3D Forward Model

Jelena Koldan

Barcelona Supercomputing Center
Centro Nacional de Supercomputación

Goals

- Numerical solution of
  - 3D CSEM induction problems
  - 3D MT problems
- Efficiency increase of numerical scheme by implementation on massively parallel computers

PHYSICAL PROBLEM FORMULATION

Physical problem formulation

- Secondary Coulomb-gauged EM potentials

  - simplify form of Maxwell's equations
  - avoid singularities introduced by sources

Gauged EM potentials

- Magnetic vector potential, $\vec{A}$
- Electric scalar potential, Φ
- EM potentials are defined by:

$$\vec{B} \equiv \nabla \times \vec{A}$$

$$\vec{E} \equiv i\omega\vec{A} - \nabla\Phi$$

Coulomb gauged formulation

- Simultaneous solution of two equations that constitute incompletely gauged coupled vector-scalar potential formulation of Maxwell's equations:

$$\nabla^2 \vec{A} + i\omega\mu_0\sigma\left(\vec{A} + \nabla\Psi\right) = -\mu_0 \vec{J}_s$$
$$\nabla \cdot \left[i\omega\mu_0\sigma\left(\vec{A} + \nabla\Psi\right)\right] = 0$$

- For unique $\vec{A}$, Coulomb gauge condition must be applied:

$$\nabla \cdot \vec{A} = 0$$

Secondary potential formulation

- Source of arbitrary shape, complexity, and orientation can be introduced by defining set of known primary potentials $\left(\vec{A}p, \Psi p\right)$

- Normally, primary potentials are analytic expressions for induction in homogeneous formation of constant electrical conductivity, σ_p

Primary potentials for MT

- EM field of electric dipole at infinity is plane wave
- Natural source is modeled as big electric dipole very far away
- Primary potentials are potentials from dipole source located in homogeneous formation of uniform electrical conductivity, σ_p:

$$\vec{A}p(\vec{r}) = \frac{Ids}{4\pi r} e^{-kr} \vec{x}$$

Secondary potentials

- $(\vec{A}s, \Psi s)$ are defined by:

$$\vec{A} \equiv \vec{A}p + \vec{A}s,\ \ \Psi \equiv \Psi p + \Psi s$$

- Finally, governing equations become:

$$\nabla^2 \vec{A}s + i\omega\mu_0\sigma\left(\vec{A}s + \nabla\Psi s\right)$$
$$= -i\omega\mu_0\Delta\sigma\left(\vec{A}p + \nabla\Psi p\right)$$
$$\nabla \cdot \left[i\omega\mu_0\sigma\left(\vec{A}s + \nabla\Psi s\right)\right]$$
$$= -\nabla \cdot \left[i\omega\mu_0\sigma\left(\vec{A}p + \nabla\Psi p\right)\right]$$

Boundary conditions

- Homogeneous Dirichlet boundary condition on outer boundary of domain:

$$\left(\vec{A}s, \Psi s\right) \equiv \left(\vec{0}, 0\right)$$

NUMERICAL SCHEME

Physical problem discretization

- Frequency-domain finite-element (FD-FE) scheme based on unstructured tetrahedral meshes

Solver for resulting system of equations

- Preconditioned complex biconjugate gradient stabilized (BCGStab) algorithm

Parallel implementation

- Domain decomposition technique using
  - master-slave strategy
  - MPI

POSTPROCESS

Postprocess

- Fields $(\vec{E}s, \overrightarrow{H}s)$ are obtained from potentials $(\vec{A}s, \Psi s)$ by numerical differentiation:
  - moving least-squares interpolation (MLSI) scheme

MLSI

- Each Cartesian component of $\vec{A}s$ and Ψs are approximated by linear functions:

$$ax + by + cz + d$$

- Spatial derivatives of potentials are first three coefficients (a, b, c)
- MLSI determination of (a, b, c) at arbitrary test point is made in weighted least-squares sense

