Dublin 3D MT Workshop 2008
3D Forward Model

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Geosystem / WesternGeco

Maxwell's Equations

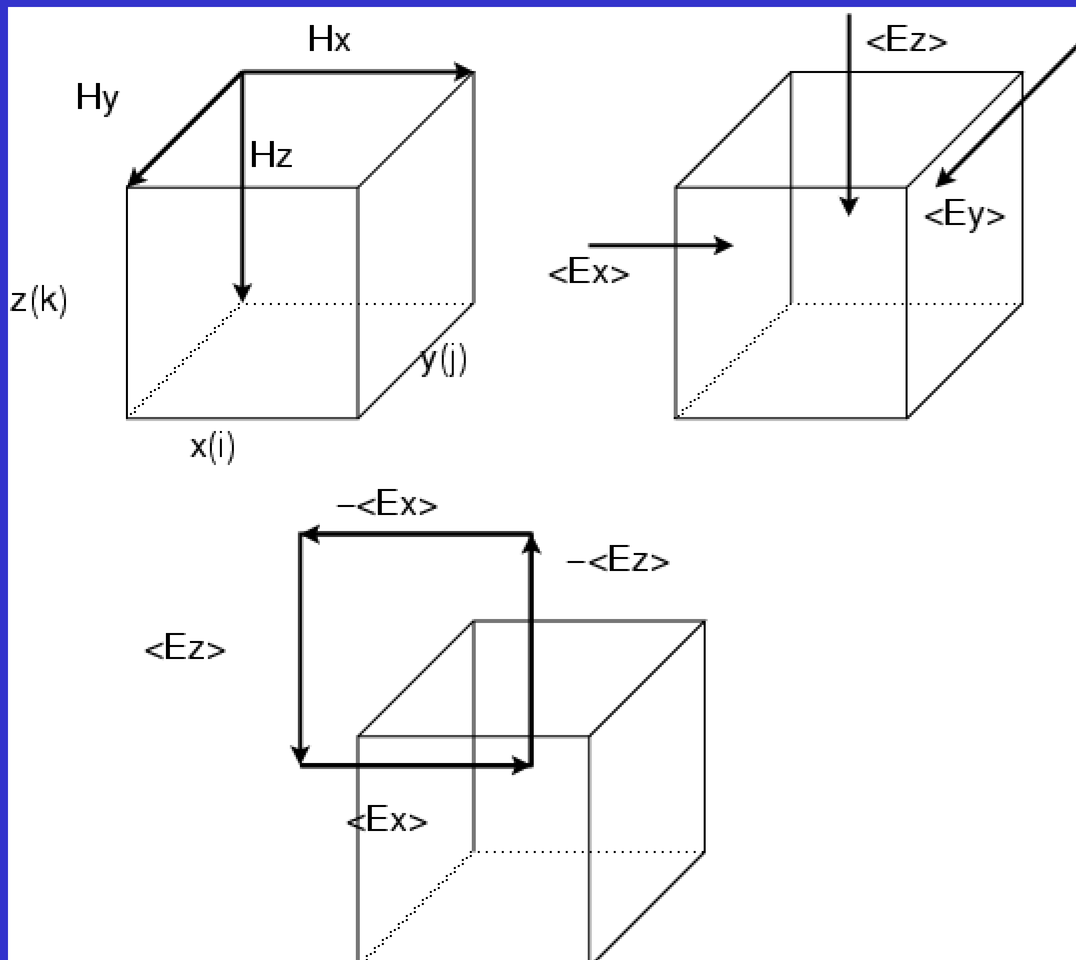
At the frequencies involved in MT exploration, conduction currents dominate over displacement currents. Therefore the integral forms of Maxwell's equations, assuming an $e^{-i\omega t}$ time dependence, are given by

$$\oint \mathbf{H} \cdot d\mathbf{l} = \iint \mathbf{J} \cdot d\mathbf{S} = \iint \sigma \mathbf{E} \cdot d\mathbf{S}$$

$$\oint \mathbf{E} \cdot d\mathbf{l} = \iint i\mu\omega \mathbf{H} \cdot d\mathbf{S}$$

where in general μ and σ are tensor quantities [*Stratton*, 1941].

