3D MT Inversion
Dublin 2008 Workshop

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Inverse Problem Formulation

The goal of CSEM/MT inversion is to infer the electrical conductivity of earth's subsurface given CSEM/MT measurements.

The problem can be expressed as an inverse problem of the form

$$d_i = F_i(m) + e_i, \qquad i = 1, 2, \ldots, N, \quad m \in M.$$

where

m=model
F=forward modeling operator
d=observed data
e=observational error

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Tikhonov Regularization

Tikhonov's method defines a regularized solution of the inverse problem to be a model m that minimizes the objective function

$$\phi(m) = (d - F(m))^T V^{-1}(d - F(m)) + \tau||Lm||^2$$

where

d=observed data vector

F=forward modeling operator

m=unknown model vector

V=error covariance matrix

L=a linear operator

τ=regularization parameter

Nonlinear Conjugate Gradients

- **Non-linear conjugate gradients** applies directly to the minimization of ϕ. The model sequence is given by

$$m_{j+1} = m_j + \alpha_{j+1} h_{j+1}$$

where h_{j+1} is a given search direction.

- Like linear CG, NLCG computes the search directions as

$$h_{j+1} = C_j g_j + \beta_j h_j$$

where C_j is the preconditioner.

- Unlike linear CG, α_{j+1} is computed to minimize ϕ along the search direction using an iterative line minimization procedure.

Nonlinear Conjugate Gradients

- In our NLCG algorithm, the line minimization automatically defaults to a one-step computation when the Forward problem can be well-approximation by it's linear expansion around the previous model.

- Operations with A and A^T are computed efficiently using reciprocity relationships. They require solving two extra forward problems per frequency.

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Preconditioner

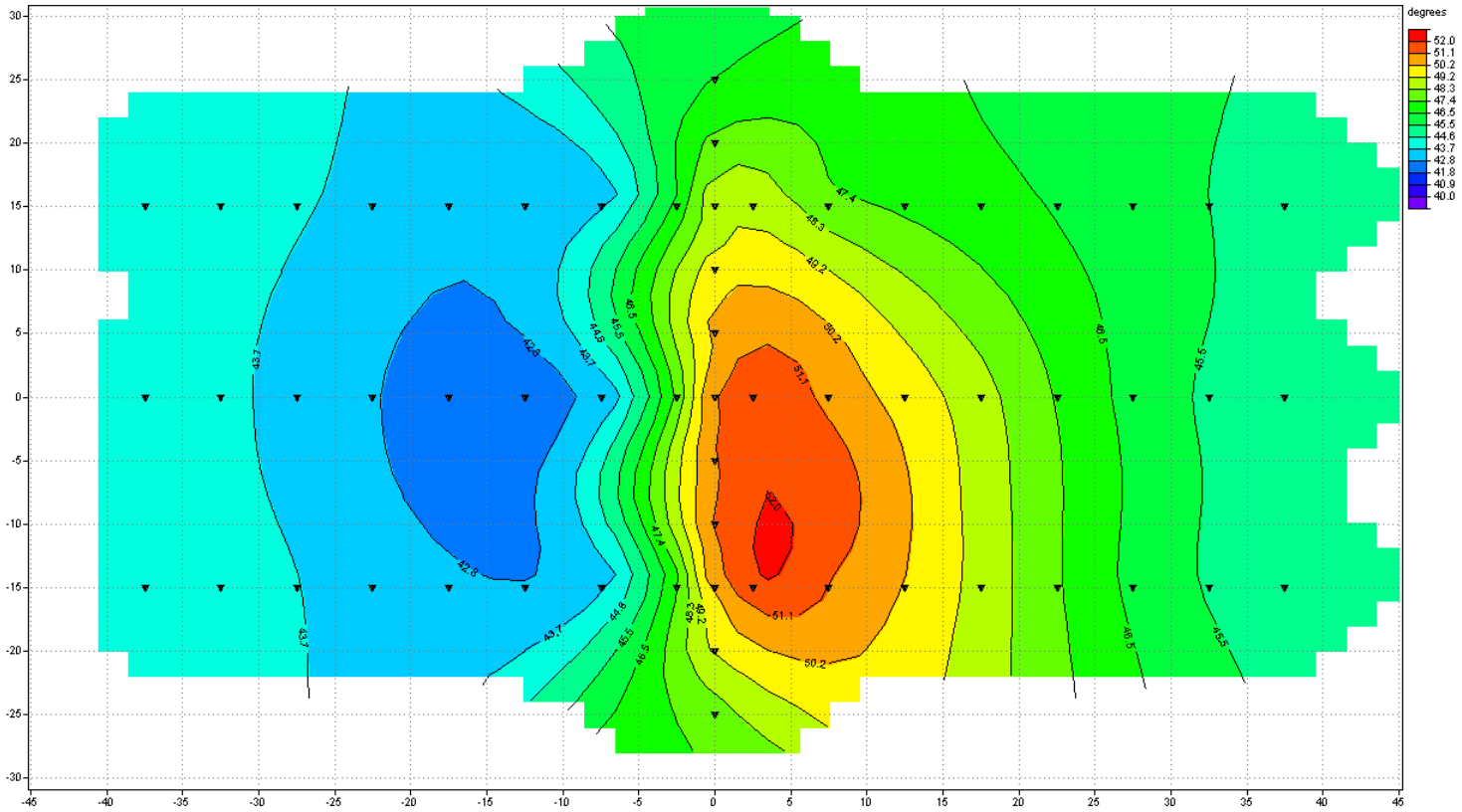
- The preconditioner is important for an efficient algorithm. The preconditioner should 'steer' the gradient in the direction of the true solution, but should not be computationally expense.

- We take the preconditioner to be the inverse of an approximation to the Hessian:

$$C = (\Lambda + \lambda L^T L)^{-1}$$

Where Λ approximates the diagonal part of $A^T V^{-1} A^T$

3D Forward Model Data



3D Phase XY Map at 30s

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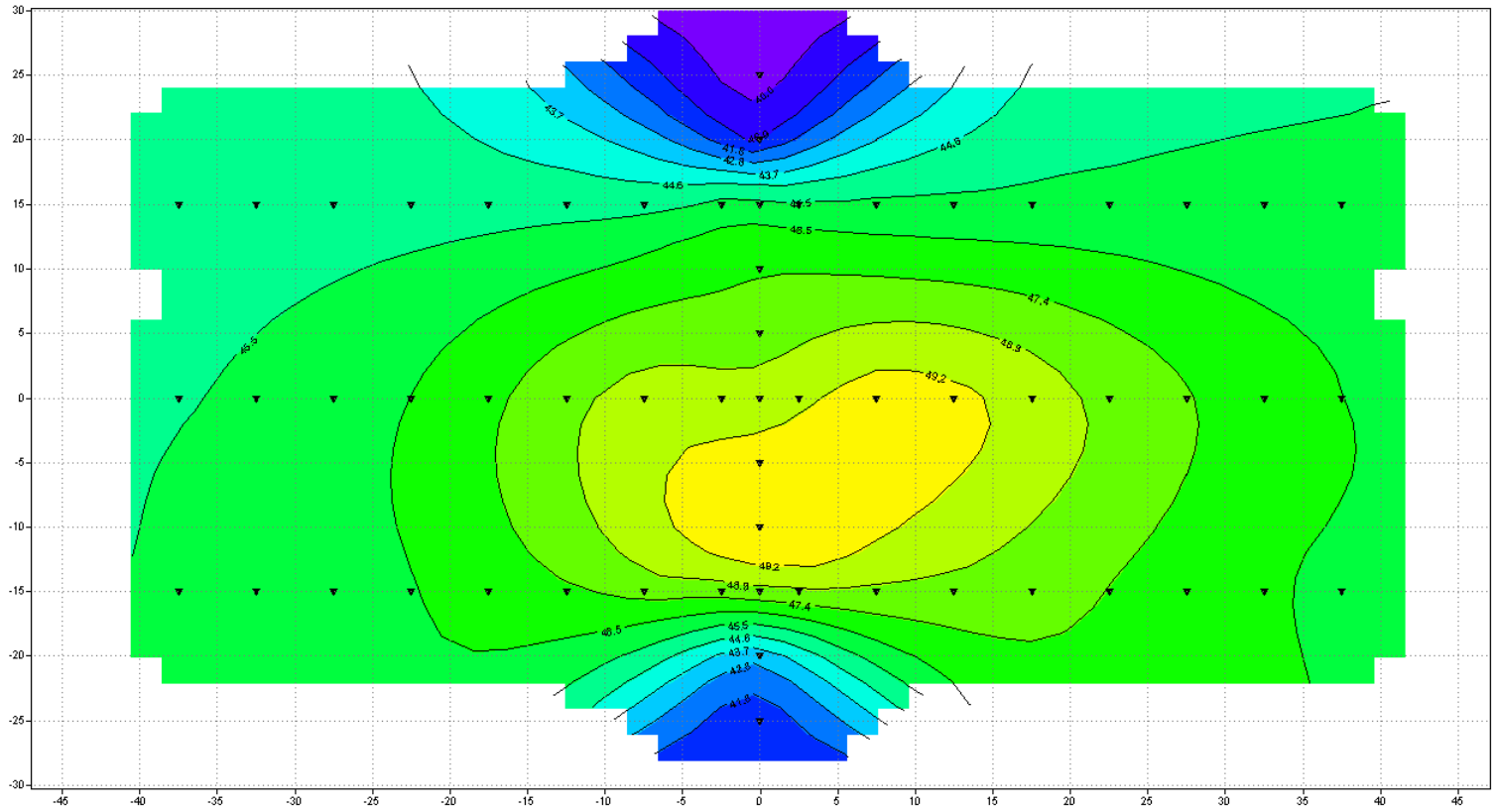
3D Forward Model Data



3D Phase YX Map at 30s

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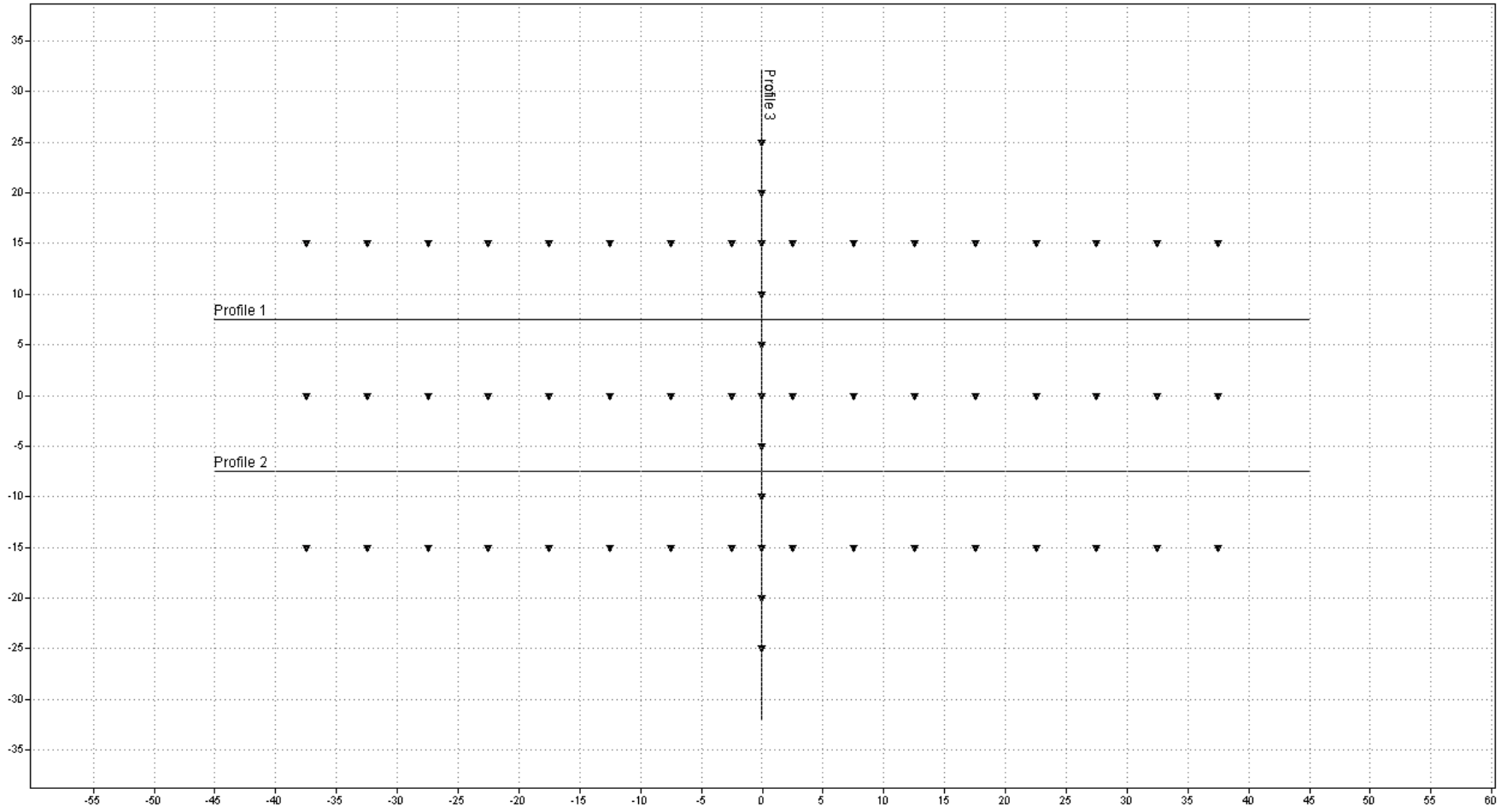
3D Forward Model Data



