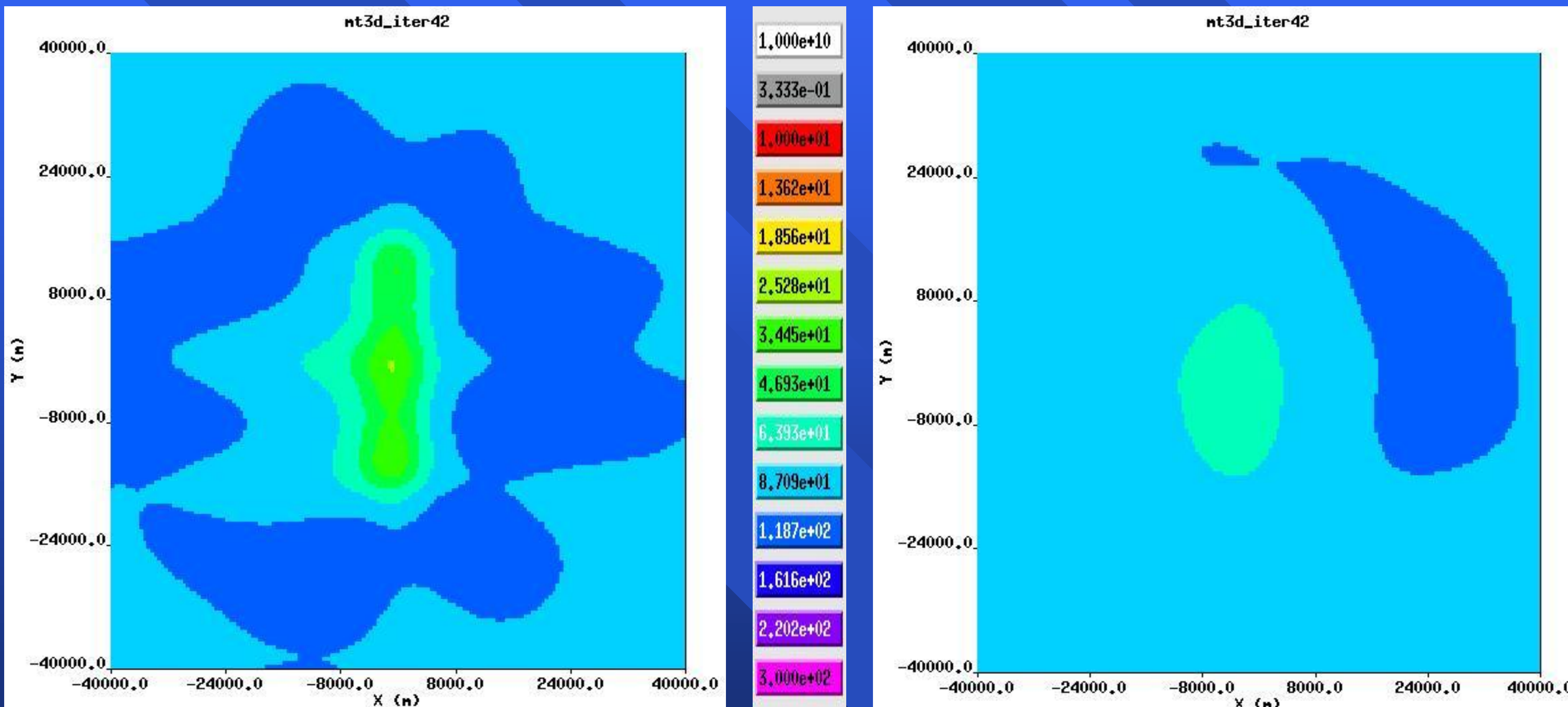
y=15 km



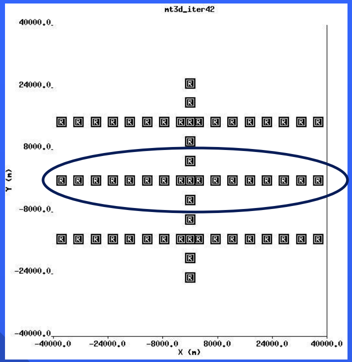
Resistivity Image in Depth Section

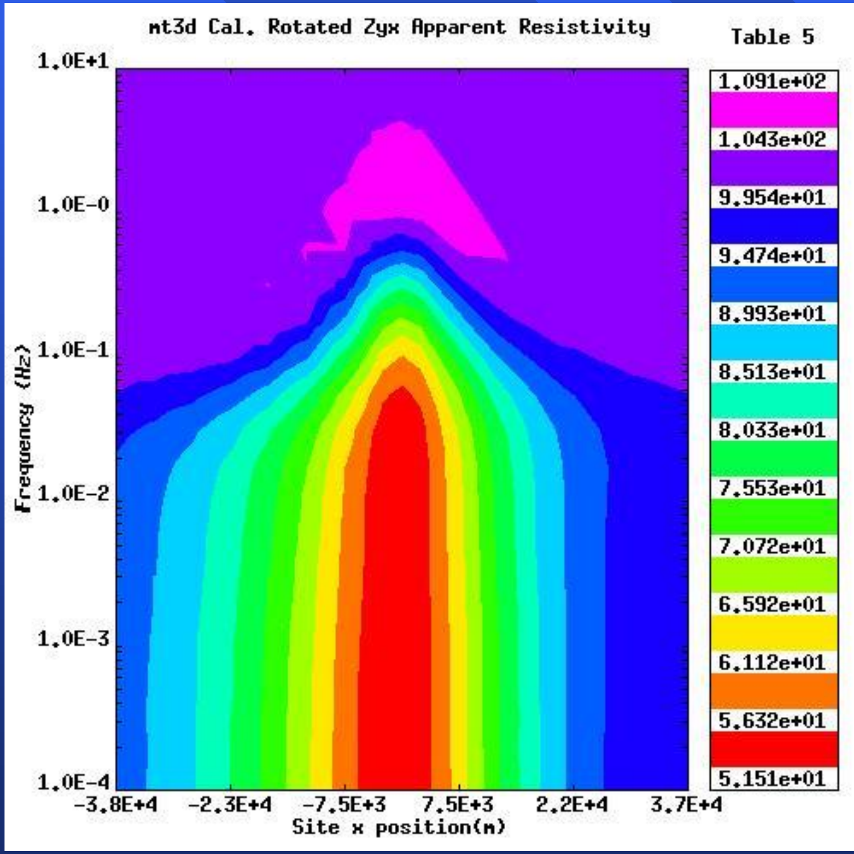
z=7.5 km

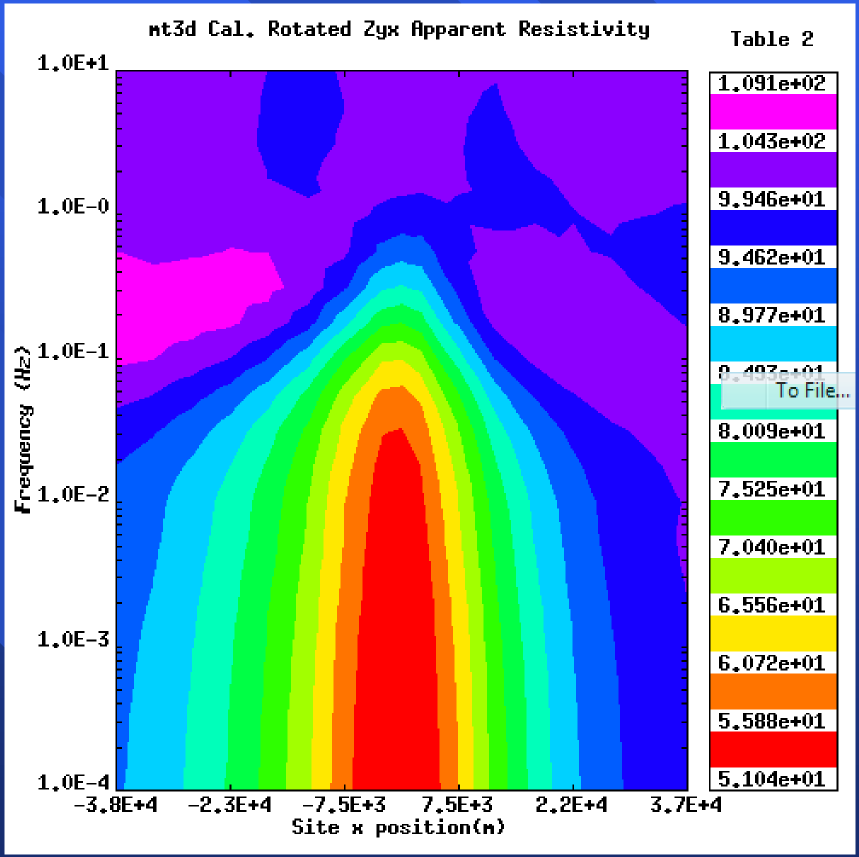
z=15 km



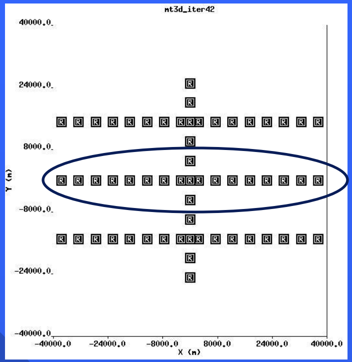
Z_{yx} Apparent Resistivity Fits

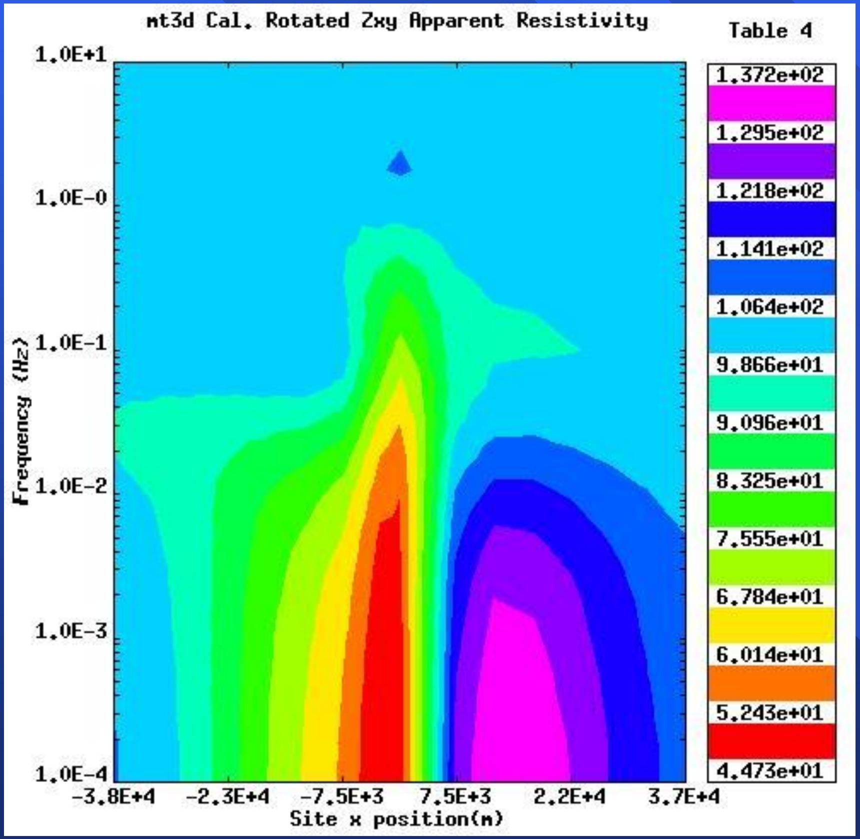


