The UBC
Geophysical Inversion Facility

MT3D: 3D Inversion of MT Data

Doug Oldenburg, Colin Farquharson, Elliot Holtham

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Earth and Ocean Sciences @ UBC

http://www.eos.ubc.ca/ubcgif

Background for MT3D

- MT3D developed as part of a GIF industry consortium (2000-2003)

- Basic aspects of forward modelling and inversion parallel those for controlled sources
  - Finite volume discretization for forward modelling
  - Gauss-Newton methodology for inversion

- References:

  - Farquharson, Oldenburg, Haber, and Shekhtman, 2002, 3D Inversion of MT Data SEG Ext. Abstract
  - Haber, Ascher, and Oldenburg, 2004, 3D Inversion of frequency and time domain data using all-at-once. Geophysics 69, p 1214
  - Haber, Ascher, Aruliah and Oldenburg, 2000, Fast simulation of 3D EM problems using potentials. J. Comp. Phys. 163, p 150.
  - Haber and Ascher: 2001 Fast finite volume simulation of 3D EM problems with highly discontinuous coefficients. SIAM J. Sci. Comp. p 1943

Forward Modelling

- **FD Maxwell's equations ($e^{-i\omega t}$)**

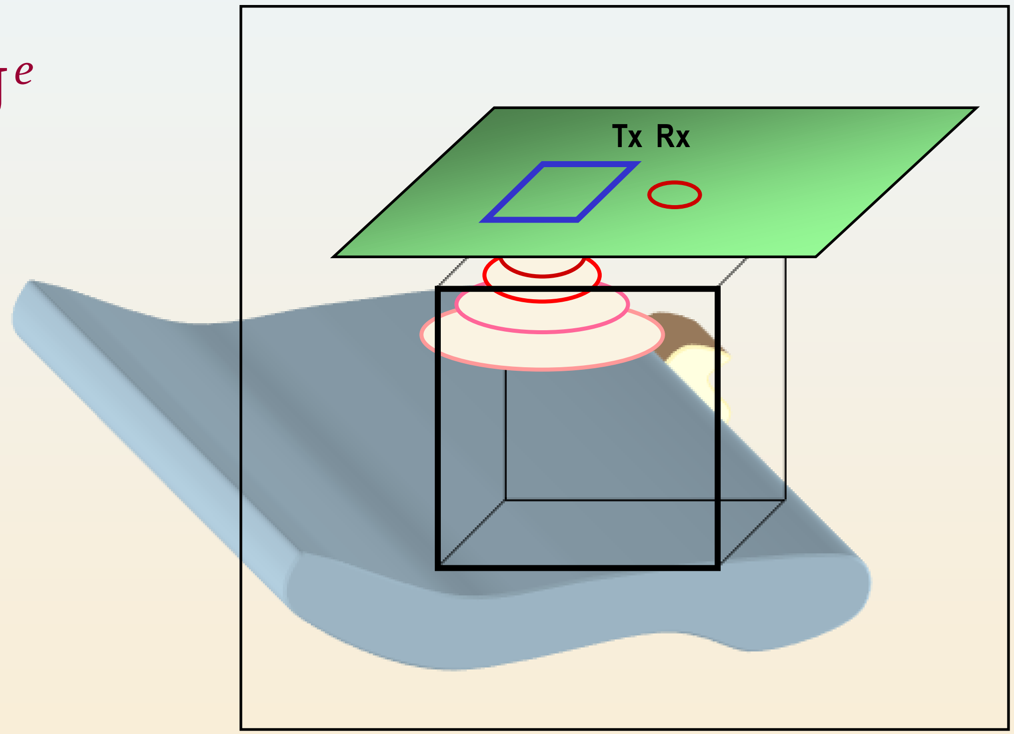
$$\nabla \times \mathbf{E} - i\omega\mu \mathbf{H} = 0$$

$$\nabla \times \mathbf{H} - (\sigma - i\omega\varepsilon)\mathbf{E} = \mathbf{J}^e$$

$$\nabla \cdot \varepsilon \mathbf{E} = 0$$

$$\nabla \cdot \mu \mathbf{H} = 0$$

Electromagnetic induction



- **Boundary condition**

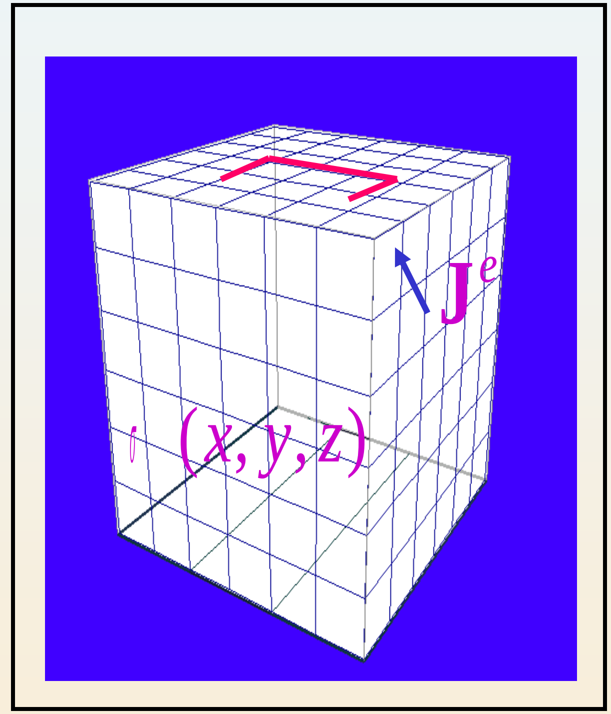
$$\mathbf{n} \times \mathbf{H} = 0$$

Forward Modelling

- **A Helmholtz decomposition with Coulomb gauge**

$$\mathbf{E} = -i\omega\mathbf{A} - \nabla\phi$$

$$\nabla \cdot \mathbf{A} = 0$$



- **System equations for A and φ**

$$\begin{pmatrix} \mathbf{L}_\mu - i\omega\hat{\sigma} & -i\omega\hat{\sigma}\nabla \\ \nabla\cdot\hat{\sigma} & \nabla\cdot\hat{\sigma}\nabla \end{pmatrix} \begin{pmatrix} \mathbf{A} \\ \phi \end{pmatrix} = \begin{pmatrix} -i\omega\mathbf{J}^e \\ -\nabla\cdot\mathbf{J}^e \end{pmatrix}$$

where

$$\mathbf{L}_\mu = \nabla\times\frac{1}{\mu}\nabla\times - \nabla\frac{1}{\mu}\nabla\cdot$$

