Radiomagnetotelluric 2D forward and inverse modelling with displacement currents

Thomas Kalscheuer[1] Laust B. Pedersen[2]
Weerachai Siripunvaraporn[3]

[1]Department of Earth Sciences
ETH Zürich, Switzerland

[2]Department of Earth Sciences
Uppsala University, Sweden

[3]Department of Physics, Faculty of Science
Mahidol University, Thailand

3/9/2009

Introduction

Goal: Inclusion of displacement currents in a 2D inverse scheme of radiomagnetotelluric (RMT) data with simplifications:

- using vertically incident plane waves and
- assuming constant dielectric permittivity in subsurface.

Method: Modification of routines for forward and sensitivity computations of an existing 2D inverse code (REBOCC) that utilizes finite-difference approach.

Outline

RMT Field Setup I

- Surface measurement of electric and magnetic field components.
- Frequency range typically 10 to 300 kHz.
- Primary signal from remote radio transmitters.
  Hence, plane-wave assumption.

RMT Field Setup II

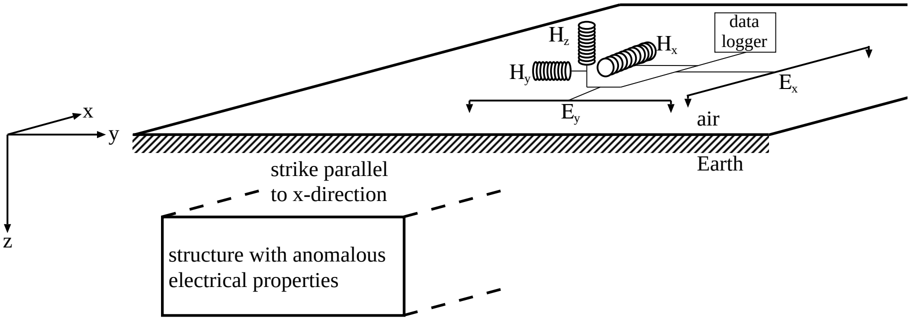


Figure: RMT field setup.

Governing equations

Governing equations:

- In frequency domain:

$$\nabla \times \mathbf{E} = -(i\omega\mu_0)\mathbf{H} \qquad \text{Faraday's law} \qquad (1)$$

$$\nabla \times \mathbf{H} = (\sigma + i\omega\epsilon)\,\mathbf{E} \qquad \text{Ampere's law} \qquad (2)$$

- Conduction currents: $\sigma\mathbf{E}$.
  Displacement currents: $i\omega\epsilon\mathbf{E}$.
- So far quasi-static assumption, i.e. displacement currents negligible: $\omega\epsilon << \sigma$.
- Assume $\epsilon_r = 5$ and $\rho = 1/\sigma = 10000\,\Omega\mathrm{m}$. Then, conduction and displacement currents are equally strong at $f = 360\,\mathrm{kHz}$.

Responses on 2D Earth for vertical incidence only (!):

- Impedance tensor **Z**:

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} 0 & Z_{xy} \\ Z_{yx} & 0 \end{bmatrix} \begin{bmatrix} H_x \\ H_y \end{bmatrix} \qquad \begin{matrix} \mathrm{TE-mode} \\ \mathrm{TM-mode} \end{matrix} \tag{3}$$

  giving
  apparent resistivities $\rho_a^{ij} = \frac{1}{\omega\mu_0}|Z_{ij}|^2$ and
  phases $\phi^{ij} = \arg\left(Z_{ij}\right)$.

- Vertical magnetic transfer function (VMT) B:

$$H_z = B \cdot H_y . \tag{4}$$

- Given error level of 2 % on elements of **Z**, a homogeneous half-space and vertically incident plane waves, effect of displacement currents on ϕ is above the error level at e.g. $f = 15\,\mathrm{kHz}$ and $\rho = 10000\,\Omega\mathrm{m}$ or $f = 170\,\mathrm{kHz}$ and $\rho = 1000\,\Omega\mathrm{m}$ for $\epsilon_r = 5$.