Difference Equation Geometry



Difference Equations

$$\begin{aligned} J_x(i,j,k)\Delta z_k \Delta y_j &= [H_z(i,j+1,k) - H_z(i,j,k)]\,\Delta z_k \\ &\quad - [H_y(i,j,k+1) - H_y(i,j,k)]\,\Delta y_j \end{aligned}$$

where Δz_k and Δy_j are the block spacings.

$$\begin{aligned} i\omega\mu H_x(i,j,k)\Delta z_{k-1/2}\Delta y_{j-1/2} &= \\ [E_z(i,j,k) \quad &- \quad E_z(i,j-1,k)]\,\Delta z_{k-1/2} \\ -\,[E_y(i,j,k) \quad &- \quad E_y(i,j,k-1)]\,\Delta y_{j-1/2} \end{aligned}$$

where $\Delta z_{k-1/2}$ and $\Delta y_{j-1/2}$ are the distances between midpoints of the blocks defined by the normals to block faces.

Difference Equations Continued

Since the **E** fields are specified as averages across block faces, they will suffer discontinuities if adjoining blocks have different conductivities. Thus we define the **E** fields as the average of the **E** fields on either side of the block face. Since **J** is continuous, this can be written for the *x* component as

$$
\begin{aligned}
E_x(i,j,k) &= \frac{\rho(i,j,k)\Delta x_i + \rho(i-1,j,k)\Delta x_{i-1}}{\Delta x_i + \Delta x_{i-1}} J_x(i,j,k) \\
&= \rho_x(i,j,k) J_x(i,j,k)
\end{aligned}
$$

where $\rho_x(i,j,k)$ refers to the average resistivity in the x direction.

Difference Equations Continued

We can eliminate the electric fields and group the resulting second-order equations together in the form

$$\mathbf{A}\mathbf{x} = \begin{bmatrix} M_{xx} & N_{xy} & N_{xz} \\ N_{yx} & M_{yy} & N_{yz} \\ N_{zx} & N_{zy} & M_{zz} \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = \mathbf{b},$$

where **b** contains the terms associated with the known boundary values and source field.

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Solution of Equations

- Indeterminancy caused by air layers removed by adding a gradient (ρ div H) term to the equations or by alternating divH updates with EM updates.
- Preconditioned by ILU decomposition of diagonal sub-blocks
- Solution by BiCGStab algorithm

Vertical Meshing Rules

- Surface layer thickness: 1/10 skin depth at shortest period in most conductive surface block

- increase the thickness of each layer by factor of 1.2 to 1.5 until the block thickness is $\geq 1/3$ skin depth in the deep model at longest period.

Horizontal Meshing Rules

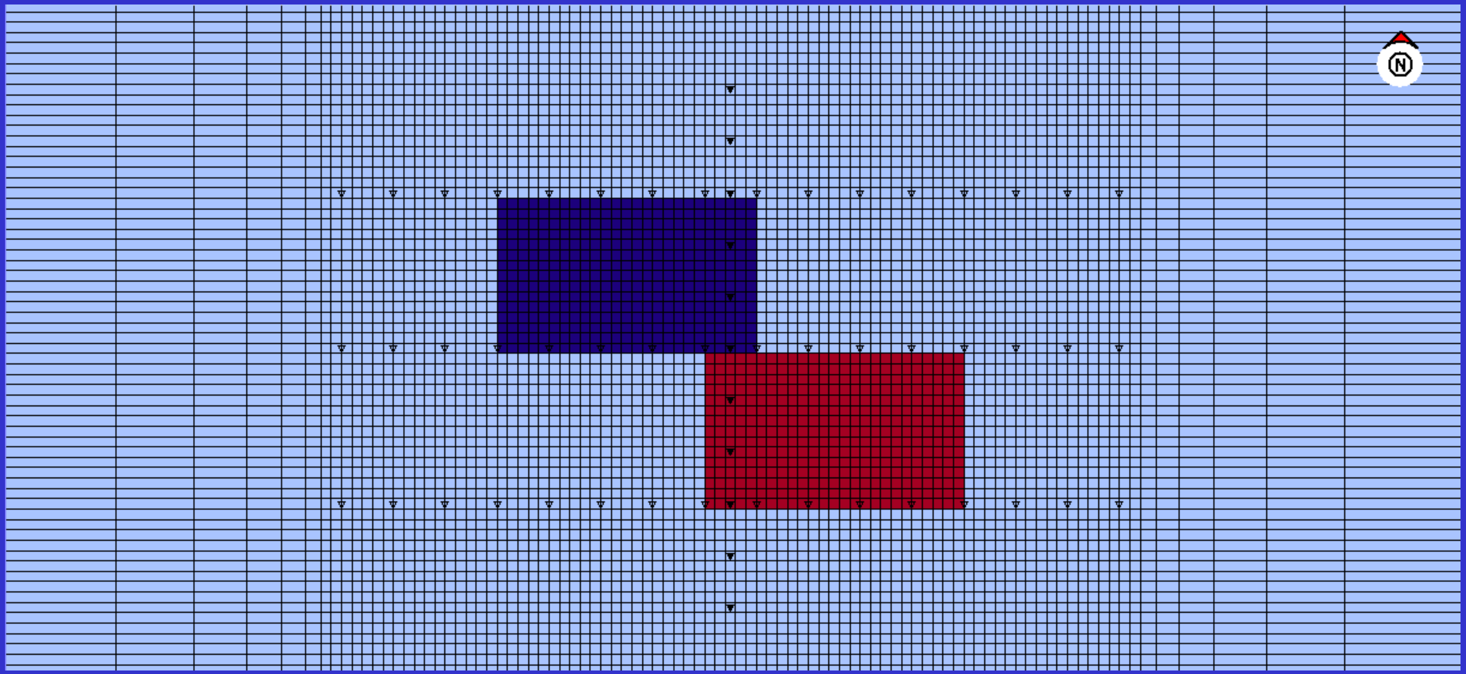
- Columns widths can increase away from the stations by a factor of 1.5 until the block width is about the same width as the thickness of the deepest layer in the model.