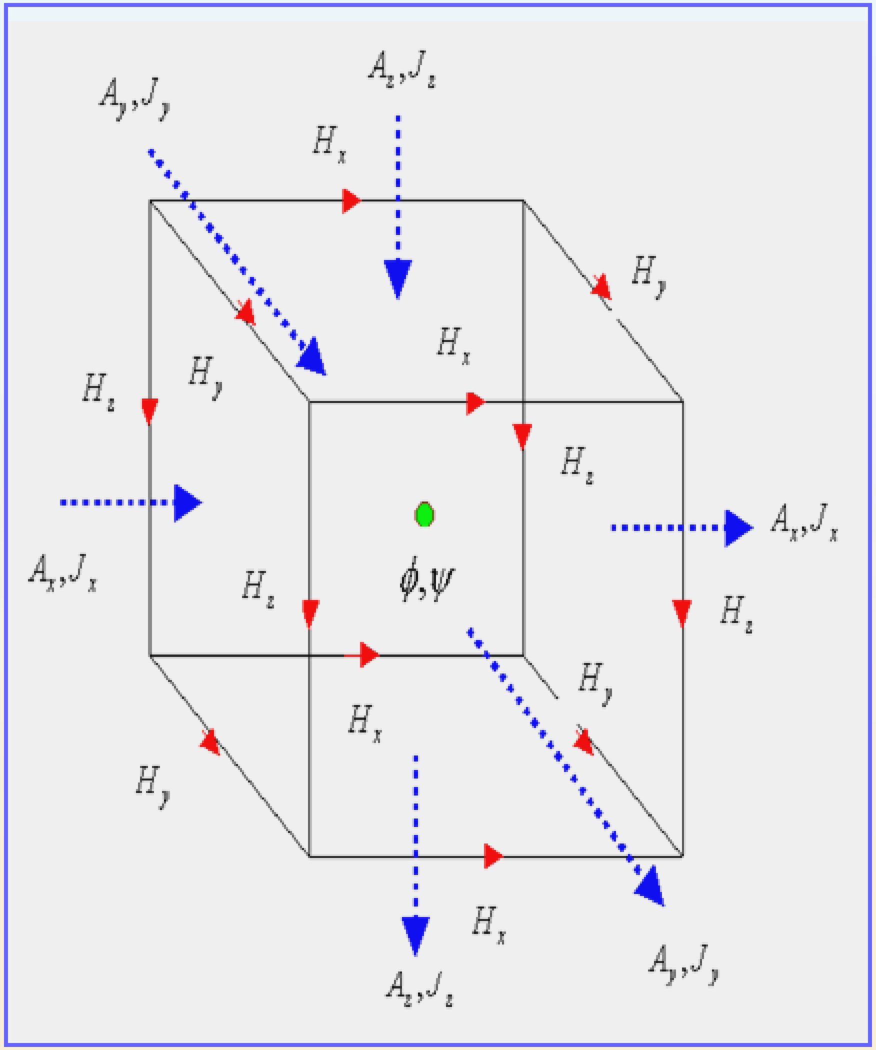
$$\hat{\sigma} = \sigma - i\omega\varepsilon$$

- **Discretization with staggered grid**

-- **A, J** are defined on the *faces*

-- ϕ is in the *centers*

-- **H** is on the *edges*



- **Discretized matrix system**

$$\begin{pmatrix} \mathbf{L}_{\mu} - i\omega \mathbf{M}_{\hat{\sigma}} & -i\omega \mathbf{M}_{\hat{\sigma}} \nabla_h \\ \nabla_h \cdot \mathbf{M}_{\hat{\sigma}} & \nabla_h \cdot \mathbf{M}_{\hat{\sigma}} \nabla_h \end{pmatrix} \begin{pmatrix} \mathbf{A} \\ \phi \end{pmatrix} = \begin{pmatrix} -i\omega \mathbf{J}^e \\ -\nabla_h \cdot \mathbf{J}^e \end{pmatrix}$$

where $\mathbf{L}_{\mu} = \nabla_h^{(e)} \times \mathbf{M}_e^{-1} \nabla_h^{(f)} \times - \nabla_h \mathbf{M}_c^{-1} \nabla_h \cdot$

$\nabla_h^{(e)} \times, \nabla_h^{(f)} \times$: Discretization of **Curl**

$\nabla_h \cdot$: **Div**

∇_h : **Grad**

$\mathbf{M}_{\hat{\sigma}}$: Conductivity

$\mathbf{M}_e, \mathbf{M}_c$: Magnetic permeability

or simplified as

$$\mathcal{A}(\mathbf{m})\mathbf{u} = \mathbf{q} \longrightarrow \mathbf{u} = \mathcal{A}(\mathbf{m})^{-1}\mathbf{q}$$

- **Solving the matrix system**

$$\mathcal{A}(\mathbf{m})\mathbf{u} = \mathbf{q}$$

- **Iterative solver BiCGSTAB**

- **Preconditioner**

Find $\mathbf{M} \cong \mathcal{A}(\mathbf{m})$

where $\mathbf{M}^{-1}$ is readily computed

Solve $\mathbf{M}^{-1}\mathcal{A}(\mathbf{m})\,\mathrm{u} = \mathbf{M}^{-1}\mathrm{q}$

$$\mathbf{M} = \begin{pmatrix} \mathbf{L} + i\omega\mathbf{M}_{\hat{\sigma}} & -i\omega\mathbf{M}_{\hat{\sigma}}\nabla_h \\ 0 & \nabla_h \cdot \mathbf{M}_{\hat{\sigma}}\nabla_h \end{pmatrix}$$

Inverse Problem

- **Minimize** $\phi = \phi_d + \beta \phi_m$

where

$$\phi_d = \left\| \mathbf{W_d}(F[\mathbf{m}] - \mathbf{d}^{obs}) \right\|^2, \quad F[\mathbf{m}] = f(\mathbf{Qu})$$

$$\phi_m = \left\| \mathbf{W}(\mathbf{m} - \mathbf{m}_{ref}) \right\|^2$$

β : Regularization parameter

$\mathbf{Q}$: Projection matrix

$\mathbf{u}$: Potentials

$\mathbf{d}^{obs}$: Observed data

$\mathbf{m}, \mathbf{m}_{ref}$: Model and Reference model

$\mathbf{W_d}, \mathbf{W}$: Data error, model weighting

• Solving the inverse problem

-- Differentiating the objective function ϕ with model **m**

$$\frac{\partial \phi}{\partial \mathbf{m}} = \mathbf{g}(\mathbf{m}) = \mathbf{J}^T \mathbf{W}_d^T [F[\mathbf{m}] - \mathbf{d}^{obs}] + \beta \mathbf{W}^T \mathbf{W}(\mathbf{m} - \mathbf{m}_{ref})$$

where sensitivity matrix

$$\mathbf{J} = \frac{\partial F[\mathbf{m}]}{\partial \mathbf{m}} = \frac{\partial f[\mathbf{Q}\mathbf{u}]}{\partial \mathbf{m}} = \mathbf{S}\mathbf{Q}\frac{\partial \mathbf{u}}{\partial \mathbf{m}} = -\mathbf{S}\mathbf{Q}\mathcal{A}(\mathbf{m})^{-1}\mathbf{G}(\mathbf{m}, \mathbf{u})$$

and

$$\mathbf{G}(\mathbf{m}, \mathbf{u}) = \partial[\mathcal{A}(\mathbf{m})\mathbf{u}]/\partial \mathbf{m} \qquad \mathbf{S} = \frac{\partial f}{\partial(\mathbf{E}, \mathbf{H})}$$

f: Function that converts E, H fields to other data types

• Gauss-Newton method

-- Solve $\mathbf{g}(\mathbf{m}) = 0$, and let $F[\mathbf{m}+\delta\mathbf{m}] = F[\mathbf{m}] + \mathbf{J}\,\delta\mathbf{m}$

$$(\mathbf{J}^T\mathbf{J} + \beta\,\mathbf{W}^T\mathbf{W})\delta\mathbf{m} = -\,\mathbf{g}(\mathbf{m})$$

The sensitivity matrix $\mathbf{J}$ has been normalized by $\mathbf{W}_d$

$$\mathbf{J} = -\mathbf{W}_d\mathbf{S}\mathbf{Q}\mathcal{A}(\mathbf{m})^{-1}\mathbf{G}(\mathbf{m},\mathbf{u}).$$

and the gradient is

$$\mathbf{g}(\mathbf{m}) = -\mathbf{G}(\mathbf{m},\mathbf{u})^T\mathcal{A}(\mathbf{m})^{-T}\mathbf{Q}^T\mathbf{S}^T\mathbf{W}_d^T\mathbf{W}_d[\mathbf{F}[\mathbf{m}] - \mathbf{b}] + \beta\mathbf{W}^T\mathbf{W}(\mathbf{m} - \mathbf{m}_{ref}) \quad .$$

Matrices W_d, W, S, Q, $\mathcal{A}(\mathbf{m})$, G(m,u) are SPARSE!

