Lecture 2: Quantum teleportation and super-dense coding

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This lecture begins our first exciting application of the postulates of the quantum theory to quantum communication. We study the fundamental, unit quantum communication protocols. These protocols involve a single sender, whom we name Alice, and a single receiver, whom we name Bob. The protocols are ideal and noiseless because we assume that Alice and Bob can exploit perfect classical communication, perfect quantum communication, and perfect entanglement. At the end of this lecture, we suggest how to incorporate imperfections into these protocols for later study.

Alice and Bob may wish to perform one of several quantum information processing tasks, such as the transmission of classical information, quantum information, or entanglement. Several fundamental protocols make use of these resources:

1. We will see that noiseless entanglement is an important resource in quantum Shannon theory because it enables Alice and Bob to perform other protocols that are not possible with classical resources only. We will present a simple, idealized protocol for generating entanglement, named *entanglement distribution*.

2. Alice may wish to communicate classical information to Bob. A trivial method, named *elementary coding*, is a simple way for doing so and we discuss it briefly.

3. A more interesting technique for transmitting classical information is *super-dense coding*. It exploits a noiseless qubit channel and shared entanglement to transmit more classical information than would be possible with a noiseless qubit channel alone.

4. Finally, Alice may wish to transmit quantum information to Bob. A trivial method for Alice to transmit quantum information is for her to exploit a noiseless qubit channel. Though, it is useful to have other ways for transmitting quantum information because such a resource is difficult to engineer in practice. An alternative, surprising method for transmitting quantum information is *quantum teleportation*. The teleportation protocol exploits classical communication and shared entanglement to transmit quantum information.

Each of these protocols is a fundamental unit protocol and provides a foundation for asking further questions in quantum Shannon theory. In fact, the discovery of these latter two protocols was the stimulus for much of the original research in quantum Shannon theory.

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We introduce the technique of *resource counting* in this lecture. This technique is of practical importance because it quantifies the communication cost of achieving a certain task. We include only nonlocal resources in a resource count—nonlocal resources include classical or quantum communication or shared entanglement.

1 Nonlocal Unit Resources

We first briefly define what we mean by a noiseless qubit channel, a noiseless classical bit channel, and noiseless entanglement. Each of these resources is a *nonlocal, unit resource.* A resource is *nonlocal* if two spatially separated parties share it or if one party uses it to communicate to another. We say that a resource is *unit* if it comes in some "gold standard" form, such as qubits, classical bits, or entangled bits. It is important to establish these definitions so that we can check whether a given protocol is truly simulating one of these resources.

A noiseless qubit channel is any mechanism that implements the following map:

$$|i\rangle^A \to |i\rangle^B, \tag{1}$$

where $i \in \{0,1\}$, $\{|0\rangle^A, |1\rangle^A\}$ is some preferred orthonormal basis on Alice's system, and $\{|0\rangle^B, |1\rangle^B\}$ is some preferred orthonormal basis on Bob's system. The bases do not have to be the same, but it must be clear which basis each party is using. The above map is linear so that it preserves arbitrary superposition states (it preserves any qubit). For example, the map acts as follows on a superposition state:

$$\alpha|0\rangle^A + \beta|1\rangle^A \to \alpha|0\rangle^B + \beta|1\rangle^B. \tag{2}$$

We can also write it as the following isometry:

$$\sum_{i=0}^{1} |i\rangle^B \langle i|^A. \tag{3}$$

Any information processing protocol that implements the above map simulates a noiseless qubit channel. We label the communication resource of a noiseless qubit channel as follows:

$$[q \to q], \tag{4}$$

where the notation indicates one forward use of a noiseless qubit channel.

A noiseless classical bit channel is any mechanism that simply measures the input state in the computational basis and forwards the result to a receiver. There is a more formal way to describe this resource in the language of density operators, but we will just use the above informal description for now.

This resource is weaker than a noiseless qubit channel because it does not require Alice and Bob to maintain arbitrary superposition states—it merely transfers classical information. Alice can of course use the above channel to transmit classical information to Bob. She can prepare either of the classical states $|0\rangle\langle 0|$ or $|1\rangle\langle 1|$, send it through the classical channel, and Bob performs a computational basis measurement to determine the message Alice transmits. We denote the communication resource of a noiseless classical bit channel as follows:

$$[c \to c], \tag{5}$$

where the notation indicates one forward use of a noiseless classical bit channel.