1D forward modelling example

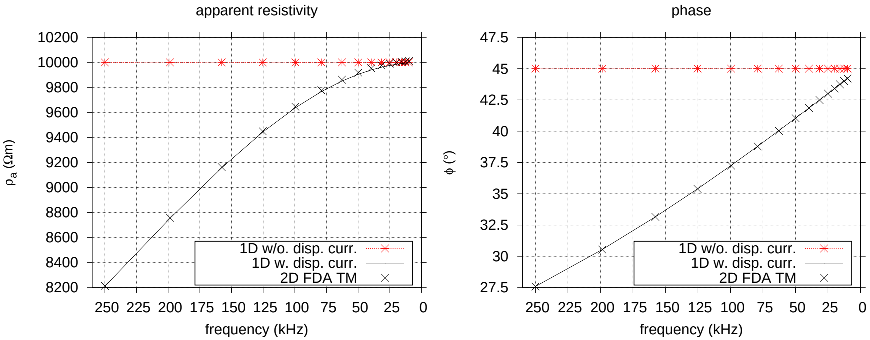


Figure: Analytic 1D solution and 2D FDA solution of apparent resistivity and phase for the TM-mode on the surface of a homogeneous half-space with $\rho = 10000\,\Omega\mathrm{m}$ and $\epsilon_r = 5$.

Effect of oblique incidence I

- As only vertically incident plane waves are considered, effect of oblique incidence needs to be estimated for typical 1D examples.
- Typical depth section of later field example:

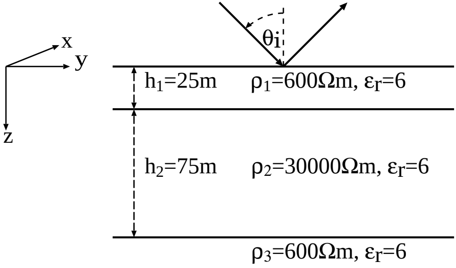
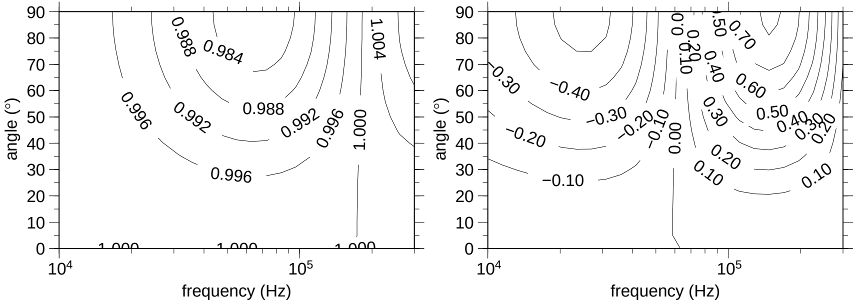


Figure: 1D model; $\theta_i = 0$ for vertical incidence.

Effect of oblique incidence II



(a) relative amplitude

(b) phase difference (°)

Figure: Relative amplitude and phase difference of TM-mode impedance w.r.t. case of normal incidence (angle$=0^\circ$) at frequencies between 10 and 300 kHz.