• Solution of the matrix system

$$(\mathbf{J}^T\mathbf{J} + \beta\,\mathbf{W}^T\mathbf{W})\delta\mathbf{m} = -\,\mathbf{g}(\mathbf{m})$$

IPCG solver with preconditioner $\mathbf{M} = (\mathbf{I} + \beta\mathbf{W}^T\mathbf{W})$

Main computations:

(1) $\mathbf{J}\,\mathbf{v} = -\mathbf{W}_\mathrm{d}\,\mathbf{S}\,\mathbf{Q}\,\mathcal{A}(\mathbf{m})^{-1}\underbrace{\mathbf{G}\,\mathbf{v}}_{\mathbf{w}}$

→ Solve $\mathcal{A}(\mathbf{m})\ \mathbf{f} = \mathbf{w}$:*Forward modelling*

(2) $\mathbf{J}^\mathrm{T}\,\mathbf{v} = -\,\mathbf{G}^\mathrm{T}\,\mathcal{A}(\mathbf{m})^{-T}\underbrace{\mathbf{Q}^\mathrm{T}\,\mathbf{S}^\mathrm{T}\,\mathbf{W}_\mathrm{d}{}^\mathrm{T}\,\mathbf{v}}_{\mathbf{w}}$

→ Solve $\mathcal{A}^T(\mathbf{m})\,\mathbf{f} = \mathbf{w}$:*Adjoint modelling*

UPDATE $\mathbf{m}_{k+1} = \mathbf{m}_k + \alpha\,\delta\mathbf{m}$

Flow chart

Recall we are solving …

$$\phi = \phi_d + \beta \phi_m$$

$$= \left\| \mathbf{W}_d (F[\mathbf{m}] - \mathbf{d}^{obs}) \right\|^2 + \beta \left\| \mathbf{W}(\mathbf{m} - \mathbf{m}_{ref}) \right\|^2$$

Choose β_0, $\mathbf{m}_{ref}$

Evaluate $\phi(\mathbf{m}_{ref})$, $g(\mathbf{m}_{ref})$, matrices $\mathbf{W}_d$, $\mathbf{W}$…

For β cooling loop

For k = 1 → max iterations

- **IPCG to solve** $(\mathbf{J}^T\mathbf{J} + \beta \mathbf{W}^T\mathbf{W})\,\delta\mathbf{m} = -\mathbf{g}(\mathbf{m})$
- **Line search for step length α**
- **Update model** $\mathbf{m}_{k+1} = \mathbf{m}_k + \alpha\,\delta\mathbf{m}$
- **Exit if** $\phi_d \approx \phi_d^{*}$ OR $\dfrac{\left\|\mathbf{g}(\mathbf{m}_{k+1})\right\|}{\left\|\mathbf{g}(\mathbf{m}_k)\right\|} < tol$

