Exact solution of simple models for quantum information

## Abstract:

The quantum Rabi model, describing the interaction between light and matter in the simplest possible way, was originally introduced as the basis to understand nuclear magnetic resonance and has since been applied to physical systems ranging from quantum optics to condensed matter, e.g. cavity and circuit quantum electrodynamics, quantum dots, trapped ions, and superconducting qubits. The quantum Rabi model is, hence, of considerable importance in quantum information theory.

In the language of quantum information theory the Rabi model describes a qubit interacting with a single bosonic degree of freedom. After a further approximation, the rotating wave approximation which is often justified in quantum optics, the model becomes exactly soluble while the full Rabi model, until recently, was believed to have no exact solution.

In these lectures, we are concerned with the exact solution of the Rabi model with and without the rotating wave approximation. In order to achieve an understanding of the exact solutions an approach from two convergent directions will be taken.

First we exploit the intimate connection between two-dimensional Ising-like lattice models of classical statistical mechanics, socalled vertex models, and quantum models in one dimension. We shall start from the six-vertex model which leads to the anisotropic Heisenberg (or $X X Z$ ) quantum spin chain. Along the way, we shall be able to understand the quantum integrability of the model and derive, in an algebraic manner, the integrability conditions: the Bethe ansatz equations which determine the energy spectrum of the spin chain.

In the next step we shall show how the Rabi model with rotating wave approximation, the Jaynes-Cummings model and, especially, its N qubit generalization, the Tavis-Cummings model, fit into the framework of the algebraic Bethe ansatz.

Lastly, we shall demonstrate that also the full Rabi model, i.e. without the rotating wave approximation, is amenable to an exact solution, albeit not by employing the Bethe ansatz.