2D forward modelling example

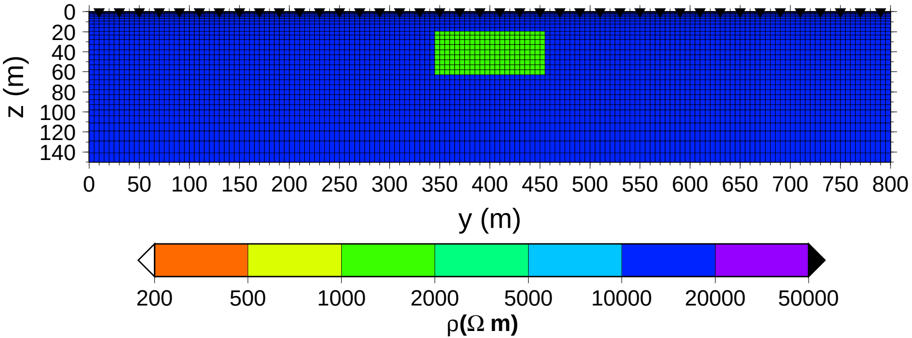
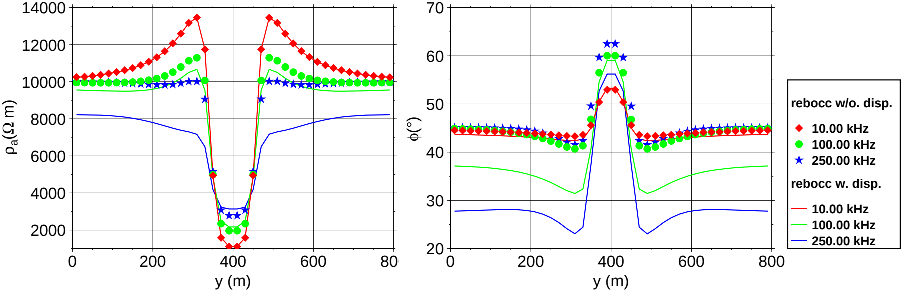


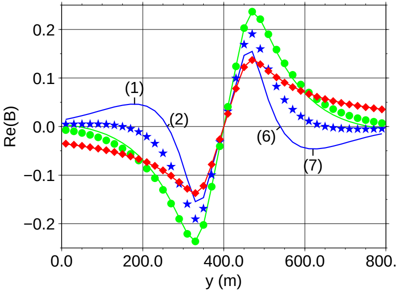
Figure: Simple 2D model with a conductive block of $\rho = 1000\,\Omega\mathrm{m}$ in a half-space with a resistivity of $\rho = 10000\,\Omega\mathrm{m}$ and $\epsilon_r = 5$ throughout. Receiver positions are indicated by black triangles.



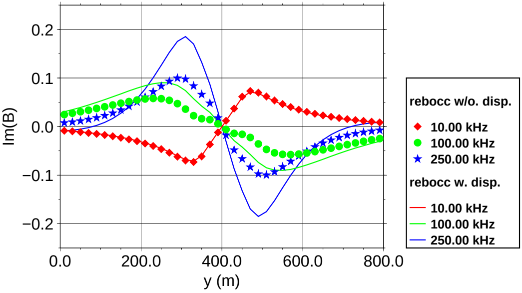
(a) ρ_a of TM-mode

(b) ϕ of TM-mode

Figure: Comparison of FDA responses of block model with displacement currents (symbols) with FDA solution computed in quasi-static approximation (dotted lines with symbols).

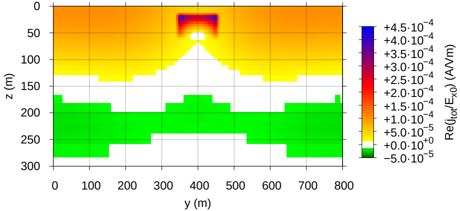


(c) $\mathfrak{Re}(B)$ of VMT

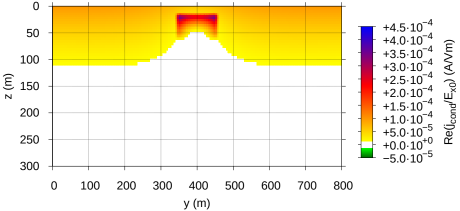


(d) $\mathfrak{Im}(B)$ of VMT

Figure: – *continued*



(a) $\Re\mathfrak{e}\,(j_x)$ with displacement currents



(b) $\Re\mathfrak{e}\,(j_x)$ of quasi-static case

Figure: Real part of current density j_x of the TE-mode at $f = 250\,\mathrm{kHz}$ for the general case with displacement currents (left) and the quasi-static case (right).