It is impossible for a noiseless classical channel to simulate a noiseless qubit channel because it cannot maintain arbitrary superposition states. Though, it is possible for a noiseless qubit channel to simulate a noiseless classical bit channel and we denote this fact with the following *resource inequality*:

$$[q \to q] \geq [c \to c]. \tag{6}$$

Noiseless quantum communication is therefore a stronger resource than noiseless classical communication.

The final resource that we consider is shared entanglement. The ebit is our "gold standard" resource for pure bipartite (two-party) entanglement. An ebit is the following state of two qubits:

$$\left|\Phi^+\right\rangle^{AB} \equiv \frac{|00\rangle^{AB} + |11\rangle^{AB}}{\sqrt{2}}, \tag{7}$$

where Alice possesses the first qubit and Bob possesses the second.

Below, we show how a noiseless qubit channel can generate a noiseless ebit through a simple protocol named *entanglement distribution*. Though, an ebit cannot simulate a noiseless qubit channel (for reasons which we explain later). Therefore, noiseless quantum communication is the strongest of all three resources, and entanglement and classical communication are in some sense "orthogonal" to one another because neither can simulate the other.

2 Protocols

2.1 Entanglement Distribution

The entanglement distribution protocol is the most basic of the three unit protocols. It exploits one use of a noiseless qubit channel to establish one shared noiseless ebit. It consists of the following two steps:

1. Alice prepares a Bell state locally in her laboratory. She prepares two qubits in the state $|0\rangle^A|0\rangle^{A'}$, where we label the first qubit as A and the second qubit as A'. She performs a Hadamard gate on qubit A to produce the following state:

$$\left(\frac{|0\rangle^A + |1\rangle^A}{\sqrt{2}}\right)|0\rangle^{A'}. \tag{8}$$

   She then performs a CNOT gate with qubit A as the source qubit and qubit A' as the target qubit. The state becomes the following Bell state:

$$\left|\Phi^+\right\rangle^{AA'} = \frac{|00\rangle^{AA'} + |11\rangle^{AA'}}{\sqrt{2}}. \tag{9}$$

2. She sends qubit A' to Bob with one use of a noiseless qubit channel. Alice and Bob then share the ebit $|\Phi^+\rangle^{AB}$.

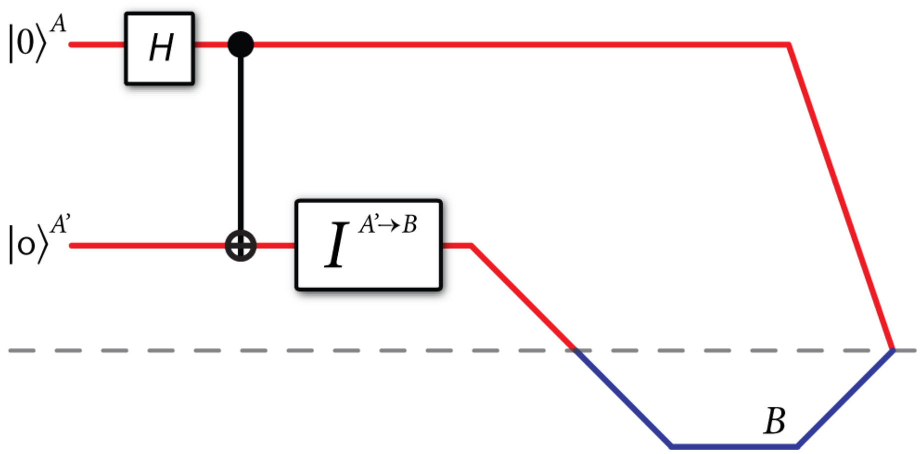

Figure 1: The above figure depicts a protocol for entanglement distribution. Alice performs local operations (the Hadamard and CNOT) and consumes one use of a noiseless qubit channel to generate one noiseless ebit $|\Phi^+\rangle^{AB}$ shared with Bob.

Figure 1 depicts the entanglement distribution protocol.

