Einstein-Yang-Mills solutions in higher dimensional de Sitter spacetime

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Abstract

We consider particle-like and black holes solutions of the Einstein-Yang-Mills system with positive cosmological constant in $d > 4$ spacetime dimensions. These configurations are spherically symmetric and present a cosmological horizon for a finite value of the radial coordinate, approaching asymptotically the de Sitter background. In the usual Yang–Mills case we find that the mass of these solutions, evaluated outside the cosmological horizon at future/past infinity generically diverges for $d > 4$. Solutions with finite mass are found by adding to the action higher order gauge field terms belonging to the Yang–Mills hierarchy. A discussion of the main properties of these solutions and their differences from those to the usual Yang–Mills model, both in four and higher dimensions is presented.

1 Introduction

Recently, there has been a significant increase in interest in the properties of gravity in more than $d = 4$ dimensions. This interest was enhanced with the development of string theories, along with the idea of large extra dimensions recently resurrected by TeV gravity models. Several solutions of higher dimensional classical general relativity have been known for some time, in particular extensions to any $d > 4$ of the Schwarzschild and Reissner-Nordström black holes by Tangherlini [1], and of the Kerr black hole by Myers and Perry [2]. Investigations over the past fifteen years have produced an impressive catalogue of solutions for various effective theories of Einstein gravity coupled to many different kinds of matter fields. These results indicate that the physics in higher-dimensional general relativity is far richer and complex than in the standard four-dimensional theory.

Solutions to the Einstein-Yang-Mills (EYM) equations in higher dimensions have recently been studied. As found in [3], for asymptotically flat solutions to the usual Yang-Mills (YM) gravitating system in five spacetime dimensions the particle spectrum obtained by uplifting the $d = 4$ flat space YM instantons become completely destroyed by gravity, as a result of their scaling behaviour. These results [3] can be systematically extended to the $d \geq 5$ case and one finds that no finite mass spherically symmetric solutions exist in EYM theory, unless one modifies the non Abelian action density by adding higher order curvature terms in the YM hierarchy. (The YM hierarchy features higher order curvature $2p$ forms $F(2p) = F \wedge F \wedge F ... \wedge F$, p times, labeled by p, the $p = 1$ term giving the usual YM system.) Without these higher order YM terms, only vortex-type finite energy solutions [3] exist, describing effective systems in three spacelike dimensions and with a number of codimensions.

Asymptotically flat, regular, static and spherically symmetric solutions of EYM equations with higher order terms in the Yang-Mills hierarchy were presented in [4] for spacetime dimensions $d = 6, 7, 8$, and for $d = 5$, both globally regular and black hole solutions were found in [5]. The properties of these solutions are rather different from the familiar Bartnik-McKinnon solutions [6] to EYM equations in $d = 4$, and are somewhat more akin to the gravitating monopole solutions to EYM-Higgs system [7]. This is because like in the latter case [7], where the vacuum expectation value of the Higgs field features as an additional dimensional constant, here also additional dimensional constants enter with each higher order YM curvature term [4]. The typical critical features discovered in [4, 5] have been analysed and explained in [8].

When a cosmological constant is added to the theory, the asymptotic behaviour of the physically relevant solutions changes from Minkowski to Anti-de Sitter (AdS) for $\Lambda < 0$ or de Sitter (dS) for $\Lambda > 0$. In the latter case ($\Lambda > 0$), to our knowledge no studies of EYM solutions in higher dimensions ($d > 4$) have been undertaken to date. To carry out this investigation is the aim for the present work.

Higher dimensional EYM-$F(2)$ solutions with a negative cosmological constant ($\Lambda < 0$) have been studied in [9, 10]. The main properties of these configurations resemble the familiar $d = 4$ AdS ones [11, 12]. These describe a continuum of solutions with arbitrary asymptotic values of the function $w(r)$ parametrising the spherically symmetric gauge field. However, the total mass-energy of the AdS$_d$ nonabelian solutions diverges for $d > 4$. Higher dimensional asymptotically AdS EYM solutions with finite mass-energy were found by augmenting the action density of the system with higher order curvature terms, consisting of $2p$-form curvatures $F(2p)$ [10]. The qualitative features of these solutions are very similar to those of the asymptotically flat case with $\Lambda = 0$; in particular, and most notably, the nonabelian fields approach asymptotically a pure gauge configuration with $w(r \to \infty) = -1$, uniquely, unlike in the $p = 1$ case.

Proceeding to EYM systems with positive cosmological constant $\Lambda > 0$, we note that at present only the $p = 1$ gravitating YM system is studied, and that, only in $d = 4$ (see [13]-[16], and also the systematic approach in [17]). The gravitating YM field equations present both black holes and solutions with a regular origin, in contrast to the case of a gravitating Abelian field. This property is shared with all cases, $\Lambda > 0$, $\Lambda < 0$ and $\Lambda = 0$. Asymptotically dS ($\Lambda > 0$) configurations present, in particular, a cosmological horizon for a finite value of the radial coordinate. As in the $\Lambda < 0$ case, the asymptotic value of the gauge field function $w(r)$ for solutions with $\Lambda > 0$ is not fixed, implying the existence of a nonvanishing magnetic charge. This contrasts with the asymptotically flat situation [6].

Our task in this paper is to examine the corresponding situation for $d > 4$ EYM solutions with positive cosmological constant. Our strategy is to first consider the usual YM model, namely the $p = 1$ member of the YM hierarchy, i.e. the square of the 2-form curvature $F(2)$. We find that although the EYM equations in this case present solutions approaching asymptotically the dS background, the mass of solutions evaluated at future/past infinity generically diverges. Like in the asymptotically flat and AdS cases, finite mass-energy EYM solutions are found by augmenting the action density of the system with higher order curvature terms, consisting of $2p$-form curvatures $F(2p)$.

2 Higher dimensional gravitating $p = 1$ YM system

2.1 The model

In this Section we shall examine the usual EYM system in a $d-$dimensional spacetime described by the following action

$$I = I_{bulk} + I_{surf} = \int_{\mathcal{M}} d^d x \sqrt{-g} \left(\frac{1}{16\pi G}(R - 2\Lambda) + \mathcal{L}_m \right) - \frac{1}{8\pi G} \int_{\partial\mathcal{M}^{\pm}} d^{d-1}x \sqrt{h} K, \tag{1}$$

where R is the Ricci scalar associated with the spacetime metric $g_{\mu\nu}$, $\Lambda = (d-1)(d-2)/(2\ell^2)$ is the cosmological constant and G is the gravitational constant (following [4, 5], we define also $\kappa = 1/(8\pi G)$). $\partial\mathcal{M}^{\pm}$ are spatial Euclidean boundaries at Euclidean surfaces at future/past timelike infinity $\mathcal{I}^{\pm}$ and $\int_{\partial\mathcal{M}^{\pm}}$ indicates the sum of the integral over the early and late time boundaries. The quantities $g_{\mu\nu}, h_{\mu\nu}$ and K are the bulk spacetime metric, induced boundary metrics and the trace of extrinsic curvatures of the boundaries respectively.

The matter term in the above relation, $\mathcal{L}_m = -\frac{1}{4}\tau_1$ tr $\{F_{\mu\nu}F^{\mu\nu}\}$ is the usual $F(2)$ nonabelian action density, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$ being the gauge field strength tensor.

The field equations are obtained by varying the action (1) with respect to the field variables $g_{\mu\nu}, A_\mu$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad D_\mu \left(\sqrt{-g}\ F^{\mu\nu}\right) = 0, \tag{2}$$

where the energy momentum tensor is defined by

$$T_{\mu\nu} = \mathrm{tr}\ \{F_{\mu\alpha} F_{\nu\beta} g^{\alpha\beta} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}\}. \tag{3}$$

For the case of a d-dimensional spacetime, we restrict to static fields that are spherically symmetric in the $d-1$ spacelike dimensions, with a metric ansatz in terms of Schwarzschild coordinates

$$ds^2 = \frac{dr^2}{N(r)} + r^2 d\Omega^2_{d-2} - \sigma^2(r) N(r) dt^2, \tag{4}$$

with $d\Omega_{d-2}$ the $d-2$ dimensional angular volume element and

$$N = 1 - \frac{2m(r)}{\kappa r^{d-3}} - \frac{r^2}{\ell^2}, \tag{5}$$

the function $m(r)$ being related to the local mass-energy density up to some $d-$dependent factor.