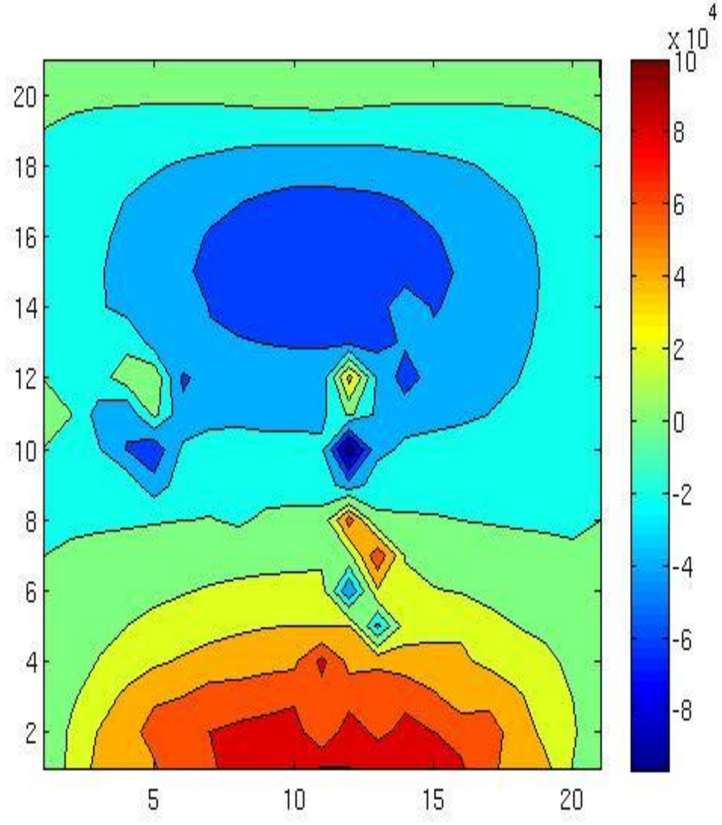
WLS

- Minimize sum of weighted squared residuals between linear function and FE-computed values of potentials at $N \sim 30$ nearest nodes to test point
- Weighting function has positive exponential form which is maximum at test point and decreases monotonically with increasing distance away from it