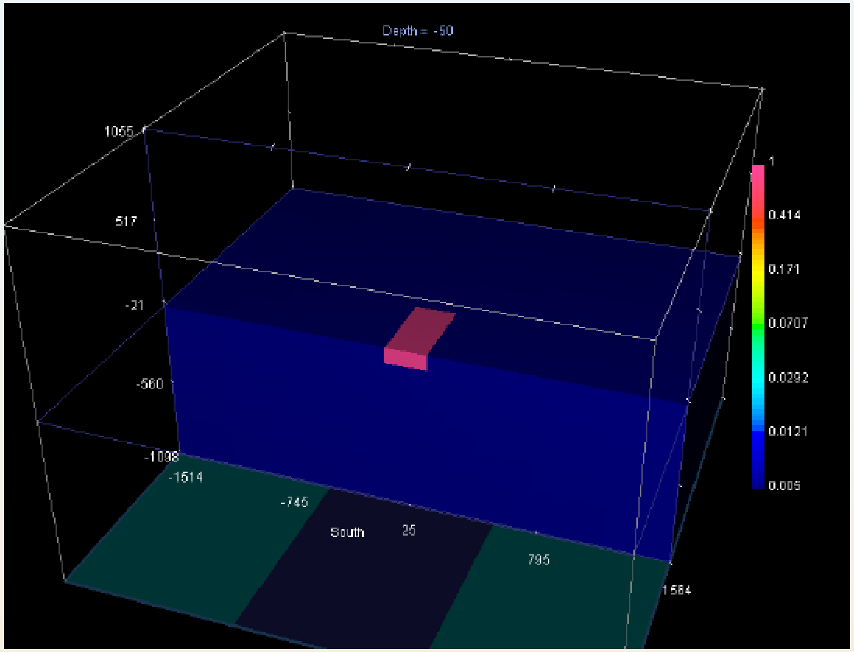
End

Reduce β

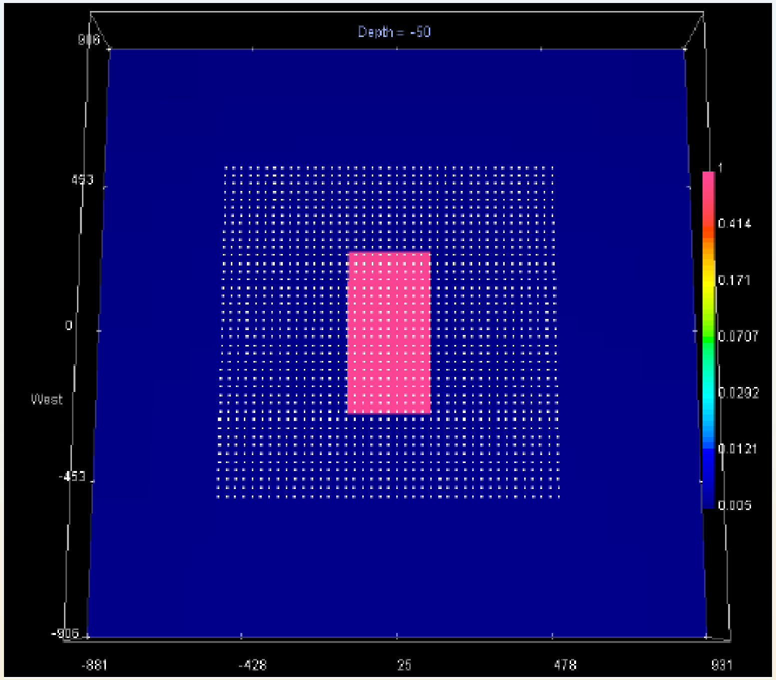
End

Synthetic Model

Cross section



Plan view showing model and station coverage



Block: 250 X 500 X 100m
Buried 50 m

Block: 1 **Ω-m**
Background 200 **Ω-m**

# of stations: 1661
# of data: 13,448

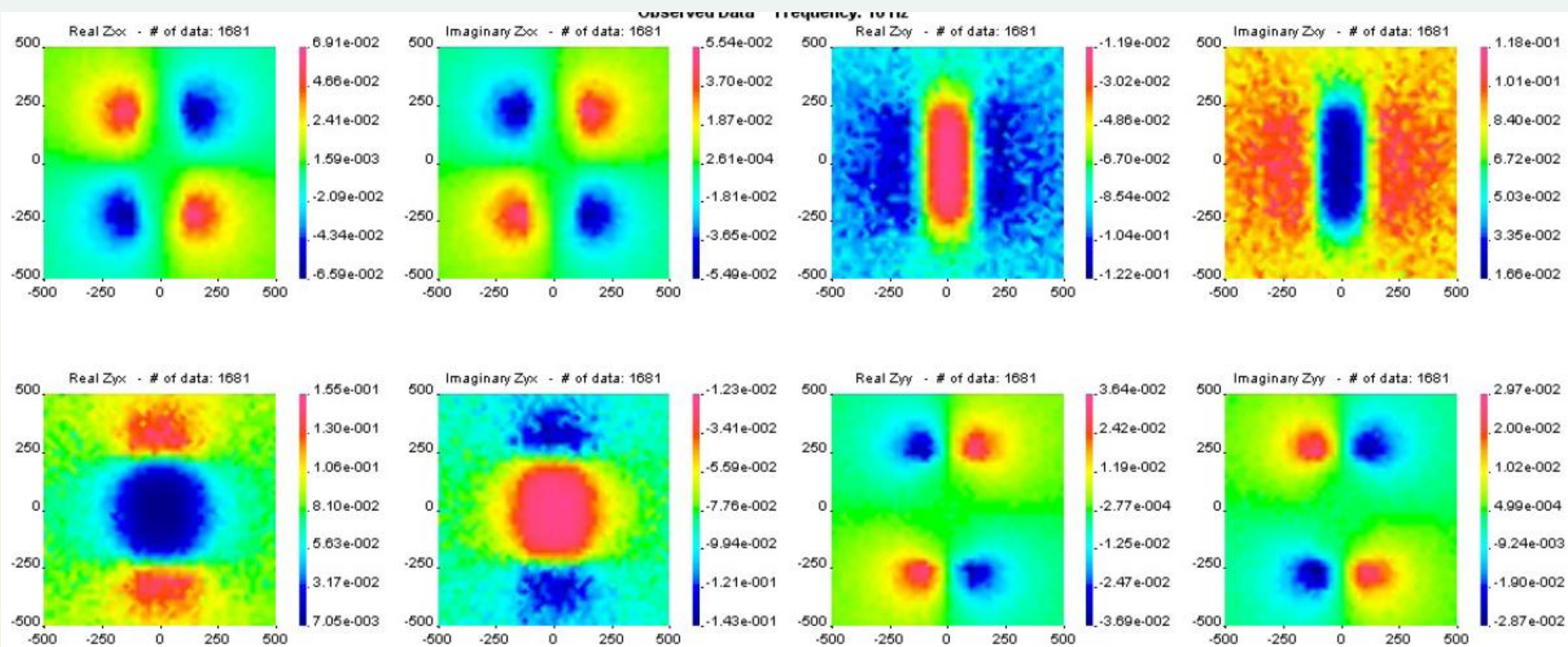
Mesh: 32 32 28 cells

Data for Inversion: Impedances 10Hz

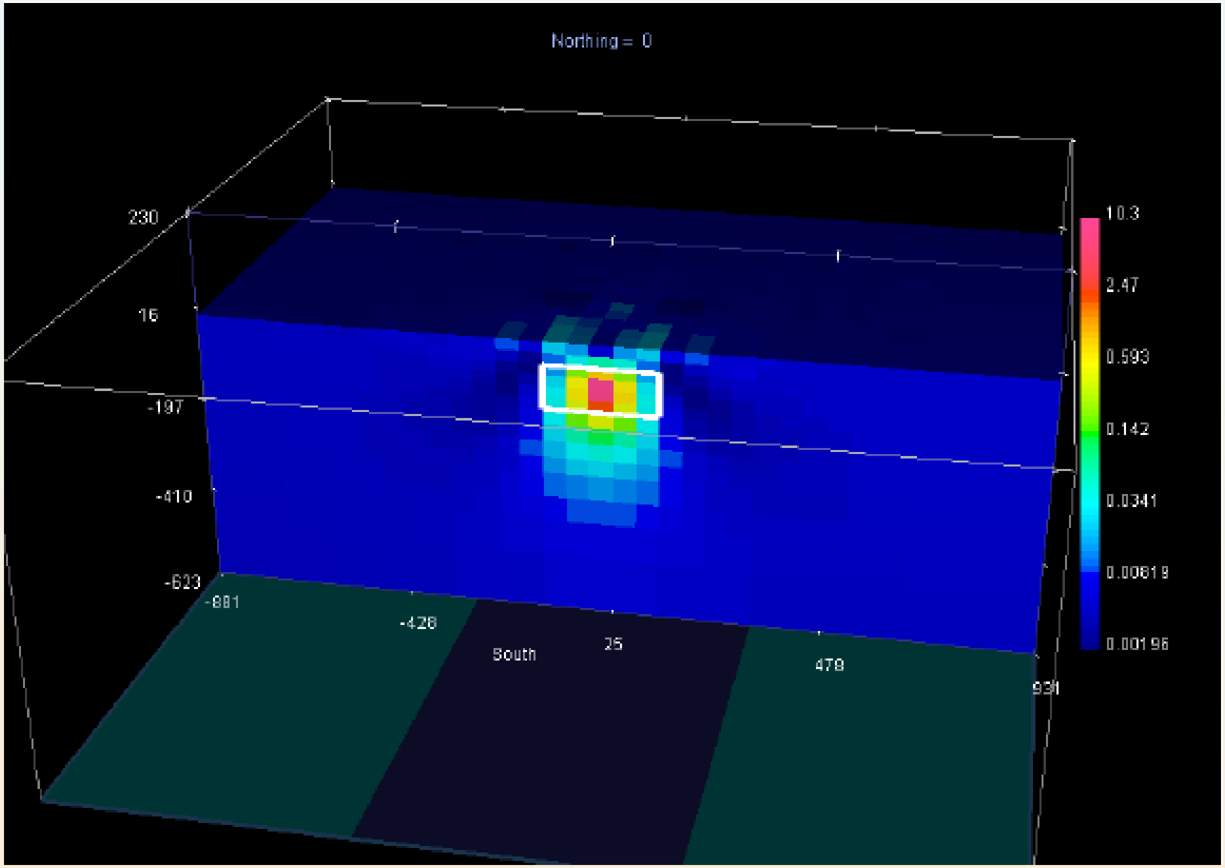
Noise is Gaussian and equivalent to:

5% of apparent resistivity

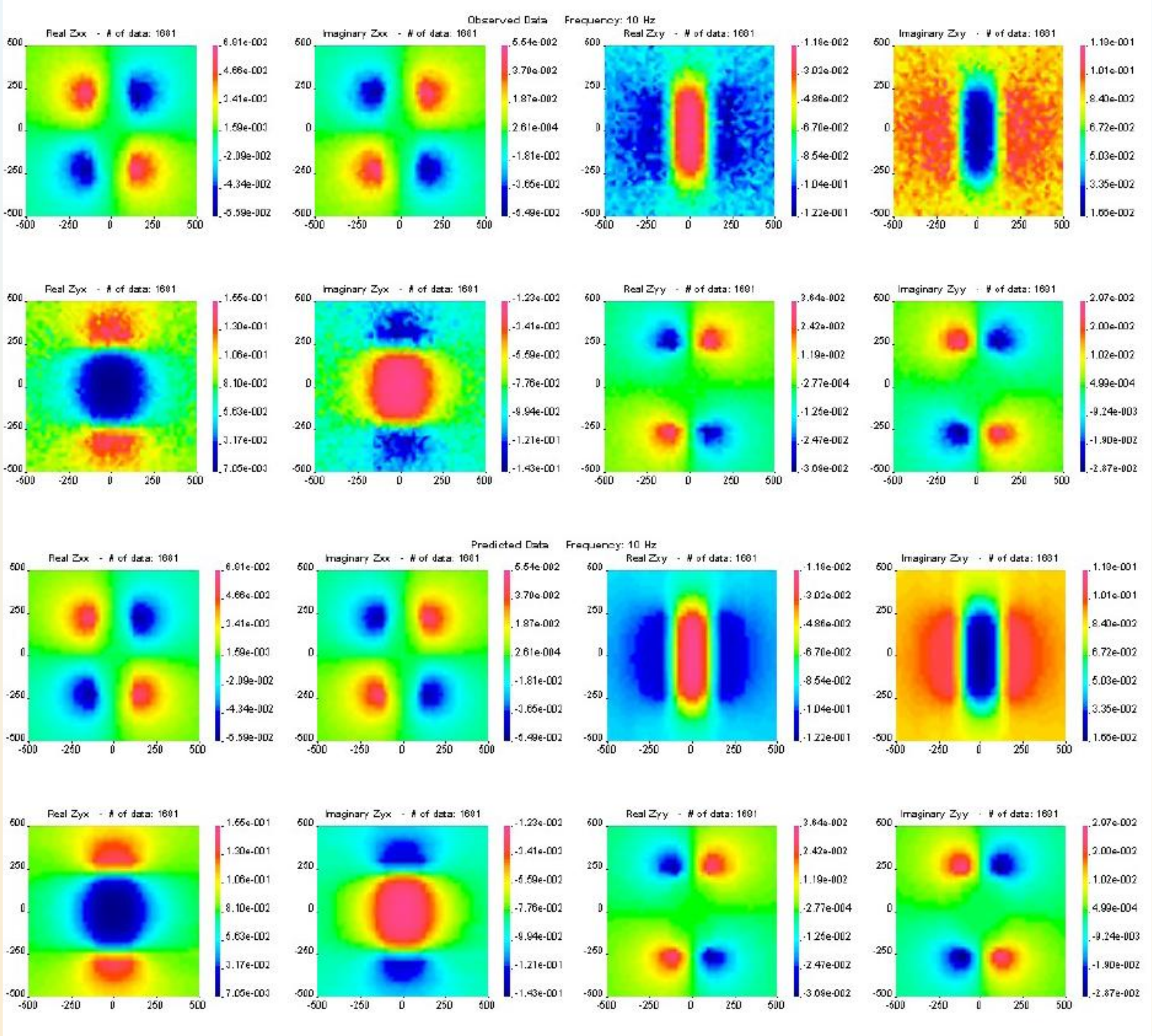
2 degrees in phase



Recovered model



Observed and predicted data

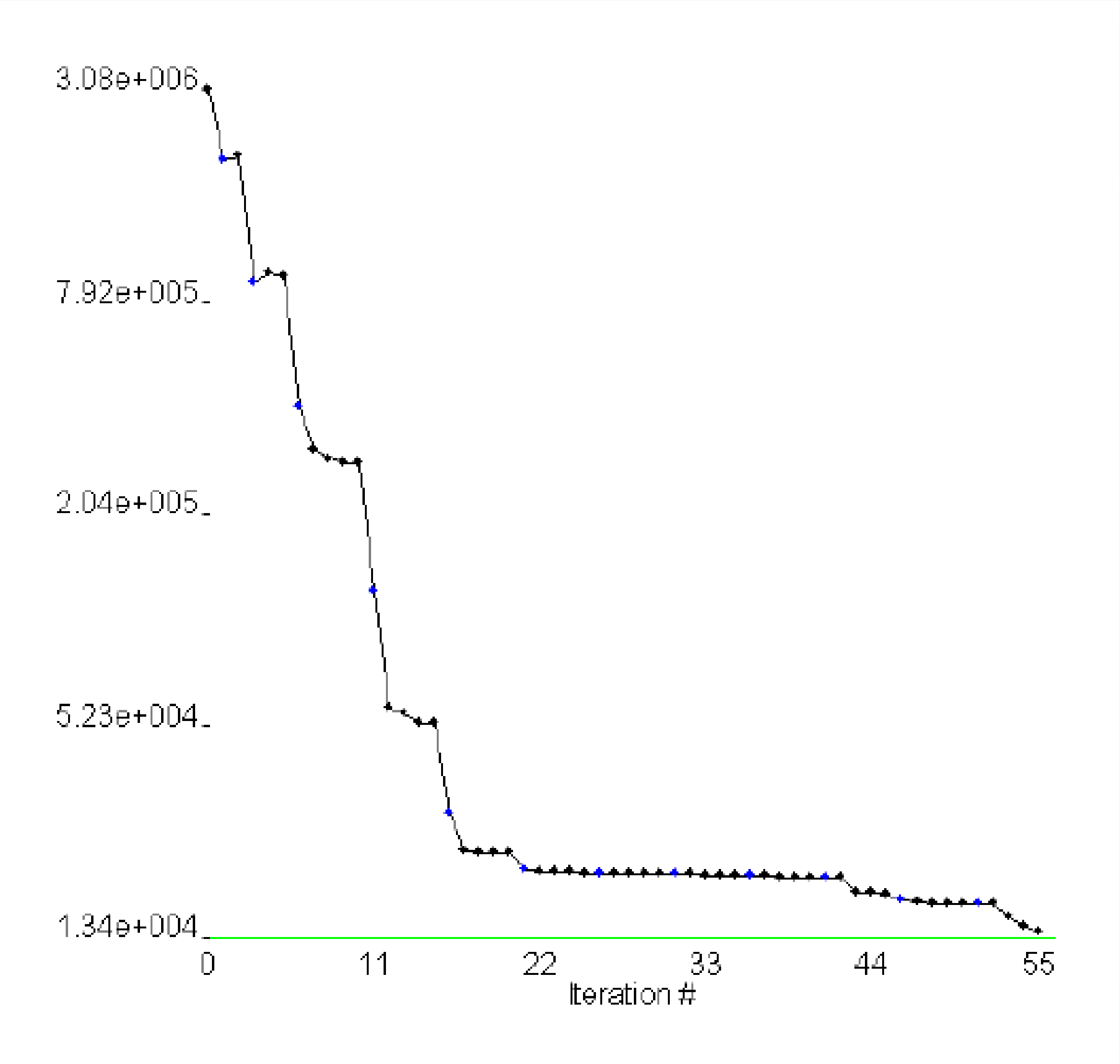


Observed

Predicted

