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General Computations Without Fixing the Gauge

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Abstract

Within the framework of a manifestly gauge invariant exact renormalization group for $SU(N)$ Yang-Mills, we derive a simple expression for the expectation value of an arbitrary gauge invariant operator. We illustrate the use of this formula by computing the $\mathrm{O}(g^2)$ correction to the rectangular, Euclidean Wilson loop with sides $T \gg L$. The standard result is trivially obtained, directly in the continuum, for the first time without fixing the gauge. We comment on possible future applications of the formalism.

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I. INTRODUCTION

The necessity of gauge fixing in order to compute in Yang-Mills theories is, in common lore, practically taken for granted and, for perturbative calculations, generally considered obligatory. This point of view is lent considerable weight both by Feynman's unitarity

argument for the existence of Faddeev-Popov ghosts [1] and by the elegance and power of the resulting BRST symmetries [2].

However, there is no reason in principle why gauge invariant quantities, as opposed to Green's functions in gauge fixed formulations, cannot be computed in a manifestly gauge invariant manner. Indeed, in a non-perturbative context, this is routinely exploited on the lattice, where calculations can be performed without gauge fixing. Excitingly, in a series of works [3–20], a formalism has been developed which allows manifestly gauge invariant computations to be performed directly in the continuum.

The benefits of this scheme are numerous. The gauge field is protected from field strength renormalization and the Ward identities take a particularly simple form since the Wilsonian effective action is built only from gauge invariant combinations of the covariant derivative, even at the quantum level [5]. In the non-perturbative domain, the difficult technical issue of Gribov copies [21] is entirely avoided. Furthermore, it should be possible to make statements about phenomena such as confinement in a completely gauge independent manner, and it is surely this which gives a manifestly gauge invariant scheme much of its appeal.

The framework developed in [3–20] is based on the exact renormalization group (ERG), the continuum version of Wilson's RG [22–24]. The essential physical idea behind this approach is that of integrating out degrees of freedom between the bare scale of the quantum theory and some effective scale, Λ. The effects of these modes are encoded in the Wilsonian effective action, S_Λ, which describes the physics of the theory in terms of parameters relevant to the effective scale.

The possibility of constructing manifestly gauge invariant ERGs arises, fundamentally, from the huge freedom inherent in the approach [25]. For any given quantum field theory, there are an infinite number of ERGs corresponding to the infinite number of different ways in which the high energy degrees of freedom can be averaged over (the continuum version of blocking on the lattice) [8, 16, 25]. In Yang-Mills theory, an infinite subset of these schemes allow the computation of the gauge invariant Wilsonian effective action, without fixing the gauge.[1]

Central to the ERG methodology is the ERG equation, which determines how the Wilso-

[1] In practise we further specialize to those ERGs which allow convenient renormalization to any loop order [15–17].

nian effective action changes under infinitesimal changes of the scale. Part of the reason for the considerable amount of work put into adapting the ERG for Yang-Mills (see [26] for a summary of the various approaches) is that the ERG equation, by relating physics at different scales, provides access to the low energy dynamics of the theory. Indeed, more generally, the ERG has proven itself to be a flexible and powerful tool for studying both perturbative and non-perturbative problems in a range of field theories (see [27–35] for reviews). A particular advantage conferred by the ERG is that renormalization is built in: solutions to the flow equation (in pretty much any approximation scheme), from which physics can be extracted, are naturally phrased directly in terms of renormalized parameters. It is thus clear that a manifestly gauge invariant formalism, based on the ERG, has considerable potential. Furthermore, an interesting link between this formalism and the AdS/CFT correspondence has recently been made [36].

The majority of the work into the scheme employed in this paper has focused on constructing and testing the formalism, culminating in the successful reproduction of the $SU(N)$ Yang-Mills two-loop β-function [15, 17]. Subsequent to this, the powerful diagrammatic techniques developed to facilitate this calculation have been refined and applied in the context of β-function coefficients at arbitrary loop order [18–20]. These substantial works have paved the way for more general application of the formalism; in this paper, we describe how to compute the expectation values of gauge invariant operators, without fixing the gauge, and illustrate the formalism with a very simple computation of the $\mathrm{O}(g^2)$ correction to the rectangular, Euclidean Wilson loop with sides $T \gg L$. [There have been attempts to compute the perturbative corrections to this Wilson loop in (gauge fixed) ERG studies, in the past. In particular, using the axial gauge flow equation proposed by [38], it was found in [39] that, whilst the $\mathrm{O}(g^2)$ result could be correctly reproduced, the formalism failed at $\mathrm{O}(g^4)$. However, the flow equation of [38] is not a flow equation in the Wilsonian sense: the implementation of the cutoff, Λ, is not sufficient to regularize the theory, and dimensional regularization has to be employed as well. This is the reason for the negative result of [39]; as recognized in [40], properly Wilsonian axial gauge flow equations can be constructed, which work perfectly well. In the formalism used in this paper, the above issues never arise, since the implementation of the (gauge invariant) cutoff is sufficient to regularize the theory, as proven in [9].]

The outline of this paper is as follows. In section II we review the setup of our mani-

festly gauge invariant ERG. Section III is devoted to the methodology for computing the expectation of gauge invariant operators. The basic idea, for which little prior knowledge is required, is detailed in the short section III A. In the remainder of section III, the machinery for performing calculations in perturbation theory is developed. This section concludes with a fantastically compact diagrammatic expression for the perturbative corrections to the expectation value of any gauge invariant operator. In section IV, we specialize to the computation of the expectation values of (renormalized) Wilson loops. After covering some general features in section IV A, in section IV B we compute the $\mathrm{O}(g^2)$ correction to the Euclidean, rectangular Wilson loop with sides $T \gg L$ and recover the standard result. We conclude in section V.

II. REVIEW

A. Elements of $SU(N|N)$ Gauge Theory

Throughout this paper, we work in Euclidean dimension, D. We regularize $SU(N)$ Yang-Mills, carried by the physical gauge field A^1_μ, by embedding it in spontaneously broken $SU(N|N)$ Yang-Mills, which is itself regularized by covariant higher derivatives [9]. The massive gauge fields arising from the spontaneous symmetry breaking play the role of gauge invariant Pauli-Villars (PV) fields, furnishing the necessary extra regularization to supplement the covariant higher derivatives. In order to unambiguously define contributions which are finite only by virtue of the PV regularization, a preregulator must be used in $D = 4$ [9]. We will use dimensional regularization, emphasising that this makes sense non-perturbatively, since it is not being used to renormalize the theory, but rather as a prescription for discarding surface terms in loop integrals [9].

