3D Forward Model

Analytic solution

MT 3D INVERSION WORKSHOP II
30 March – 1 April 2011
DIAS, Dublin

Grup d'Exploració Electromagnètica i Sísmica
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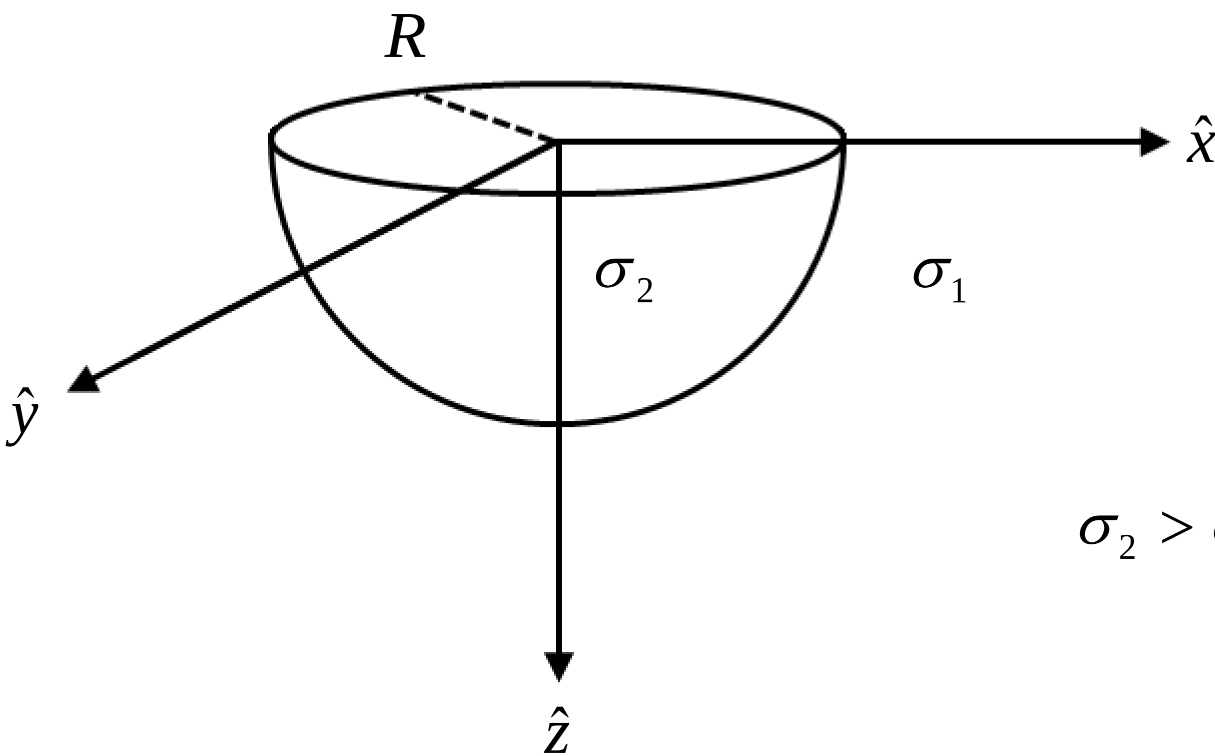
The model: a conducting hemisphere



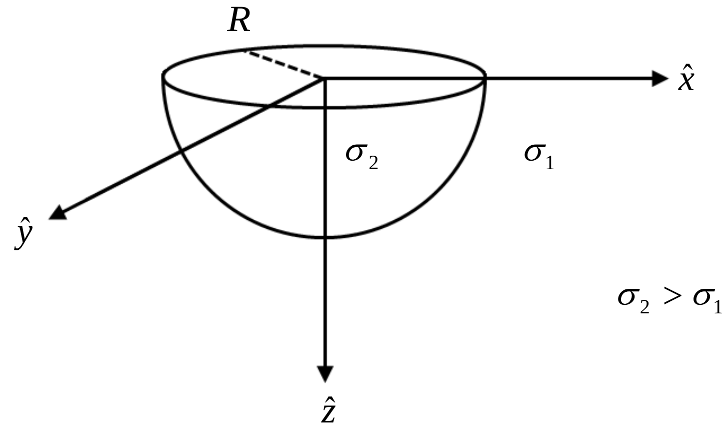
$$\sigma_2 > \sigma_1$$

$R = 5\ \text{km}$

$\rho_1 = 300\ \Omega{\cdot}\text{m}$

$\rho_2 = 10\ \Omega{\cdot}\text{m}$

Electric distortion tensor



Following Groom & Bailey (1991):

$$\mathbf{E}(x_0, y_0, 0) = \mathbf{E_0} + \mathbf{E_a} = \tilde{C}\mathbf{E_0}$$

- Inside the hemisphere:

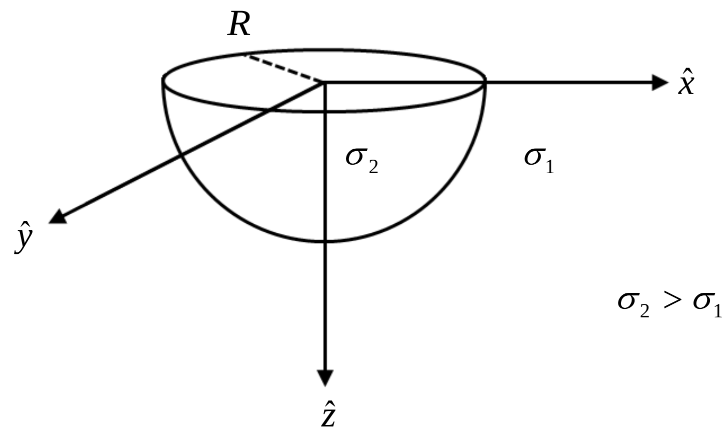
$$\tilde{C}_{in}(x_0, y_0, 0) = \begin{pmatrix} \dfrac{3\sigma_1}{\sigma_2 + 2\sigma_1} & 0 \\ 0 & \dfrac{3\sigma_1}{\sigma_2 + 2\sigma_1} \end{pmatrix}$$

- Outside the hemisphere:

$$\tilde{C}_{ex}(x_0, y_0, 0) = \begin{pmatrix} 1 + P\dfrac{(2x_0^2 - y_0^2)}{r_0^5} & \dfrac{P3x_0y_0}{r_0^5} \\ \dfrac{P3x_0y_0}{r_0^5} & 1 + P\dfrac{(2y_0^2 - x_0^2)}{r_0^5} \end{pmatrix}, \quad r_0^2 = x_0^2 + y_0^2$$

$$P = R^3 \frac{(\sigma_2 - \sigma_1)}{(\sigma_2 + 2\sigma_1)}$$

Magnetic distortion tensor



$$\mathbf{H}(x_0, y_0, 0) = \mathbf{H_0} + \mathbf{H_a}$$

$$\mathbf{H_a}(x_0, y_0, 0) = \frac{1}{4\pi}\int \frac{\mathbf{J_a}(x, y, z)\times\hat{\mathbf{r}}}{r^2}\, dV \;,\quad \mathbf{r} = (x_0 - x)\hat{\mathbf{x}} + (y_0 - y)\hat{\mathbf{y}} + z\hat{\mathbf{z}}$$