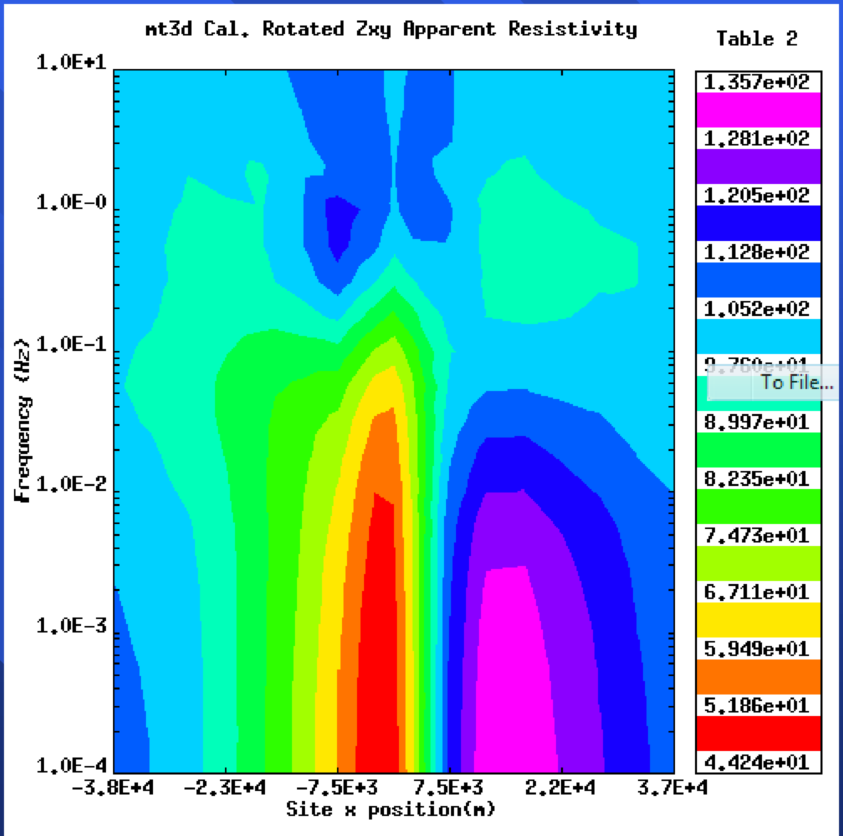




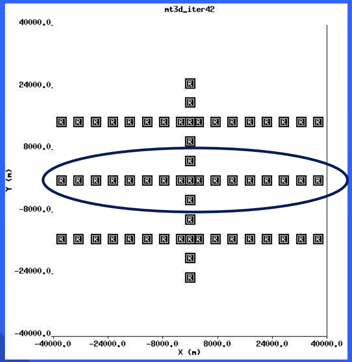
Z_{xy} Apparent Resistivity Fits

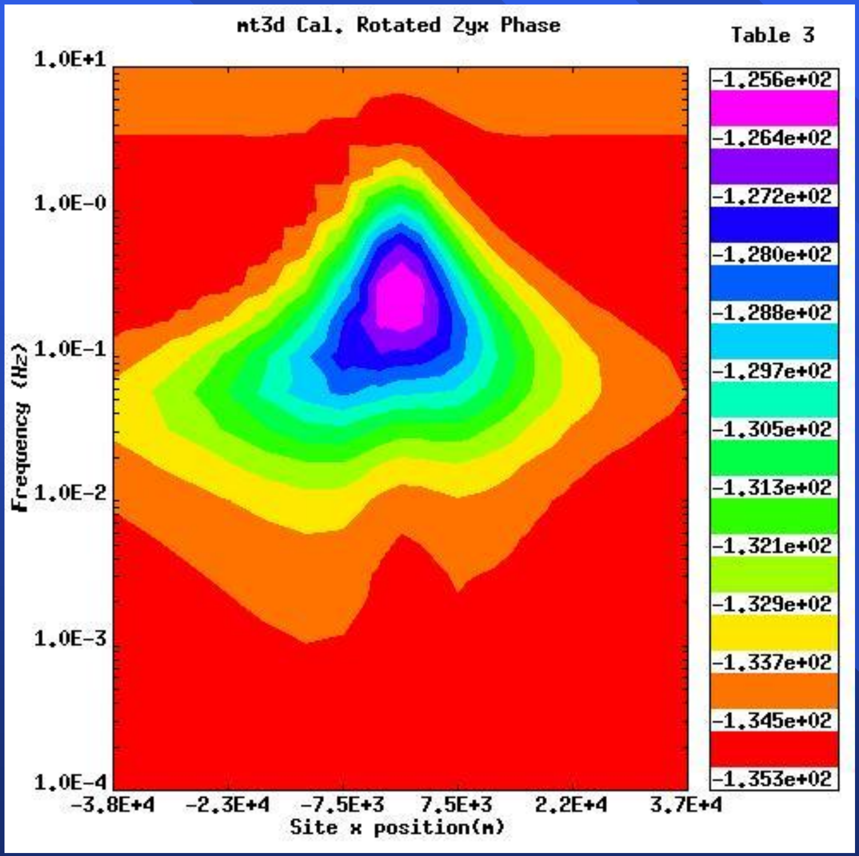


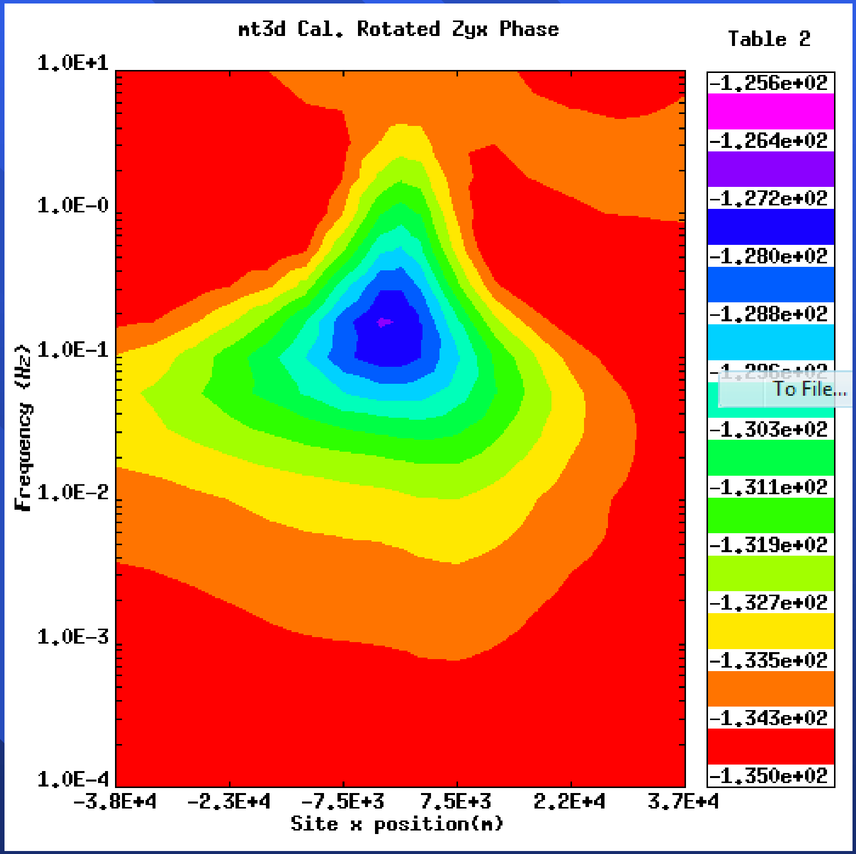




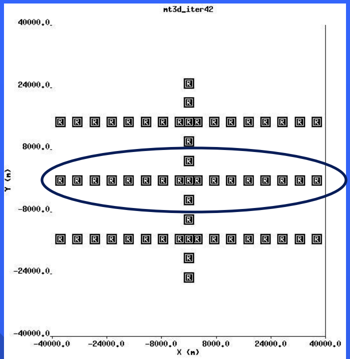
Z_{yx} Impedance Phase Fits

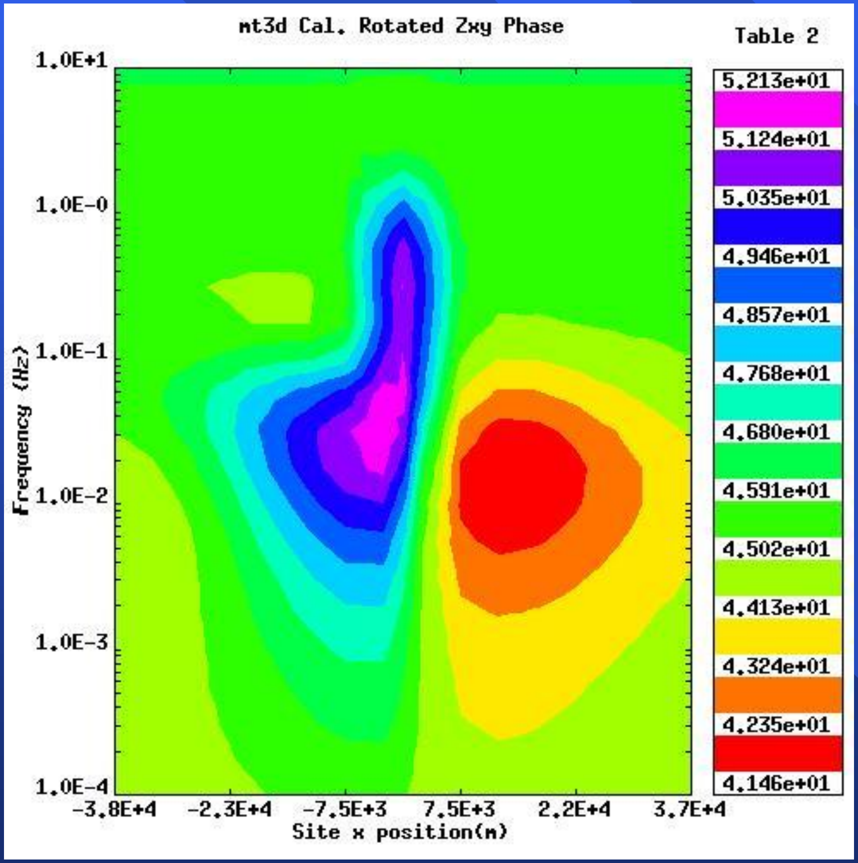


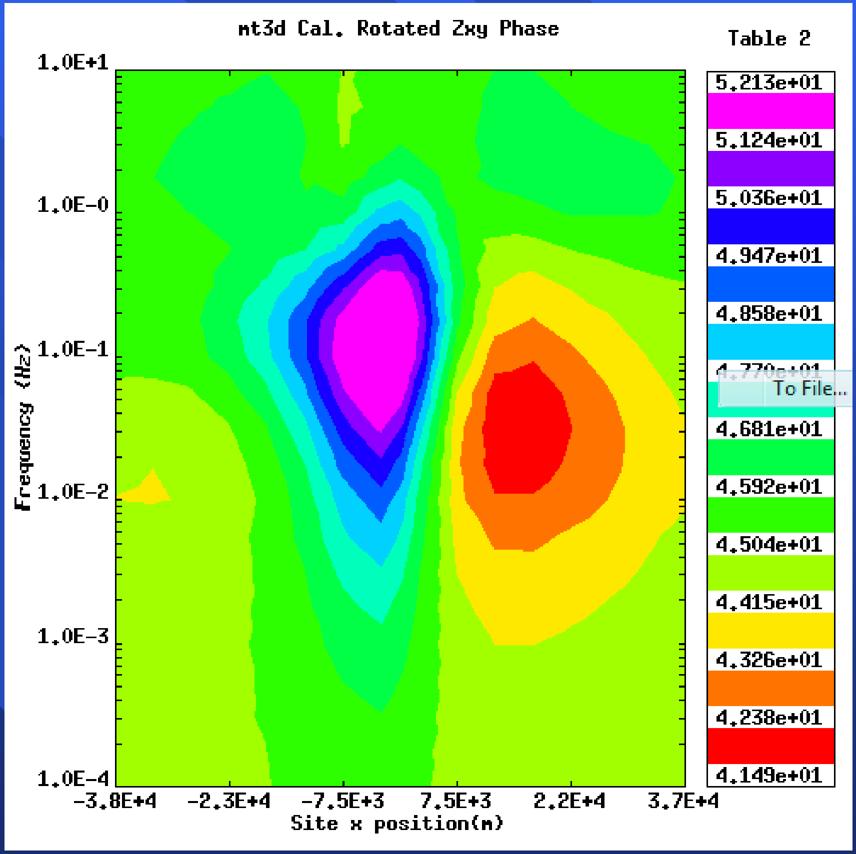