The following resource inequality quantifies the nonlocal resources consumed or generated in the above protocol:

$$[q \to q] \geq [qq], \tag{10}$$

where $[q \to q]$ denotes one forward use of a noiseless qubit channel and $[qq]$ denotes a shared, noiseless ebit. The meaning of the resource inequality is that there exists a protocol that consumes the resource on the left in order to generate the resource on the right. The best analogy is to think of a resource inequality as a "chemical reaction"-like formula, where the protocol is like a chemical reaction that transforms one resource into another.

There are several subtleties to notice about the above protocol and its corresponding resource inequality:

1. We are careful with the language when describing the resource state. We described the state $|\Phi^+\rangle$ as a Bell state in the first step because it is a local state in Alice's laboratory. We only used the term "ebit" to describe the state after the second step, when the state becomes a nonlocal resource shared between Alice and Bob.

2. The resource count involves nonlocal resources only—we do not factor any local operations, such as the Hadamard gate or the CNOT gate, into the resource count. This line of thinking is different from the theory of computation, where it is of utmost importance to minimize the number of steps involved in a computation. In this book, we are developing a theory of quantum communication and we thus count nonlocal resources only.

3. We are assuming that it is possible to perform all local operations perfectly. This line of thinking is another departure from practical concerns that one might have in fault tolerant quantum computation, the study of the propagation of errors in quantum operations. Performing a CNOT gate is a highly nontrivial task at the current stage of experimental

development in quantum computation, with most implementations being far from perfect. Nevertheless, we proceed forward with this communication-theoretic line of thinking.

The following exercises outline classical information processing tasks that are analogous to the task of entanglement distribution.

Exercise 1 *Outline a protocol for common randomness distribution. Suppose that Alice and Bob have available one use of a noiseless classical bit channel. Give a method for them to implement the following resource inequality:*

$$[c \to c] \geq [cc], \tag{11}$$

where $[c \to c]$ *denotes one forward use of a noiseless classical bit channel and* $[cc]$ *denotes a shared, nonlocal bit of common randomness.*

Exercise 2 *Consider three parties Alice, Bob, and Eve and suppose that a noiseless private channel connects Alice to Bob. Privacy here implies that Eve does not learn anything about the information that traverses the private channel—Eve's probability distribution is independent of Alice and Bob's:*

$$p_{A,B,E}(a,b,e) = p_A(a)p_{B|A}(b|a)p_E(e). \tag{12}$$

For a noiseless private bit channel, $p_{B|A}(b|a) = \delta_{b,a}$*. A noiseless secret key corresponds to the following distribution:*

$$p_{A,B,E}(a,b,e) = \frac{1}{2}\delta_{b,a}p_E(e), \tag{13}$$

where $\frac{1}{2}$ *implies that the key is equal to '0' or '1' with equal probability,* $\delta_{b,a}$ *implies a perfectly correlated secret key, and the factoring of the distribution* $p_{A,B,E}(a,b,e)$ *implies the secrecy of the key (Eve's information is independent of Alice and Bob's). The difference between a noiseless private bit channel and a noiseless secret key is that the private channel is a dynamic resource while the secret key is a shared, static resource. Show that it is possible to upgrade the protocol for common randomness distribution to a protocol for secret key distribution, if Alice and Bob share a noiseless private bit channel. That is, show that they can achieve the following resource inequality:*

$$[c \to c]_{priv} \geq [cc]_{priv}, \tag{14}$$

where $[c \to c]_{priv}$ *denotes one forward use of a noiseless private bit channel and* $[cc]_{priv}$ *denotes one bit of shared, noiseless secret key.*

2.1.1 Entanglement and Quantum Communication

Can entanglement enable two parties to communicate quantum information? It is natural to wonder if there is a protocol corresponding to the following resource inequality:

$$[qq] \geq [q \to q]. \tag{15}$$

Unfortunately, it is physically impossible to construct a protocol that implements the above resource inequality. The argument against such a protocol arises from the theory of relativity. Specifically, the theory of relativity prohibits information transfer or signaling at a speed greater than the speed of light. Suppose that two parties share noiseless entanglement over a large distance.

That resource is a static resource, possessing only shared quantum correlations. If a protocol were to exist that implements the above resource inequality, it would imply that two parties could communicate quantum information faster than the speed of light, because they would be exploiting the entanglement for the instantaneous transfer of quantum information.