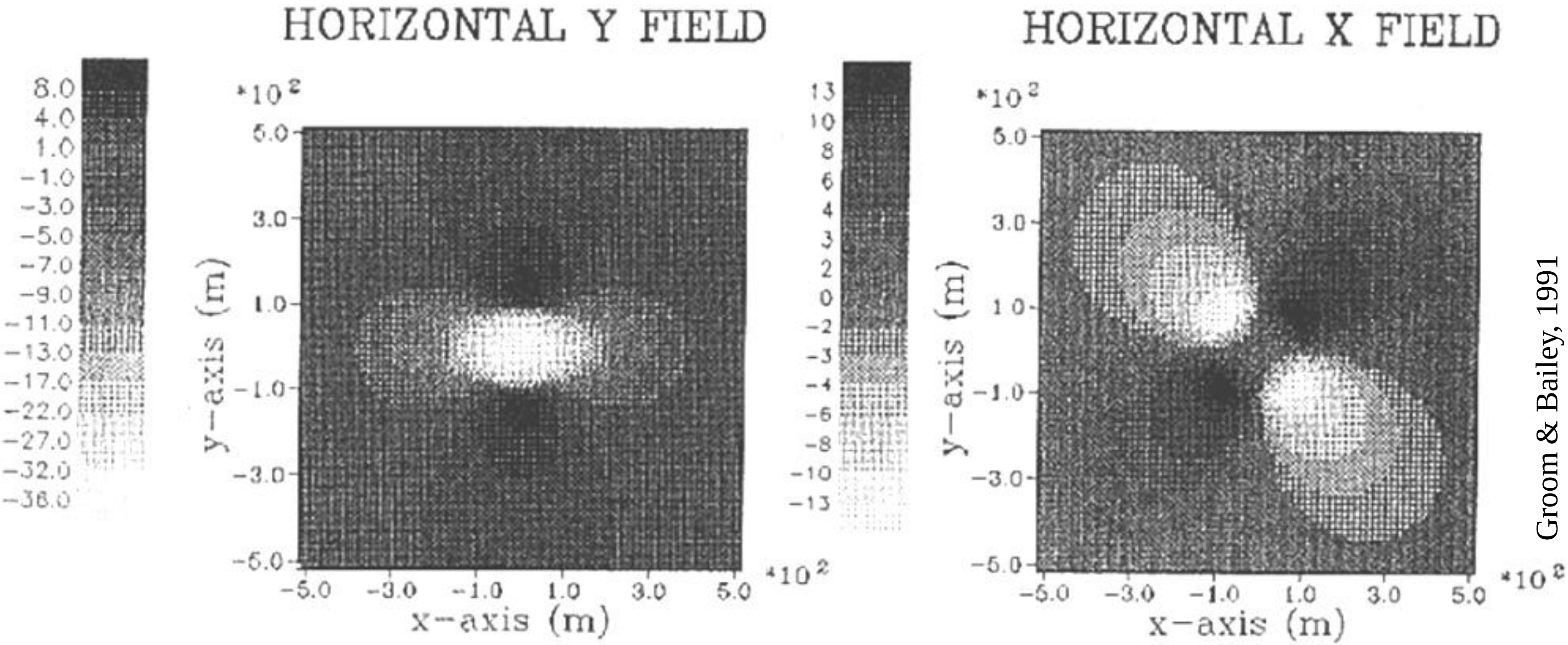
The problem reduces to solve numerically five triple integrals (some of them with singularities).

$$\mathbf{H_a^h}(x_0, y_0, 0) = \begin{pmatrix} \gamma & -\beta \\ \alpha & -\gamma \end{pmatrix}\mathbf{E_0} = \tilde{D}\mathbf{E_0}$$

$$\mathbf{H^h}(x_0, y_0, 0) = \mathbf{H_0} + \tilde{D}\mathbf{E_0} = \left(I + \tilde{D}Z_0\right)\mathbf{H_0} \;,\; \mathbf{E_0} = Z_0\mathbf{H_0} \;,\; Z_0$$: regional 1D impedance

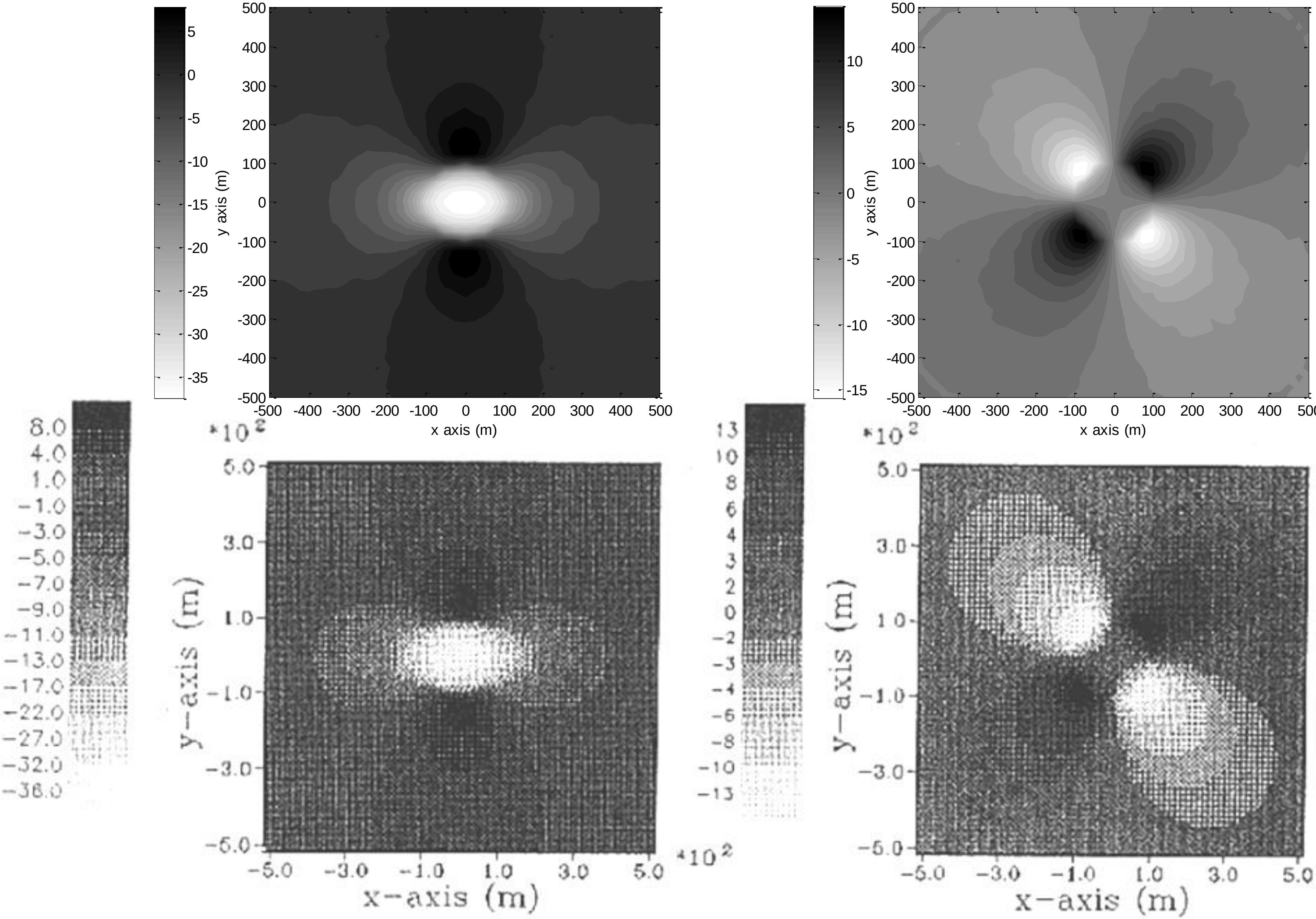
$$\mathbf{E} = \tilde{C}\mathbf{E_0} = \tilde{C}Z_0\mathbf{H_0} = \tilde{C}Z_0\left(I + \tilde{D}Z_0\right)^{-1}\mathbf{H^h} \Rightarrow \boxed{Z = \tilde{C}Z_0\left(I + \tilde{D}Z_0\right)^{-1}}$$

Validation: reproducing Groom and Bailey's results



Anomalous magnetic field components (normalized by $\beta = \sigma_1(\sigma_2 - \sigma_1)/(\sigma_2 + 2\sigma_1)$) for an electrostatic excitation of an hemisphere (R = 100 m, $\rho_1 = 300\,\Omega\text{m}, \rho_2 = 10\,\Omega\text{m}$) due to an uniform electric field in the **x** direction.