The choice of the gauge group compatible with the symmetries of the line element (4) is discussed in [4]. This choice implies the use of the representation matrices $SO_\pm(\bar{d})$, where $\bar{d} = d$ and $\bar{d} = d-1$ for *even* and *odd* d respectively. In this unified notation (for odd and even d), the spherically symmetric Ansatz for the $SO_\pm(\bar{d})$-valued gauge fields then reads [4]

$$A_0 = 0\ , \quad A_i = \left(\frac{1-w(r)}{r}\right) \Sigma^{(\pm)}_{ij} \hat{x}^j\ , \quad \Sigma^{(\pm)}_{ij} = -\frac{1}{4}\left(\frac{1 \pm \Gamma_{\bar{d}+1}}{2}\right) [\Gamma_i, \Gamma_j]\ . \tag{6}$$

The Γ's denote the $\bar{d}$-dimensional gamma matrices and $1,\ j = 1, 2, ..., d-1$ for both cases; $\hat{x}^j = x^j/r$, with $r^2 = x_i x^i$.

Inserting this ansatz into the action (1), the EYM field equations reduce to

$$\left(r^{d-4}\sigma N w'\right)' - (d-3) r^{d-6} \sigma (w^2-1) w = 0, \quad m' = \frac{\tau_1}{2} r^{d-4} \left(N w'^2 + (d-3)\frac{(w^2-1)^2}{2r^2}\right), \quad \frac{\sigma'}{\sigma} = \frac{\tau_1}{\kappa} \frac{w'^2}{r}, \tag{7}$$

where the prime denotes the derivative with respect to the radial coordinate r

The above differential equations have two analytic solutions. For a pure gauge field $w(r) = \pm 1$, one finds $m(r) = M,\ \ \sigma(r) = 1$, M being a constant, which corresponds to Schwarzschild-dS spacetime. For $w(r) = 0$ we find a non Abelian generalisation of the magnetic- Reissner-Nordstrøm-dS (RNdS) solution with $\sigma(r) = 1$ and

$$m(r) = M_0 + \frac{\tau_1}{2} \log(\frac{r}{\ell}) \ \text{ for } \ d = 5, \ \ \text{and} \quad m(r) = M_0 + \frac{\tau_1(d-3)}{4(d-5)} r^{d-5} \ \text{ for } \ d \neq 5, \tag{8}$$

M_0 being an arbitrary constant. Some properties of these solutions are discussed in [18]. We can see e.g. that, for a suitable range of $(M_0,\ \Lambda)$ it describes a cosmological black hole, the horizons being located at the zeros of N. Also, although these solutions are asymptotically dS, their total masses/energies defined outside the cosmological horizon at future/past infinity ($r \to \infty$), diverge.

2.2 Numerical solutions

We want the generic line element (4) to describe a nonsingular, asymptotically de Sitter spacetime outside a cosmological horizon located at $r = r_c > 0$. Here $N(r_c) = 0$ is only a coordinate singularity where all curvature invariants are finite.

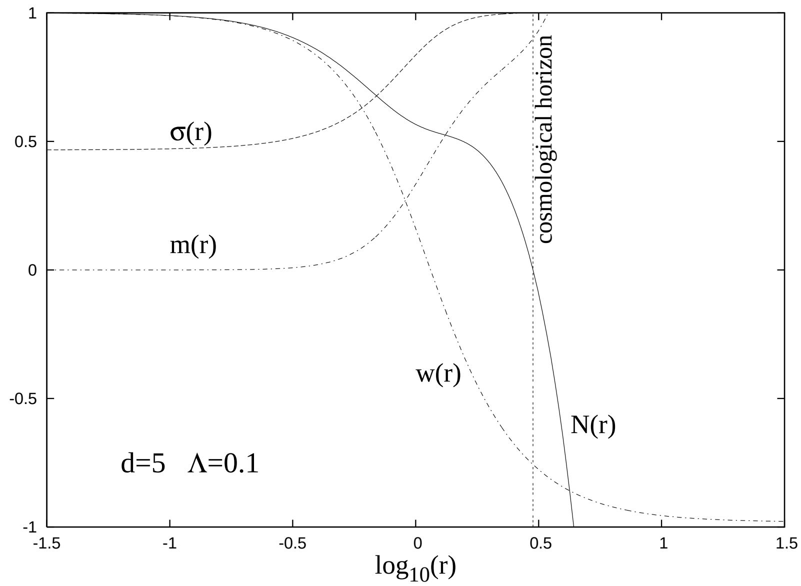

Figure 1. The functions $\sigma(r)$, $w(r)$, $N(r)$ and $m(r)$ are plotted as functions of radius for a typical one-node $d = 5$ regular solutions in a $F(2)$ EYM-dS theory.

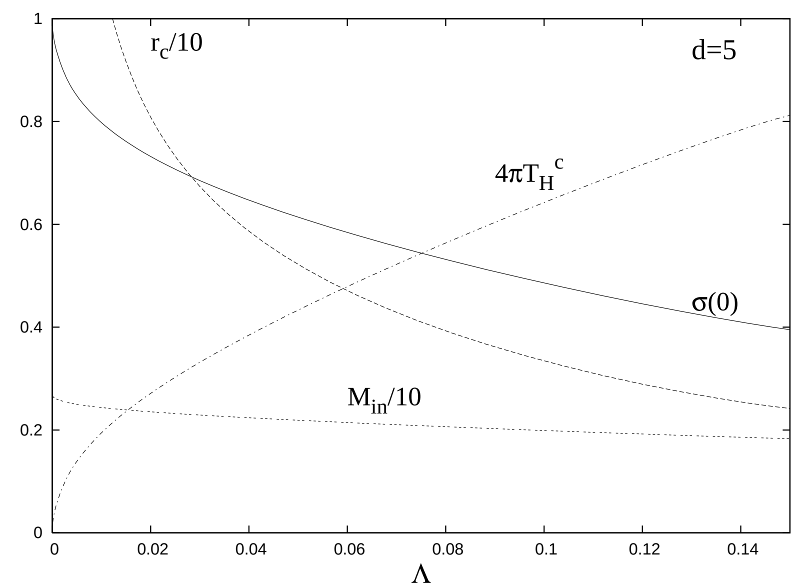

Figure 2. The value of the cosmological horizon radius r_c, the value M_{in} of the mass function $m(r)$ at $r = r_c$, the Hawking temperature associated with the cosmological horizon as well as the value $\sigma(0)$ of the metric function σ at the origin, are shown as functions of Λ for $d = 5$ particle-like solutions of $F(2)$ theory.

Outside the cosmological horizon r and t changes the character (i.e. r becomes a timelike coordinate for $r > r_c$). A nonsingular extension across this null surface can be found just as at the event horizon of a black hole. The regularity assumption implies that all curvature invariants at $r = r_c$ are finite. Also, all matter functions and their first derivatives extend smoothly through the cosmological horizon, e.g. in a similar way as the U(1) electric potential of a RNdS solution.

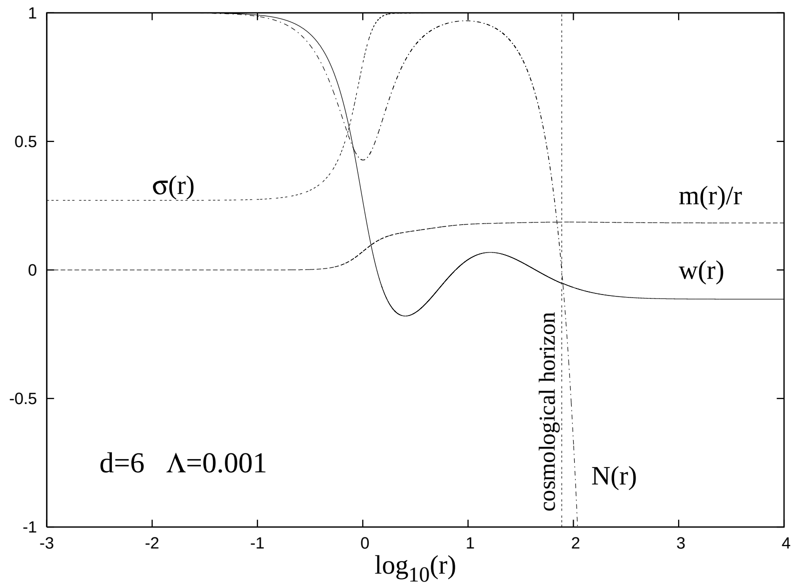

Figure 3. The functions $\sigma(r)$, $w(r)$, $N(r)$ and $m(r)/r$ are plotted as functions of radius for a typical $d = 6$ regular solutions in a $F(2)$ EYM-dS theory.