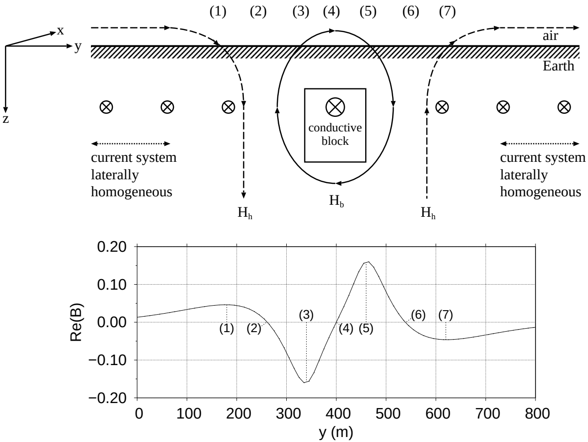


Figure: Real part of current density j_x with real part of the magnetic field **H** (top) and the real part of the VMT response $\mathfrak{Re}(B)$ (bottom).

2D inverse modelling example

- Simple model with block in a two-layer host.
- Synthetic data for TE- and TM-mode were computed and contaminated with 5% Gaussian noise on the impedances.
- Results from inversions with and without displacement currents are compared.

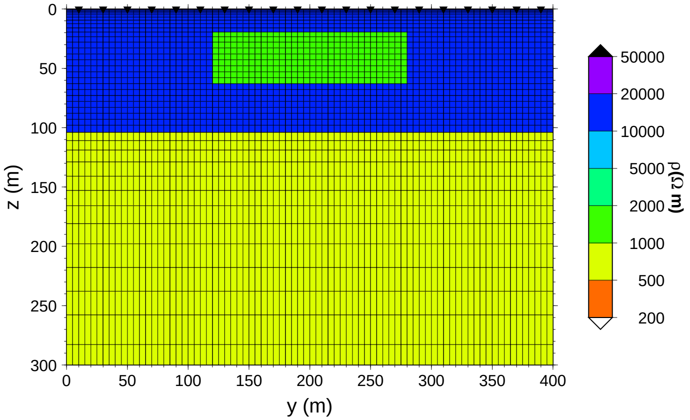


Figure: Model with a buried elongated block of a resistivity of 1000 Ωm in a resistive layer of 10000 Ωm underlain by a half-space of 500 Ωm and $\epsilon_r = 5$ throughout. Receiver positions are indicated by black triangles.

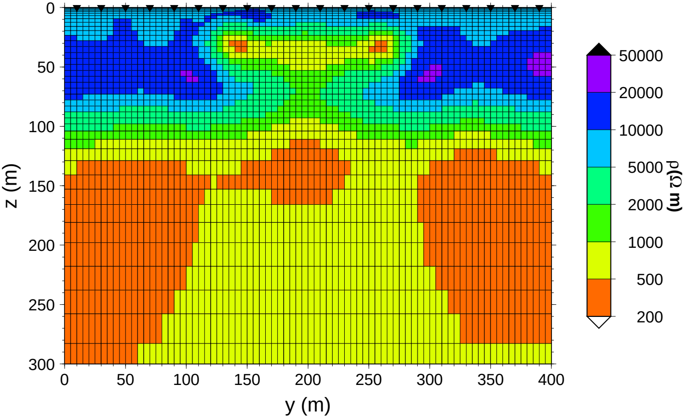


Figure: 2D REBOCC inversion result of synthetic data from block model, displacement currents were allowed for; RMS = 1.04.

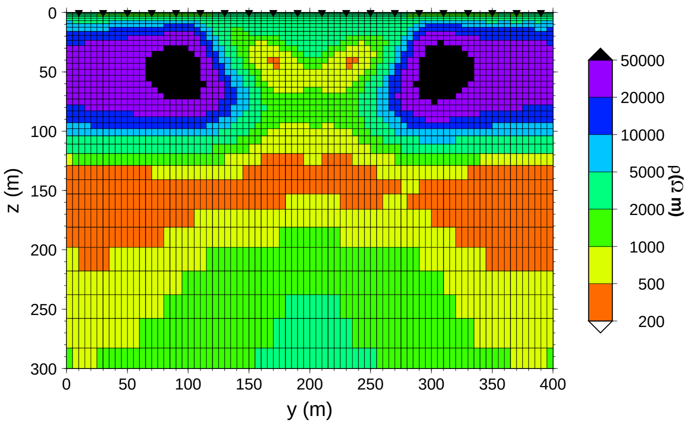


Figure: 2D REBOCC inversion result of synthetic data from block model, displacement currents were not allowed for; RMS = 1.95.