Validation: reproducing Groom and Bailey's results



Filled contour plot (25x25m grid, 20 contours)

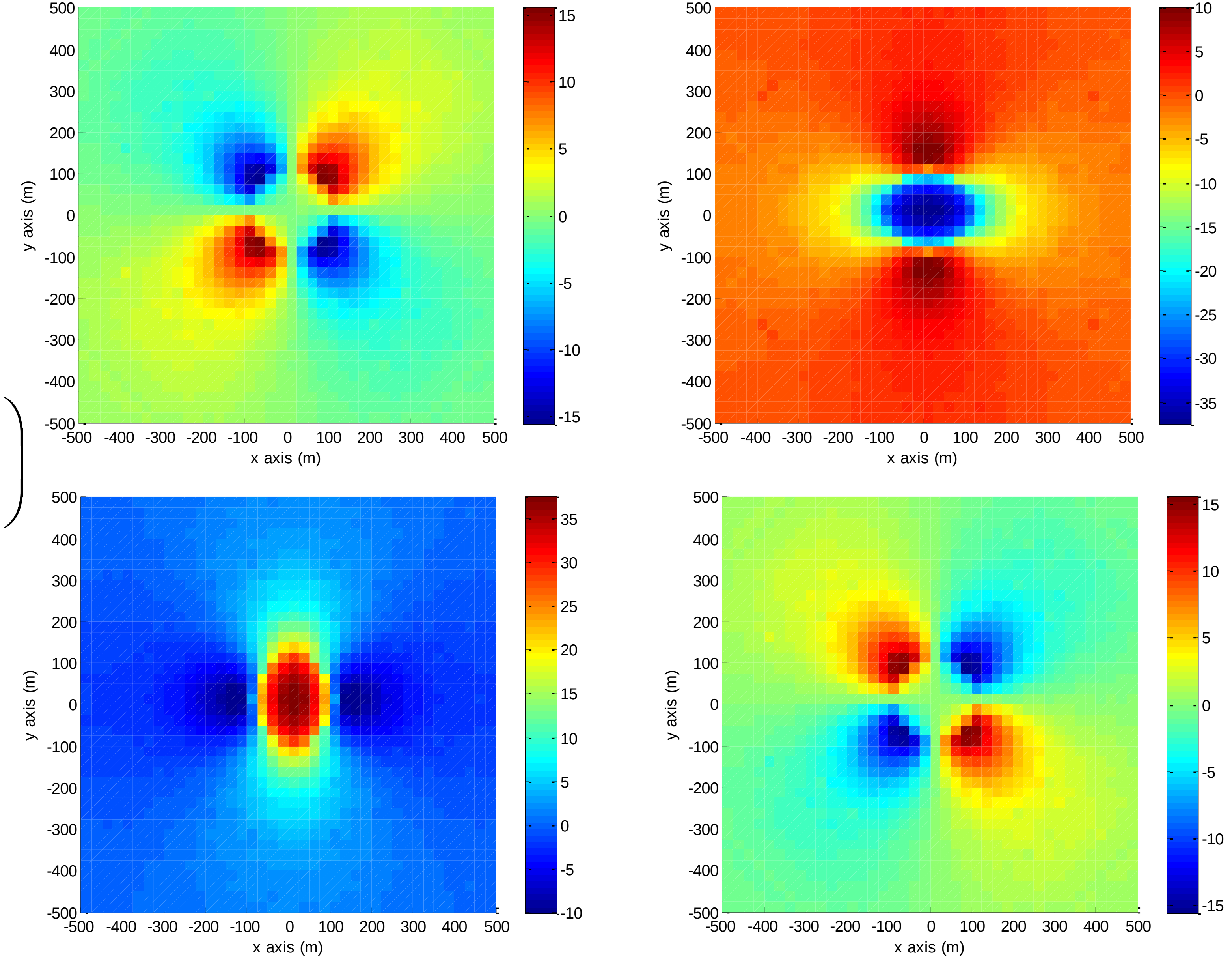
Groom & Bailey, 1991

Magnetic distortion tensor elements:

Excitating **x** electric field

$$\tilde{D} = \begin{pmatrix} \gamma & -\beta \\ \alpha & -\gamma \end{pmatrix}$$