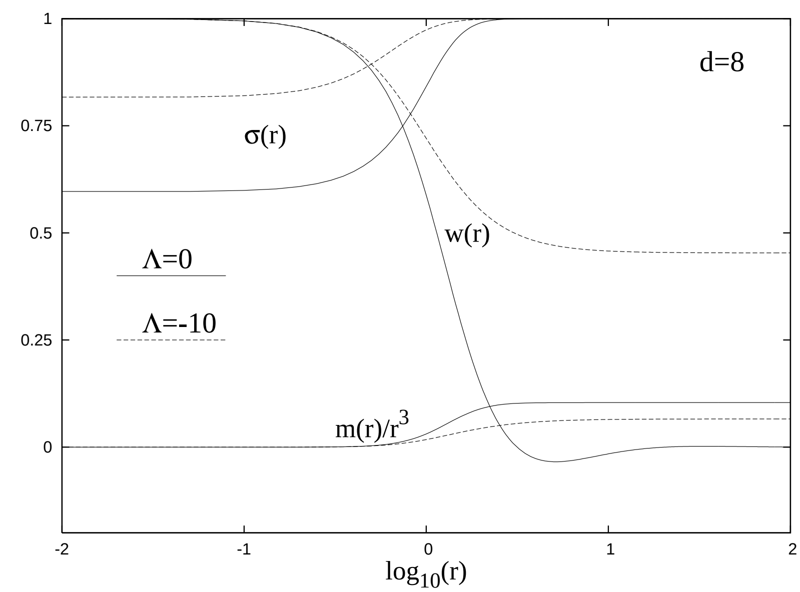

Figure 4. Typical EYM-$F(2)$ solutions in $d = 8$ asymptotically flat and AdS spacetimes.

As in the $\Lambda \leq 0$ case, it is natural to consider two types of configurations, corresponding in the usual terminology to cosmological *particle-like* and *black hole* solutions. The black hole configurations possess an event horizon located at some intermediate value of the radial coordinate $0 < r_h < r_c$, all curvature invariants being finite as $r \to r_h$. Both the event and the cosmological horizons have their own surface gravity κ given by

$$\kappa_{h,c}^2 = -\frac{1}{4} g^{tt} g^{rr} (\partial_r g_{tt})^2 \Big|_{r=r_h,r_c} \ ,$$

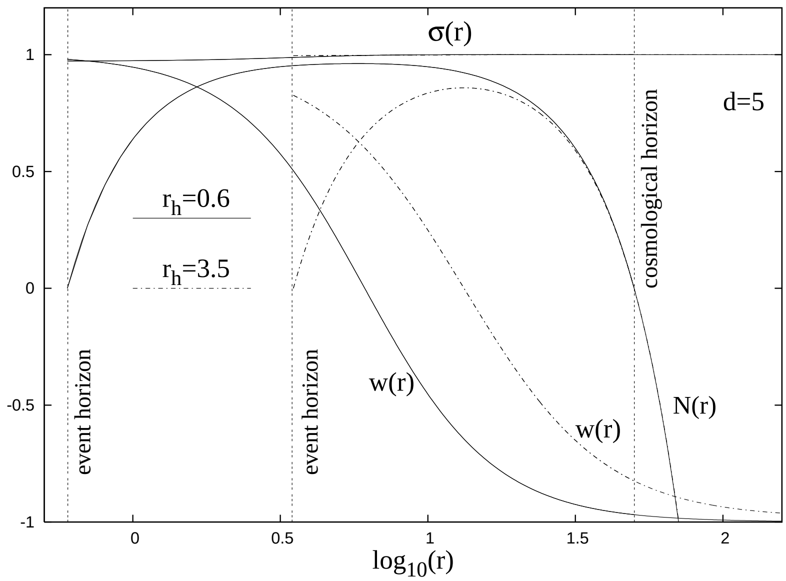

Figure 5. The profiles of two different $d = 5$ EYM-$F(2)$ black hole solution are presented for the same value of the cosmological horizon radius.

the associated Hawking temperatures being $T_H^{h,c} = |\kappa_{h,c}|/(2\pi)$.

The corresponding boundary conditions at the origin and cosmological/event horizon are found by taking $\tau_2 = 0$ in the general relations (18), (19) given in Section 3.

To integrate the field equations, we used in this work the differential equation solver COLSYS which involves a Newton-Raphson method [19]. The case $\Lambda > 0$ leads to the occurrence of a cosmological horizon at $r = r_c$ with $N(r_c) = 0$. As in the U(1) case, the cosmological horizon radius r_c is a function of Λ. In practice, we solved the equations first on the interval $[0, r_c]$ (or $[r_h, r_c]$), choosing r_c by hand and imposing regularity conditions of the solutions at $r = r_c$. This allows to determine the values of the functions (m, σ, w, w') at $r = r_c$ as well as the numerical value of Λ corresponding to the choice of cosmological horizon. The integration on $[r_c, \infty)$ can then be performed as a second step, leading to the knowledge of the solution on the full r-axis and regular at $r = r_c$.

For a $F(2)$ theory, the constants κ and τ_1 can always be absorbed by rescaling $r \to cr$, $\Lambda \to \Lambda/c^2$ and $m \to m\tau_1 c^{d-5}$, with $c = \sqrt{\tau_1/\kappa}$, the only remaining parameter being the cosmological constant. We have numerically integrated Eqs. (7) for $d = 5, 6, 7, 8$, and for several values of Λ.

All numerical solutions we found have $w^2(r) \leq 1$, which implies a nonzero node number of the gauge potential w. This can be proven by using the sum rule

$$-\left(\frac{r^{d-4}N\sigma w'}{w}\right)\Big|_{r_0}^{r_c} = \int_{r_0}^{r_c} dr\ \sigma\left((d-3)r^{d-6}(1-w^2) + r^{d-4}N\frac{w'^2}{w^2}\right), \tag{9}$$

which follows directly from the YM equations (r_0 here is $r_0 = 0$ for solutions with regular origin or $r_0 = r_h$, for black holes). Suppose that $w(r)$ never vanishes and $w^2 \leq 1$ for $r_0 < r < r_c$. Then the l.h.s. of the above relation vanishes, while the integrand of the r.h.s. is positive definite. Therefore the gauge potential of the nontrivial YM configurations with $w^2 \leq 1$ must vanish at least once in the region inside the cosmological horizon. For solutions with dS asymptotics, the asymptotic value of the gauge potential $w(\infty) = w_0$ appears as a result of the numerical integration. As a general feature, all configurations we have considered have $w_0^2 < 1$.

Considering first the case $d = 5$, we were able to construct a numerical solution for each value of the cosmological constant $\Lambda \leq 0.5$, the numerics becoming more difficult for larger values of Λ. As in the $d = 4$

case [13], we expect there to be a maximal value Λ_c of the cosmological constant, such that for $\Lambda > \Lambda_c$ the gravitational interaction becomes too strong for the solutions with dS asymptotics to exist. The profile of a typical one-node solution corresponding to $\Lambda = 0.1$ and $r_c = 3$ is given on Figure 1 (multi-node solutions have been found as well). One can see that the gauge field function approaches asymptotically a finite value, $w_0 \simeq -0.98$ as a consequence of which the mass function diverges logarithmically.

The corresponding numerical value of $r_c, \sigma(0)$, the Hawking temperature T_H of the cosmological horizon and the value of the mass function on the cosmological horizon $M_{in} = m(r_c)$ are reported on Figure 2 as functions of the cosmological constant for one-node configurations. One can see that, similar to the case of a RNdS solution, the Hawking temperature increases with Λ.

Extending these solutions outside the cosmological horizon reveals, however, that the value w_0 differs slightly from one. As a general feature, the Einstein equations imply that the mass function $m(r)$ develops a logarithmic term which makes the mass divergent as $r \to \infty$,

$$m(r) = M_0 + \frac{\tau_1}{2}(w_0^2 - 1)^2 \log(\frac{r}{\ell}). \tag{10}$$

A similar result holds for $d = 5$ asymptotically AdS solutions [10], the $\Lambda = 0$ picture being more complex [3].

The results we found by solving numerically the field equations for $d = 6, 7, 8$ confirm that this is a generic behaviour of the higher dimensional EYM solutions. For $d > 5$ the mass function diverges for large r according to

$$m(r) = M_0 + \frac{\tau_1(d-3)}{4(d-5)}(w_0^2 - 1)^2 r^{d-5}, \tag{11}$$

other properties of these solutions being very similar to the the $d = 5$ case. Despite this divergence, these solutions are still asymptotically dS (as $r \to \infty$ one finds $\sigma \simeq 1 + O(1/r^6)$). A typical $d = 6$ configurations with a regular origin is presented in Figure 3, for $\Lambda = 0.0001$ and $r_c = 77.45$. One can see that the mass function diverges linearly, while the gauge potential $w(r)$ presents three nodes.