Field example from Ävrö, Sweden

- RMT profile: 96 Rx and frequencies from 14 to 226 kHz.
- Linde and Pedersen [2004] restricted data for inversion to lower frequencies up to 56 kHz.
- Only determinant inversion due to 3D effects at ends of profile.
- Comparison of models from inversions in quasi-static approximation and with displacement currents (with $\epsilon_r = 6$ throughout) for both data set restricted to lower frequencies and full data set.
- Comparison of models with seismic reflectors C and D by Juhlin and Palm [1999] and a normal-resistivity log of borehole KAV01 by Gentzschein et al. [1987].

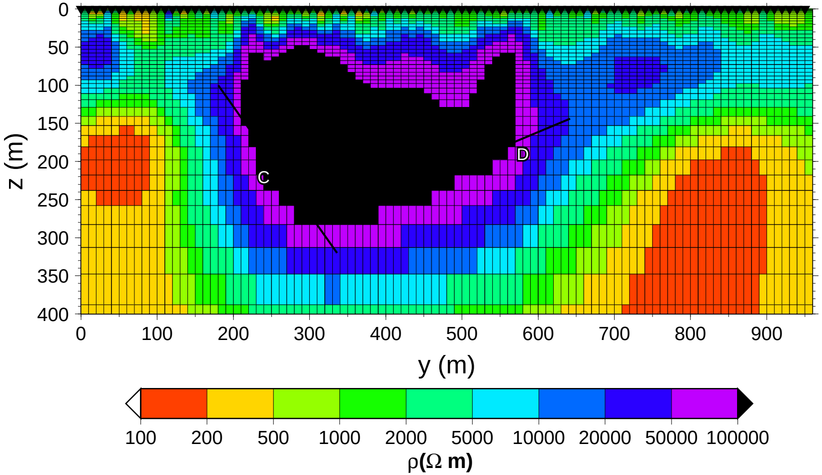


Figure: Model QL for low-frequency data set w/o. disp. curr.; RMS = 1.56.

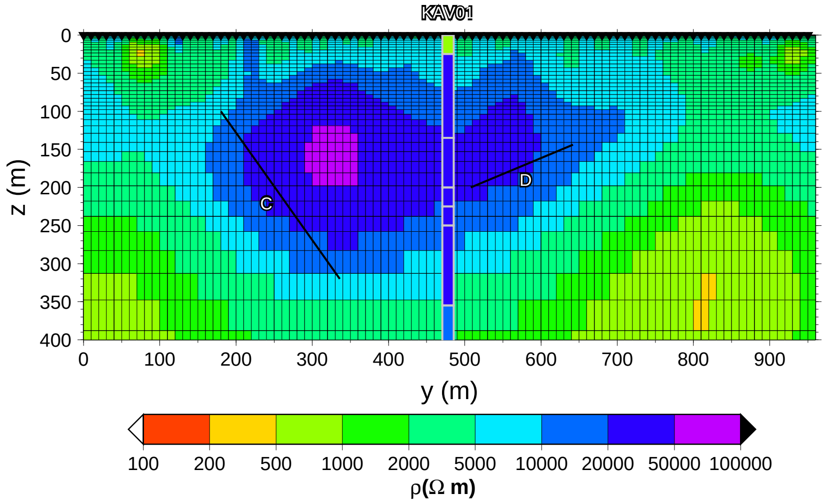


Figure: Model DL for low-frequency data set w. disp. curr.; RMS = 2.03.