Z_{xy} Impedance Phase Fits

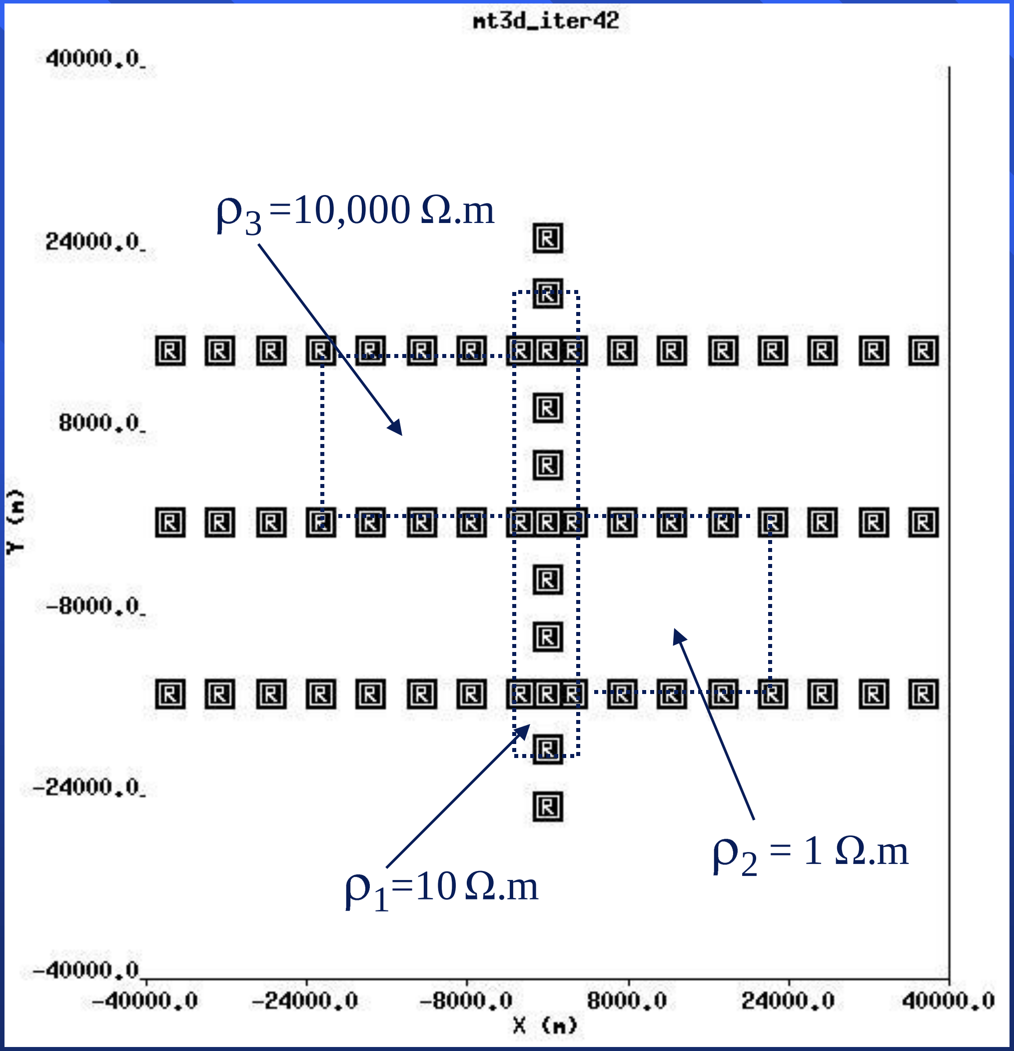






Survey Layout

in my coordinate system



The Forward Problem

The electric field vector equation for MT:

$$\nabla \times \nabla \times \mathbf{e} + i\omega\mu\sigma\ \mathbf{e} = \mathbf{0}.$$

equation approximated on staggered FD grid

The Krylov Solver