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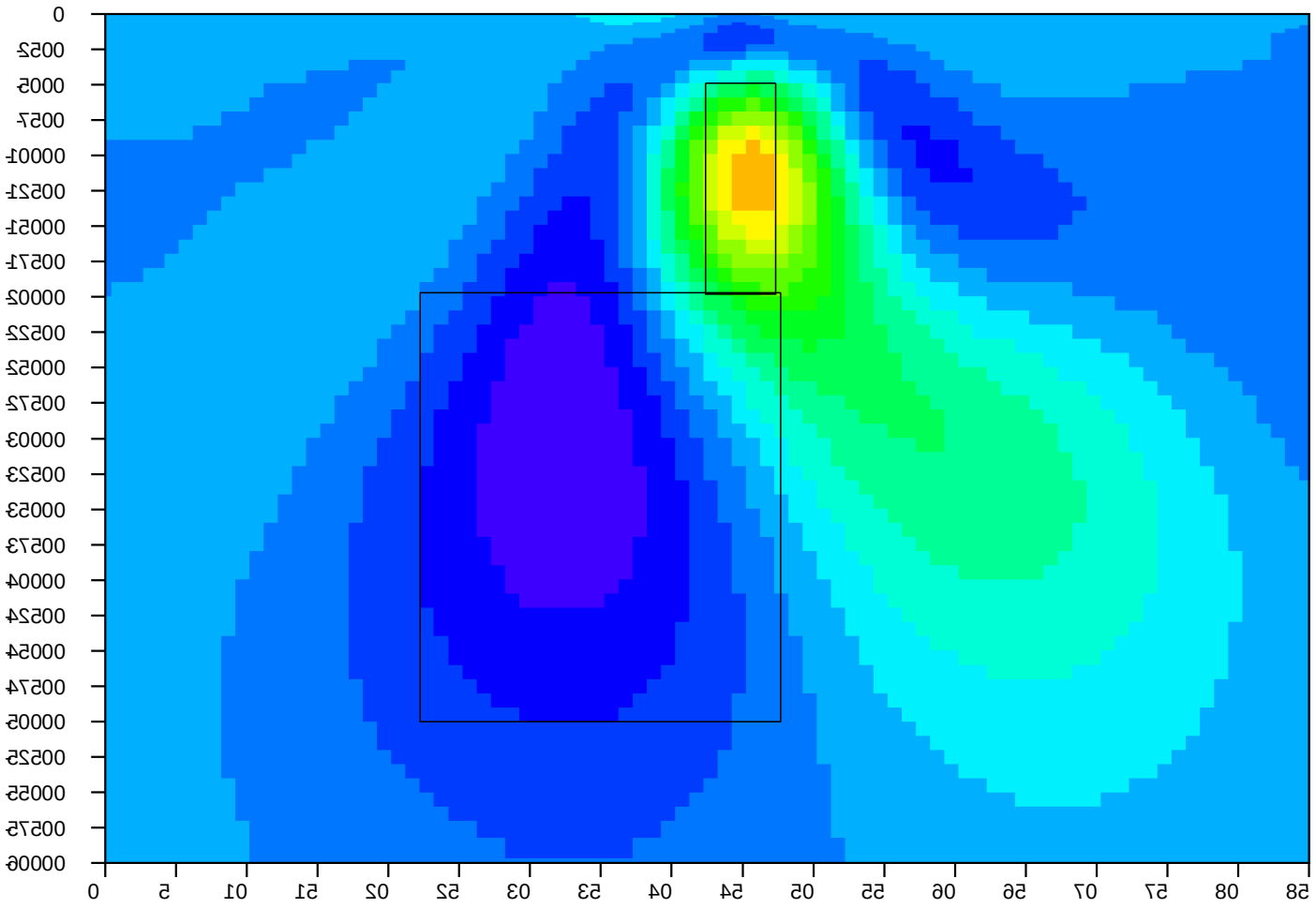
Profile 1: Inversion Result from FWD data



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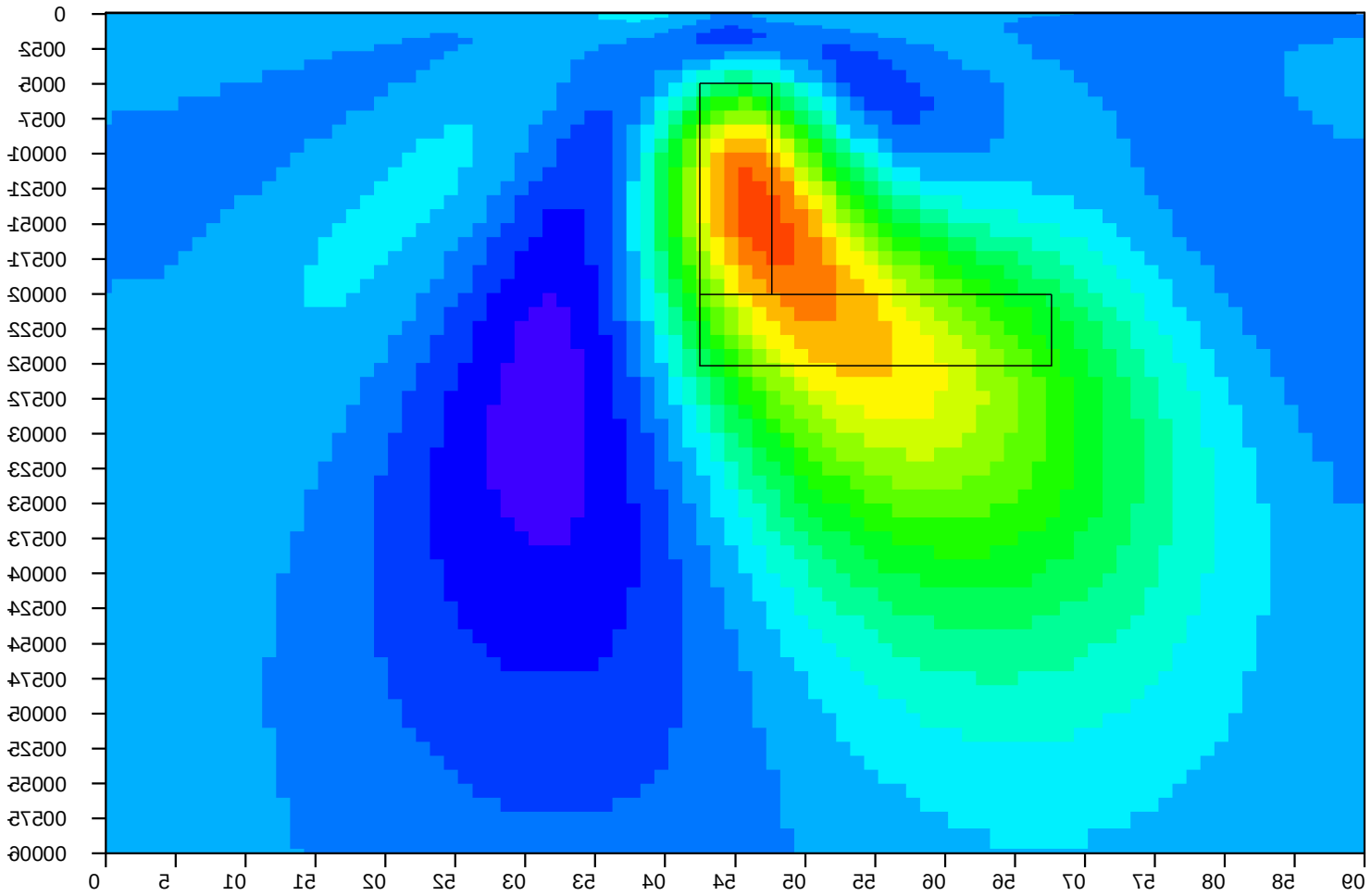
Profile 2: Inversion Result from FWD data