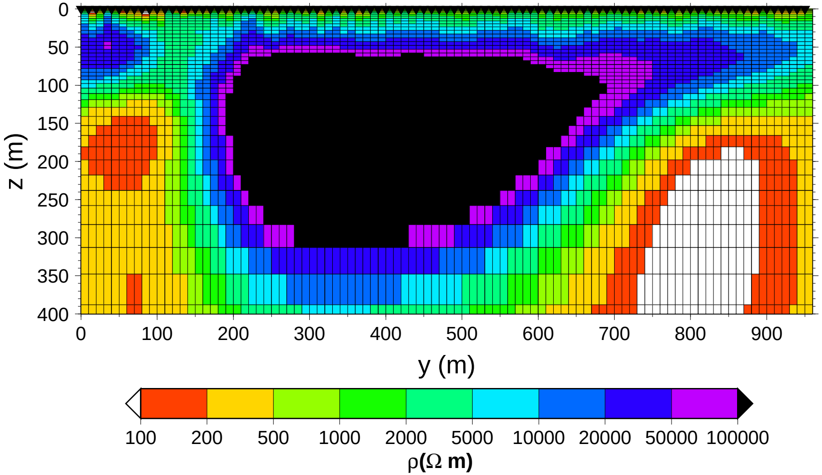


Figure: Model QF for full set of frequencies w/o. disp. curr.; RMS = 3.16.

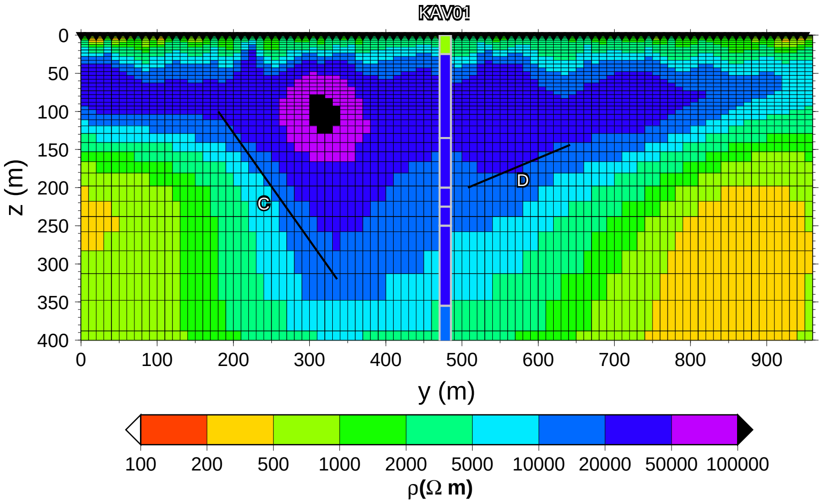


Figure: Model DF for full set of frequencies w. disp. curr.; RMS = 2.60.

Conclusions

1. If displacement currents are present,
   - apparent resistivity and phase responses are smaller than in quasi-static approximation,
   - tipper responses can show more distinct sign reversals, and
   - quasi-static inverse models are prone to artefactual structures.
2. If displacement currents are accounted for, field example shows better agreement with
   - seismic reflectors and
   - a normal-resistivity borehole log.
3. Even if restricted to lower frequencies quasi-static inversion might give strongly distorted models.

References I

B. Gentzschein, G. Nilsson, and L. Steinberg. Preliminary investigations of fracture zones at Ävrö - Results from investigations performed July 1986 - May 1987. SKB Progress Report 25-87-16, SKB, 1987.

C. Juhlin and H. Palm. 3-D structure below Ävrö island from high-resolution reflection seismic studies, southeastern Sweden. *Geophysics*, 64(3):662–667, 1999.

N. Linde and L. B. Pedersen. Characterization of a fractured granite using radio magnetotelluric (RMT) data. *Geophysics*, 69(5): 1155–1165, 2004.

A dark chapter ...

... with figures that should not be shown to a critical audience ...