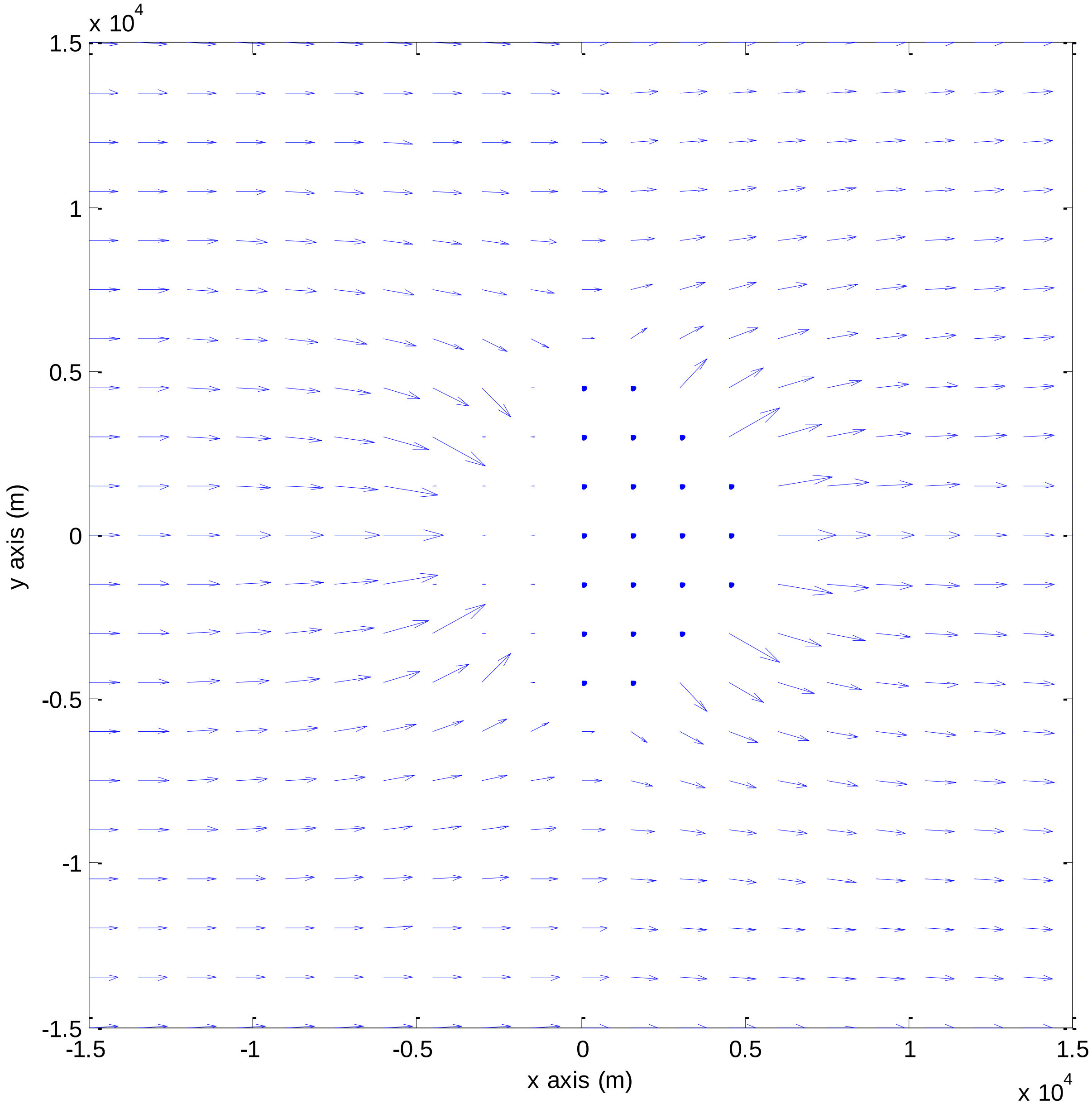
Excitating **y** electric field



Plotting the fields: electric field

$$\mathbf{E} = \widetilde{C}\mathbf{E_0}$$

$$\mathbf{E_0} = (1,0)$$

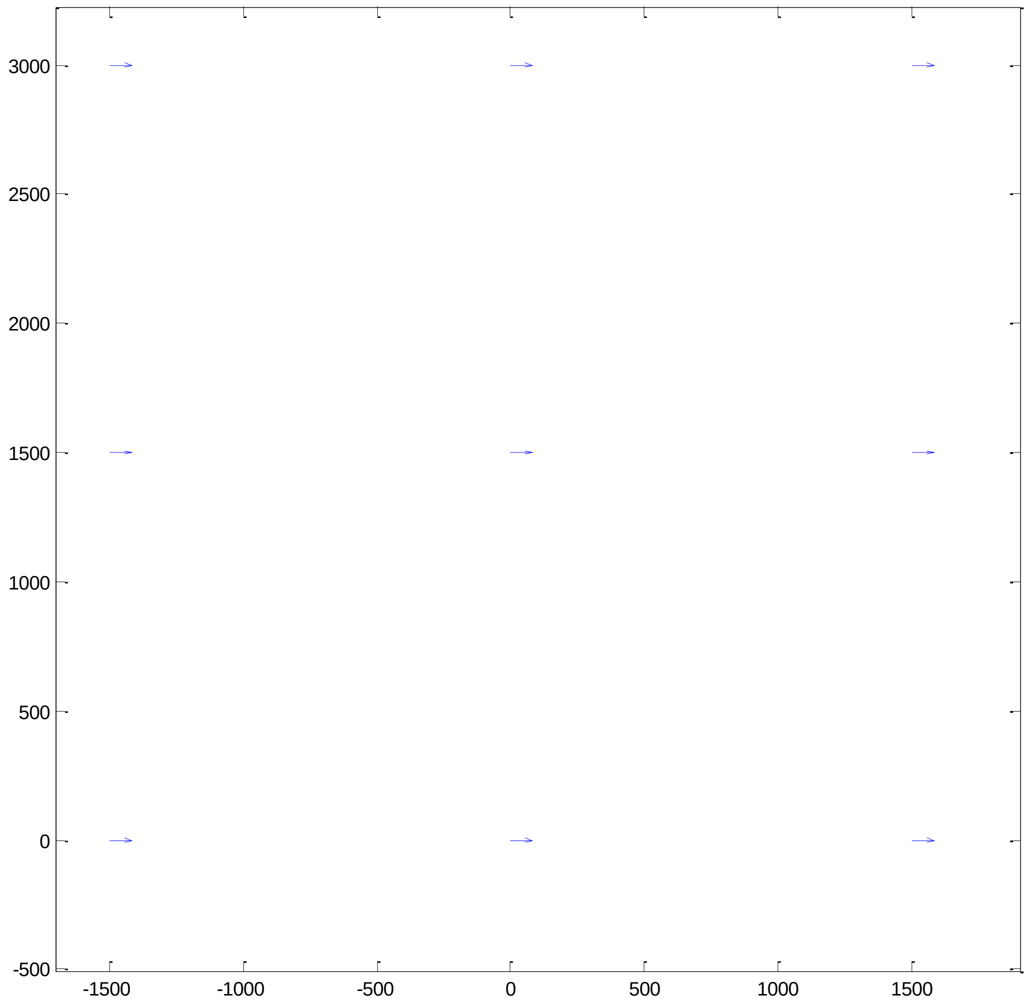


Plotting the fields: electric field

Zoom inside the hemisphere: constant in direction and magnitude

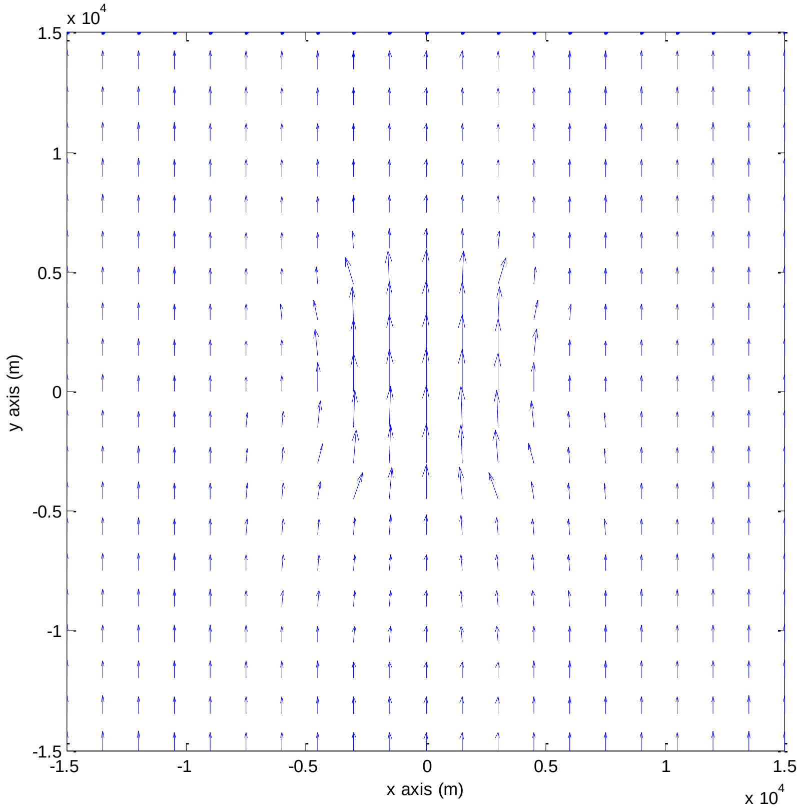
$$\mathbf{E} = \tilde{C}\mathbf{E_0}$$

$$\mathbf{E_0} = (1,0)$$

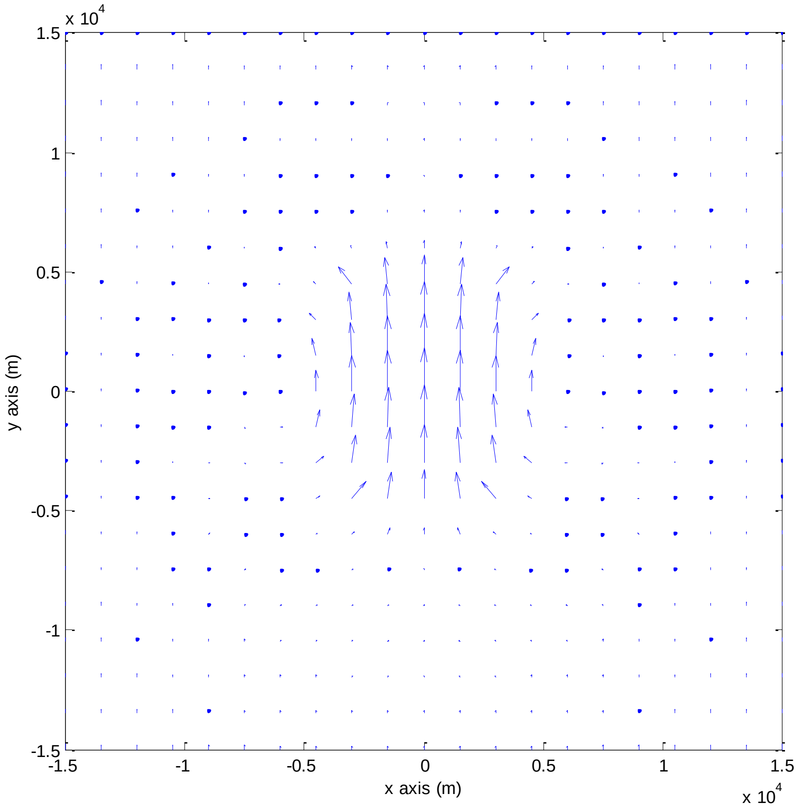


Plotting the fields: magnetic field

$\mathrm{Re}(\mathbf{H^h})$ $\qquad \mathbf{H^h} = \mathbf{H_0} + \tilde{D}\mathbf{E_0} = \left(Z_0^{-1} + \tilde{D}\right)\mathbf{E_0}\,,\ \ \mathbf{E_0} = (1{,}0)$