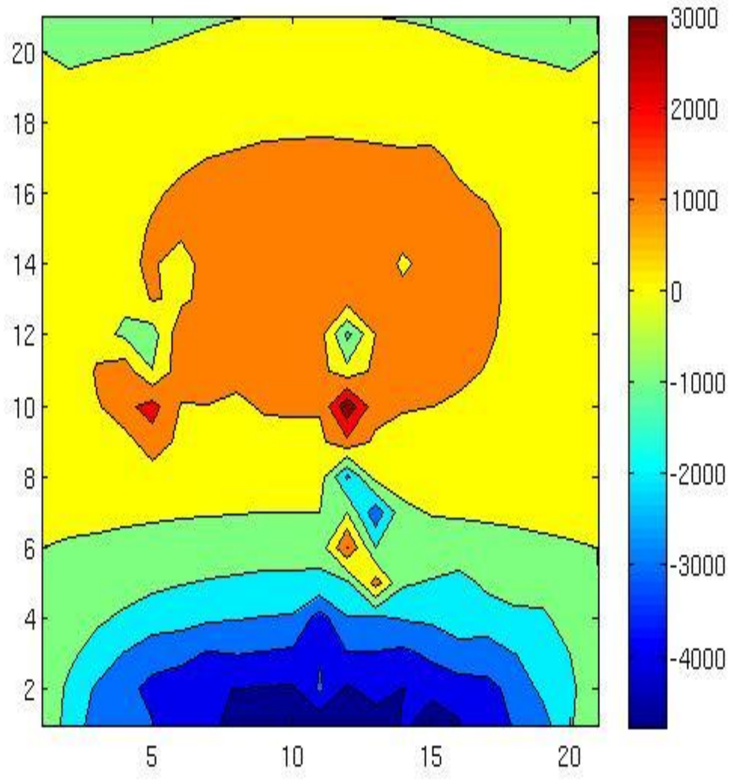
Simple homogeneous half space

RESULTS

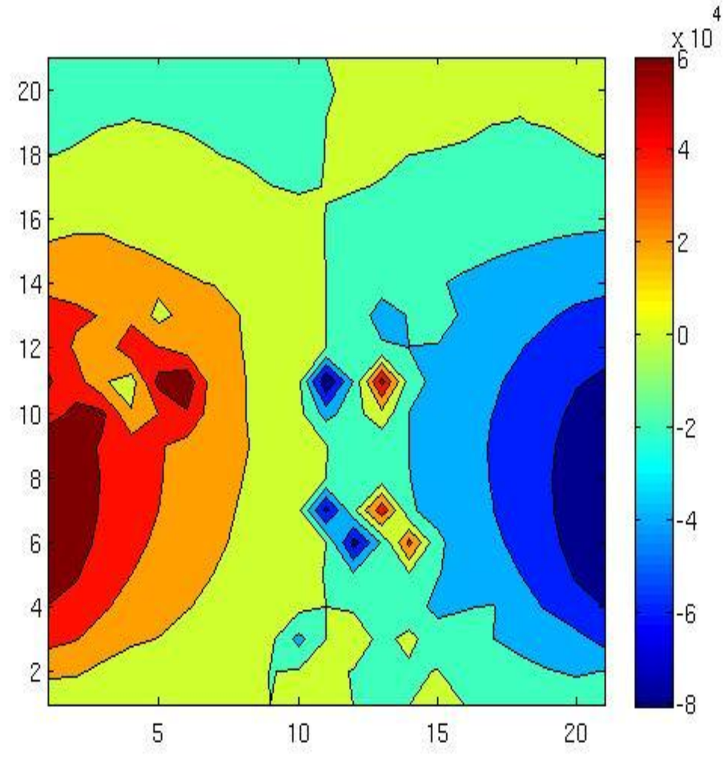
Real part of Ex



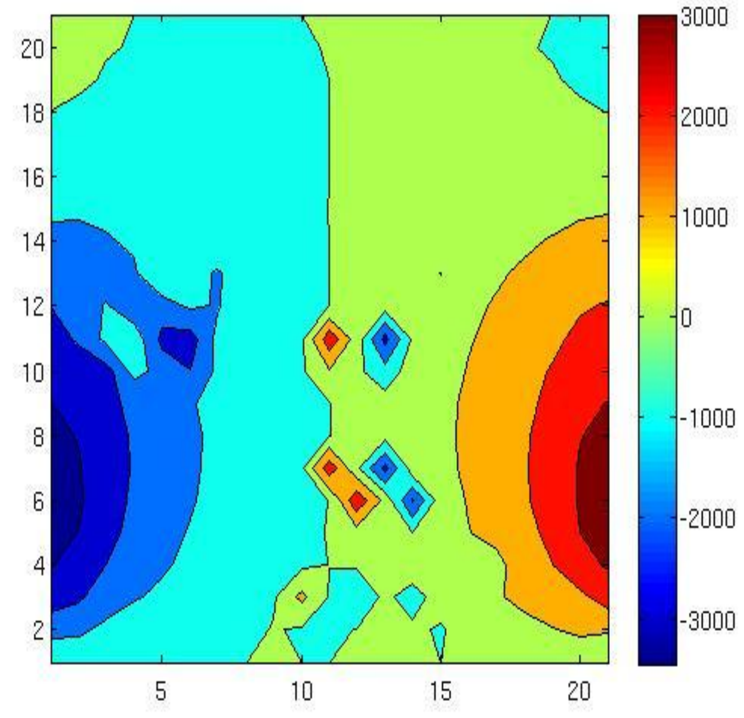