For completeness, we give in Figure 4 the profiles of typical EYM-F(2) $d = 8$ configurations in asymptotically flat and AdS spacetimes (the solutions for other dimensions $d > 5$ have the same qualitative features). The solutions we have found for $\Lambda \leq 0$ exist for a compact interval $0 < b < b_{max}$, b being the parameter in the expansion of the gauge function at the origin $w(r) = 1 - br^2 + O(r^4)$, their masses diverging again according to (11), since $w_0^2 \neq 1$. For the AdS case, the asymptotic value of the gauge potential may take arbitrary values [10], being fixed by the parameter b (with $b = 0.5$ for the solutions in Figure 4). All asymptotically flat solutions we have studied have $w_0 = 0$ and present at least one node of the gauge function $w(r)$. Also, a critical solution is approached as $b \to b_{max}$, with $\sigma(0) \to 0$ in this limit.

Apart from particle-like solutions, we have found black hole solutions as well. Restricting to the $d = 5$ case, our numerical analysis suggests that any asymptotically dS particle-like solution presents black hole counterparts. Imposing the condition of a regular horizon at $r = r_h$, we obtained a family of black hole solutions for any $r_c > r_h > 0$. When the value r_h increases, the values $|w(r_h)|$ and $|w(r_c)|$ slowly decrease. The asymptotics of the cosmological black hole solutions are similar to the particle-like case. In particular one finds $w_0^2 < 1$, which implies a divergent value of the mass-function as $r \to \infty$ according to (10). The profiles of the solutions corresponding two different values of r_h are reported on Figure 5 (these configurations have the cosmological horizon at $r_c = 50$).

We see that the mass function of both regular and black hole solutions diverges asymptotically, yielding infinite total mass. The situation is exactly the same for the $\Lambda \leq 0$ too. Not having a finite value for the mass in the $\Lambda > 0$ case however may not be regarded as quite as serious a physical disadvantage, if one takes the view that the mass is nonetheless finite inside the cosmological horizon.

For $\Lambda \leq 0$, the non-existence of $d > 4$ spherically symmetric EYM-$F(2)$ configurations with finite mass could be proven in a rigurous way. However, the arguments in [9, 10] fail to apply for a positive cosmological constant, a different approach being necessary in this case.

3 Higher dimensional gravitating nonabelian p YM hierarchies

3.1 The equations and boundary conditions

A simple way to find nontrivial solutions with a finite mass is to modify the matter Lagrangean by adding higher order terms in the YM hierarchy, constructed exclusively from YM curvature $2p$-forms. Such terms are also predicted by the low energy string theory (see e.g. [20]-[22]).

The definition we use for superposed YM hierarchy is

$$\mathcal{L}_m = -\sum_{p=1}^{P} \frac{1}{2(2p)!} \, \tau_p \, \sqrt{-g} \, \mathrm{Tr} \, \{F(2p)^2\} \, , \tag{12}$$

where $F(2p)$ is the $2p$-form p-fold totally antisymmetrised product of the $SO(d)$ YM curvature 2-form $F(2)$

$$F(2p) \equiv F_{\mu_1\mu_2\ldots\mu_{2p}} = F_{[\mu_1\mu_2}F_{\mu_3\mu_4}\ldots F_{\mu_{2p-1}\mu_{2p}]} \, . \tag{13}$$

Even though the $2p$-form (13) is dual to a total divergence, namely the divergence of the corresponding Chern-Simons form, the density (12) is never a total divergence since it is the square of one. But the $2p$-form (13) vanishes by (anti)symmetry for $d < 2p$ so that the upper limit in the summation in (12) is $P = \frac{d}{2}$ for even d and $P = \frac{d-1}{2}$ for odd d.

We define the p-stress tensor pertaining to each term in (12) as

$$T_{\mu\nu}^{(p)} = \mathrm{Tr}\{F(2p)_{\mu\lambda_1\lambda_2\ldots\lambda_{2p-1}}F(2p)_{\nu}{}^{\lambda_1\lambda_2\ldots\lambda_{2p-1}} - \frac{1}{4p} g_{\mu\nu} \, F(2p)_{\lambda_1\lambda_2\ldots\lambda_{2p}}F(2p)^{\lambda_1\lambda_2\ldots\lambda_{2p}}\}. \tag{14}$$

We shall restrict in this work to solutions in dimensions less than nine, in which case it is sufficient to consider the first two terms in the YM hierarchy, i.e. a $F(2) + F(4)$ model (see [8, 10] for the equations of the general (P, d) model). As in the previous section, we restrict our attention to static spherically solutions given by the Ansätze (4) and (6), with exactly the same choices for the gauge group as in Section **2.1**.

The field equations of the $F(2) + F(4)$ model are

$$\begin{aligned} &\tau_1 \left(\left(r^{d-4}\sigma N w' \right)' - (d-3) r^{d-6} \sigma (w^2-1) w \right) + \\ &+ \frac{\tau_2}{6}(d-3)(d-4)(w^2-1) \left(\left(r^{d-8}\sigma N (w^2-1) w' \right)' - (d-5) r^{d-10} \sigma (w^2-1)^2 w \right) = 0, \end{aligned} \tag{15}$$

$$\begin{aligned} m' = {}& \frac{1}{2} r^{d-4} \Bigg(\tau_1 \left[N w'^2 + \frac{1}{2}(d-3) \left(\frac{w^2-1}{r} \right)^2 \right] \\ &+ \frac{1}{6} \frac{\tau_2}{r^2}(d-3)(d-4) \left(\frac{w^2-1}{r} \right)^2 \left[N w'^2 + \frac{1}{4}(d-5) \left(\frac{w^2-1}{r} \right)^2 \right] \Bigg), \end{aligned} \tag{16}$$

$$\kappa \left(\frac{\sigma'}{\sigma} \right) = \frac{1}{r} \left[\tau_1 + \frac{1}{6} \frac{\tau_2}{r^2}(d-3)(d-4) \left(\frac{w^2-1}{r} \right)^2 \right] w'^2 \, . \tag{17}$$

The corresponding expansion of the gauge potential and metric functions as $r \to 0$ is

$$\begin{aligned} w(r) &= 1 - b r^2 + O(r^4), \quad m(r) = \left(\tau_1 + \frac{\tau_2}{3}(d-3)(d-4) b^2 \right) b^2 r^{d-1} + O(r^{d+1}), \\ \sigma(r) &= \sigma_0 + \frac{2 b^2 \sigma_0}{\kappa} \left(\tau_1 + \frac{2\tau_2}{3}(d-3)(d-4) b^2 \right) r^2 + O(r^4), \end{aligned} \tag{18}$$

and contains one essential parameter b (the value of σ_0 can be fixed by rescaling the time coordinate).

Assuming the existence of a regular, nonextremal event horizon at $r = r_0$ (with $r_0 = r_h$ or $r_0 = r_c$), the approximate expression of the solution near the event horizon is

$$m(r) = (1 - \frac{r_0^2}{\ell^2})\frac{\kappa}{2}r_0^{d-3} + m'(r_0)(r - r_0) + O(r - r_0)^2, \tag{19}$$

$$\sigma(r) = \bar{\sigma}_0 + \sigma_0'(r - r_0) + O(r - r_0)^2, \quad w(r) = w_0 + w'(r_0)(r - r_0) + O(r - r_0)^2,$$

where

$$m'(r_0) = \frac{r_0^{d-6}}{2}(d-3)(w_0^2 - 1)^2\Big(\tau_1 + \tau_2(d-4)(d-5)\frac{(w_0^2-1)^2}{24r_0^4}\Big), \quad N_0' = \frac{d-3}{r_0} - \frac{(d-1)r_0}{\ell^2} - \frac{2m'(r_0)}{\kappa r_0^{d-3}},$$

$$\sigma_0' = \frac{\bar{\sigma}_0 w_0'^2}{\kappa r_0}\Big(\tau_1 + \tau_2(d-3)(d-4)\frac{(w_0^2-1)^2}{6r_0^4}\Big), \quad w_0' = \frac{1}{N_0'}\frac{w_0(w_0^2-1)}{r_0^2}(d-3)\Big(\frac{\tau_1 + \tau_2(d-4)(d-5)\frac{(w_0^2-1)^2}{r_0^4}}{\tau_1 + \tau_2(d-3)(d-4)\frac{(w_0^2-1)^2}{6r_0^4}}\Big),$$

with two free parameters, w_0, $\bar{\sigma}_0$. Also, since the field equations are invariant under $w \to -w$, one can take $w(0) = 1$ and $w(r_h) > 0$ without any loss of generality.