- For typical MT surveys, column widths should be on the order of a few hundred meters. If you have enough sites so as not to alias the lateral response variation, then one block/site should be fine.

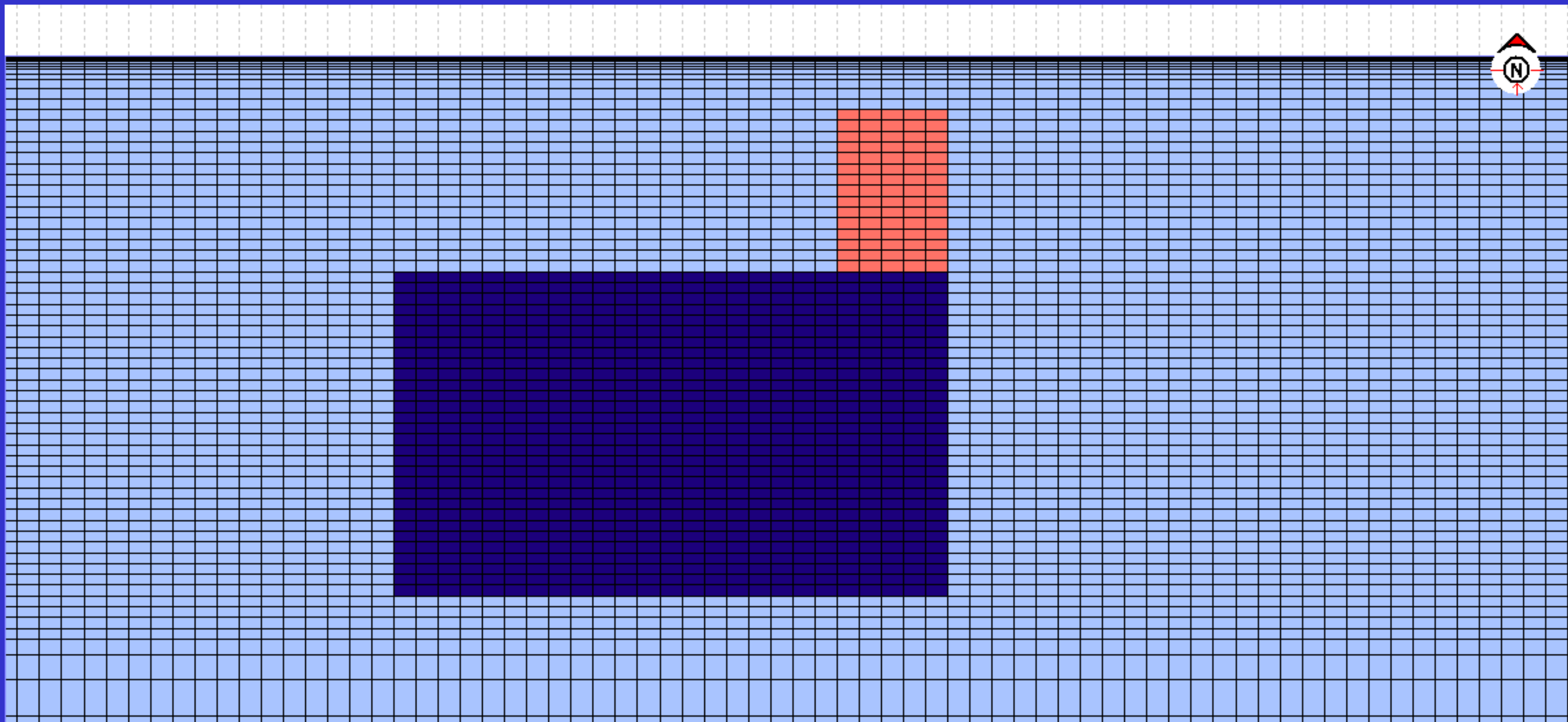
- Block widths can increase by a factor of 1.2 to 1.5 in between stations depending on the spacing.