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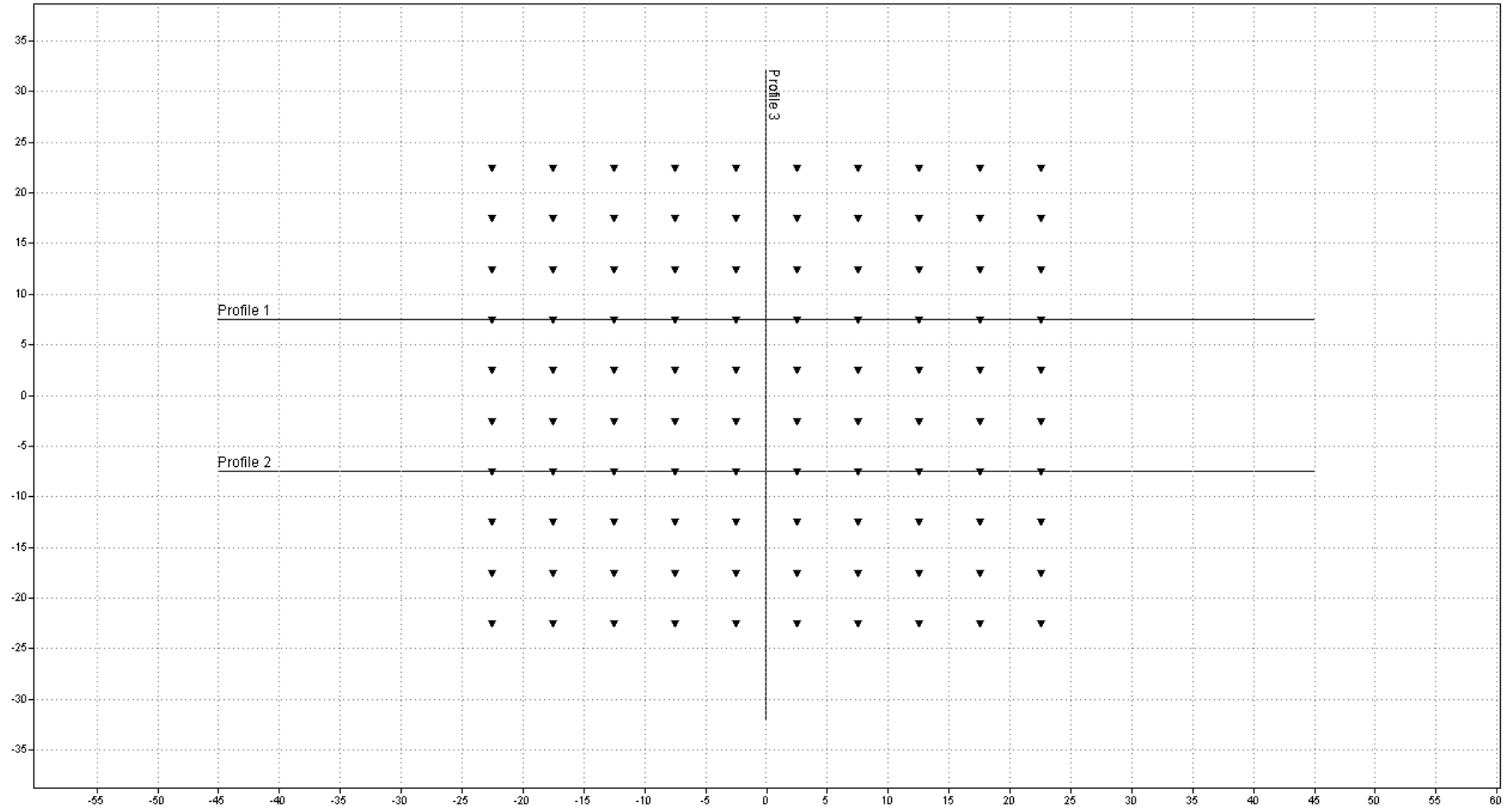
Secret Data Site Plan



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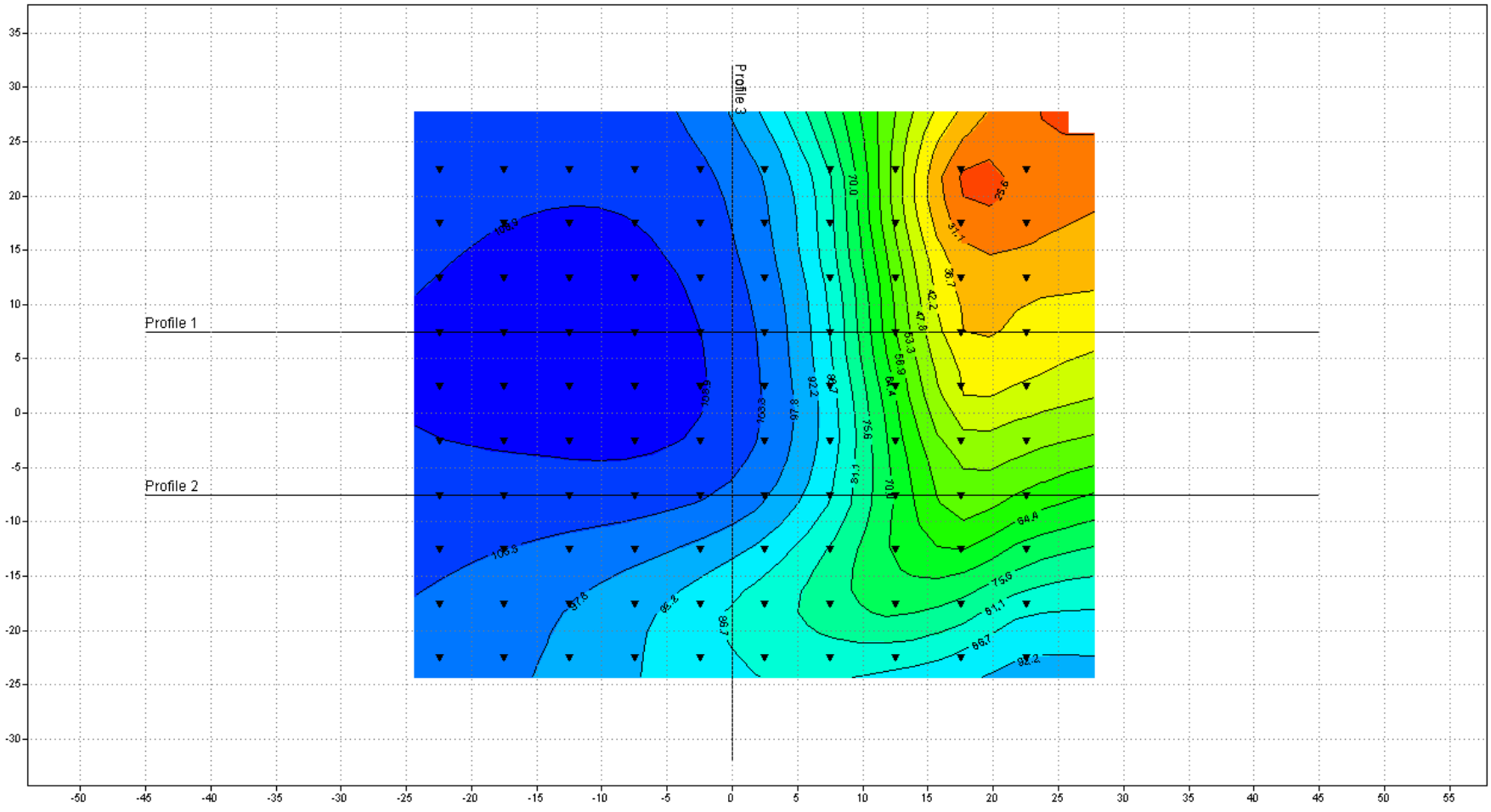
App Res XY at 10 sec