For $r \to \infty$ we find for both regular and black hole solutions

$$w(r) = \pm 1 + \frac{w_1}{r^{d-3}} + \dots, \quad m(r) = M - \frac{\tau_1(d-3)w_1^2}{8\ell^2}\frac{1}{r^{d-3}} + \dots, \quad \sigma(r) = \quad 1 - \frac{w_1^2(d-3)^2\tau_1}{2\kappa(d-2)}\frac{1}{r^{d-4}} + \dots.$$

These boundary conditions are also shared by the asymptotically flat solutions (with a different decay of the mass function $m(r)$, however), $w = \pm 1$ being again the only allowed values of the gauge function as $r \to \infty$. As a general feature, all solutions discussed in the rest of this section present only one node in the gauge function $w(r)$, i.e. $w(\infty) = -1$. As in the $\Lambda \le 0$ cases, we could not find multi-node solutions in the $F(2) + F(4)$ model.

3.2 Numerical solutions

In the presence of higher oder terms in the YM action, dimensionless quantities are obtained by rescaling

$$r \to (|\tau_2/\tau_1|)^{1/4}r, \quad \Lambda \to (|\tau_1/\tau_2|)^{1/2}\Lambda, \quad m(r) \to \kappa(|\tau_1/\tau_2|)^{(d-3)/4}m(r). \tag{20}$$

This reveals the existence of one fundamental parameter which gives the strength of the gravitational interaction $\alpha^2 = |\tau_1|^{3/2}/(\kappa|\tau_2|^{1/2})$. The solutions can then be constructed in terms of α^2 and Λ. Most of the numerical work was carried out for positive coupling constants in the YM hierarchy (12), having set $\tau_1 = \tau_2 = 1$ without loss of generality[1]. After presenting our main results however, we briefly consider at the end of this section, the case where τ_2 is taken to be negative.

Starting again with particle-like solutions in $d = 5$, we have solved equations (16) for several values of Λ, varying α. The pattern of these solutions is illustrated by Figure 6a, where the quantity $\sigma(0)$ is reported as a function of α^2 for three different values of Λ.

Here it is useful to recall the situation corresponding to $\Lambda = 0$ (see [5]). In this case, several branches of solutions exist, depending on the parameter α^2, as illustrated on Figure 6a. The first (or main) branch exists for $\alpha^2 \in [0, 0.2824]$. Then another branch of solutions (with larger masses than the corresponding one on the main branch) exists for $\alpha^2 \in [0.1749, 0.2824]$. Several other branches of solutions further exist on smaller intervals centered on the critical value $\alpha^2_{\rm cr} \sim 0.1749$. It is clear from Figure 6a that this oscillatory behaviour converging on $\alpha^2_{\rm cr}$ is a common feature of $\Lambda \ge 0$ solutions. (This is also the case with $\Lambda < 0$ solutions [10], which is not displayed on Figure 6a since its branch structure is nearly identical to that of $\Lambda = 0$.) This phenomenon, which occurs in appropriately similar models in all $4p + 1$ dimensions, was exhaustively analysed for the $\Lambda = 0$ case in [8], where this $\alpha^2_{\rm cr}$ was called a *conical* critical point. In the present paper, we limit ourselves to a qualitative discussion only.

For positive values of the cosmological constant, our numerical analysis in the case $0 < \Lambda < 0.001$ reveals that an extra branch of solutions exists, as shown on Figure 6a for $\Lambda = 0.0004$.

[1] The coupling constant τ_1 equals the inverse of the square of the gauge coupling constant and is strictly positive. Also, string theory predicts a positive value for the coefficient of the $F(4)$ term in the YM hierarchy (see [20]-[22]).

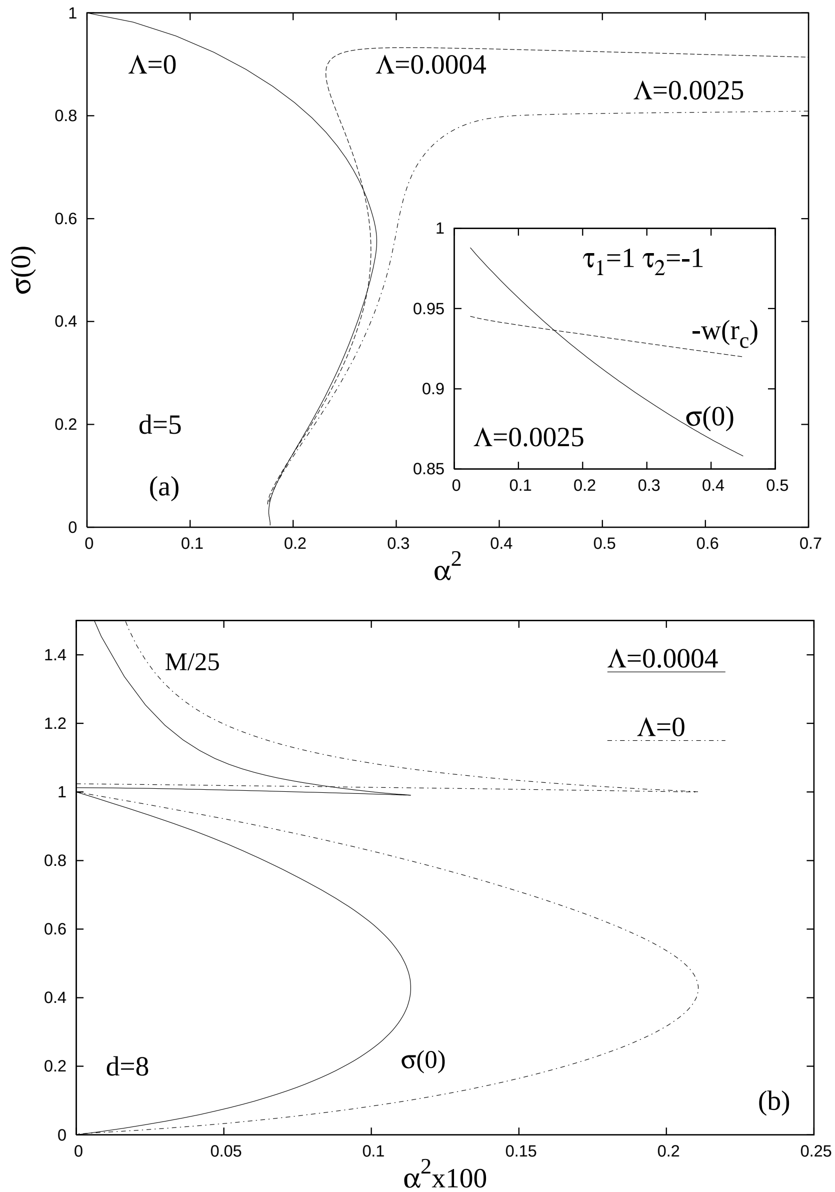

Figure 6. Several global quantities are shown as functions of α^2 and several values of Λ for $d = 5,\ 8$ solutions of $F(2) + F(4)$ theory.

Our numerical analysis suggests that this branch, which has no counterpart in the $\Lambda = 0$ case, exists for $\alpha > \alpha_0^2$, with $\alpha_0^2 \approx 0.21$ for $\Lambda = 0.0004$. This branch, appears to survive for arbitrarily large values of α (although the numerical accuracy deteriorates for α larger than one). Decreasing the cosmological constant, the critical value α_0^2 decreases and the pattern of the $\Lambda = 0$ case is approached. For larger values of Λ the pattern simplifies and a single branch persists, as illustrated for $\Lambda = 0.0025$ on Figure 6a, for $\alpha^2 > 0.1749$. Also, only a small variation of the asymptotic value M of the mass function was noticed when varying the parameter α.

Three remarks can be made to summarise this description : (i) For $d = 5$, $\Lambda > 0$ solutions do not exist for arbitrarily small values of α; (ii) Non trivial cosmological solutions seem to exist for large values of α^2, and (iii) The multiple branche phenomenon converging on the *conical* fixed point $\alpha^2 \sim 0.1749$, observed for

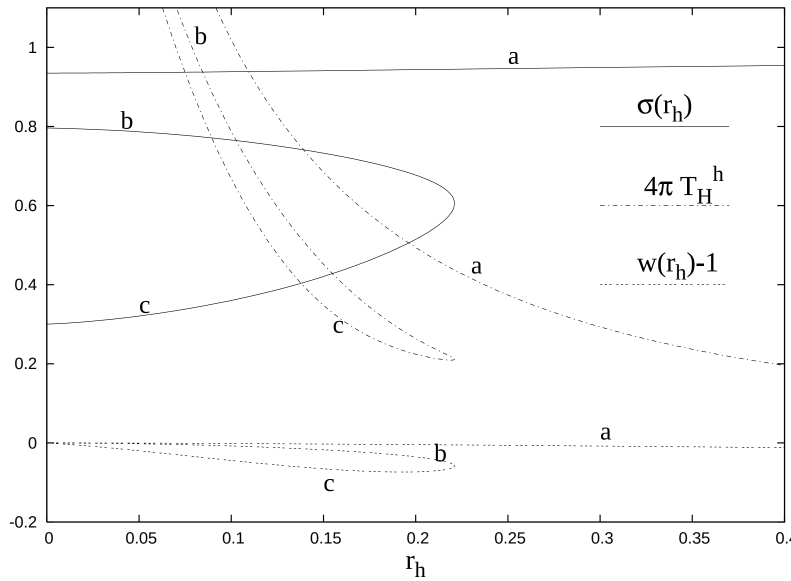

Figure 7. The event horizon Hawking temperature, the value of the gauge potential on the event horizon w_h as well as the value of the metric function σ at the event horizon, are shown as functions of the event horizon radius for $d = 5$ black hole solutions of $F(2) + F(4)$ theory with $\alpha^2 = 0.25$, $\Lambda = 0.0004$.