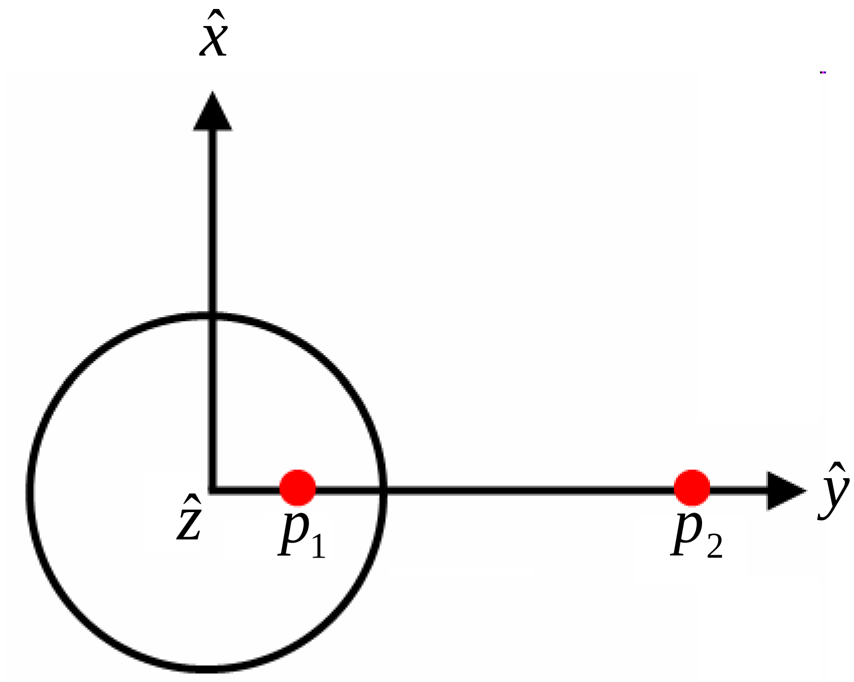


$f = 10\ \mathrm{Hz}$



$f = 100\ \mathrm{Hz}$

Some results: inside and outside the hemisphere



$$p_1 = (0, R/2, 0)$$

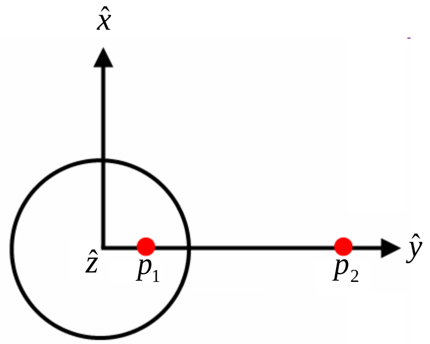
$$p_2 = (0, 3R, 0)$$

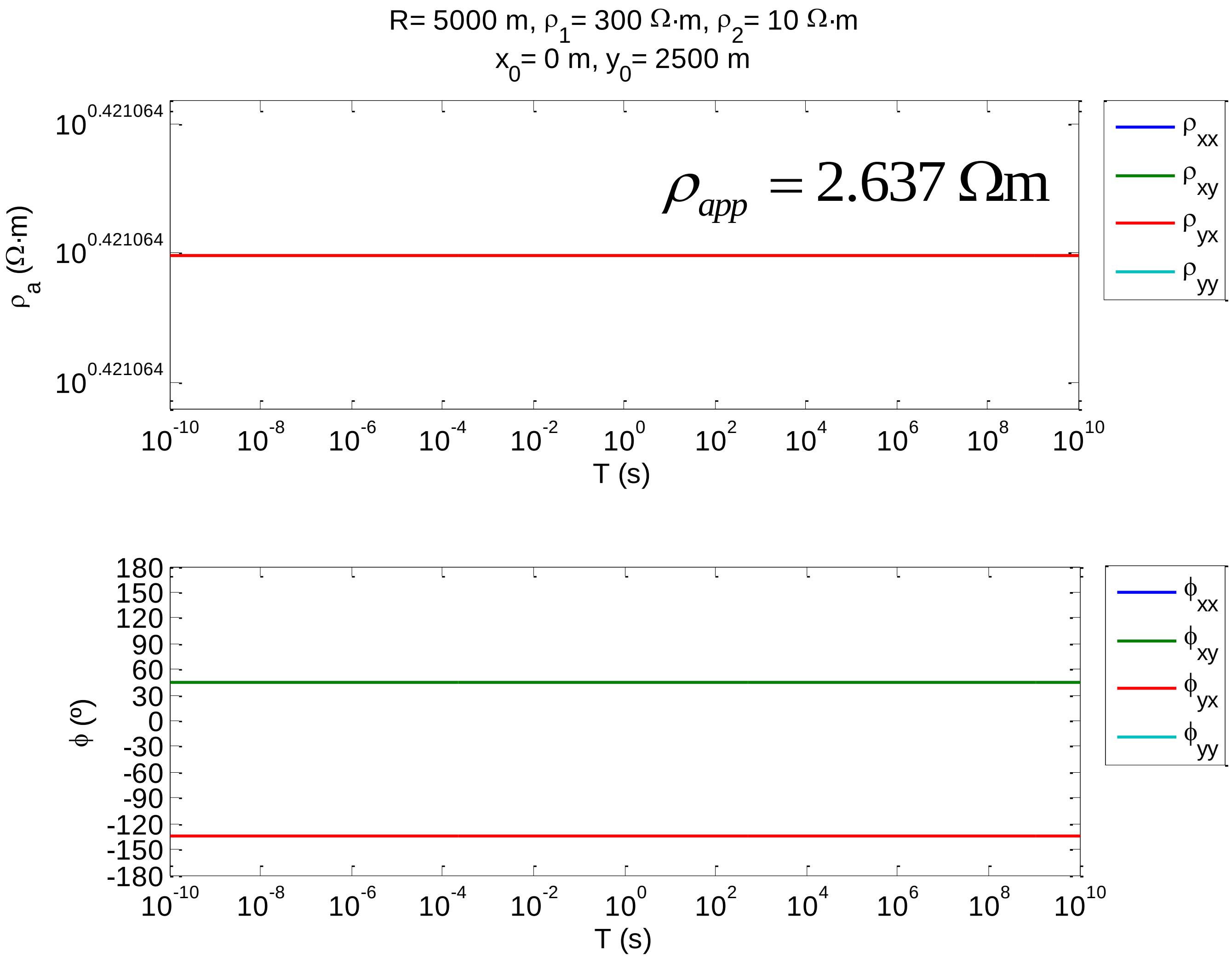
Some results: inside the hemisphere

$p_1 = (0, R/2, 0)$

Ignoring $\tilde{D}$

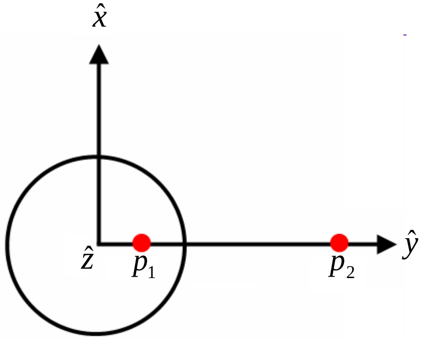
$$Z = \tilde{C} Z_0$$

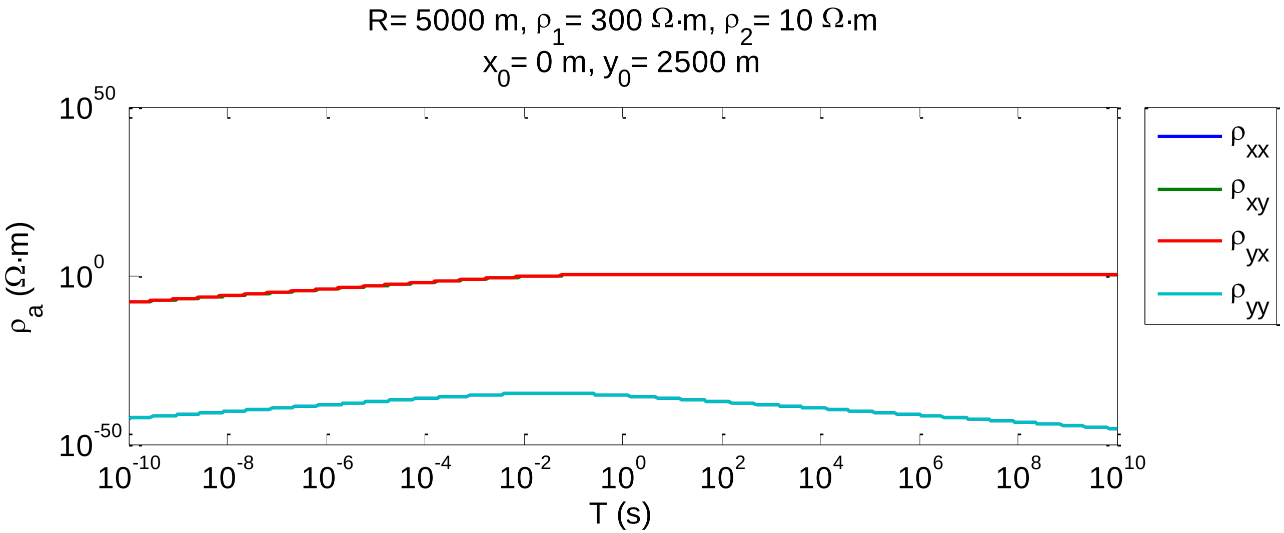