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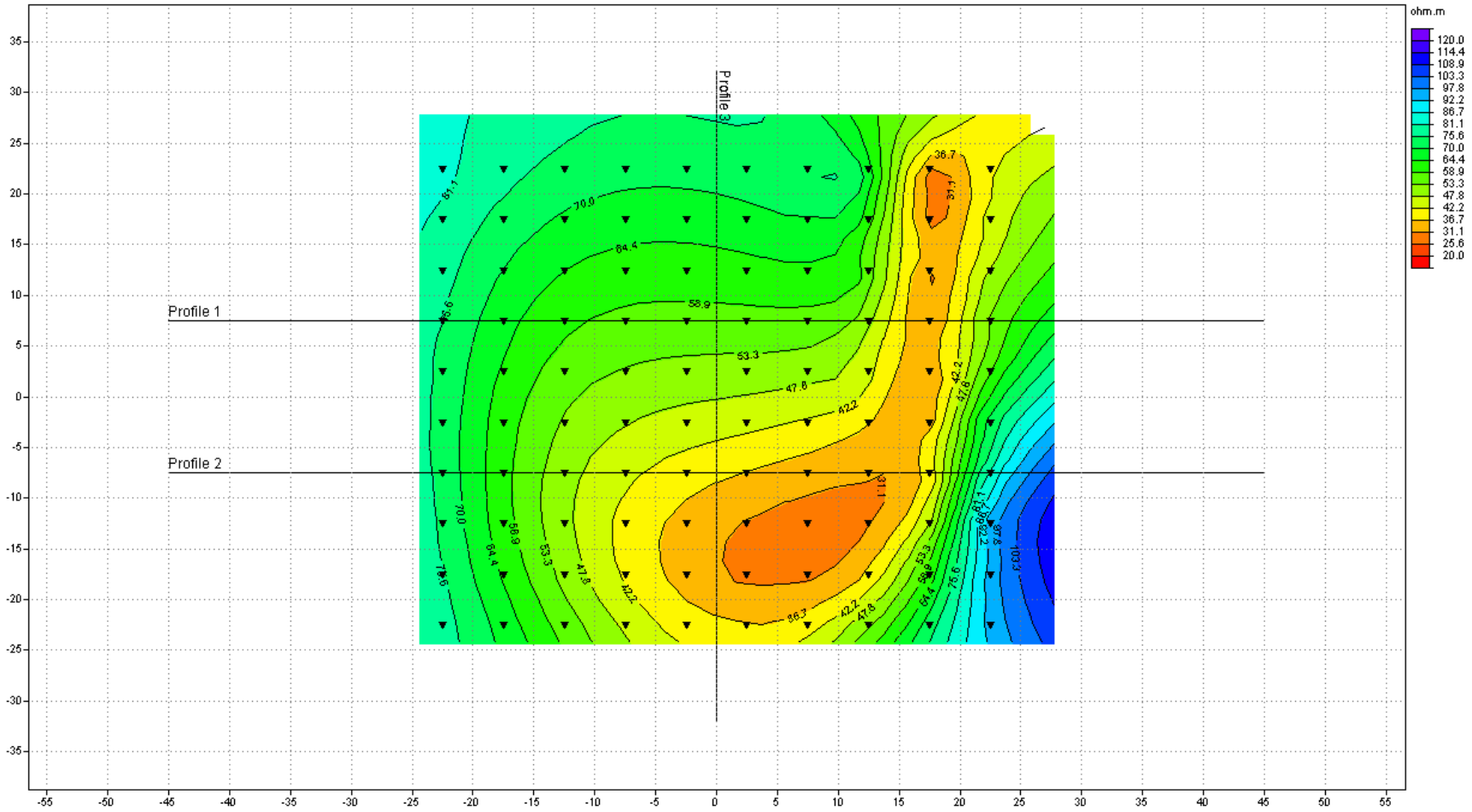
App Res XY at 100 sec



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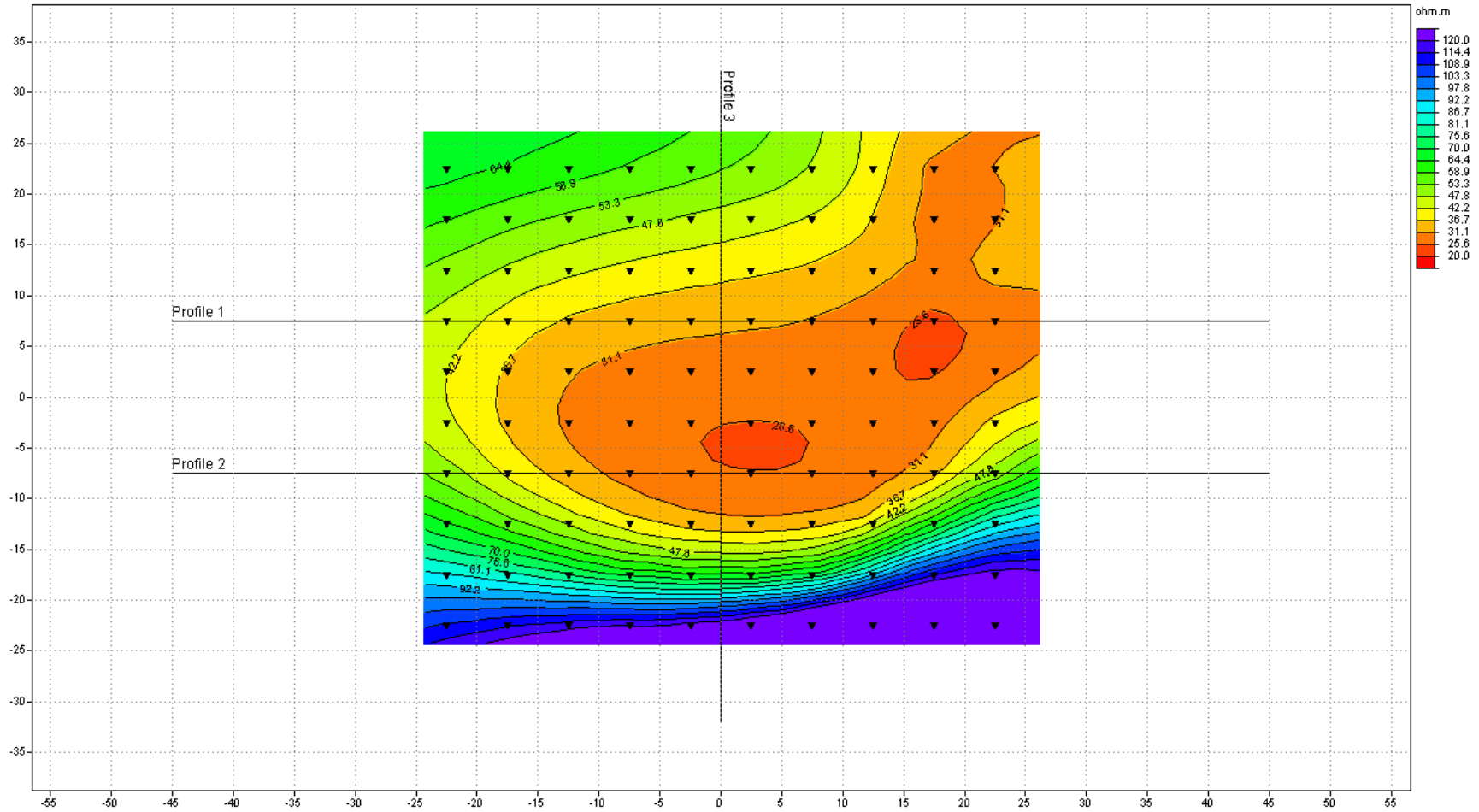
App Res XY at 1000 sec