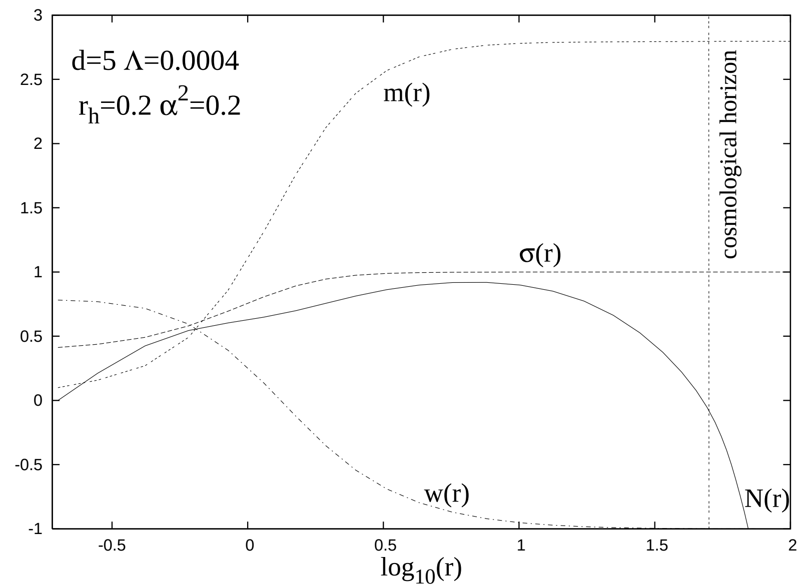

Figure 8. The functions $\sigma(r)$, $w(r)$, $N(r)$ and $m(r)$ are plotted as functions of radius for a typical $d = 5$ black hole solution in a $F(2) + F(4)$ EYM-dS theory.

$\Lambda = 0$, seems to persist for $\Lambda > 0$.

The $F(4)$ term allows also for the existence of $d = 6, 7, 8$ configurations with finite mass. The solutions available in the absence of a cosmological constant ($\Lambda = 0$) feature two branches which exist for $\alpha^2 \in [0, \alpha_m^2]$, where the maximal value α_m depends on d (see [4]). Integrating the equations for $\Lambda > 0$ reveals that the cosmological solutions also feature two branches leading to a pattern very similar to the $\Lambda = 0$ case. The maximal values α_m decrease with increasing cosmological constant. Also, with increasing α the masses of the gravitating solutions decrease. Along the second branch the value of $\sigma(0)$ decreases monotonically with α. The mass of a second branch solution is always larger than the corresponding mass (for the same value of α) on the first branch. Some relevant quantities for $d = 8$ solutions are plotted in Figure 6b.

So far we discussed solutions which are regular at the origin. However the equations also admit black hole solutions with a regular event horizon occuring at r_h with $r_h < r_c$. The study of the domain of black hole solutions in the space of the parameters (α, Λ, r_h) is a considerable task which is beyond the scope of this paper. We present however a few features of these solutions which reflect the general pattern, limiting our investigation to the $d = 5$ case.

As we have seen in the previous section, several branches of regular solutions exist, according to the value of Λ. When imposing a regular horizon at $r = r_h$, our numerical analysis indicates that the regular solutions are deformed into black hole solutions. We have analysed in detail the evolution of the black holes solutions in the case $\alpha^2 = 0.25, \Lambda = 0.0004$ (corresponding to $r_c \simeq 50.0$). In this case there are three different regular solutions (see Figure 6) distinguished namely by the value of the metric function σ at the origin (for instance $\sigma(0) \approx 0.93, 0.79, 0.30$, let us call them a, b and c respectively). The evolution of these solutions into black holes is summarized on Figure 7. We see clearly that the solutions b and c are deformed into black holes up to a rather small value of r_h, namely for $r_h < 0.227$. In the limit $r_h \to 0.227$, the two branches merge into a single solution. A typical profile of a black hole solution corresponding to the solution c is presented on Figure 8.

The scenario is completely different for the solution a. Indeed, it seems that it can be deformed into a black hole with large event horizon. When the value r_h increases, we observe that the corresponding function $w(r)$ has a tendency to spead over the interval $r \in [r_h, r_c]$ and that the combination of $w(r), w'(r)$ appearing in the right hand side of the equation for $m'(r)$ becomes uniformly very small. As a consequence, the equation for $m(r)$ can be simplified and leads to the following approximate solution

$$N(r) \simeq \frac{1}{r^2}\left(r^2 - r_h^2 - \frac{r^2 + r_h^2}{r_c^2 + r_h^2}(r^2 - r_h^2)\right) \quad , \quad \sigma(r) \simeq 1 \quad , \quad w(r) \simeq \pm 1 + \frac{w_1}{r^2} \ , \tag{21}$$

for the region between the event and cosmological horizons. The metric functions $N(r)$, $\sigma(r)$ above turn out to be a very good approximations of the numerical solution a obtained for $r_h > 1$ (although $w(r)$ is non constant). Due to the presence of this solution, the numerical integration of the equation with the non trivial $w(r)$ becomes increasingly difficult while increasing r_h.

The Hawking temperature associated with the cosmological horizon is almost constant on the branch (a), and stronly decreases with r_h for the other two branches. As a generic feature, the Hawking temperatures of the event and cosmological horizons are different (this holds also for the black hole solutions of the $F(2)$-theory). Therefore, the energy flows from the hotter horizon to the cooler one and the black hole will gain or lose mass.

Before concluding this section, we allude to the unusual situation where the coupling constant τ_2 associated with the $F(4)$ term is taken to be negative [2]. We have managed to construct such solutions, which satisfy the same set of boundary conditions as the usual ones above with $\tau_2 > 0$. Physically, this would imply a negative contribution to the mass-energy density of the solutions, but in practice this possibility does not seem to obtain and the solutions we constructed lead to finite masses.

Since we do not have any existence proof for the type of EYM solutions found in this section, we are unable to exclude the possibility of solutions with $\tau_2 < 0$. For this reason we have extended our numerical analysis to $\tau_2 < 0$ models with $\Lambda \leq 0$ as well as the $\Lambda > 0$ model at hand. Our numerical results indicate the absence of such configurations approaching asymptotically the AdS or flat background in all dimensions between five and eight. Surprisingly enough, we found finite mass solutions with $\tau_2 < 0$ in the dS case studied here. The results for $d = 5$ indicate the existence of a branch of solutions emerging at $\alpha = 0$ from a configuration in fixed dS background and extending in α up to a maximal value α_{max}. This behaviour

[2] Ascribing a negative value to the square of the gauge coupling constant has been considered in the case of gravitating electromagnetism in [29], where explicit solutions are found. We found however that in the corresponding situation when the Abelian field in [29] is replaced by a non-Abelian $SU(2)$ gauge field, no numerical solutions with right asymptotics appear to exist.

strongly constrasts with what we have found for $\tau_2 > 0$. However, the profile of a typical $F(2) + F(4)$ solution does not depend on the sign of τ_2. As $\alpha \to \alpha_{max}$ the numerics deteriorates and the solver fails to converge (although all metric and matter functions stay finite there), a different approach being necessary. Along this branch, the value at the origin of the metric function σ decreases and also the mass-parameter M (note that all solutions we found with $\tau_2 < 0$ have $M > 0$, with a small variation when increasing α, however). The corresponding picture for $\tau = 1$, $\tau_2 = -1$ and $\Lambda = 0.0075$ is plotted in the inlet of Figure 6a.