The supergauge invariant Wilsonian effective action has an expansion in terms of supertraces and products of supertraces [11]:

$$\begin{aligned} S \;=\; & \sum_{n=1}^{\infty} \frac{1}{s_n} \int d^D x_1 \cdots d^D x_n \, S^{X_1 \cdots X_n}_{a_1 \cdots a_n}(x_1, \cdots, x_n) \mathrm{str} X_1^{a_1}(x_1) \cdots X_n^{a_n}(x_n) \\ & + \frac{1}{2!} \sum_{m,n=0}^{\infty} \frac{1}{s_n s_m} \int d^D x_1 \cdots d^D x_n \, d^D y_1 \cdots d^D y_m \, S^{X_1 \cdots X_n, Y_1 \cdots Y_m}_{a_1 \cdots a_n \, , b_1 \cdots b_m}(x_1, \cdots, x_n; y_1 \cdots y_m) \\ & \qquad \mathrm{str} X_1^{a_1}(x_1) \cdots X_n^{a_n}(x_n) \, \mathrm{str} Y_1^{b_1}(y_1) \cdots Y_m^{b_m}(y_m) \\ & + \ldots \end{aligned} \tag{2.1}$$

where the $X_i^{a_i}$ and $Y_j^{b_j}$ are embeddings of broken phase fields into supermatrices. We take only one cyclic ordering for the lists $X_1 \cdots X_n$, $Y_1 \cdots Y_m$ in the sums over n, m. If any term is invariant under some nontrivial cyclic permutations of its arguments, then s_n (s_m) is the order of the cyclic subgroup, otherwise $s_n = 1$ ($s_m = 1$).

The momentum space vertices are written

$$S^{X_1 \cdots X_n}_{a_1 \cdots a_n}(p_1, \cdots, p_n)\,(2\pi)^D\,\delta\left(\sum_{i=1}^{n} p_i\right) = \int d^D x_1 \cdots d^D x_n\, e^{-i\sum_i x_i \cdot p_i} S^{X_1 \cdots X_n}_{a_1 \cdots a_n}(x_1, \cdots, x_n),$$

where all momenta are taken to point into the vertex. We will employ the shorthand

$$S^{X_1 X_2}_{a_1\, a_2}(p) \equiv S^{X_1 X_2}_{a_1\, a_2}(p, -p).$$

In addition to the coupling, g, of the physical gauge field, there is a second dimensionless coupling, g_2, associated with one of the unphysical regulator fields, A^2_μ [11, 15–17]. For convenience, we work not with g_2 directly but with

$$\alpha \equiv g_2^2/g^2. \tag{2.2}$$

The coupling g (similarly α) is defined through its renormalization condition:

$$S[A^1] = \frac{1}{2g^2}\,\mathrm{tr}\int d^D x\,\left(F^1_{\mu\nu}\right)^2 + \cdots, \tag{2.3}$$

where the ellipses stand for higher dimension operators and the ignored vacuum energy. Equation (2.3) constrains the classical two-point vertex of the physical field, $S_{0\mu}{}^{A^1}{}_{\nu}^{A^1}(p) \equiv S_{0\mu\nu}{}^{1\,1}(p)$, as follows:

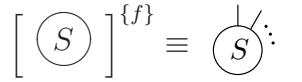
$$S_{0\mu\nu}{}^{1\,1}(p) = 2(p^2\delta_{\mu\nu} - p_\mu p_\nu) + \mathrm{O}(p^4) \equiv 2\Box_{\mu\nu}(p) + \mathrm{O}(p^4). \tag{2.4}$$

B. Diagrammatics

In this section, we introduce and describe the diagrammatics necessary for this paper. For a comprehensive description of the diagrammatics see [15, 16].

1. *Diagrammatics for the Action*

The *vertex coefficient functions* belonging to the action (2.1) have a simple diagrammatic representation:



(2.5)

represents all vertex coefficient functions corresponding to all cyclically independent orderings of the set of broken phase fields, $\{f\}$, distributed over all possible supertrace structures. For example,

$$\Big[\ \textcircled{S}\ \Big]^{A^1A^1} \tag{2.6}$$

represents the coefficient functions $S^{A^1A^1}$ which, from (2.1), is associated with the (super)trace structure $\mathrm{tr}A^1A^1$. This diagram would also represent the coefficient function S^{A^1,A^1}, were it not for the fact that this does not exist, on account of $\mathrm{tr}A^1 = 0$.

2. Diagrammatics for the Exact Flow Equation

The diagrammatic representation of the flow equation is shown in figure 1 [15, 16].

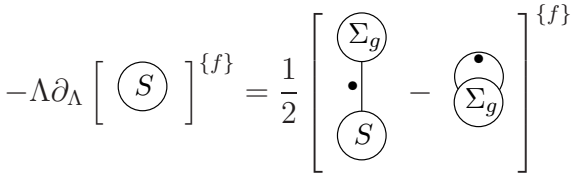


FIG. 1: The diagrammatic form of the flow equation.

The left-hand side just depicts the flow of all cyclically independent Wilsonian effective action vertex coefficient functions. The objects on the right-hand side of figure 1 have two different types of component. The lobes represent vertices of action functionals, where $\Sigma_g \equiv g^2 S - 2\hat{S}$, $\hat{S}$ being the seed action [10–12, 14–17]: a functional which respects the same symmetries as the Wilsonian effective action, S, and has the same structure. Physically, the seed action can be thought of as (partially) parameterizing a general Kadanoff blocking in the continuum [16, 25].

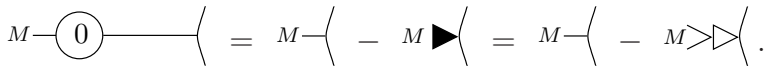
The object attaching to the various lobes, $\overset{\bullet}{\text{———}}$, is the sum over vertices of the covariantized ERG kernels [5, 11] and, like the action vertices, can be decorated by fields belonging to $\{f\}$. The fields of the action vertex (vertices) to which the vertices of the kernels attach act as labels for the ERG kernels. We loosely refer to both individual and summed over vertices of the kernels simply as a kernel.