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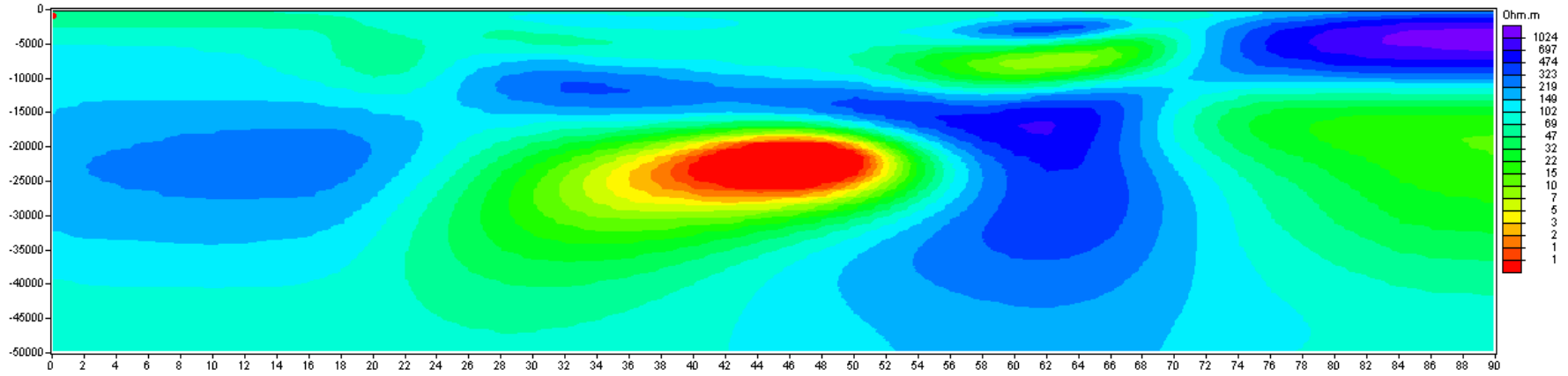
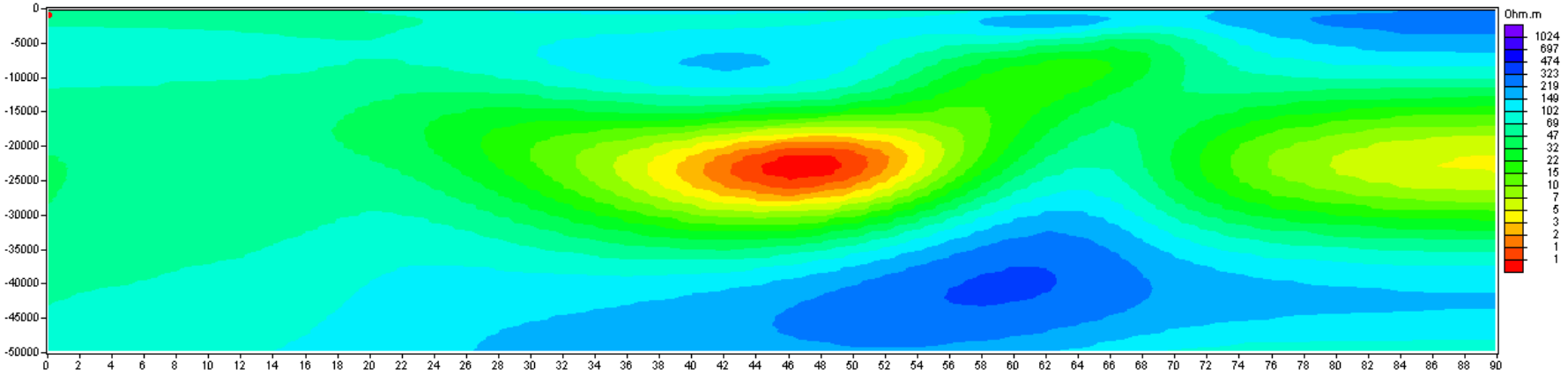
Secret Data: Profile 1





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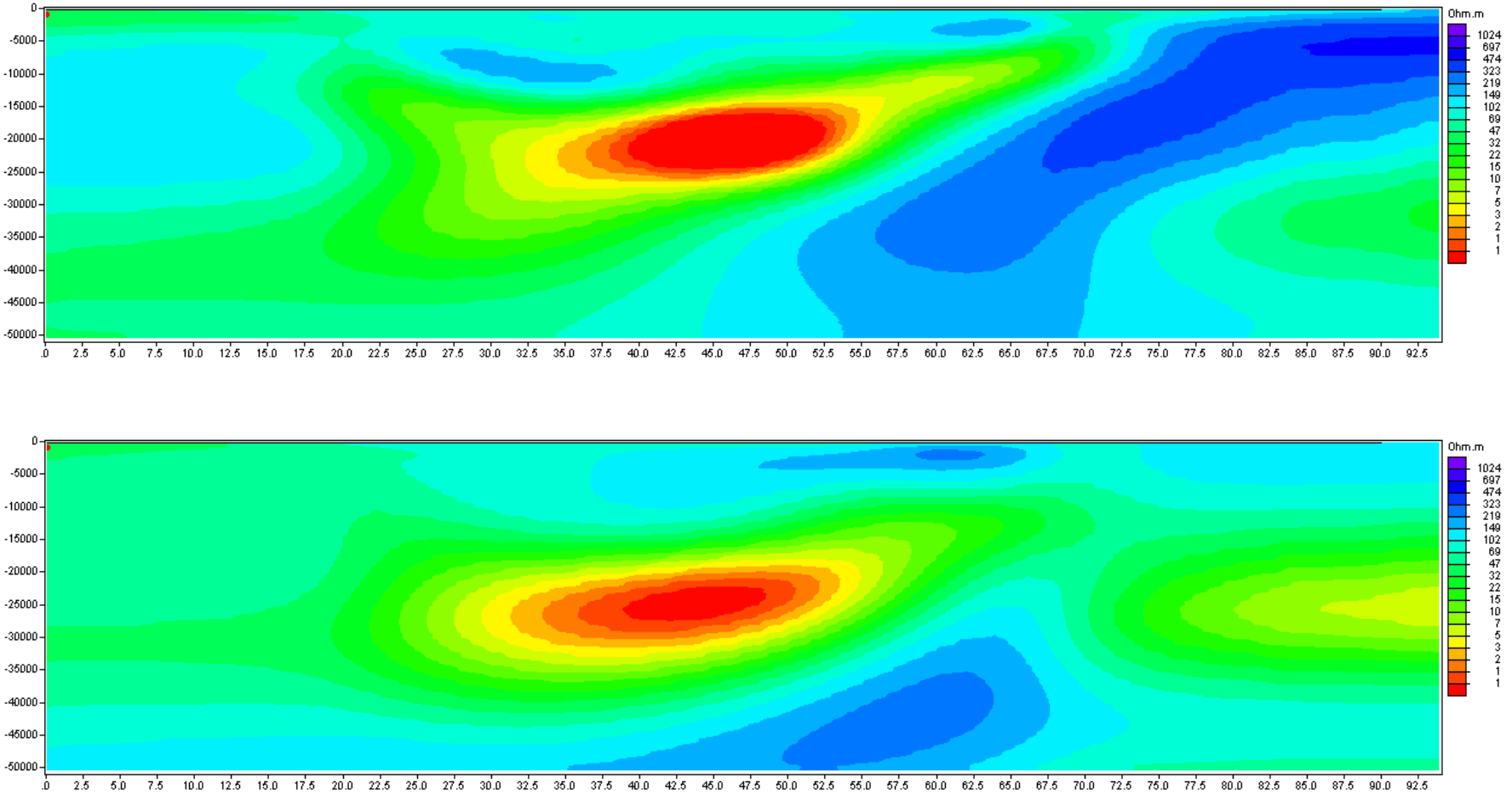
Secret Data: Profile 2