The entanglement distribution resource inequality is only "one-way," as in (10). Quantum communication is therefore strictly stronger than shared entanglement when no other nonlocal resources are available.

2.2 Elementary Coding

We can also send classical information with a noiseless qubit channel. A simple protocol for doing so is *elementary coding.* This protocol consists of the following steps:

1. Alice prepares either $|0\rangle$ or $|1\rangle$, depending on the classical bit that she would like to send.

2. She transmits this state over the noiseless qubit channel and Bob receives the qubit.

3. Bob performs a measurement in the computational basis to determine the classical bit that Alice transmitted.

Elementary coding succeeds without error because Bob's measurement can always distinguish the classical states $|0\rangle$ and $|1\rangle$. The following resource inequality applies to elementary coding:

$$[q \to q] \geq [c \to c]. \tag{16}$$

Again, we are only counting nonlocal resources in the resource count—we do not count the state preparation at the beginning or the measurement at the end.

If no other resources are available for consumption, the above resource inequality is optimal—one cannot do better than to transmit one classical bit of information per use of a noiseless qubit channel. This result may be a bit frustrating at first, because it may seem that we could exploit the continuous degrees of freedom in the probability amplitudes of a qubit state for encoding more than one classical bit per qubit. Unfortunately, there is no way that we can access the information in the continuous degrees of freedom with any measurement scheme.

2.3 Quantum Super-Dense Coding

We now outline a protocol named *super-dense coding.* It is named such because it has the striking property that noiseless entanglement can double the classical communication ability of a noiseless qubit channel. It consists of three steps:

1. Suppose that Alice and Bob share an ebit $|\Phi^+\rangle^{AB}$. Alice applies one of four unitary operations $\{I, X, Z, XZ\}$ to her side of the above state. The state becomes one of the following four Bell states (up to a global phase), depending on the message that Alice chooses:

$$\left|\Phi^+\right\rangle^{AB}, \qquad \left|\Phi^-\right\rangle^{AB}, \qquad \left|\Psi^+\right\rangle^{AB}, \qquad \left|\Psi^-\right\rangle^{AB}. \tag{17}$$

   The definitions of these Bell states are in the first lecture.

2. She transmits her qubit to Bob with one use of a noiseless qubit channel.

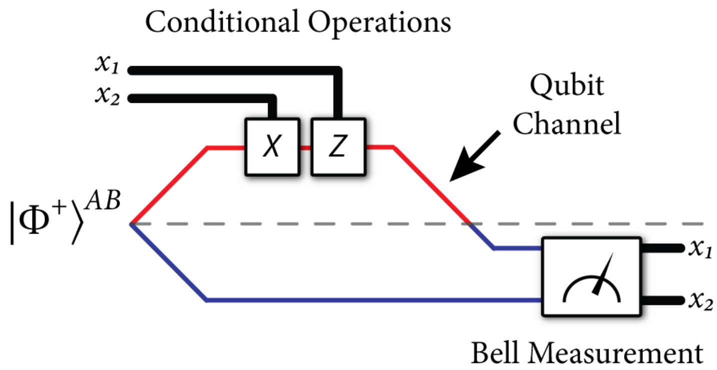

Figure 2: The above figure depicts the dense coding protocol. Alice and Bob share an ebit before the protocol begins. Alice would like to transmit two classical bits x_1x_2 to Bob. She performs a Pauli rotation conditional on her two classical bits and sends her half of the ebit over a noiseless qubit channel. Bob can then recover the two classical bits by performing a Bell measurement.

3. Bob performs a Bell measurement (a measurement in the basis $\{|\Phi^+\rangle^{AB}$, $|\Phi^-\rangle^{AB}$, $|\Psi^+\rangle^{AB}$, $|\Psi^-\rangle^{AB}\}$) to distinguish perfectly the four states—he can distinguish the states because they are all orthogonal to each other.

Thus, Alice can transmit two classical bits (corresponding to the four messages) if she shares a noiseless ebit with Bob and uses a noiseless qubit channel. Figure 2 depicts the protocol for quantum super-dense coding.

The super-dense coding protocol implements the following resource inequality:

$$[qq] + [q \to q] \geq 2[c \to c]. \tag{18}$$

Notice again that the resource inequality counts the use of nonlocal resources only—we do not count the local operations at the beginning of the protocol or the Bell measurement at the end of the protocol.