The rule for decorating the diagrams on the right-hand side is simple: the set of fields, $\{f\}$, are distributed in all independent ways between the component objects of each diagram.

Following [4–6, 11, 12, 15, 16], it is technically conventient to use the freedom inherent in $\hat{S}$ by choosing the two-point, classical seed action vertices equal to the corresponding Wilsonian effective action vertices. The effect of this is that the kernels, integrated with respect to $\ln\Lambda$ (at constant α), turn out to the the inverses of the classical, two-point vertices in the transverse space. For example, in the A^1-sector we find that

$$S^{11}_{0\mu\alpha}(p)\Delta^{11}_{\alpha\nu}(p) = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}, \tag{2.7}$$

where Δ^{11} in the integrated A^1 sector kernel. It is apparent that Δ^{11} is the inverse of the corresponding classical, two-point vertex up to a remainder term which, since it is forced to be there as a consequence of the manifest gauge invariance, we call a 'gauge remainder'. In recognition of the similarities of the integrated kernels to propagators, in both form and diagrammatic role, we refer to them as effective propagators [11]. However, we emphasise that at no point is gauge fixing required in their definition and that our diagrams to not correspond, in any way, to conventional Feynman diagrams. Equation (2.7) can be diagrammtically generalized to hold in all sectors:



. (2.8)

We have attached the effective propagator, denoted by a solid line, to an arbitrary structure since it only ever appears as an internal line. The field labelled by M can be any of the broken phase fields. The object $\blacktriangleright \equiv >\triangleright$ is a gauge remainder. The individual components of $>\triangleright$ will often be loosely referred to as gauge remainders; where it is necessary to unambiguously refer to the composite structure, we will use the terminology 'full gauge remainder'. Equation (2.8) is referred to as the effective propagator relation. From (2.4) and (2.7), it follows that

$$\Delta^{11}_{\rho\sigma}(p) = \frac{\delta_{\rho\sigma}}{2p^2}(1 + \mathrm{O}(p^2/\Lambda^2)), \tag{2.9}$$

which we will use later.

Embedded within the diagrammatic rules is a prescription for evaluating the group theory factors. Suppose that we wish to focus on the flow of a particular vertex coefficient function which, necessarily, has a unique supertrace structure.

On the right-hand side of the flow equation, we must focus on the components of each diagram with precisely the same supertrace structure as the left-hand side, noting that the kernel, like the vertices, has multi-supertrace contributions (for more details see [15, 16]). In

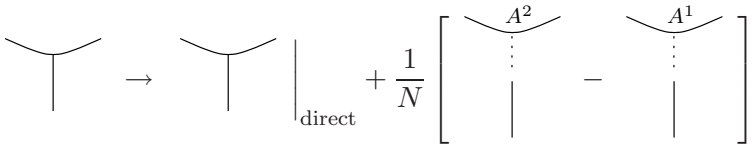


FIG. 2: The $1/N$ corrections to the group theory factors.

this more explicit diagrammatic picture, the kernel is to be considered a double sided object. Thus, whilst the dumbbell like term of figure 1 has at least one associated supertrace, the next diagram has at least two, on a account of the loop (this is strictly true only in the case that kernel attaches to fields on the same supertrace). If a closed circuit formed by a kernel is devoid of fields then it contributes a factor of $\pm N$, depending on the flavours of the fields to which the kernel forming the loop attaches. This is most easily appreciated by defining the projectors

$$\sigma_{\pm} \equiv \frac{1}{2}(\mathbb{1} \pm \sigma)$$

and noting that $\mathrm{str}\sigma_{\pm} = \pm N$. In the counterclockwise sense, either a σ_+ or σ_-, as appropriate, can always be inserted for free after any of the broken phase fields.

The above prescription for evaluating the group theory factors receives $1/N$ corrections in the A^1 and A^2 sectors. If a kernel attaches to an A^1 or A^2, it comprises a direct attachment and an indirect attachment. In the former case, one supertrace associated with some vertex coefficient function is 'broken open' by an end of a kernel: the fields on this supertrace and the single supertrace component of the kernel are on the same circuit. In the latter case, the kernel does not break anything open and so the two sides of the kernel pinch together at the end associated with the indirect attachment. This is illustrated in figure 2; for more detail, see [15, 16].

We can thus consider the diagram on the left-hand side as having been unpackaged, to give the terms on the right-hand side. The dotted lines in the diagrams with indirect attachments serve to remind us where the loose end of the kernel attaches in the parent diagram.

As an example, which will be of use later, consider the group theory factor of the diagram on the left-hand side of figure 3, where we suppose that the kernel forming the loop is in the A^1 sector.

On the right-hand side, we have unpackaged the parent diagram and explicitly indicated,

FIG. 3: An example showing how to evaluate the group theory factor of a diagram in which the kernel is taken to be in the A^1 sector.

in red, how many circuits each diagram has. To evaluate the corresponding group theory factors, we simply take each circuit to contribute $\mathrm{str}\sigma_+$ (σ_+ because we are taking the kernel to be in the A^1 sector). Therefore, the overall group theory factor is

$$\frac{1}{N}\left(N^2 - 2\frac{1}{N}N + \frac{1}{N^2}N^2\right) = \frac{1}{N}(N^2-1) = 2C_2(F),$$

where $C_2(F)$ is the quadratic Casimir operator for the fundamental representation of $SU(N)$.

III. METHODOLOGY

A. Basics

In this section we describe the strategy for computing the renormalized expectation value of the gauge invariant operator, $\mathcal{O}$. Denoting the set of (dynamical) broken phase fields by Φ, we aim to compute

$$\langle\mathcal{O}\rangle_R = \frac{1}{Z_0}\int_{\Lambda_0}\mathcal{D}\Phi\,\mathcal{O}_{\Lambda_0}[A^1]\,e^{-S_{\Lambda_0}[\Phi]}, \tag{3.1}$$

where the subscript R stands for renormalized and $Z_0 = \int_{\Lambda_0}\mathcal{D}\Phi\,e^{-S_{\Lambda_0}[\Phi]}$. Notice that we have explicitly tagged the functional integral, action and $\mathcal{O}$ with Λ_0. This is to remind us that the expression is defined at the bare scale, Λ_0. At this scale, the gauge invariant operator is taken to be a functional of just the physical gauge field, A^1. The limit $\Lambda_0 \to \infty$, which essentially corresponds to the continuum limit [28], is taken at the end of a calculation.