4 A computation of the mass in the boundary counterterm method

In evaluating expressions like the mass and action, one usually encounters divergencies coming from integration over the infinite volume of spacetime. In the case of AdS gravity, a regularization procedure was proposed in [23], that consists in adding to (1) counterterms constructed from local curvature invariants of the boundary. These counterterms, which are esentially unique, can be easily generalized to the case of a positive Λ [24]. The following counterterms are sufficient to cancel divergences in a pure dS gravity theory for $d \leq 7$

$$I_{\rm ct} = -\frac{1}{8\pi G}\int_{\partial\mathcal{M}^{\pm}} d^{d-1}x\sqrt{-h}\left[-\frac{d-2}{\ell} + \frac{\ell\Theta(d-4)}{2(d-3)}\mathcal{R} - \frac{\ell^3\Theta(d-6)}{2(d-5)(d-3)^2}\Big(\mathcal{R}_{AB}\mathcal{R}^{AB} - \frac{d-1}{4(d-2)}\mathcal{R}^2\Big)\right]. \quad (22)$$

Here $\mathcal{R}_{AB}$, $\mathcal{R}$ are the Ricci tensor and the Ricci scalar for the boundary metric and $\Theta(x)$ is the step function, which is equal to 1 for $x \geq 0$ and zero otherwise; $A, B, \ldots$ indicate the intrinsic coordinates of the boundary. $\int_{\partial\mathcal{M}^{\pm}}$ indicates the sum of the integral over the early and late time boundaries. In what follows, to simplify the picture, we will consider the $\mathcal{I}^+$ boundary only, dropping the $^{\pm}$ indices (similar results hold for $\mathcal{I}^-$).

Using $I = I_{bulk} + I_{surf} + I_{ct}$, one can construct a boundary stress tensor, which is given by the variation of the total action at the boundary with respect to h_{AB} (its explicit expression is given in Ref. [24]).

The next step is to write the boundary metric in a ADM-like general form

$$ds^2 = h_{AB}dx^A dx^B = N_t^2 dt^2 + \sigma_{ab}\left(d\psi^a + N^a dt\right)\left(d\psi^b + N^b dt\right), \quad (23)$$

where N_t and N^a are the lapse function and the shift vector respectively and the ψ^a are the intrinsic coordinates on the closed surfaces Σ (a $d-2$ dimensional sphere in our case). In this approach, the conserved quantity associated with a Killing vector ξ^i on the $\mathcal{I}^+$ boundary is given by

$$\mathfrak{Q}_\xi = \oint_\Sigma d^{d-2}\psi\sqrt{\sigma}n^A T_{AB}\xi^B, \quad (24)$$

where n^A is an outward-pointing unit vector, normal to surfaces of constant r. Physically, this means that a collection of observers, on the hypersurface with the induced metric h_{AB}, would all measure the same value of $\mathfrak{Q}_\xi$ provided this surface has an isometry generated by ξ^i. If $\partial/\partial t$ is a Killing vector on Σ, then the conserved mass is defined to be the conserved quantity $\mathfrak{M}$ associated with it.

We have applied this approach to compute at the far future boundary (outside the cosmological horizon) the mass of the solutions of the $F(2)+F(4)$ model. The crucial point here is that these solutions approaches asymptotically a Schwarzschild-dS background, the YM manifesting only in the next to leading order of the T_{AB} expression. As a result, one finds

$$\mathfrak{M} = -\frac{(d-2)\Omega_{d-2}}{8\pi G}M + E_0(d), \quad (25)$$

where M is asymptotic value of the mass function $m(r)$ and $\Omega_{d-2} = 2\pi^{(d-1)/2}/\Gamma((d-1)/2)$ is the area of a unit $(d-2)$-dimensional sphere. The additional term $E_0(d)$ appearing in (25) for $d = 5, (7)$ is the mass of pure global $dS_{5,(7)}$ spacetime and is usually interpreted as the energy dual to the Casimir energy of the CFT defined on a four (six) dimensional Euclidean Einstein universe [24] (i.e. $E_0 = 3\Omega_3\ell^2/(64\pi G)$ for $d = 5$

and $E_0 = 5\Omega_5 \ell^4/(128\pi G)$ in the seven dimensional case). One should also remark that all solutions of the $F(2)+F(4)$ model we found have $M > 0$ (both black holes and particle like solutions). Thus $\mathfrak{M} - E_0(d)$ is negative, consistent with the expectation [25] that pure dS spacetime has the largest mass for a singularity-free spacetime.

However, one can easily see that this approach fails to assign a finite mass to the solution of the EYM-$F(2)$ model, despite their asymptotically dS behaviour. (Note that this prescription regularizes the mass and action of the embedded abelian solutions [26]). Asymptotically AdS solutions with a diverging ADM mass have been considered by many authors, mainly for a scalar field in the bulk (see e.g. [27]). In this case it might be possible to relax the standard asymptotic conditions without loosing the original symmetries, but modifying the charges in order to take into account the presence of matter fields. For $\Lambda < 0$, the Ref. [10] suggested that it is still possible to obtain a finite mass of EYM solutions in a $F(2)$ theory by allowing I_{ct} to depend not only on the boundary metric h_{AB}, but also on the gauge field strength tensor. This means that the quasilocal stress-energy tensor also acquires a contribution coming from the matter fields.

A similar approach holds also for dS solutions in $d > 4$ dimensions and we find that by adding to the expression (22) a supplementary matter counterterm of the form

$$I_{\rm ct}^{(m)} = -\frac{\tau_1}{4}\int_{\partial M} d^4x\sqrt{h}\log(\frac{r}{\ell})\ {\rm tr}\{F_{AB}F^{AB}\}, \qquad (26)$$

for $d = 5$ and

$$I_{ct}^{(m)} = -\frac{\tau_1}{(d-5)}\int_{\partial M} d^{d-1}x\sqrt{h}\ {\rm tr}\{F_{AB}F^{AB}\}, \qquad (27)$$

for $d > 5$, the mass divergence disappears. This yields a supplementary contribution to the boundary stress-tensor

$$T_{AB}^{(m)} = -\frac{\tau_1 \log(r/\ell)}{32\pi G}h_{AB}{\rm tr}\{F_{CD}F^{CD}\},\ \ \text{if } d=5,\ \text{and}\ \ T_{AB}^{(m)} = -\frac{1}{8\pi G}\frac{1}{d-5}h_{AB}\ {\rm tr}\{F_{CD}F^{CD}\},\ \ \text{for } d>5.$$

The mass of the $d > 5$ solutions computed in this way is finite

$$\mathfrak{M} = -\frac{(d-2)\Omega_{d-2}}{8\pi G}M_0 + E_0(d), \qquad (28)$$

where M_0 is the constant appearing in the asymptotic expansion (11).

5 Conclusions

This work was primarily motivated by the question of how a positive cosmological constant will affect the properties of the gravitating nonabelian field solutions in a higher dimensional spacetime. To the best of our knowledge, this question has not yet been addressed in the literature. Apart from this motivation, the study of gravitating matter field configurations in asymptotically dS space may help a better understanding of the conjectured dS/CFT correspondence as well as act as a probe for the brane-world scenario.

Our findings have completed our qualitative understanding of gravitating nonabelian solutions in higher ($d \geq 5$) dimensions, encompassing all possible values of the cosmological constant $\Lambda < 0$, $\Lambda = 0$ and $\Lambda > 0$, the last being the object of the present investigation. Certain features of these solutions are shared, while others differ. At the most basic level, we have seen to varying degrees of rigour, that spherically symmetric solutions to the usual ($p = 1$), $F(2)$ YM model in all these three cases have infinite mass in higher ($d \geq 5$) dimensions. By contrast, finite masses are obtained when the YM sector of the model is augmented by the appropriate higher order ($p \geq 2$), $F(2p)$ members of the YM hierarchy in all three cases. (We have also considered the alternative option of using the counterterm method, avoiding the use of $p \geq 2$, $F(2p)$ terms.) It can therefore be stated that, to construct finite mass solutions of gravitating nonabelian matter, the YM sector of the theory must be an appropriate superposition of members of the YM hierarchy, beyond the usual

YM term. This statement can be qualified in the present context, namely when $\Lambda > 0$, by adding that even when the total mass diverges the mass *inside* the cosmological horizon is finite. In addition to this salient feature of higher dimensional EYM solutions, our results reveal detailed qualitative properties occurring in all three cases.

In the context of the $p = 1$, $F(2)$ YM model with $\Lambda > 0$ in $d \geq 5$ dimensional spacetime, the most remarkable property is that the asymptotics of the solutions lead to monopole-like configurations with nonvanishing magnetic flux. Indeed in those cases the solutions *never* have instanton like asymptotics. This circumstance makes it very easy to conclude that the total mass of these solutions is divergent.