assemble complex-symmetric sparse linear system

$$\mathbf{Ke} = \mathbf{s}$$

s depends on MT source polarization

(two for each frequency)

solve system with iterative Krylov methods

(also employ static divergence correction to improve convergence)

magnetic field then determined from Faraday's law

Computation of the Gradients

Evaluation of $\nabla\varphi_m$ leads to

$$\nabla\varphi_m = 2\,\mathbf{W}^t\,\mathbf{W}\,\mathbf{m}.$$

Computation of the Gradients ….

As for $\nabla\varphi_d$, let $\Delta Z_j = \{(Z_j^{obs} - Z_j^{p})/\varepsilon^2_j\}$ be complex,

$$\partial\varphi_d/\partial m_k = -2\,\mathrm{Re}\sum_{j=1}^{N} (\Delta Z_j)^* \,\partial Z^p_j/\partial m_k,$$

or

$$\partial\varphi_d/\partial m_k = 2\,\mathrm{Re}\sum_{j=1}^{N} (\Delta Z_j)^* \,{}^1\mathbf{g}_j^t\,\mathbf{K}^{-1}(\partial\mathbf{K}/\partial m_k\,\mathbf{E}_1) +$$

$$2\,\mathrm{Re}\sum_{j=1}^{N} (\Delta Z_j)^* \,{}^2\mathbf{g}_j^t\,\mathbf{K}^{-1}(\partial\mathbf{K}/\partial m_k\,\mathbf{E}_2).$$

* stands for complex conjugation.

The Non Linear CG Algorithm