Introducing the source, J, we rewrite (3.1) in the usual way:

$$\langle\mathcal{O}\rangle_R = -\left.\frac{\partial}{\partial J}\ln Z_J\right|_{J=0}, \tag{3.2}$$

where

$$Z_J = \int_{\Lambda_0}\mathcal{D}\Phi\,e^{-S_{\Lambda_0}[\Phi]-J\mathcal{O}_{\Lambda_0}[A^1]}.$$

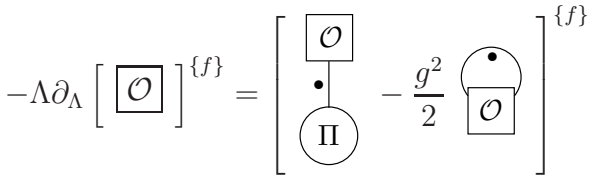


FIG. 4: The flow of $\mathcal{O}_\Lambda$.

The key step now is to integrate out modes between the bare scale and the effective scale, Λ, to yield:

$$Z_J = \int_\Lambda \mathcal{D}\Phi\, e^{-S_\Lambda[\Phi]-\mathcal{O}_\Lambda[\Phi,J]}. \tag{3.3}$$

Since both S_Λ and $\mathcal{O}_\Lambda$ are gauge invariant, the division of terms between these two functionals is of course arbitrary. For example, for some other definition of S_Λ and $\mathcal{O}_\Lambda$, we could have written the argument of the exponential in (3.3) as $S'_\Lambda[\Phi,J]-\mathcal{O}'_\Lambda[\Phi,J]$ or even just $S''_\Lambda[\Phi,J]$. However, we choose to define things such that S_Λ is independent of J. Given that the only dependence on J at the bare scale is linear, it follows from the flow equation that the dependence at the effective scale has a Taylor expansion in J and so we can write:

$$\mathcal{O}_\Lambda[\Phi,J] = \sum_{i=1}^{\infty} J^i \mathcal{O}^i_\Lambda[\Phi]. \tag{3.4}$$

The real point now is that, from (3.2), we are pulling out the $\mathrm{O}(J)$ part, only, when computing the expectation value. Therefore, it makes sense to work at small J, in which case we can take the effect of introducing the gauge invariant operator at the bare scale as inducing an infinitesimal, linear perturbation to the Wilsonian effective action at the effective scale [5]:

$$S_\Lambda \to S_\Lambda + J\mathcal{O}_\Lambda + \mathrm{O}(J^2) \tag{3.5}$$

where, since we are henceforth only interested in the $\mathrm{O}(J)$ part of (3.4), we have dropped the superscript index of $\mathcal{O}^1_\Lambda$.

By performing the shift (3.5) in the flow equation, we see that the flow of the Wilsonian effective action is still given as in figure 1 and the flow of $\mathcal{O}_\Lambda$ is given in figure 4, where we define

$$\Pi \equiv g^2 S - \hat{S}, \tag{3.6}$$

take squares to represent vertices of $\mathcal{O}_\Lambda$ and have dropped the subscript Λ.

Next, consider how Z_J evolves as we integrate out all the modes *i.e.* as we take the limit $\Lambda \to 0$. Let us start with the behaviour of the gauge invariant operator, $\mathcal{O}$. Like the

Wilsonian effective action, $\mathcal{O}$ has an expansion in terms of fields. However, unlike in (2.1), it is crucial that we retain the field-independent part (*i.e.* the vacuum energy-like term). As we integrate out modes, so this term receives quantum corrections. What of the field dependent parts? Clearly, once we have integrated out all modes, there cannot be any field dependent terms remaining which are multiplied by a finite coefficient. There are two choices: either the coefficients diverge, in which case $e^{-\mathcal{O}_{\Lambda\to 0}} \to 0$, or each coefficient corresponding to a field dependent term in the expansion of $\mathcal{O}$ vanishes. We assume that the latter is the case.

In the case of the Wilsonian effective action, matters are simple. The structure of (3.1) ensures that, when computing $\langle\mathcal{O}\rangle$, the factor of $e^{-S_{\Lambda\to 0}}$ in the numerator is cancelled out by the Z_0 in the denominator.[2]

Therefore, from (3.2), (3.3), (3.4) and (3.5) we deduce the beautifully simple equation

$$\langle\mathcal{O}\rangle_R = \mathcal{O}_{\Lambda=0}. \tag{3.7}$$

To find $\langle\mathcal{O}\rangle_R$, we can use figure 4 to compute the flow of $\mathcal{O}_\Lambda$, figure 1 to compute the flow of S (which is buried in Π) and thereby determine $\mathcal{O}_{\Lambda=0}$, in some approximation. For the remainder of this paper, we will work in the perturbative domain. Dropping the Λ, which we now take to be implicit, we take the following weak coupling expansions. The Wilsonian effective action is given by

$$S = \sum_{i=0}^{\infty} g^{2(i-1)} S_i = \frac{1}{g^2} S_0 + S_1 + \cdots, \tag{3.8}$$

where S_0 is the classical effective action and the $S_{i>0}$ the ith-loop corrections; $\mathcal{O}$ is given by

$$\mathcal{O} = \sum_{i=0}^{\infty} g^{2(i-1)} \mathcal{O}_i. \tag{3.9}$$

The seed action has an expansion consistent with the fact that S appears in the flow equation multiplied by an extra power of g^2, compared to $\hat{S}$:

$$\hat{S} = \sum_{i=0}^{\infty} g^{2i} \hat{S}_i. \tag{3.10}$$

[2] In fact, there are terms in the Wilsonian effective action which do diverge as $\Lambda \to 0$ [9]. This is easy to see: in order for the regularization scheme to work, the effective propagator in the A^1 sector dies off, for $p^2/\Lambda^2 \gg 1$, at least as fast as $(p^2/\Lambda^2)^{-r}$ (for $r > 2$). This means that the kinetic term must be modified by a term which behaves as $(p^2/\Lambda^2)^{+r}$ for $p^2/\Lambda^2 \gg 1$. Such terms in the effective action clearly diverge as $\Lambda \to 0$.