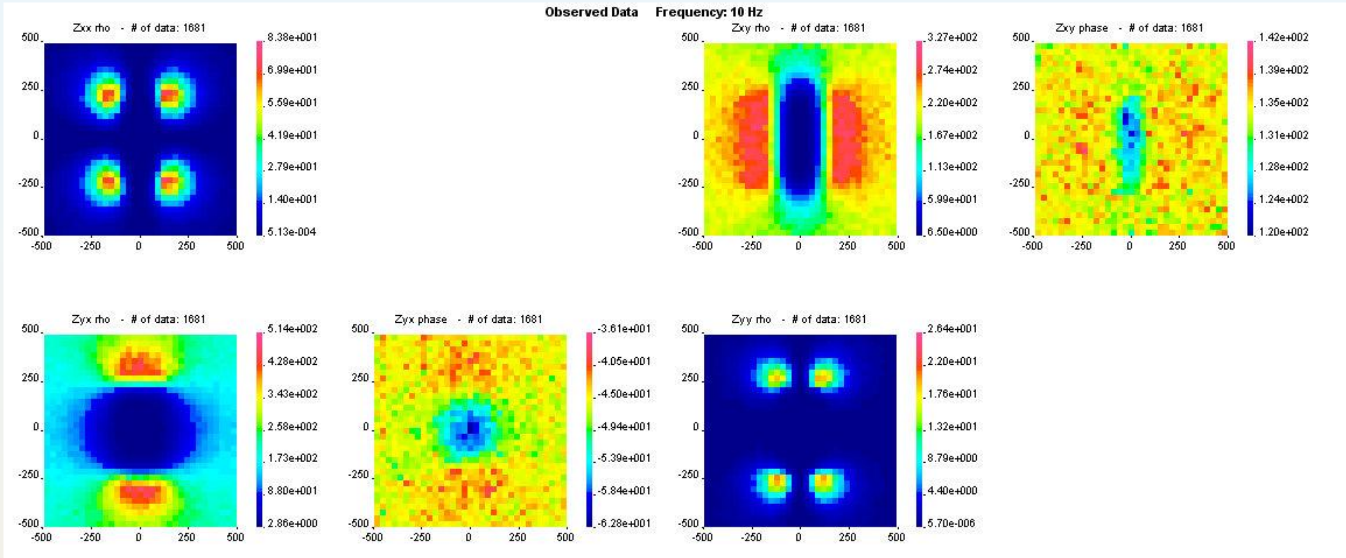
Convergence curve

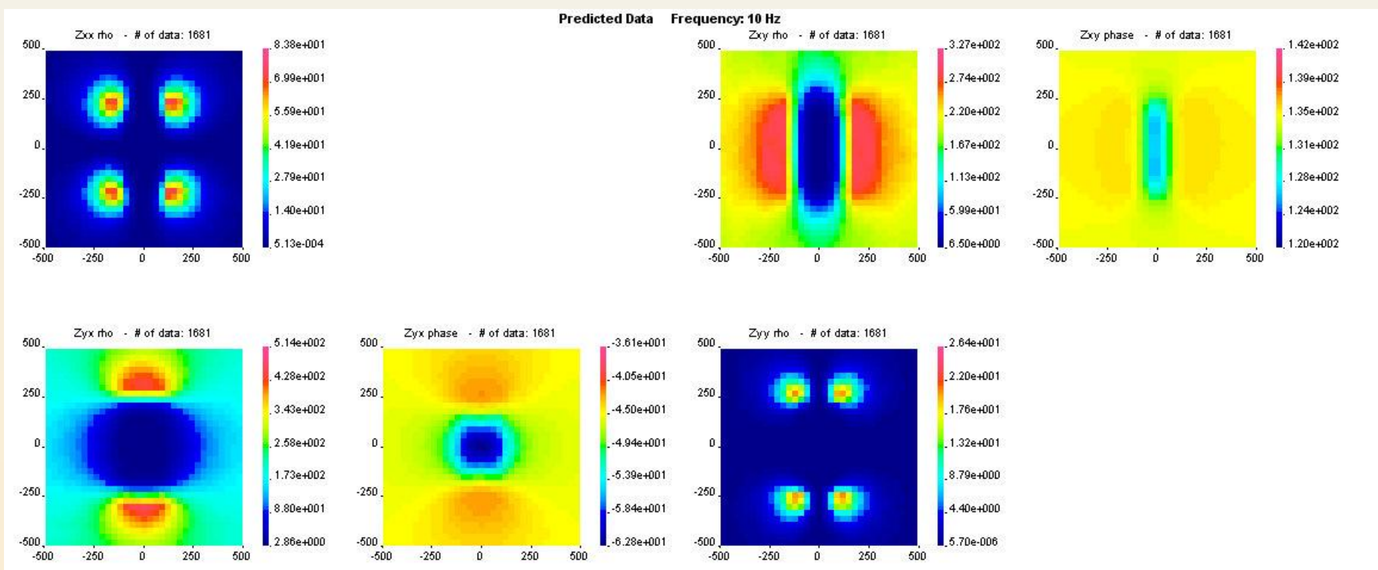
Misfit



Extra slides: apparent resistivity inversion

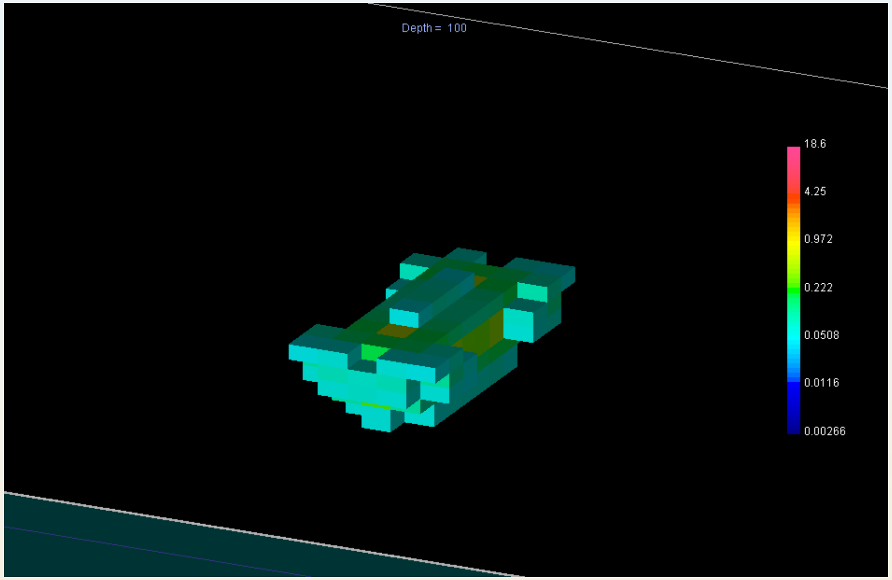
Invert apparent resistivities and phases