Imaginary part of Ex



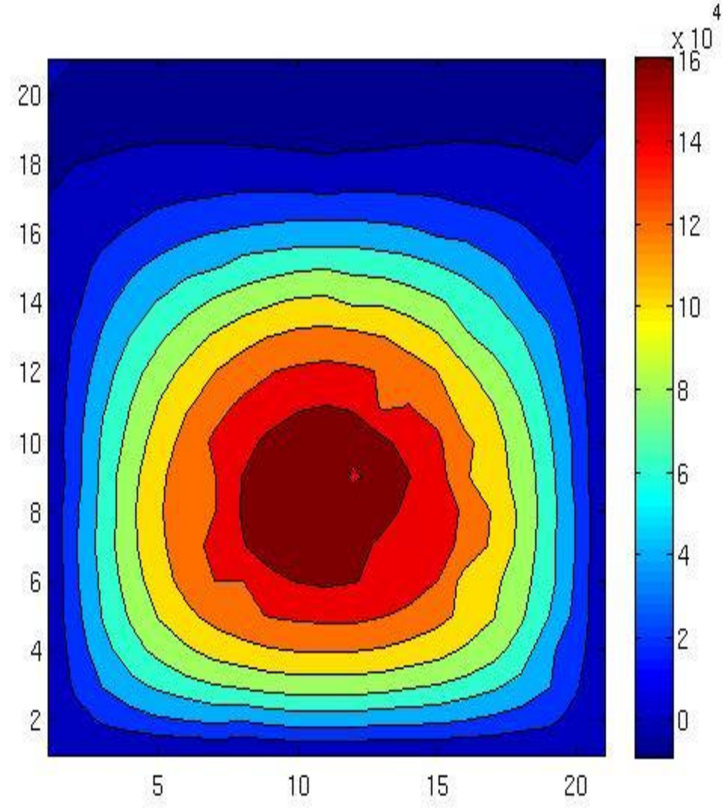
Real part of Ey



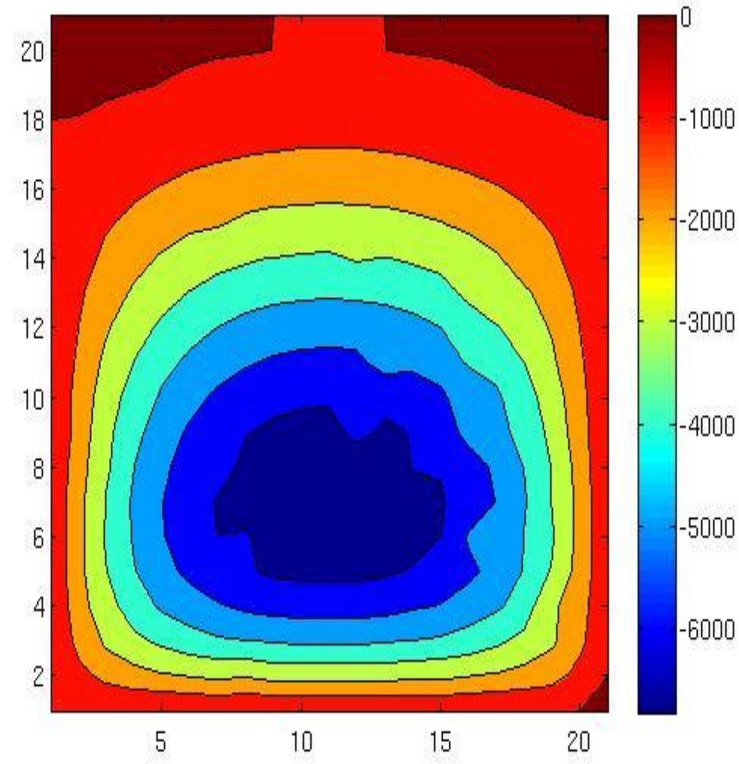
Imaginary part of Ey



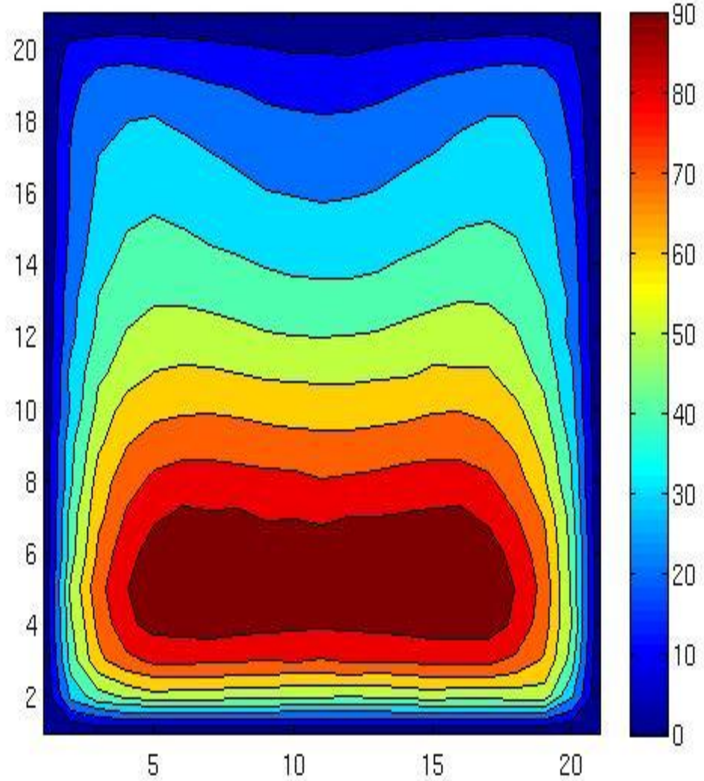