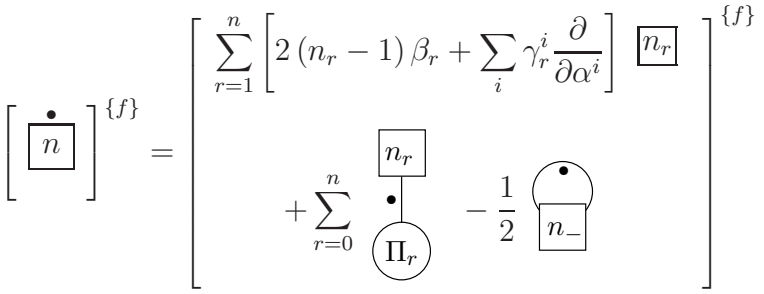


FIG. 5: The weak coupling flow equations for $\mathcal{O}$.

Recalling (2.2) we have:

$$\beta \equiv \Lambda\partial_\Lambda g \;=\; \sum_{i=1}^{\infty} g^{2i+1}\beta_i(\alpha) \tag{3.11}$$

$$\gamma \equiv \Lambda\partial_\Lambda \alpha \;=\; \sum_{i=1}^{\infty} g^{2i}\gamma_i(\alpha). \tag{3.12}$$

To obtain the weak coupling flow equation for $\mathcal{O}$ we substitute (3.8)–(3.12) into figure 4, but do not preclude the possibility that, in addition to g and α, $\mathcal{O}_\Lambda$ also depends on the dimensionless, running couplings $\alpha^{i>1}$ (we identify α^1 with α). This anticipates our treatment of Wilson loops in section IV. The weak coupling flow equations for $\mathcal{O}$ are shown in figure 5, where $\dot{X} \equiv -\Lambda\partial_\Lambda|_\alpha X$, $n_r \equiv n - r$, $n_- \equiv n - 1$ and $\Pi_r \equiv S_r - \hat{S}_r$.

B. Additional Notation

In the diagrammatic flow equation—be it the exact form or the perturbative expansion—we have considered decoration by the set of fields $\{f\}$. However, only on the right-hand side of (2.5) have we actually converted the fields $\{f\}$ into explicit decorations. Before such decoration, we consider $\{f\}$ to be implicit, or unrealized, decorations [15, 18]. Just as it is useful to consider fields as implicit decorations, so too is it useful to construct rules for decoration with implicit effective propagators and instances of the gauge remainder component $>$.

1. Gauge Remainders

Instances of $>$ arise from diagrams in which the effective propagator relation (2.8) has been applied, generating a full gauge remainder. The $\triangleright$ part of the gauge remainder can be processed, using the Ward identities [5, 11, 15, 16], leaving behind a $>$. If one of the vertices generated by the $\triangleright$ is a classical, two-point vertex, then in the case where this vertex attaches to an effective propagator, a further full gauge remainder is generated. Processing this gauge remainder using the Ward identities allows us to iteratively generate structures containing an arbitrary number, m, of $>$s. We denote m implicit instances of $>$ by

$$[\,]^{>^m\cdots},$$

where the square brackets could enclose some diagrammatic structure, but need not. The ellipsis represents any additional implicit decorations, so long as they are not further instances of $>$. The superscript notation $>^m$ simply tells us that there are m instances of $>$.

For the purposes of this paper, we do not require the rules for turning gauge remainders appearing as implicit decorations into explicit structures. The details can be found in [20].

2. Effective Propagators

The rule for explicit decoration with implicit effective propagators is as follows. If we wish to join two objects (say two vertices) together with j' out of a total of j effective propagators, then there are ${}^jC_{j'}\ 2^{j'}$ ways to do this. Intuitively, the first factor captures the notion that, so long as they are implicit decorations, the effective propagators are indistinguishable. The factor of $2^{j'}$ allows for the fact that we can interchange the two ends of an effective propagator. If these effective propagators were instead used to form j' loops on a single vertex, then the factor of $2^{j'}$ would disappear, since the vertices are defined such that all cyclically independent arrangement of their decorative fields are summed over.

3. Vertices

When analysing the perturbative flow of $\mathcal{O}$, we will find that vertices (of the Wilsonian effective action and $\mathcal{O}$) always occur in a very particular way. To introduce compact notation

for this, we start by introducing a set of vertex arguments, v^j, where the upper roman index acts as a label. Thus, the v^j are integers, denoting the loop orders of some set of vertices. In the case that a vertex argument labels a Wilsonian effective action vertex, we define the reduction of v^j, $v^{j;R}$, such that a reduced vertex does not have a classical, two-point component.

Next, we define

$$\begin{aligned} v^{j,j_+} &\equiv v^j - v^{j+1}, \\ v^{j,j_+;R} &\equiv v^{j;R} - v^{j+1;R}. \end{aligned}$$

and use this notation to construct

$$\left[\begin{array}{c} \boxed{v^j} \\ \\ \left(n_s, j\right) \end{array}\right] \equiv \prod_{i=0}^{j-1} \sum_{v^{i+}=0}^{v^i} \left[\begin{array}{c} \boxed{v^j} \\ \\ \left(v^{i,i_+;R}\right) \end{array}\right], \tag{3.13}$$

where n_s gives the value of v^0, which is the only vertex argument not summed over on the right-hand side. Notice that the sum over all vertex arguments is trivially n_s:

$$\sum_{i=0}^{j-1} v^{i,i_+} + v^j = \sum_{i=0}^{j-1} \left(v^i - v^{i+1}\right) + v^j = v^0 = n_s. \tag{3.14}$$

The structure defined by (3.13) possesses a single vertex belonging to $\mathcal{O}$ and j (reduced) Wilsonian effective action vertices. This allows us to usefully define (3.13) for $j = 0$:

$$\left[\begin{array}{c} \boxed{v^j} \\ \\ \left(n_s, j\right) \end{array}\right]_{j=0} \equiv \boxed{n_s}.$$

C. The Diagrammatic Function, $\mathcal{Q}_n$

We introduce the functions

$$\mathcal{Q}_n \equiv \boxed{n} - 2 \sum_{s=1}^{n} \sum_{m=0}^{2s-1} \sum_{j=0}^{n+s-m-1} \frac{\Upsilon_{j+s,j}}{m!} \left[\begin{array}{c} \boxed{v^j} \\ \\ \left(n_s, j\right) \end{array}\right]^{\Delta^{j+s}>^m} \tag{3.15}$$

$$\bar{\mathcal{Q}}_n \equiv \boxed{n} - \mathcal{Q}_n \tag{3.16}$$

where, for the non-negative integers a and b, we define

$$\Upsilon_{a,b} = \frac{(-1)^{b+1}}{a!b!}\left(\frac{1}{2}\right)^{a+1}. \tag{3.17}$$

In the case that either a or b are negative, $\Upsilon_{a,b}$ is null. As part of the definition of $\bar{\mathcal{Q}}$, we insist that, upon explicit decoration, all fully fleshed out diagrams must be connected.