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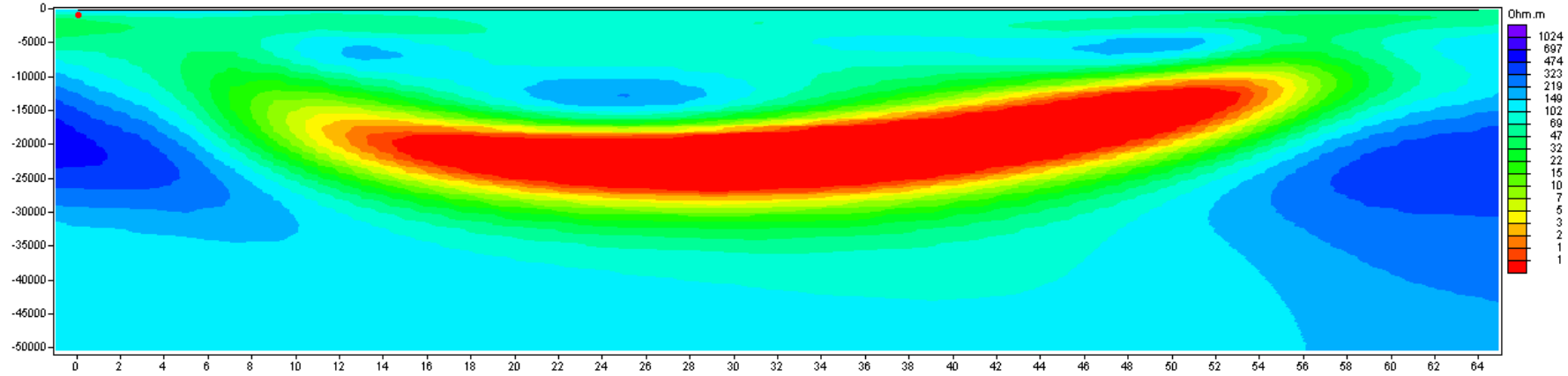
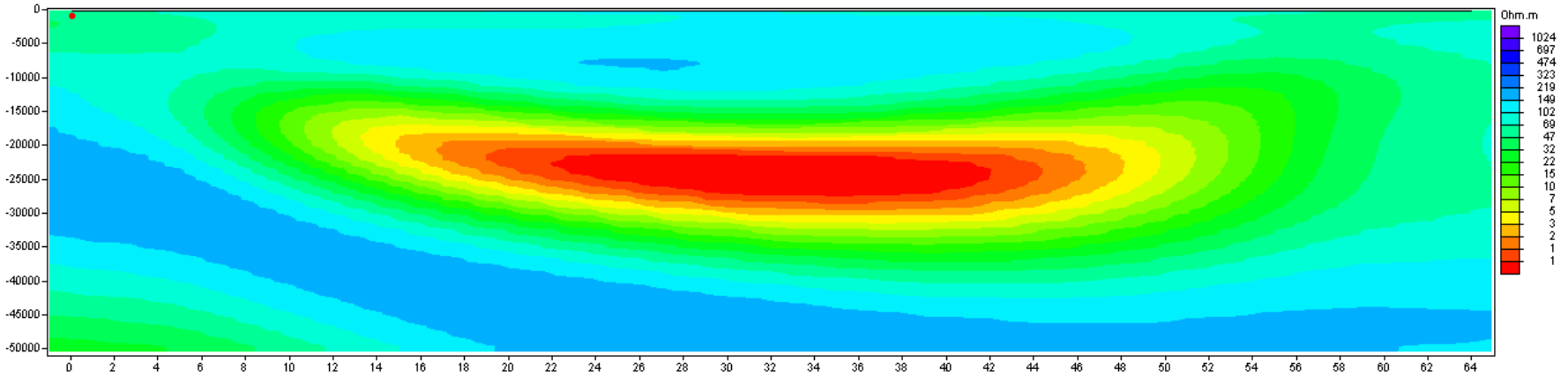
Secret Data: Profile 3





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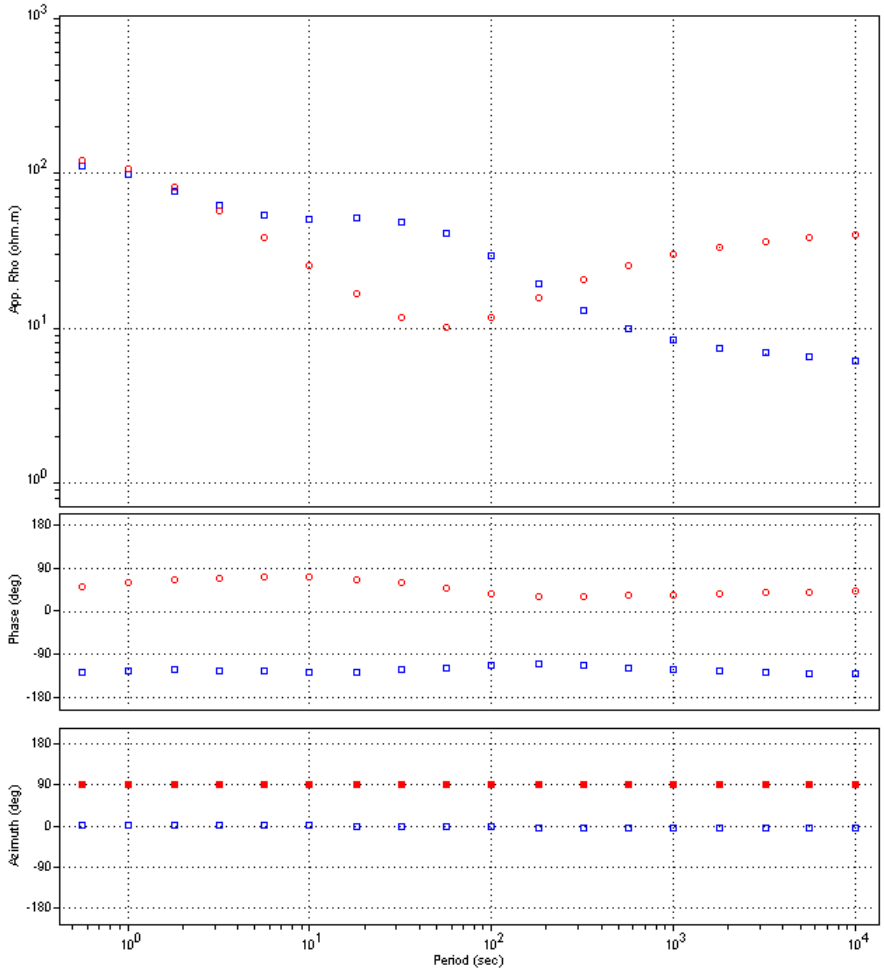
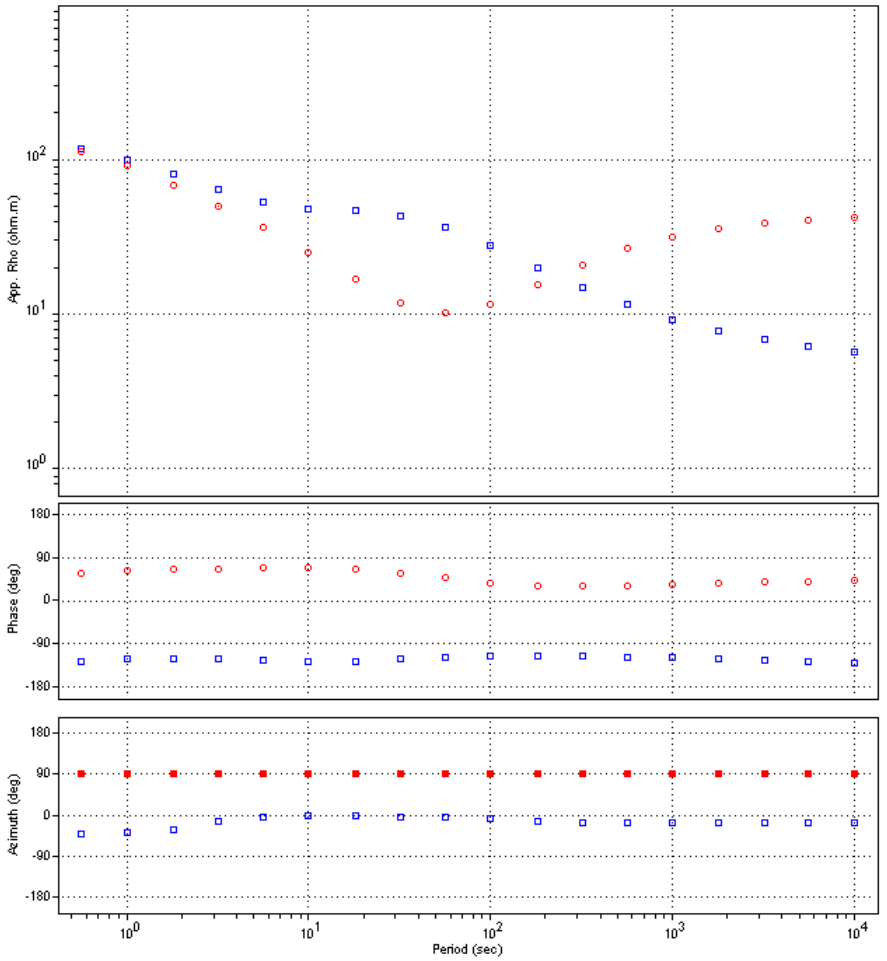
Data Fits: A09