Also, notice that we could have implemented two noiseless classical bit channels with two instances of elementary coding:

$$2[q \to q] \geq 2[c \to c]. \tag{19}$$

Though, this method is not as powerful as the super-dense coding protocol—in super-dense coding, we consume the weaker resource of an ebit to help transmit two classical bits, instead of consuming the stronger resource of an extra noiseless qubit channel.

The dense coding protocol also transmits the classical bits *privately*. Suppose a third party intercepts the qubit that Alice transmits. There is no measurement that the third party can perform to determine which message Alice transmits because the local density operator of all of the Bell states is the same and equal to the maximally mixed state π^A (the information for the eavesdropper is constant for each message that Alice transmits). The privacy of the protocol is due to Alice and Bob sharing maximal entanglement.

2.4 Quantum Teleportation

Perhaps the most striking protocol in noiseless quantum communication is the *quantum teleportation protocol*. The protocol destroys the quantum state of a qubit in one location and recreates it on a qubit at a distant location, with the help of shared entanglement. Thus, the name "teleportation" corresponds well to the mechanism that occurs.

The teleportation protocol is actually a flipped version of the super-dense coding protocol, in the sense that Alice and Bob merely "swap their equipment." The first step in understanding teleportation is to perform a few algebraic steps using the tricks of the tensor product and the Bell state substitutions from the previous lecture. Consider a qubit $|\psi\rangle^{A'}$ that Alice possesses, where

$$|\psi\rangle^{A'} \equiv \alpha|0\rangle^{A'} + \beta|1\rangle^{A'}. \tag{20}$$

Suppose she shares a maximally entangled state $|\Phi^+\rangle^{AB}$ with Bob. The joint state of the systems A', A, and B is as follows:

$$|\psi\rangle^{A'}\left|\Phi^+\right\rangle^{AB}. \tag{21}$$

Let us first explicitly write out this state:

$$|\psi\rangle^{A'}\left|\Phi^+\right\rangle^{AB} = \left(\alpha|0\rangle^{A'} + \beta|1\rangle^{A'}\right)\left(\frac{|00\rangle^{AB} + |11\rangle^{AB}}{\sqrt{2}}\right). \tag{22}$$

Distributing terms gives the following equality:

$$= \frac{1}{\sqrt{2}}\left[\alpha|000\rangle^{A'AB} + \beta|100\rangle^{A'AB} + \alpha|011\rangle^{A'AB} + \beta|111\rangle^{A'AB}\right]. \tag{23}$$

We use the relations from the previous lecture to rewrite the joint system $A'A$ in the Bell basis:

$$= \frac{1}{2}\left[\begin{array}{c} \alpha\left(|\Phi^+\rangle^{A'A} + |\Phi^-\rangle^{A'A}\right)|0\rangle^B + \beta\left(|\Psi^+\rangle^{A'A} - |\Psi^-\rangle^{A'A}\right)|0\rangle^B \\ +\alpha\left(|\Psi^+\rangle^{A'A} + |\Psi^-\rangle^{A'A}\right)|1\rangle^B + \beta\left(|\Phi^+\rangle^{A'A} - |\Phi^-\rangle^{A'A}\right)|1\rangle^B \end{array}\right] \tag{24}$$

Simplifying gives the following equivalence:

$$= \frac{1}{2}\left[\begin{array}{c} |\Phi^+\rangle^{A'A}\left(\alpha|0\rangle^B + \beta|1\rangle^B\right) + |\Phi^-\rangle^{A'A}\left(\alpha|0\rangle^B - \beta|1\rangle^B\right) \\ +|\Psi^+\rangle^{A'A}\left(\alpha|1\rangle^B + \beta|0\rangle^B\right) + |\Psi^-\rangle^{A'A}\left(\alpha|1\rangle^B - \beta|0\rangle^B\right) \end{array}\right]. \tag{25}$$

We can finally rewrite the state as four superposition terms, with a distinct Pauli operator applied to Bob's system B for each term in the superposition:

$$= \frac{1}{2}\left[\left|\Phi^+\right\rangle^{A'A}|\psi\rangle^B + \left|\Phi^-\right\rangle^{A'A}Z|\psi\rangle^B + \left|\Psi^+\right\rangle^{A'A}X|\psi\rangle^B + \left|\Psi^-\right\rangle^{A'A}XZ|\psi\rangle^B\right]. \tag{26}$$