Real part of Ez



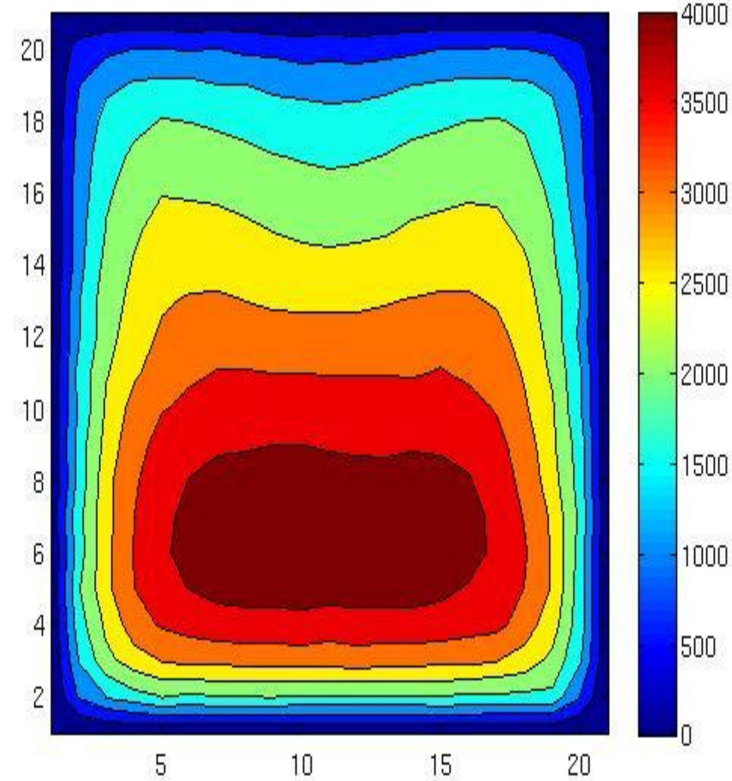
Imaginary part of Ez



Real part of Hx

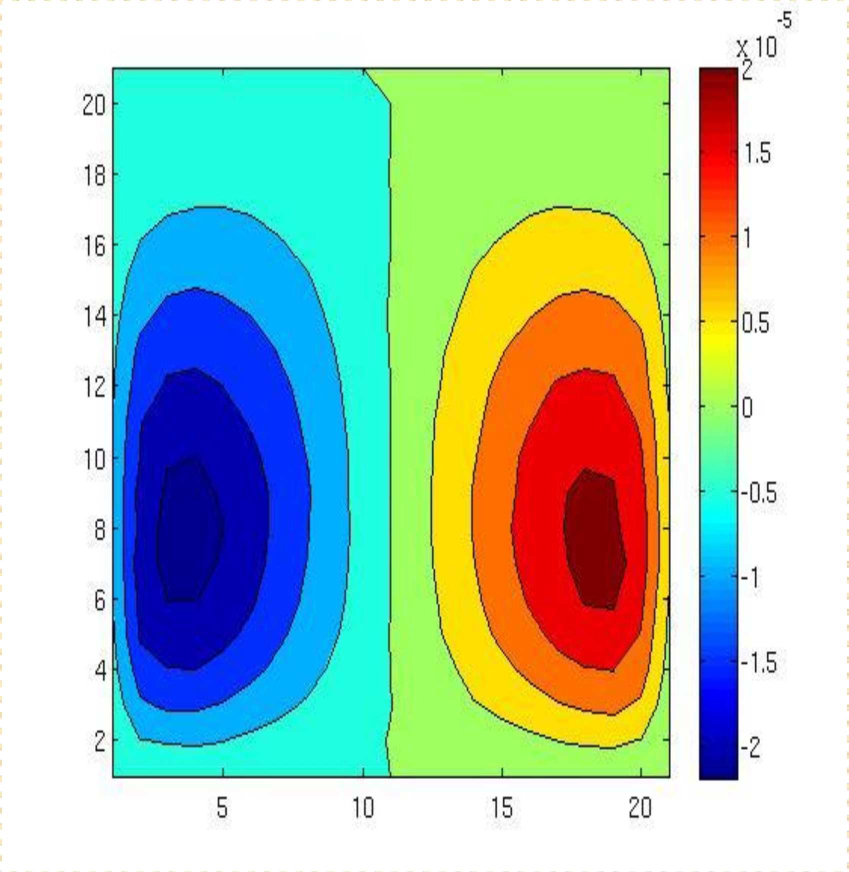