There is a simple, intuitive explanation for the relationship between the total number of vertices, the number of effective propagators and the sum over the vertex arguments. This is most simply put by taking $m = 0$ (the following argument is easily generalized). From (3.14), we know that the sum of the vertex arguments is $n - s$. Now, given $j + 1$ vertices, exactly j effective propagators are required to create a connected diagram. This leaves over s effective propagators, each of which must create a loop. Therefore, the loop order of the diagram is $n - s + s = n$, as must be the case.

The maximum values of the sums over m and j follow from the constraint that all fully fleshed out diagrams are connected [20]. The maximum value of s clearly follows from the requirement that the loop order of the diagram is ≥ 0. The minimum value of s ensures that, in $\mathcal{Q}$, we do not double count the contribution $\boxed{n}$.

D. Expectation Values in Perturbation Theory

The key to computing expectation values is to consider the flow of $\mathcal{Q}_n$. It can be shown [20], by using the flow equation, that this yields[3]:

$$\Lambda\partial_\Lambda \mathcal{Q}_n + 2\sum_{n'=1}^{n}(n - n' - 1)\beta_{n'}\mathcal{Q}_{n-n'} = 0, \tag{3.18}$$

from which it follows that

$$\Lambda\frac{d}{d\Lambda}\sum_{n=0}^{\infty}\left[g^{2(n-1)}\mathcal{Q}_n\right] = 0.$$

Integrating between $\Lambda = \mu$ and $\Lambda = \Lambda_0$ gives

$$\sum_{n=0}^{\infty}\left[g^{2(n-1)}\left(\boxed{n} - \bar{\mathcal{Q}}_n\right)\right]_{\Lambda=\mu} = \sum_{n=0}^{\infty}\left[g^{2(n-1)}\left(\boxed{n} - \bar{\mathcal{Q}}_n\right)\right]_{\Lambda=\Lambda_0}, \tag{3.19}$$

where we have used (3.16) and we aim to take the limits $\mu \to 0$ and $\Lambda_0 \to \infty$.

[3] This result, though intuitive, is far from straightforward to derive, afresh. However, the more difficult case of deriving similar diagrammatic expressions for the perturbative β-function coefficients is comprehensively illustrated in [20]. Given this derivation, (3.18) follows, essentially trivially.

The crucial point to recognize now is that (in perturbation theory, at any rate)

$$\lim_{\Lambda\to 0}\left[g^{2(n-1)}\bar{\mathcal{Q}}_n\right]=0. \tag{3.20}$$

We can argue this as follows. Consider $\bar{\mathcal{Q}}_n$, which is both UV and IR finite, in the limit of small Λ. At the level of the diagrammatic components out of which $\bar{\mathcal{Q}}_n$ is built, all contributions for which the loop momenta $k^i \gg \Lambda$ are suppressed by the UV regularization. (We might worry that this suppression does not occur in diagrams possessing classical vertices which diverge in this limit. However, these divergences are always overcompensated.) Thus, in the limit $\Lambda \to 0$, the loop integrals have no support and $\bar{\mathcal{Q}}_n$ vanishes.

To complete the argument, all that remains to be done is to show that the behaviour of $\lim_{\Lambda\to 0} g^{2(n-1)}(\Lambda)$ is sufficiently good. It should be emphasised that we are applying this limit to quantities computed in perturbation theory. Introducing the arbitrary scale, M, we can write

$$g^2(\Lambda)=\sum_{i=1}^{\infty} g^{2i}(M)a_i(M/\Lambda).$$

Differentiating both sides with respect to M yields the set of relationships:

$$0=2\sum_{j=1}^{n-1}(n-j)\beta_j a_{n-j}(M/\Lambda)+\frac{da_n(M/\Lambda)}{d\ln M/\Lambda}.$$

Given that $a_1 = 1$, it follows that every a_i must be a function of $\ln M/\Lambda$. Therefore, in the $\Lambda \to 0$ limit, $g(\Lambda)$ diverges, at worst, as powers of $\ln \Lambda$. This growth is slower than the rate at which the UV regularization kills $\bar{\mathcal{Q}}_n$ in the limit that $\Lambda \to 0$ [9] and so we have demonstrated (3.20).

From (3.19) and (3.20), we arrive at the central result of our perturbative treatment:

$$\sum_{n=0}^{n'}\left[g^{2(n-1)}\boxed{n}\right]_{\Lambda=0}=\sum_{n=0}^{n'}\left[g^{2(n-1)}\left(\boxed{n}-\bar{\mathcal{Q}}_n\right)\right]_{\Lambda=\Lambda_0}+\mathrm{O}(g^{2n'}). \tag{3.21}$$

Notice that we have replaced the upper limits of the sums over n with the finite n'. By taking n' to infinity, (3.21) becomes exact. However, the form given above is suitable for the order-by-order computation of corrections to $\langle\mathcal{O}\rangle_R$.

IV. WILSON LOOPS

In this section, we will illustrate (3.21) by using it to compute perturbative corrections to Wilson loops. Before taking the explicit case of the rectangular Wilson loop with sides $T \gg L$, we discuss some general features of Wilson loop calculations, within our framework.

A. General Considerations

For some closed path, Γ, the path ordered phase factor, *a.k.a.* the Wilson loop, is defined to be

$$\phi(\Gamma) = \frac{1}{N}\mathrm{tr}P\exp\left[i\oint_\Gamma dx_\mu A^1_\mu(x)\right].$$

It is well known [37, 41, 42] that the expectation value of this object,

$$W(\Gamma) = \langle\phi(\Gamma)\rangle\,, \tag{4.1}$$

is divergent even after renormalization of the coupling and, in the case of a gauge fixed formulation, field strength renormalization. In our manifestly gauge invariant formulation, where the gauge field does not suffer from field strength renormalization, (4.1) is defined such that the renormalization of the coupling has been done.