- You should try to make the block widths as uniform as possible within the area bounded by the stations.

What do these meshes look like?



Horizontal Slice

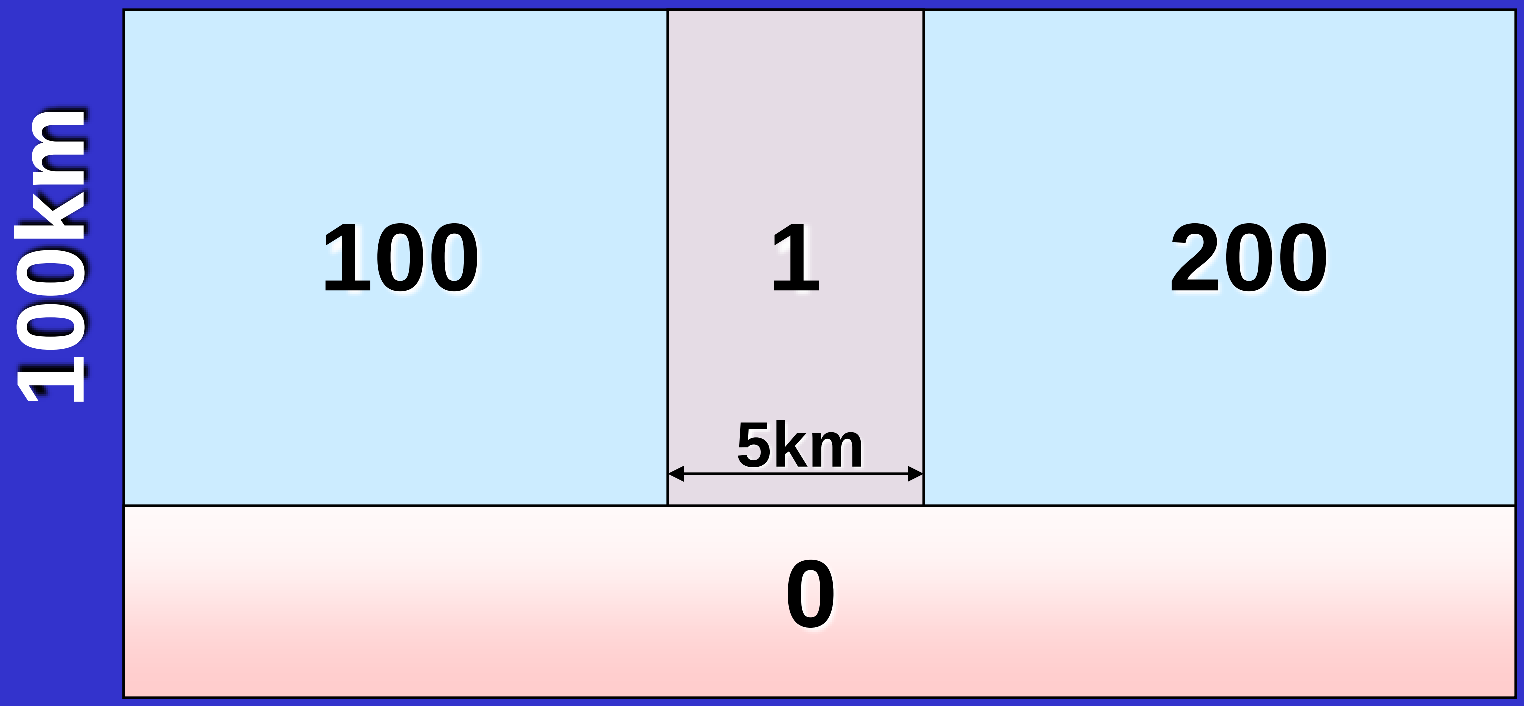


Vertical Slice

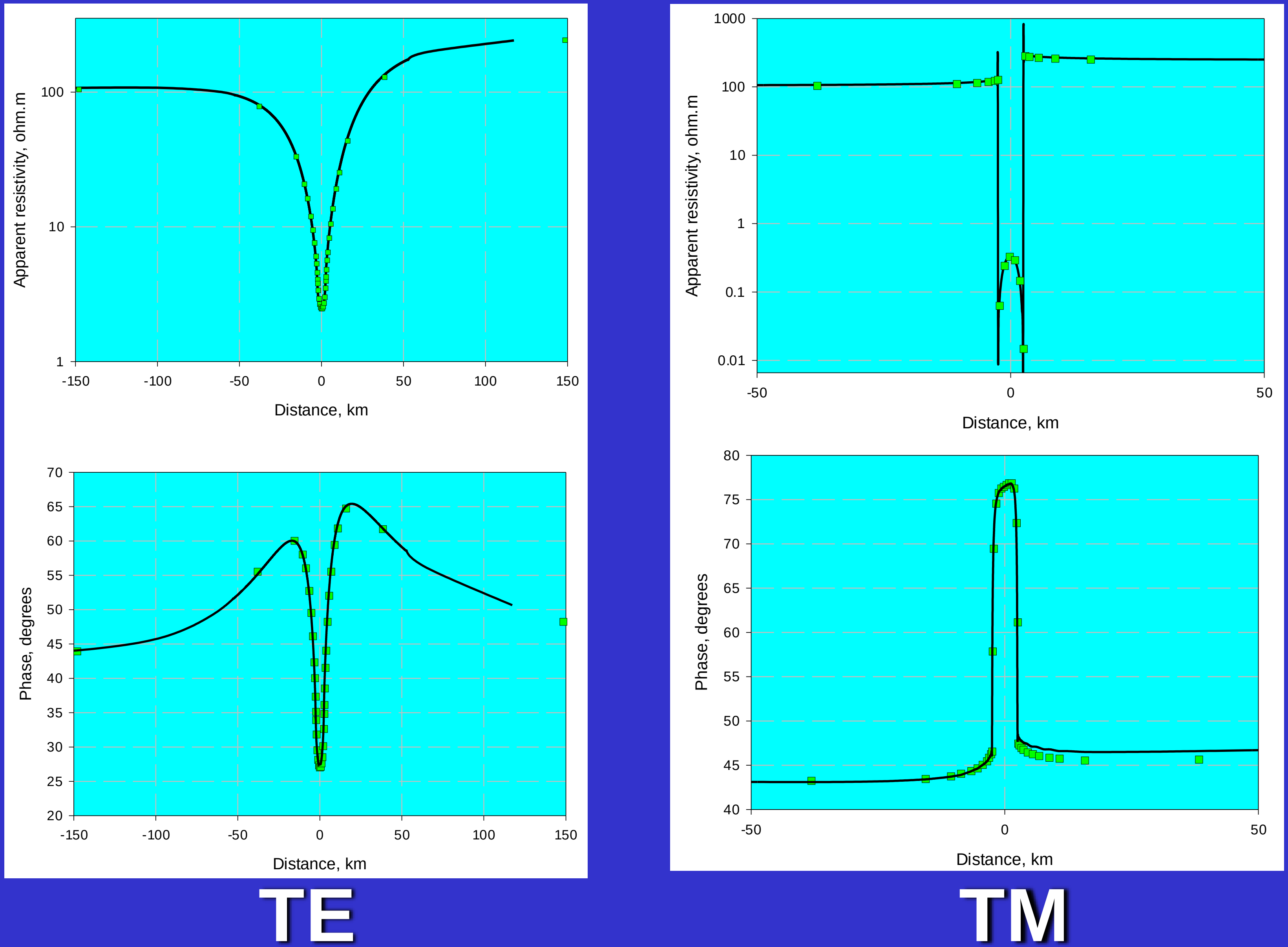
How good are these finite difference solutions?

We can test them by comparing to analytic solutions

2D ANALYTIC MODEL (WEAVER)



ANALYTIC (line) v. MACKIE (points)



Computational Considerations

- Serial or parallel
- Compute times
  - The Dublin 3D forward model I ran was 95x95x73 (not including air layers)
  - I ran it on our small cluster using 72 nodes to a preconditioned residual of 1e-10.
  - 21 frequencies in 30 minutes.