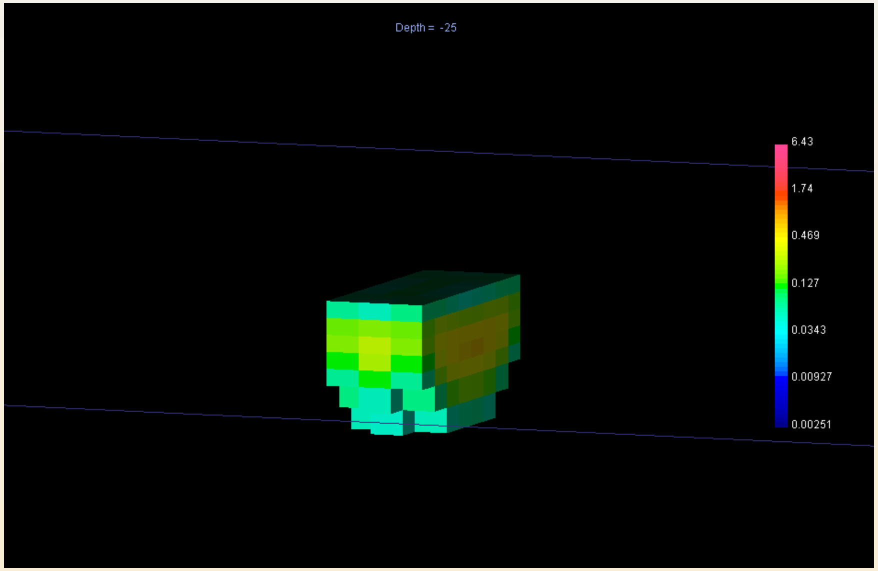
Inversion Comparisions

Apparent resistivity and phase



Cutoff 0.05

Real Imaginary Impedances

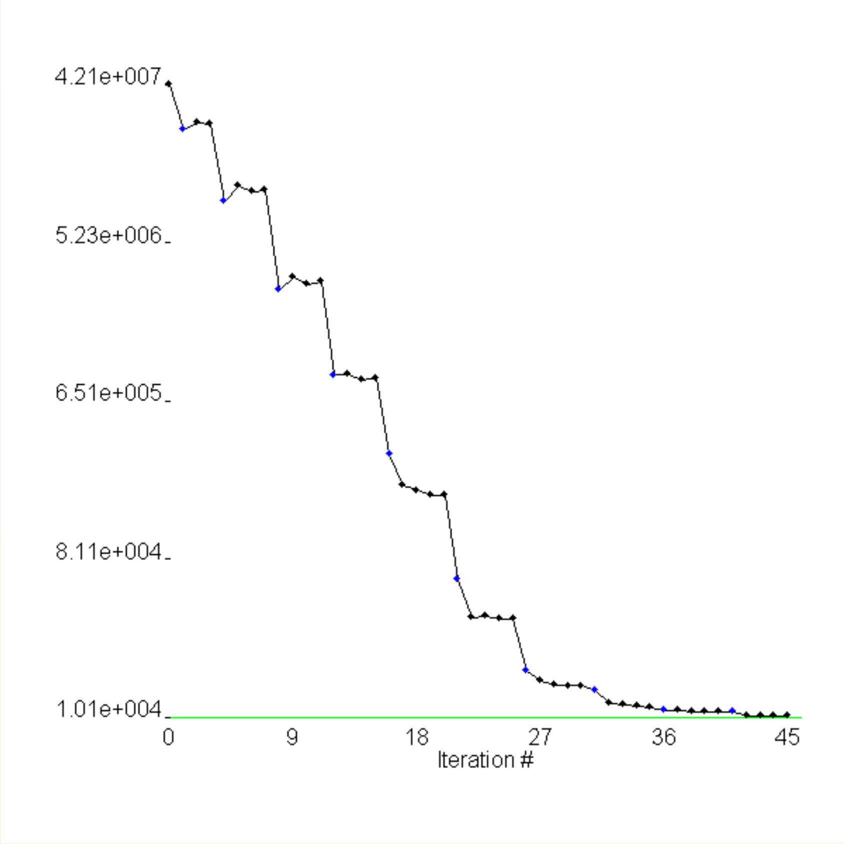


Convergence

Resistivity and phase

target misfit: 1.00860E+04
final misfit: 1.01970E+04

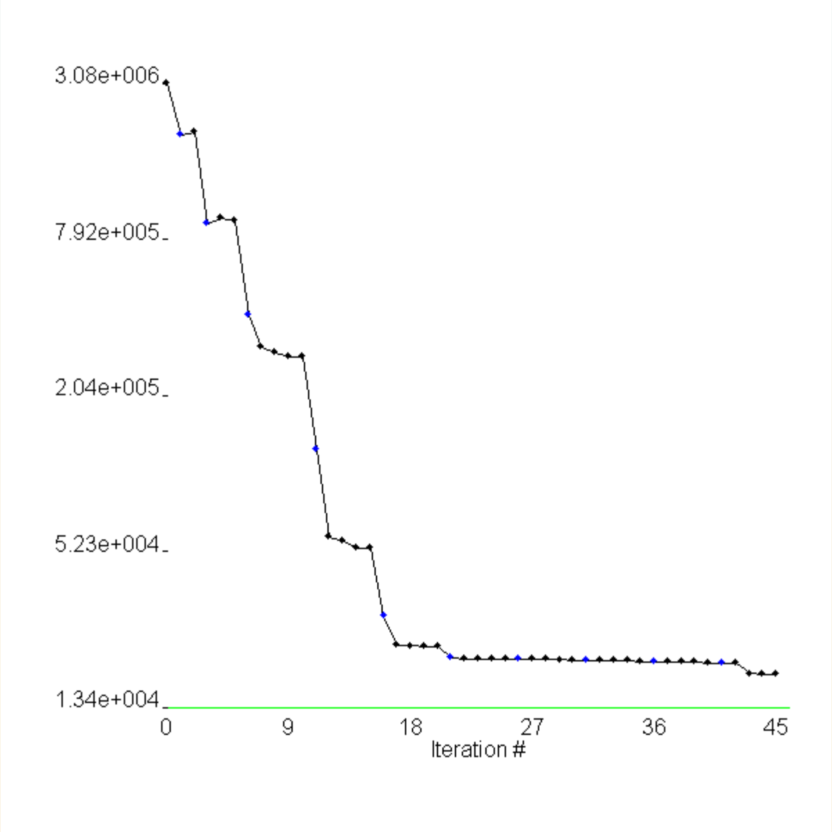
TOTAL cpu time: 12:57:01.78



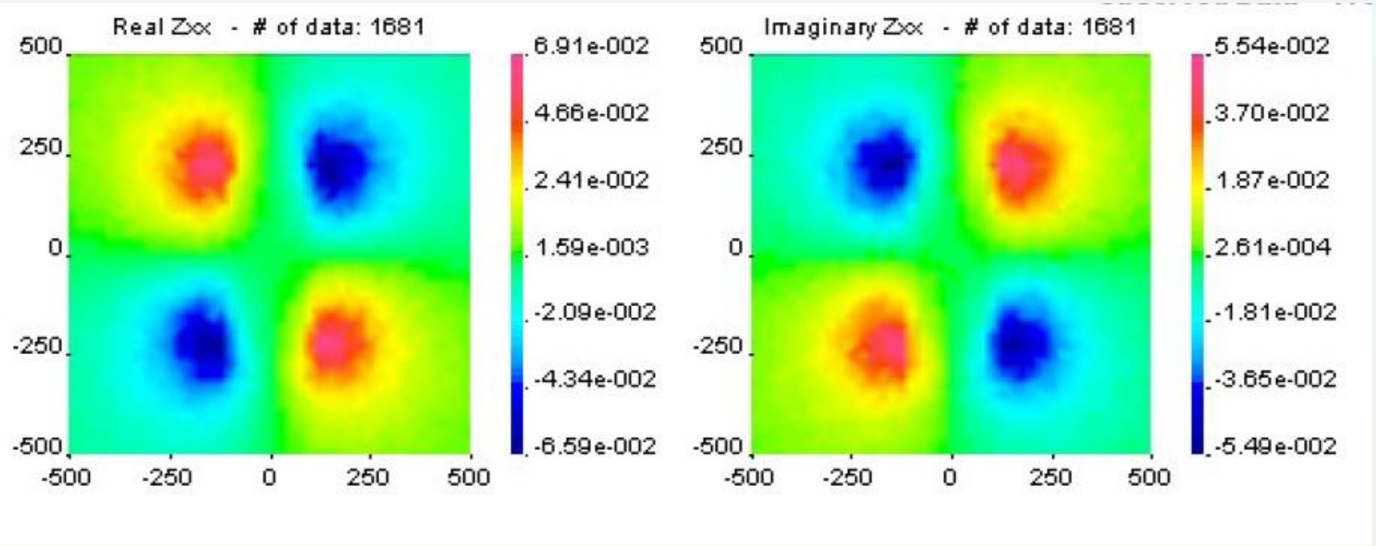
Real and Imaginary

target misfit: 1.34480E+04
final misfit: 1.78897E+04

TOTAL cpu time: 14:13:20.26

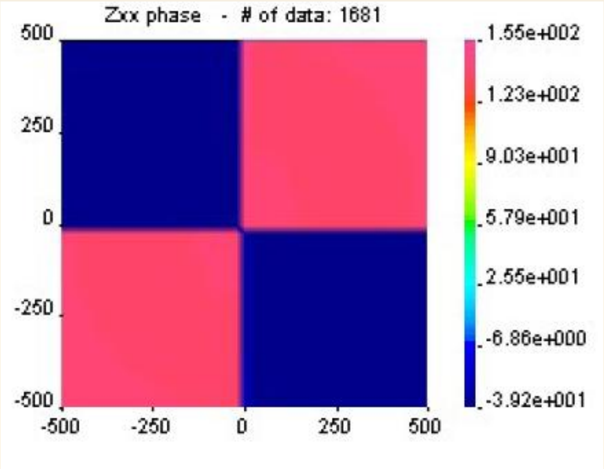


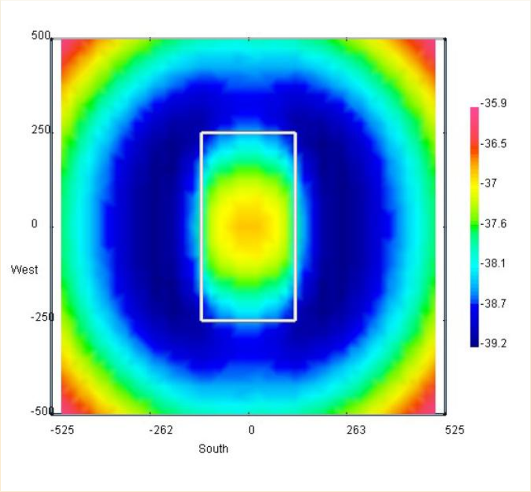
Phases for Zxx, Zyy



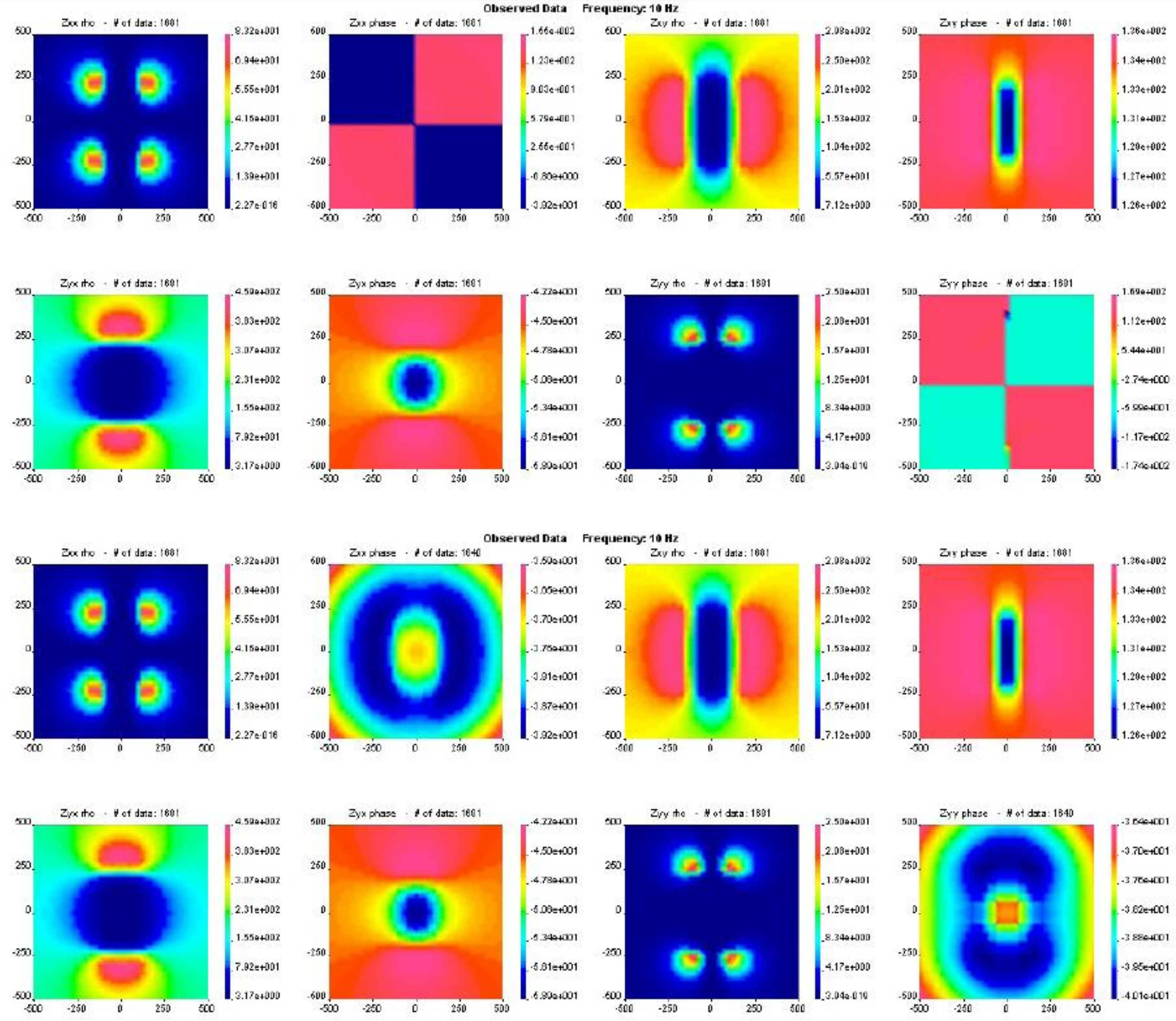
Phase evaluated with atan2
Lies in [-40, 155] degrees

Phase evaluated with atan
Lies in [-40, -35]





Block data with altered phases



True rho and phase data.

Improved Zxx and Zyy phase.

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Summary Discussion

Doug Oldenburg and Greg Newman

March 2008
Dublin

Earth and Ocean Sciences @ UBC

Summary Comments:

- Finite difference, finite element, IE modelling
- Synthetic model generated by Marion and Pilar:
- Range of times for running
- Major difficulties sorting out the sign conventions of the impedance were of major difficulties (conjugate gradients, rotations) should be able to make comparisons.
- Results were quite comparable (major discrepancies were for diagonal components when impedance values were low).

MT3D Workshop: Wednesday : Forward Modelling

- Discussion points:
  - Need compatibility for comparison of results. (EDI format?)
  - What accuracy is required by the modelling?
  - How should workshop forward modelling results be archived?
  - What other models should be considered? (topography, complicated structure, near surface heterogeneity, multi-scale)
  - Modelling large scale feature (galvanic distortions).