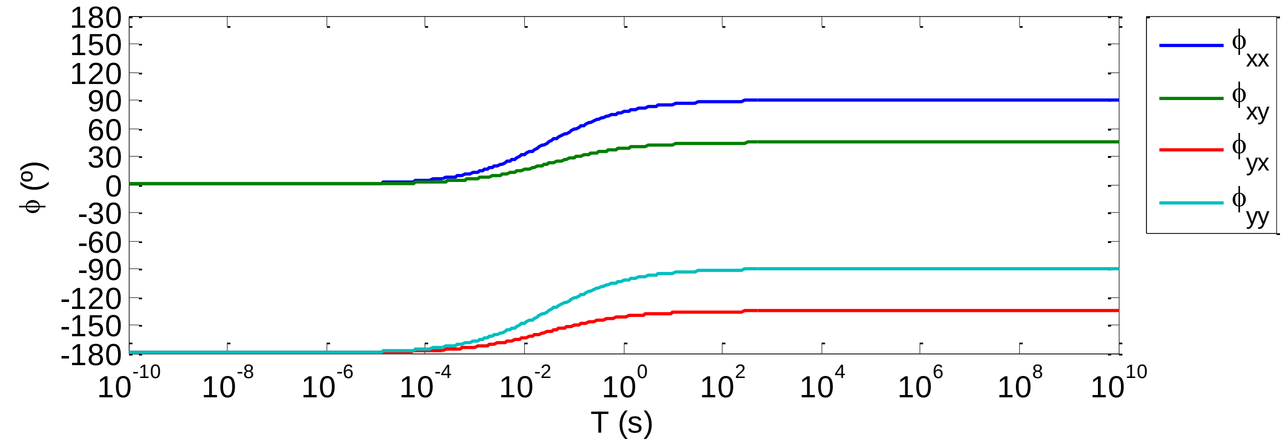


Some results: inside the hemisphere

$p_1 = (0, R/2, 0)$ Considering $\tilde{D}$

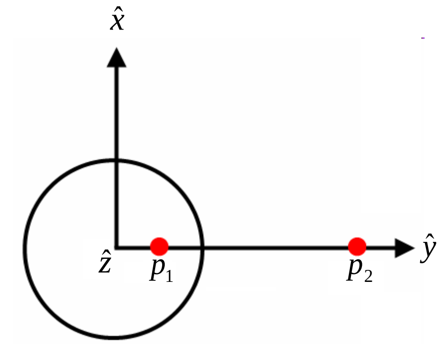




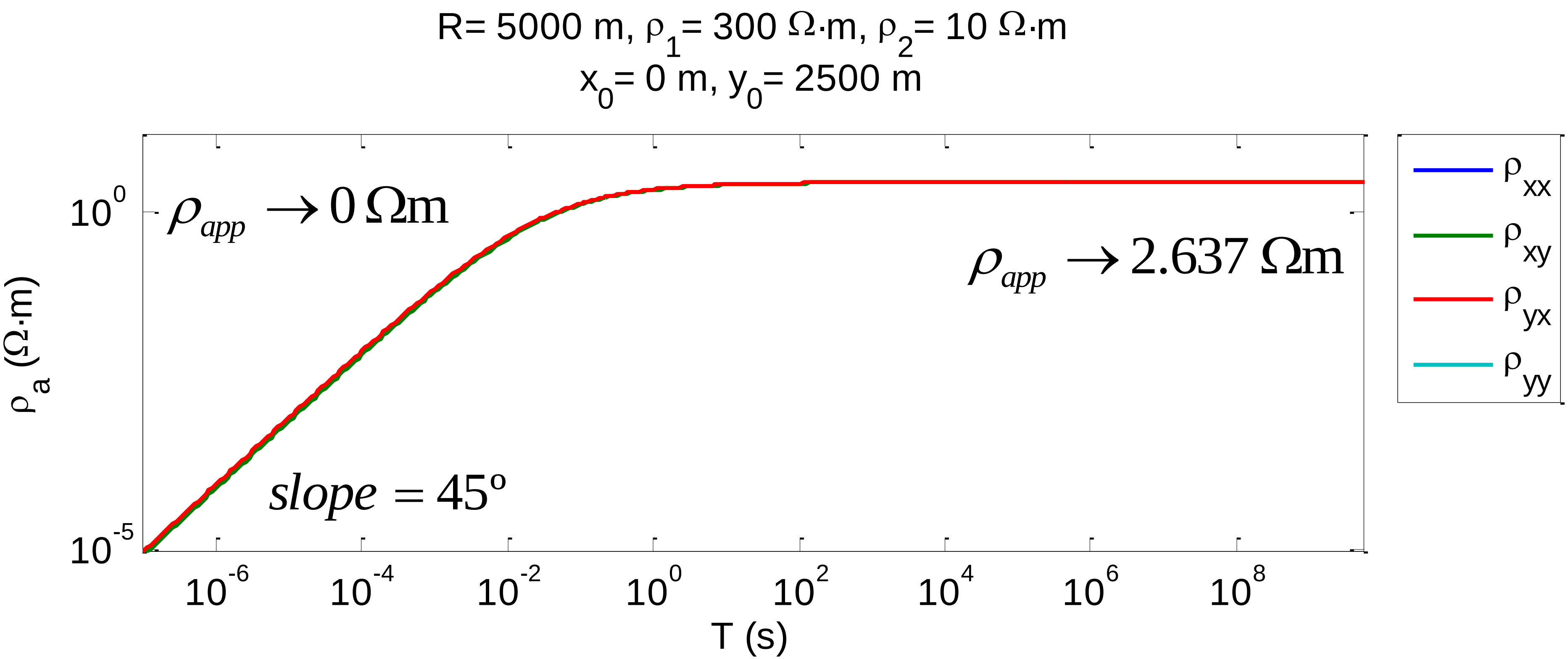


Some results: inside the hemisphere

$$p_1 = (0, R/2, 0)$$

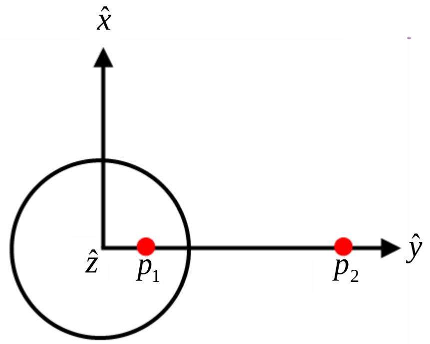


Zoom



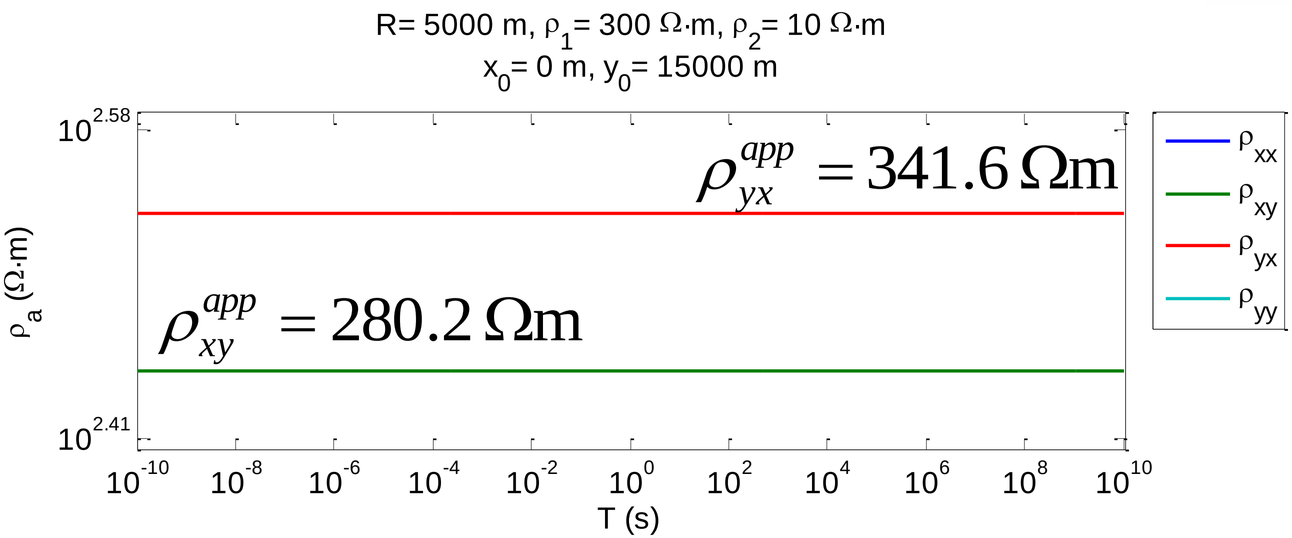
Period at which the skin depth equals the distance to the edge of the hemisphere (2.5 km): 2.47 s