We now outline the three steps of the teleportation protocol (Figure 3 depicts the protocol):

1. Alice performs a Bell measurement on her systems $A'A$. The state collapses to one of the following four states with uniform probability:

$$\left|\Phi^+\right\rangle^{A'A}|\psi\rangle^B, \tag{27}$$
$$\left|\Phi^-\right\rangle^{A'A}Z|\psi\rangle^B, \tag{28}$$
$$\left|\Psi^+\right\rangle^{A'A}X|\psi\rangle^B, \tag{29}$$
$$\left|\Psi^-\right\rangle^{A'A}XZ|\psi\rangle^B. \tag{30}$$

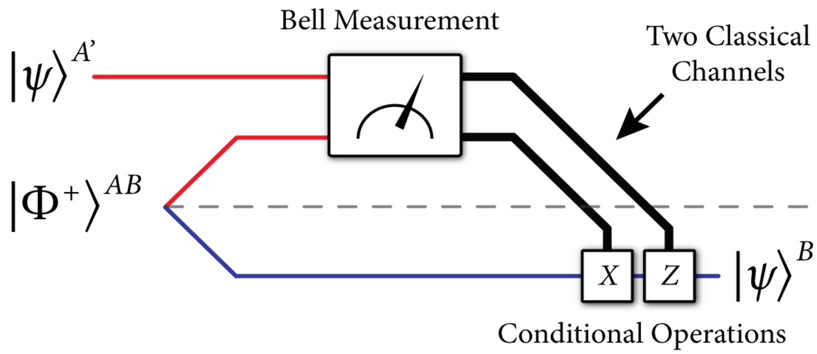

Figure 3: The above figure depicts the teleportation protocol. Alice would like to transmit an arbitrary quantum state $|\psi\rangle^{A'}$ to Bob. Alice and Bob share an ebit before the protocol begins. Alice can "teleport" her quantum state to Bob by consuming the entanglement and two uses of a noiseless classical bit channel.

Notice that the state resulting from the measurement is a product state with respect to the cut $A'A - B$, regardless of the outcome of the measurement. At this point, Alice knows whether Bob's state is $|\psi\rangle^B$, $Z|\psi\rangle^B$, $X|\psi\rangle^B$, or $XZ|\psi\rangle^B$ because she knows the result of the measurement. On the other hand, Bob does not know anything about the state of his system B—one can show that his local state is a completely random state just after Alice performs the measurement. Thus, there is no teleportation of quantum information at this point because Bob's state is completely independent of the original state $|\psi\rangle$. In other words, teleportation cannot be instantaneous.

2. Alice transmits two classical bits to Bob that indicate which of the four measurement results she obtains. After Bob receives the classical information, he is immediately certain which operation he needs to perform in order to restore his state to Alice's original state $|\psi\rangle$. Notice that he does not need to have knowledge of the state in order to restore it—he only needs knowledge of the restoration operation.

3. Bob performs the restoration operation: one of the identity, a Pauli X operator, a Pauli Z operator, or the Pauli operator XZ, depending on the classical information that he receives from Alice.

Teleportation is an *oblivious* protocol because Alice and Bob do not require any knowledge of the quantum state being teleported in order to perform it. We might also say that this feature of teleportation makes it universal—it works independent of the input state.

You might think that the teleportation protocol violates the no-cloning theorem because a "copy" of the state appears on Bob's system. But this violation does not occur at any point in the protocol because the Bell measurement destroys the information about the state of Alice's original information qubit while recreating it somewhere else. Also, notice that the result of the Bell measurement is independent of the particular probability amplitudes α and β corresponding to the state Alice wishes to teleport.

The teleportation protocol is not an instantaneous teleportation, as portrayed in the television episodes of Star Trek. There is no transfer of quantum information instantaneously after the Bell measurement because Bob's local description of the B system is the maximally mixed state π. It is only after he receives the classical bits to "telecorrect" his state that the transfer occurs. It must be this way—otherwise, they would be able to communicate faster than the speed of light, and superluminal communication is not allowed by the theory of relativity.