  - How do we handle unknown boundary conditions?

Summary comments:

- Most inversion codes gave similar results on Dublin test

Discussion points:

- Archive results for Dublin test?
- Assignment of errors to impedance.Workflows and methodologies.
- Incorporation of tipper.
- Grid design (forward modelling and inversion meshes; frequency dependent grids; validation of grid, unstructured meshes)

MT3D Workshop: Thursday Inversion: More Discussion

– Regularization functionals: reference models, inclusion of a priori information.

– Including other information (bound constraints, tears, other geophysical measurement, geologic information, petrophysical constraints)

– Ways to ameliorate high sensitivity due to receiver locations. (filtering, weighting)

– Static Shift:
  - don't worry—just model them out;
  - distortion process then invert;
  - invert for conductivity and distortion

MT3D Workshop: Thursday Afternoon

Secret Model

- Most inversion results provided similar “shoe” structure

- Discussion points:
  - Why didn’t inversions detect the inconsistent data?
  - Other approaches for delineating inconsistencies. (e.g. coordinate systems)
  - Anything more to be done with “old secret model”?

- Another Secret Model?
  - Same model but true station locations
  - Data set contaminated with noise.
  - A more complicated model?

MT3D Workshop: Thursday Afternoon: more discussion

Model assessment or Appraisal (next workshop?)

- What are the approaches for ground-truth
- Constructing other inverse solutions
- Resolution estimates

- Single RMS misfit value is not enough!
- Can we map fractures (eg for geothermal energy)
- Further ways for exploiting marine MT for hydrocarbons
- …... ???? ….

MT3D Workshop: Dublin

Thanks to Alan Jones

Thanks to Marion Miensopust