Observed



Predicted

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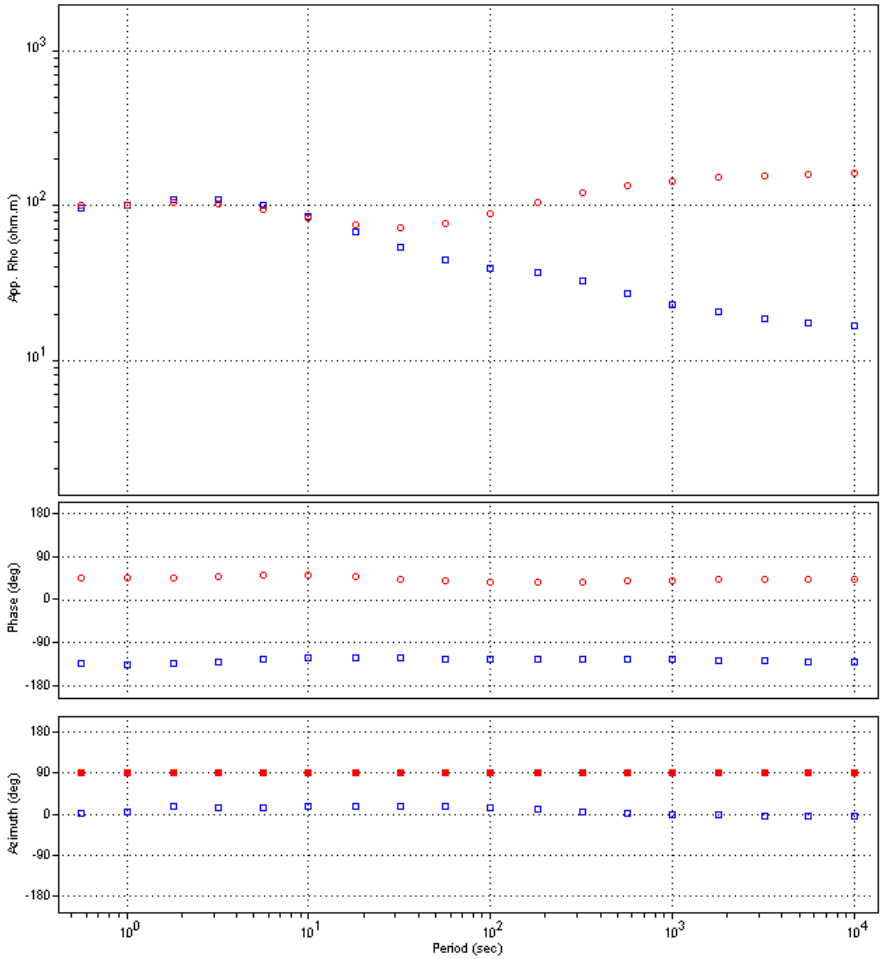
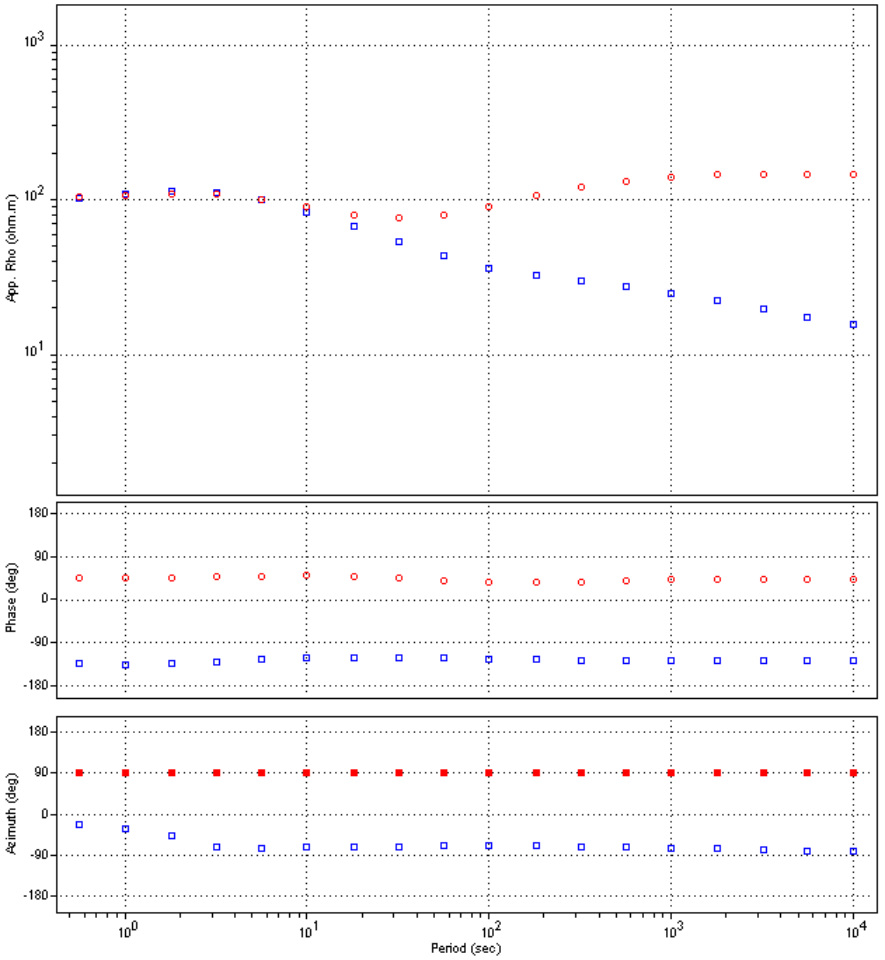
Data Fits: Site J07





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