(1) Choose $\mathbf{m}_{(1)}$ and select $\mathbf{p}_{(1)} = -M_{(1)}{}^{-1} \nabla\varphi\,(\mathbf{m}_{(1)})$

(2) find $\alpha(i)$ that minimizes $\varphi\,(\mathbf{m}_{(i)} + \alpha(i)\,\mathbf{p}_{(i)})$

(3) set $\mathbf{m}_{(i+1)} = \mathbf{m}_{(i)} + \alpha(i)\,\mathbf{p}_{(i)}$ and $\mathbf{r}_{(i+1)} = -\nabla\varphi\,(\mathbf{m}_{(i+1)})$

(4) $\beta(i+1) = \{(\mathbf{r}_{(i+1)}{}^{t} M_{(i+1)}{}^{-1}\,\mathbf{r}_{(i+1)} - \mathbf{r}_{(i+1)}\,M_{(i+1)}{}^{-1\,t}\,\mathbf{r}_{(i)}) / \mathbf{r}_{(i)}{}^{t} M_{(i+1)}{}^{-1}\,\mathbf{r}_{(i)}\,\}$

(5) $\mathbf{p}_{(i+1)} = M_{(i+1)}{}^{-1}\,\mathbf{r}_{(i+1)} + \beta_{(i+1)}\,\mathbf{p}_{(i)}$

(6) stop when $|\,\mathbf{r}_{(i+1)}\,| < \delta$, otherwise go to (2).