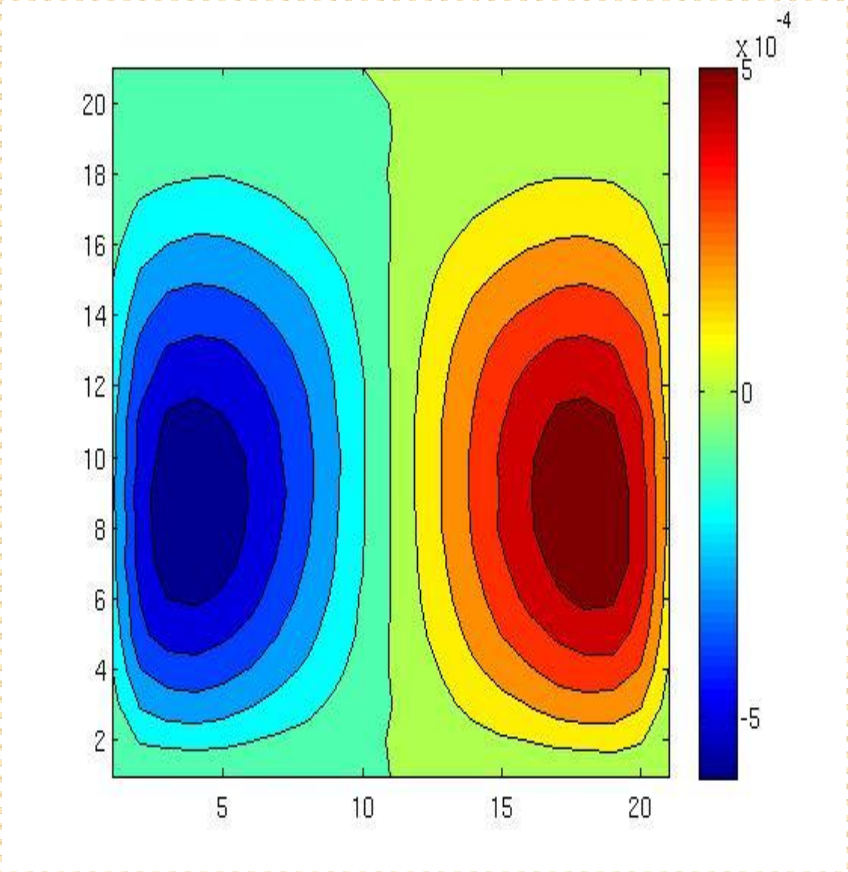
Imaginary part of Hx



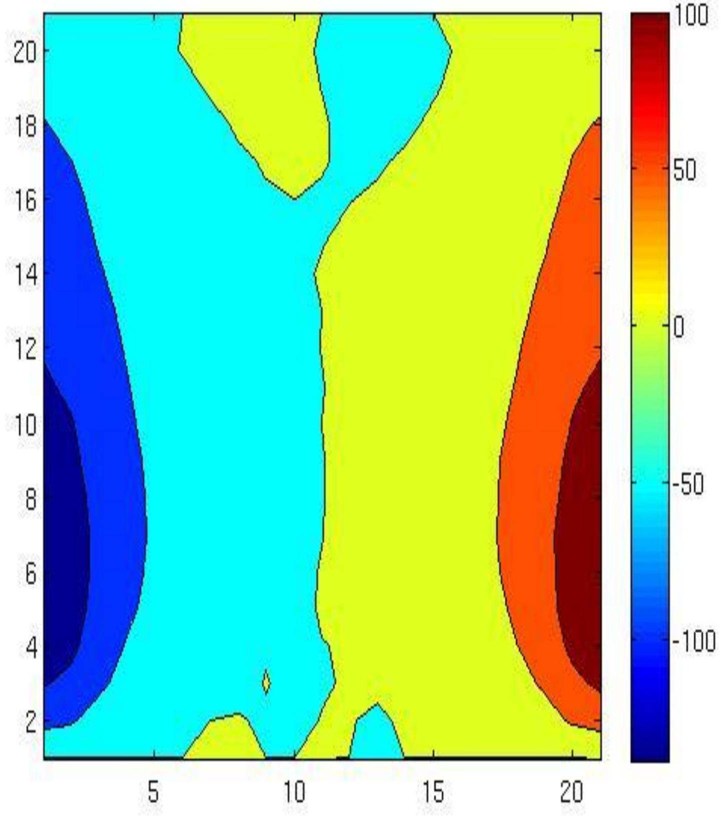
Real part of Hy

Imaginary part of Hy





Real part of Hz



Imaginary part of Hz