In the context of the $F(2) + F(4)$ model studied in this paper, which is the simplest case of models with superposed $F(2p)$ terms, our results have led to an overview of the main qualitative features common to EYM solutions in all three cases ($\Lambda < 0,\ \Lambda = 0,\ \Lambda > 0$), apart from their masses being finite. These confirm and expand on our knowledge of the branch patterns of these solutions in various dimensions, learnt from the results of [4, 5] for $\Lambda = 0$, and [10] for $\Lambda < 0$. The results of $\Lambda = 0$ solutions were analytically analysed in [8] leading to a patterns that repeat modulo $d = 4p + 1$ dimensions. In particular there arise two types of patterns, those in dimensions $d = 4p + 1$ and the rest in $4p + 2 \leq d \leq 4p + 4$. In the restricted context here, these are the dimensions $d = 5$ on the one hand, and $6 \leq d \leq 8$ on the other. We have learnt here that these patterns arise also for the $\Lambda > 0$ case. In particular for the second case, solutions in $d = 8$ conform to the pattern displayed on Figure 6b in both the other cases. Much more interestingly the situation in $d = 5$, which is qualitatively the same for $\Lambda < 0$ and $\Lambda = 0$ models, strongly departs for $\Lambda > 0$, from the patterns of the former. These features are displayed on Figure 6a, and described in Section 3.2.

What has not been studied quantitatively here, and in the $\Lambda = 0$ [4, 5, 8] and $\Lambda < 0$ [10] cases is the question of the stability of the solutions. We expect that in all even spacetime dimensions, as well as all odd dimensions $d = 4p + 1$, the solutions will be sphalerons like the four dimensional Bartnik-McKinnon solutions [6]. This is because in all these cases there is no topologically stable soliton in the gravity decoupling limit. In all other odd spacetime dimensions however, a stable soliton will survaive in the gravity decoupling limit, stabilised by the $\frac{d-1}{2}$-th Chern-Pontryagin (CP) charge, rather like the monopole charge in the case of the gravitating monopole [30], stabilised by the monopole charge (that descends from the 2-nd CP charge).

As an additional remark, we allude to the case when the coupling strength of the $F(4)$ term, τ_2, takes on a negative value. In the absence of existence proofs for solutions to models with higher order YM terms, one cannot *a priori* exclude this possibility, especially in view of the discovery of such explicit solutions in the usual Einstein–Maxwell theory [29]. We have tried to construct such solutions numerically and failed for models with $\Lambda \leq 0$, but surprisingly find them for the $\Lambda > 0$ case at hand.

While the present work is concerned with higher dimensional EYM solutions with positive cosmological constant, we have at every stage compared our results to the corresponding ones pertaining to the asymptotically flat [4, 5] and the asymptotically AdS [10] counterparts. The general characteristics of both $\Lambda \leq 0$ solutions, at least of the important finite mass ones, are quantitatively similar, while the corresponding characteristics of $\Lambda > 0$ case differ from the former strikingly, in a systematic way. The numerically discovered features for $\Lambda = 0$ solutions are explained analytically in [8], which can be adapted systematically to the $\Lambda < 0$ case which is qualitatively similar. But an analytic study like [8] for the $\Lambda > 0$ case, using the methods of [28] is outstanding and is desirable to complete the overall comparative study of all three types ($\Lambda > 0, \Lambda = 0$ and $\Lambda < 0$) of higher dimensional EYM solutions.

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References

[1] F. R. Tangherlini, Nuovo Cim. **27** (1963) 636.

[2] R. C. Myers and M. J. Perry, Annals Phys. **172** (1986) 304.

[3] M. S. Volkov, Phys. Lett. B **524** (2002) 369 [arXiv:hep-th/0103038].

[4] Y. Brihaye, A. Chakrabarti and D. H. Tchrakian, Class. Quant. Grav. **20** (2003) 2765 [arXiv:hep-th/0202141].

[5] Y. Brihaye, A. Chakrabarti, B. Hartmann and D. H. Tchrakian, Phys. Lett. B **561** (2003) 161 [arXiv:hep-th/0212288].

[6] R. Bartnik and J. McKinnon, Phys. Rev. Lett. **61** (1988) 141.

[7] P. Breitenlohner, P. Forgacs and D. Maison, Nucl. Phys. **B 383** (1992) 357; *ibid.* **442** (1995) 126

[8] P. Breitenlohner, D. Maison and D. H. Tchrakian, Class. Quant. Grav. **22** (2005) 5201 [arXiv:gr-qc/0508027].

[9] N. Okuyama and K. i. Maeda, Phys. Rev. D **67** (2003) 104012 [arXiv:gr-qc/0212022].

[10] E. Radu and D. H. Tchrakian, Phys. Rev. D **73** (2006) 024006 [arXiv:gr-qc/0508033].

[11] E. Winstanley, Class. Quant. Grav. **16** (1999) 1963 [arXiv:gr-qc/9812064].

[12] J. Bjoraker and Y. Hosotani, Phys. Rev. D **62** (2000) 043513 [arXiv:hep-th/0002098].

[13] M. S. Volkov, N. Straumann, G. V. Lavrelashvili, M. Heusler and O. Brodbeck, Phys. Rev. D **54** (1996) 7243 [arXiv:hep-th/9605089].

[14] A. N. Linden, Commun. Math. Phys. **221** (2001) 525 [arXiv:gr-qc/0005004].

[15] Y. Brihaye, B. Hartmann, E. Radu and C. Stelea, arXiv:gr-qc/0607078.

[16] T. Torii, K. i. Maeda and T. Tachizawa, Phys. Rev. D **52** (1995) 4272 [arXiv:gr-qc/9506018].

[17] P. Breitenlohner, P. Forgacs and D. Maison, Commun. Math. Phys. **261** (2006) 569 [arXiv:gr-qc/0412067].

[18] G. W. Gibbons and P. K. Townsend, Class. Quant. Grav. **23** (2006) 4873 [arXiv:hep-th/0604024].

[19] U. Ascher, J. Christiansen, R. D. Russell, Math. of Comp. **33** (1979) 659;
U. Ascher, J. Christiansen, R. D. Russell, ACM Trans. **7** (1981) 209.

[20] A.A. Tseytlin, *Born–Infeld action, supersymmetry and string theory*, in *Yuri Golfand memorial volume*, ed. M. Shifman, World Scientific (2000).

[21] E. Bergshoeff, M. de Roo and A. Sevrin, Fortsch.Phys. 49 (2001) 433-440; Nucl.Phys.Proc.Suppl. 102 (2001) 50-55. E. A. Bergshoeff, M. de Roo and A. Sevrin, Fortsch. Phys. **49** (2001) 433 [Nucl. Phys. Proc. Suppl. **102** (2001) 50] [arXiv:hep-th/0011264].

[22] M. Cederwall, B. E. W. Nilsson and D. Tsimpis, JHEP **0106** (2001) 034 [arXiv:hep-th/0102009].

[23] V. Balasubramanian and P. Kraus, Commun. Math. Phys. **208** (1999) 413 [arXiv:hep-th/9902121].

[24] A. M. Ghezelbash and R. B. Mann, JHEP **0201** (2002) 005 [arXiv:hep-th/0111217].

[25] V. Balasubramanian, J. de Boer and D. Minic, Phys. Rev. D **65** (2002) 123508 [arXiv:hep-th/0110108].

[26] D. Astefanesei, R. Mann and E. Radu, JHEP **0401** (2004) 029 [arXiv:hep-th/0310273].

[27] S. Hollands, A. Ishibashi and D. Marolf, Class. Quant. Grav. **22** (2005) 2881 [arXiv:hep-th/0503045]; T. Hertog and K. Maeda, JHEP **0407** (2004) 051 [arXiv:hep-th/0404261]; T. Hertog and K. Maeda, Phys. Rev. D **71** (2005) 024001 [arXiv:hep-th/0409314]; M. Henneaux, C. Martinez, R. Troncoso and J. Zanelli, Phys. Rev. D **70** (2004) 044034 [arXiv:hep-th/0404236]; E. Radu and D. H. Tchrakian, Class. Quant. Grav. **22** (2005) 879 [arXiv:hep-th/0410154]; J. T. Liu and W. A. Sabra, Phys. Rev. D **72** (2005) 064021 [arXiv:hep-th/0405171].

[28] P. Breitenlohner, P. Forgacs and D. Maison, Commun. Math. Phys. **163** (1994) 141.

[29] G. W. Gibbons and D. A. Rasheed, Nucl. Phys. B **476** (1996) 515 [arXiv:hep-th/9604177].

[30] P. Breitenlohner, P. Forgacs and D. Maison, Nucl. Phys. B **442** (1995) 126 [arXiv:gr-qc/9412039].