The remaining divergences have two sources. For smooth, simple loops, there is a divergence $e^{-\kappa\Lambda_0 l(\Gamma)}$, where κ is a dimensionless parameter and $l(\Gamma)$ is the length of Γ. The linearly divergent $K \equiv \kappa\Lambda_0$ can be interpreted as a mass divergence. The other divergences come from any (finite) number of cusps and intersections, parameterised by the angles θ^i and ϑ^i, respectively. The renormalized expectation value of the Wilson loop with cusps but no intersections is defined to be [42]

$$W_R(\Gamma_{\theta^i}) = Z(\theta^i)e^{-\overline{m}_0\Lambda_0 l(\Gamma)}W(\Gamma_{\theta^i}),$$

where we have used powers of Λ_0 to replace the bare mass, m_0, with a dimensionless parameter, $\overline{m}_0$. The renormalized mass, m, is

$$m = K + m_0$$

and the multiplicative renormalization constant factorizes:

$$Z(\theta^i) = Z(\theta^1)Z(\theta^2)\cdots.$$

(In the case that Γ includes intersections, $W_R(\Gamma)$ no longer renormalizes by itself, and must be considered together with expectation values of a family of other loop functions.)

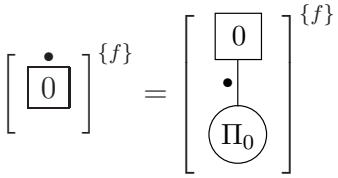
$Z(\theta)$ and $\overline{m}$ have the following expansions:

$$\begin{aligned} Z(\theta, g) &= 1 + \sum_{i=1}^{\infty} g^{2i} Z_i(\theta) \\ \overline{m}(g) &= \sum_{i=1}^{\infty} g^{2i}\overline{m}_i. \end{aligned}$$

With these points in mind, we identify the boundary value of our gauge invariant operator with

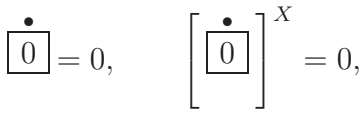
$$\mathcal{O}_{\Lambda_0} = \frac{1}{N}\mathrm{tr}P\exp\left[i\oint_\Gamma dx_\mu A^1_\mu(x)\right]e^{-\overline{m}_0\Lambda_0 l(\Gamma)}Z(\theta^i). \tag{4.2}$$

We can use the fact that $\mathcal{O}_{\Lambda_0}$ does not possess an $\mathrm{O}(1/g^2)$ component to simplify the following analysis. To this end, consider the classical flow of $\mathcal{O}$:



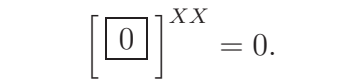
Now, for the right-hand side not to vanish, the Π_0 vertex must be decorated by at least two fields belonging to $\{f\}$. This is because both seed action and Wilsonian effective action one-point vertices vanish at tree level [11, 16] and $\Pi_{0RS}^{XX}(k) = 0$ due to our choice to set the seed action, two-point, classical vertices equal to their Wilsonian effective action counter parts.

From this it follows that

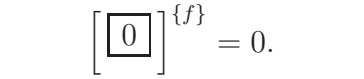


where X is any field. Integrating up and using the fact that all classical vertices vanish at the boundary (see (3.9) and (4.2)) we find that

But, these relationships, together with the vanishing of $\Pi_{0RS}^{XX}(k)$ and the boundary condition imply that

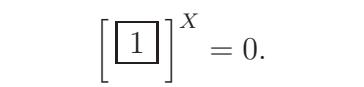


Iterating this argument, it is clear that, in fact,



Given that the $\mathcal{O}$ vertex of $\bar{\mathcal{Q}}$ must, therefore, have an argument of at least one, this allows us to reduce the maximum value of j by unity [20].

In a similar fashion, we can demonstrate that



With these points in mind, let us apply (3.21) for $n' = 1$. Using the boundary condition, we obtain the expected (trivial) result that

$$\Big[\boxed{1}\Big]_{\Lambda=0} = 1. \tag{4.3}$$

[Note that (4.3) is in fact exact, not requiring supplementation at $\mathrm{O}(g^2)$. This follows because, in the weak coupling expansion, the vertex $\boxed{1}$ multiplies g^0.] At the next order we find

$$\Big[g^2\ \boxed{2}\ \Big]_{\Lambda=0} = \Bigg[g^2\left(\ \boxed{2}\ - 2\Upsilon_{1,0}\ \begin{array}{c}\cup\\ \boxed{1}\end{array}\ \right)\Bigg]_{\Lambda=\Lambda_0} + \mathrm{O}(g^4). \tag{4.4}$$

Equation (4.4) gives the first non-trivial correction to the renormalized Wilson loop parameterized by a contour with an arbitrary (finite) number of cusps (generalization to include intersections is straightforward, as indicated earlier). To evaluate (4.4) we feed in the boundary condition (4.2). The first term on the right-hand side possesses precisely those contributions necessary to cancel the divergences in the second term. With these divergences cancelled, we can safely take the continuum limit, $\Lambda_0 \to \infty$.

B. The Rectangular Wilson Loop with sides $T \gg L$

To illustrate the application of (4.4) in a way which will allow us to compare directly with known results, we must compute a quantity which is independent of the renormalization prescription. To this end, we focus on the rectangular Wilson loop, $\bar{\Gamma}$, with sides T and L, in the limit where $T \gg L$. The leading order contribution in this limit is universal, being directly related to the lowest order Coulomb potential of the physical $SU(N)$ Yang-Mills theory.