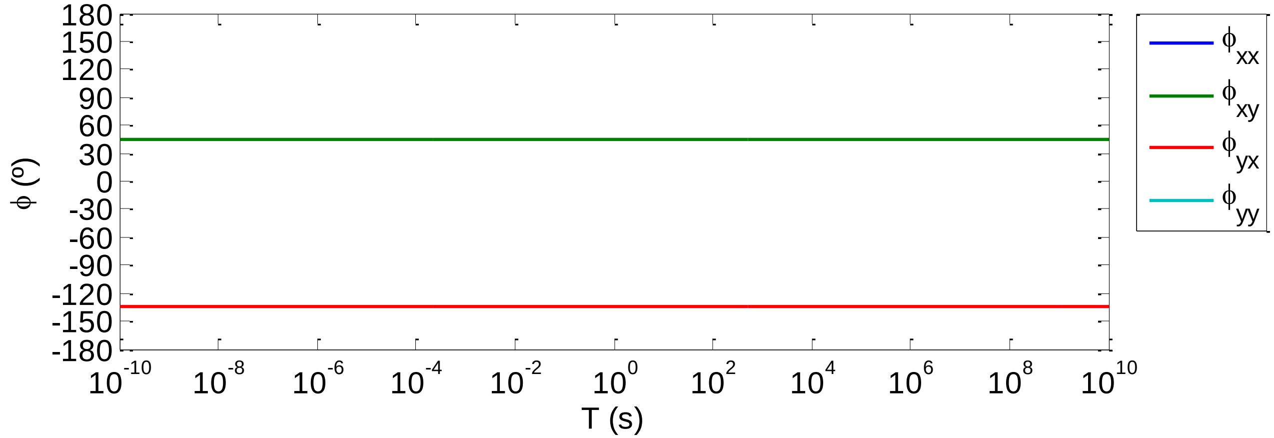
Some results: outside the hemisphere



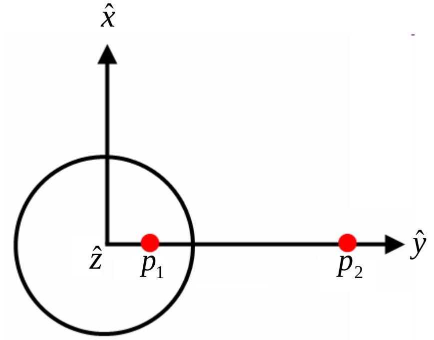
$p_2 = (0, 3R, 0)$

Ignoring $\tilde{D}$



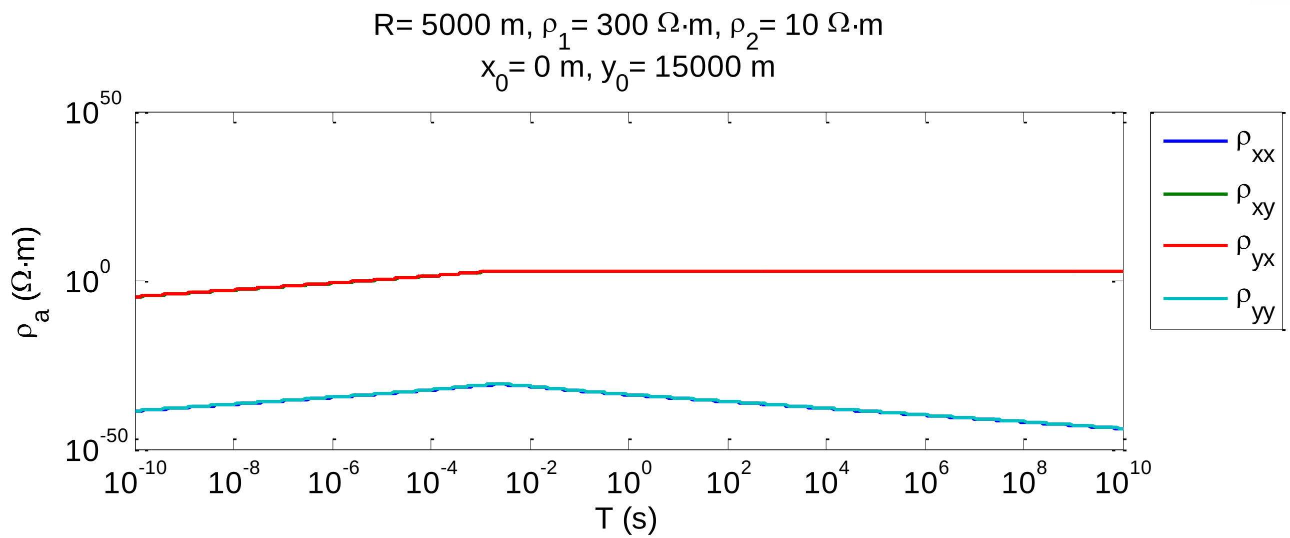


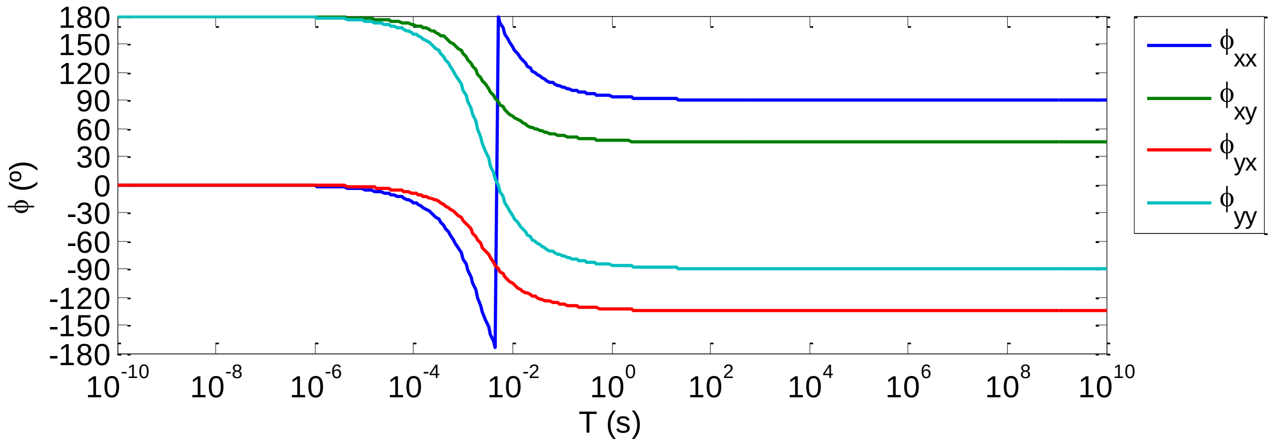
Some results: outside the hemisphere



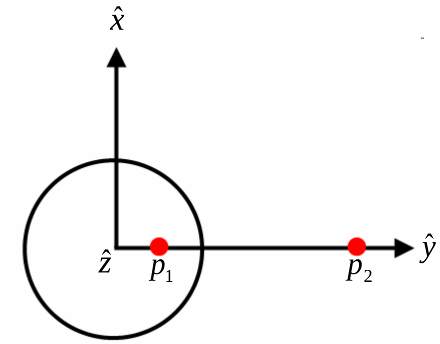
$$p_2 = (0, 3R, 0)$$

Considering $\tilde{D}$



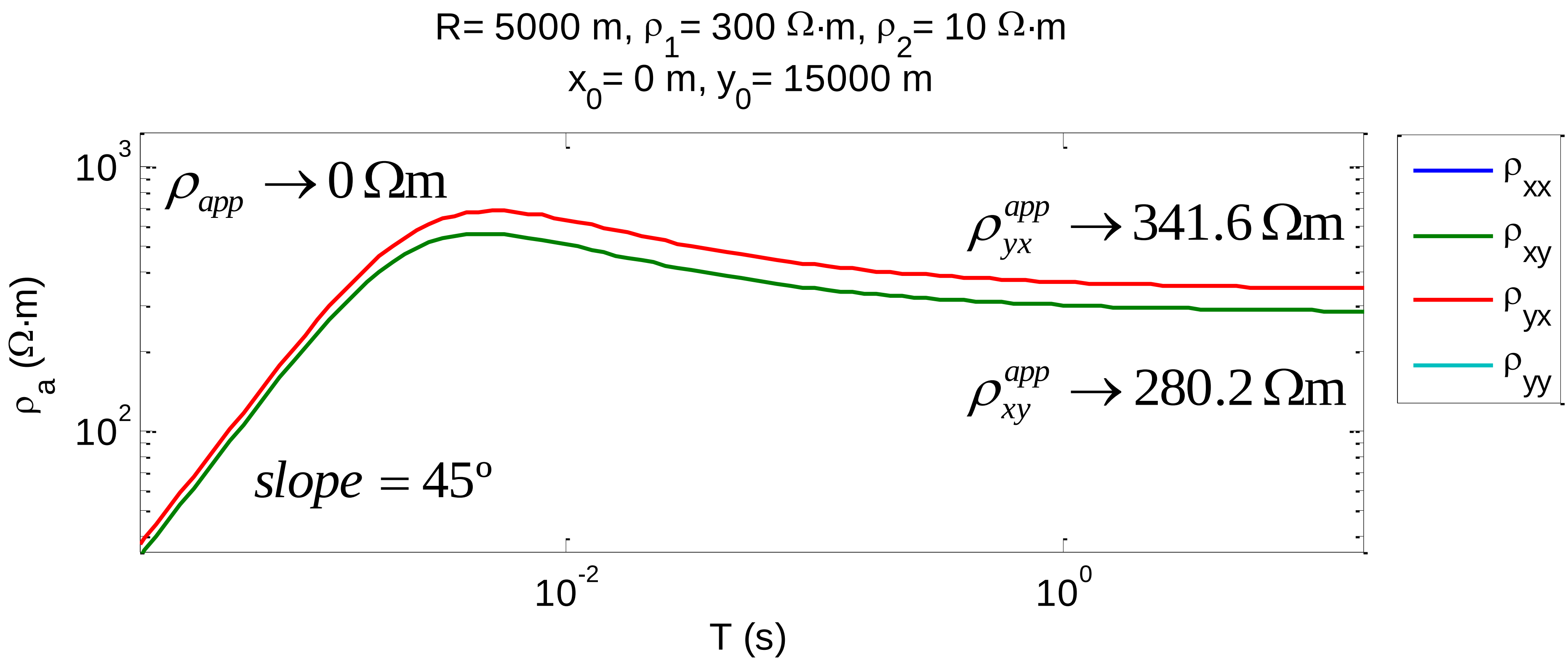


Some results: outside the hemisphere



$$p_2 = (0, 3R, 0)$$

Zoom



Period at which the skin depth equals the distance to the edge of the hemisphere (10 km): 1.32 s

Assymptotic behaviour of the MT curves:

A simpler (and equivalent) expression for the impedance is

$$Z = \tilde{C}\left(Z_0^{-1} + \tilde{D}\right)^{-1}$$

It has been obtained as follows:

$$\mathbf{H^h}\left(x_0, y_0, 0\right) = \mathbf{H_0} + \tilde{D}\mathbf{E_0}$$

$$\mathbf{E} = \tilde{C}\mathbf{E_0} = Z\mathbf{H^h} = Z\left(\mathbf{H_0} + \tilde{D}\mathbf{E_0}\right) = Z\left(Z_0^{-1} + \tilde{D}\right)\mathbf{E_0}$$

Therefore,

$$\tilde{C}\mathbf{E_0} = Z\left(Z_0^{-1} + \tilde{D}\right)\mathbf{E_0} \Rightarrow Z = \tilde{C}\left(Z_0^{-1} + \tilde{D}\right)^{-1}$$

Now we can calculate the assymptotic limit in an easier way.