Finally, we can phrase the teleportation protocol as a resource inequality:

$$[qq] + 2[c \to c] \geq [q \to q]. \tag{31}$$

Again, we include only nonlocal resources into the resource count. The above resource inequality is perhaps the most surprising of the three unit protocols we have studied so far. It combines two resources, noiseless entanglement and noiseless classical communication, that achieve noiseless quantum communication even though they are both individually weaker than it. This protocol and super-dense coding are two of the most fundamental protocols in quantum communication theory because they sparked the notion that there are clever ways of combining resources to generate other resources.

In Exercise 3 below, we discuss a variation of teleportation called *remote state preparation*, where Alice possesses a classical description of the state that she wishes to teleport. With this knowledge, it is possible to reduce the amount of classical communication necessary for teleportation.

Exercise 3 *Remote state preparation is a variation on the teleportation protocol. We consider a simple example of a remote state preparation protocol. Suppose Alice possesses a classical description of a state* $|\psi\rangle \equiv \left(|0\rangle + e^{i\phi}|1\rangle\right)/\sqrt{2}$ *(on the equator of the Bloch sphere) and she shares an ebit* $|\Phi^+\rangle^{AB}$ *with Bob. Alice would like to prepare this state on Bob's system. Show that Alice can prepare this state on Bob's system if she measures her system* A *in the* $\left\{|\psi^*\rangle, |\psi^{\perp *}\rangle\right\}$ *basis, transmits one classical bit, and Bob performs a recovery operation conditional on the classical information. (Note that* $|\psi^*\rangle$ *is the conjugate of the vector* $|\psi\rangle$*).*

Exercise 4 *Third-party controlled teleportation is another variation on the teleportation protocol. Suppose that Alice, Bob, and Charlie possess a GHZ state:*

$$|\Phi_{GHZ}\rangle \equiv \frac{|000\rangle^{ABC} + |111\rangle^{ABC}}{\sqrt{2}}. \tag{32}$$

Alice would like to teleport an arbitrary qubit to Bob. She performs the usual steps in the teleportation protocol. Give the final steps that Charlie should perform and the information that he should transmit to Bob in order to complete the teleportation protocol. (Hint: The resource inequality for the protocol is as follows:

$$[qqq]_{ABC} + 2[c \to c]_{A \to B} + [c \to c]_{C \to B} \geq [q \to q]_{A \to B}, \tag{33}$$

where $[qqq]_{ABC}$ *represents the resource of the GHZ state shared between Alice, Bob, and Charlie, and the other resources are as before with the directionality of communication indicated by the corresponding subscript.)*

Exercise 5 *Gate teleportation is yet another variation of quantum teleportation that is useful in fault-tolerant quantum computation. Suppose that Alice would like to perform a single-qubit gate* U *on a qubit in state* $|\psi\rangle$*. Suppose that the gate* U *is difficult to perform, but that* $U\sigma_i U^\dagger$*, where*

σ_i is one of the single-qubit Pauli operators, is much less difficult to perform. A protocol for gate teleportation is as follows. Alice and Bob first prepare the ebit $U^B|\Phi^+\rangle^{AB}$. Alice performs a Bell measurement on her qubit $|\psi\rangle^{A'}$ and system A. She transmits two classical bits to Bob and Bob performs one of the four corrective operations $U\sigma_i U^\dagger$ on his qubit. Show that this protocol works, i.e., Bob's final state is $U|\psi\rangle$.

Exercise 6 *Show that it is possible to simulate a dephasing qubit channel by the following technique. First, Alice prepares a maximally entangled Bell state $|\Phi^+\rangle$. She sends half of it to Bob through a dephasing qubit channel. She and Bob perform the usual teleportation protocol. Show that this procedure gives the same result as sending a qubit through a dephasing channel. (Hint: This result holds because the dephasing channel commutes with all Pauli operators.)*

Exercise 7 *Construct an entanglement swapping protocol from the teleportation protocol. That is, suppose that Charlie and Alice possess a bipartite state $|\psi\rangle^{CA}$. Show that if Alice teleports her half of the state $|\psi\rangle^{CA}$ to Bob, then Charlie and Bob share the state $|\psi\rangle^{CB}$. A special case of this protocol is when the state $|\psi\rangle^{CA}$ is an ebit. Then the protocol is equivalent to an entanglement swapping protocol.*