At the boundary, the expression for the first term on the right-hand side of (4.4) follows, directly, from (4.2) upon expanding the exponentials and identifying the $\mathrm{O}(g^2)$, field independent contribution. For the second term we must work a little harder, since we need to relate the two-point vertex to the boundary condition. To do this, we expand the exponential of (4.2) and focus on the coefficient of $\mathrm{tr}A^1_\mu A^1_\nu$ at $\mathrm{O}(g^0)$:

$$-\frac{1}{2N}\oint_{\bar{\Gamma}} dx_\mu \oint_{\bar{\Gamma}} dy_\nu = -\frac{1}{2N}\int d^D x \int dt \frac{dx_\mu(t)}{dt}\delta(x - x(t)) \int d^D y \int ds \frac{dy_\nu(s)}{ds}\delta(y - y(s)).$$

The recasting on the right-hand side allows us to directly compare this expression with the field expansion of $\mathcal{O}$, given by the analogue of (2.1) with S replaced by $\mathcal{O}$. Therefore,

$$\left.\begin{array}{c} x\ \mu \\ \boxed{1} \\ y\ \nu \end{array}\right|_{\Lambda_0} = -\frac{1}{N}\int dt \frac{dx_\mu(t)}{dt}\delta(x-x(t))\int ds \frac{dy_\nu(s)}{ds}\delta(y-y(s)).$$

The other components of the second diagram on the right-hand side of (4.4) are the effective propagator, $\Delta^{1\,1}_{\mu\nu}(x,y)$[4], the group theory factor (which can be evaluated according to figure 3) and an integral over the undetermined coordinates, x and y.

Using (3.17), equation (4.4) becomes

$$\lim_{T/L\to\infty} W_R^{(2)}(\bar{\Gamma}) = \lim_{T/L\to\infty}\lim_{\Lambda_0\to\infty}\left[g^2\left(\boxed{2} - \frac{N^2-1}{2N}\oint_{\bar{\Gamma}} dx_\mu \oint_{\bar{\Gamma}} dx_\nu \Delta^{1\,1}_{\mu\nu}(x,y,\Lambda_0)\right)\right], \qquad (4.5)$$

where we have changed notation slightly to make the path dependence of the left-hand side explicit. Since we are taking the $T/L\to\infty$ limit, we do not need to be too precise about our renormalization prescription: the associated finite terms are sub-leading and so we have:

$$\lim_{T/L\to\infty} W_R^{(2)}(\bar{\Gamma}) = \lim_{T/L\to\infty}\left[-g^2 C_2(F)\oint_{\bar{\Gamma}} dx_\mu \oint_{\bar{\Gamma}} dx_\nu \Delta^{1\,1}_{\mu\nu}(x,y)\Big|_{\text{finite}}\right]. \qquad (4.6)$$

Writing

$$\Delta^{1\,1}_{\mu\nu}(x,y) = \int \frac{d^Dp}{(2\pi)^D}\, e^{ip\cdot(x-y)}\Delta^{1\,1}_{\mu\nu}(p^2/\Lambda^2)$$

and recalling (2.9), it is clear that our expression corresponds to the usual one. However, we emphasise once again that, despite obvious similarities, the object $\Delta^{1\,1}_{\mu\nu}(p^2/\Lambda^2)$ is not a (regularized) Feynman propagator and that at no stage have we fixed the gauge. Notice that we can immediately take $D\to 4$, since preregularization plays no role here. Indeed, this highlights the fact that we only ever use dimensional regularization as a prescription for removing finite surface terms present as a consequence of the Pauli-Villars regularization provided by the $SU(N|N)$ scheme [9]. All necessary UV regularization in (4.5) and (4.6) is provided by the cutoff functions buried in the effective propagator.

Explicitly evaluating the contour integrals we find that

$$\lim_{T/L\to\infty} W_R^{(2)}(\bar{\Gamma}) = g^2\frac{C_2(F)T}{4\pi L},$$

recovering the standard result.

[4] The fields must be in the A^1 sector at the bare scale, since this is the only sector in which the vertex to which the effective propagator attaches has support.

V. CONCLUSION

We have described how to compute the expectation values of renormalized gauge invariant operators in a manifestly gauge invariant way, within the framework of the exact renormalization group. The methodology has been illustrated with a computation of the $\mathrm{O}(g^2)$ correction to the rectangular Wilson loop with sides $T \gg L$.

The key elements of the methodology are as follows. Given our regularized $SU(N)$ gauge theory defined at the bare scale, Λ_0, we add a source term $J\mathcal{O}_{\Lambda_0}$ for the gauge invariant operator, $\mathcal{O}_{\Lambda_0}$. As we integrate out modes, so the source term evolves. Although this generates a Taylor series in J, the only term which contributes to $\langle\mathcal{O}\rangle_R$ is the one linear in J which, after specializing to the small J limit, we denote by $\mathcal{O}_\Lambda$. Figure 4 gives the flow of this component.

We then derived equation (3.7), which states that the expectation value of our gauge invariant operator is simply given by $\mathcal{O}_{\Lambda=0}$. Thus, in conjunction, figures 1 and 4 and equation (3.7) allow us to compute the expectation value of an arbitrary gauge invariant operator (in some approximation scheme).

The rest of the paper was devoted to exploring the formalism in the perturbative domain. It was here that the considerable effort invested in [15, 18–20] to understand the structure of perturbative β-function coefficients really paid off. The associated developments allowed us to directly obtain (3.21), which gives an extremely compact diagrammatic expression for the perturbative corrections to $\langle\mathcal{O}\rangle_R$. We note that this expression makes use of the diagrammatic function, $\bar{\mathcal{Q}}$, given by (3.15) and (3.16). This function depends only on Wilsonian effective action vertices, effective propagators and (components of) gauge remainders. There is no explicit dependence on either the seed action or the covariantization of the ERG kernels.

Whilst the perturbative treatment is useful both to gain experience with the techniques and also to demonstrate that practical calculations can be straightforwardly (and correctly) performed, the real challenge is to apply the formalism non-perturbatively. Of course, the key results figures 1 and 4 and equation (3.7) are defined non-perturbatively. The main difficulty is deciding how best to approximate the flow equation where there is no obviously small parameter in which to expand (for speculations on whether it might be possible to perform a strong coupling expansion in the inverse of the *renormalized* coupling see [43]). However, some inspiration for this may be provided by the perturbative treatment. We

know that for operators which correspond to physical observables, the expression for $\mathcal{O}_{\Lambda=0}$ must be universal. Obviously, such an expression is independent of the details of the seed action or the covariantization of the ERG kernels. Thus, it is natural speculate whether, non-perturbatively, $\mathcal{O}_{\Lambda=0}$ can be written in terms of a generalization of $\bar{\mathcal{Q}}$; indeed, this generalization has now been found [44]. Nevertheless, this generalized diagrammatic function possesses an infinite number of vertices and so much work remains to be done to extract useful information. However, this surely represents a desirable, direct starting point for attacking non-perturbative problems within the ERG formalism.

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