Assymptotic behaviour of the MT curves:

Recall $Z = \tilde{C}\left(Z_0^{-1} + \tilde{D}\right)^{-1}$

We have that $Z_0^{-1} = \frac{1}{k}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad k = \sqrt{i\omega\mu_0\rho_1}$

Then, $\lim_{\omega\to\infty} Z_0^{-1} = \lim_{\omega\to\infty} \frac{1}{\sqrt{i\omega\mu_0\rho_1}}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = 0I$

So, $\lim_{\omega\to\infty} Z = \lim_{\omega\to\infty} \tilde{C}\left(Z_0^{-1} + \tilde{D}\right)^{-1} = \tilde{C}\tilde{D}^{-1}$ is finite and real ($\lim_{\omega\to\infty} \phi_{ij} = \begin{cases} 0 \\ \pi \end{cases}$).

Finally, $\lim_{\omega\to\infty} \rho_{ij}^{app} = \lim_{\omega\to\infty} \frac{1}{\omega\mu_0}\left|Z_{ij}\right|^2 = 0$

As $\lim_{\omega\to\infty}\left|Z_{ij}\right|^2$ is independent of frequency, apparent resistivity becomes proportional to the period. This explains the slope of 45º.

Thank you for your attention

Bibliography:

Groom, R.W. and Bailey, R. C. (1991) Analytical investigations of the effects of near surface three-dimensional galvanic scatterers on MT tensor decomposition, Geophys., v. 56, pp. 496-518.

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