Fit of model QF I

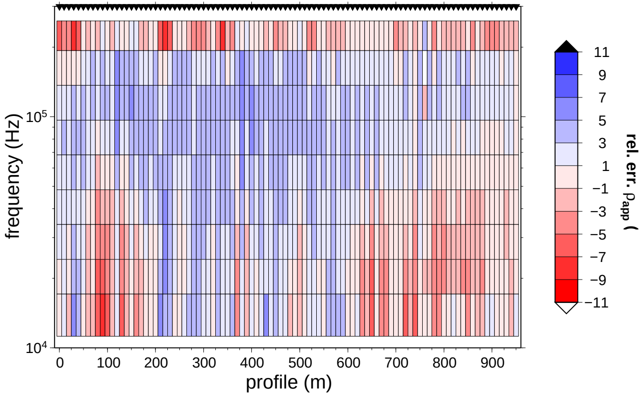


Figure: Data fit of model QF to apparent resistivity of determinant.

Fit of model QF II

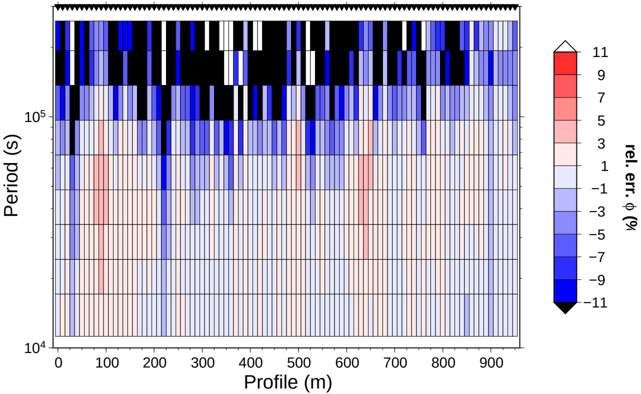


Figure: Data fit of model QF to phase of determinant.

Fit of model DF I

Model DF fits

- apparent resistivity rather well at all frequencies, some 3D effects are visible at the beginning and end of profile and
- phase quite badly at high frequencies, might be an effect of local sources (nuclear power plant), i.e. strong near-field effects in phase.

Fit of model DF II

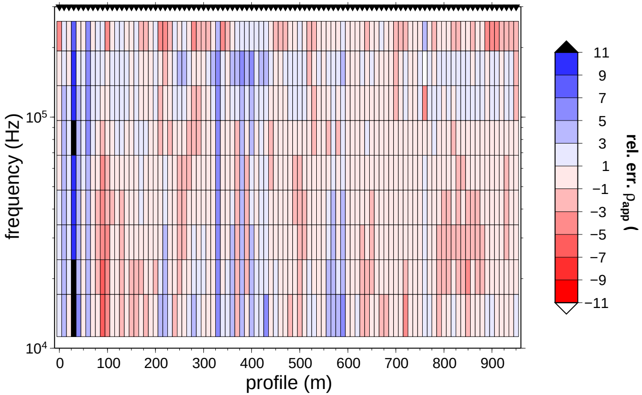


Figure: Data fit of model DF to apparent resistivity of determinant.

Fit of model DF III

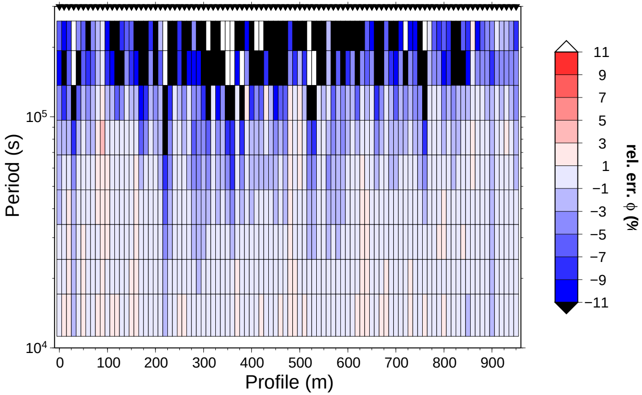


Figure: Data fit of model DF to phase of determinant.