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The Gravity Base Stations for Ireland

by

THOMAS MURPHY

Scoil na Fisice Cosmaí, Institiúid Árd-Léinn Bhaile Átha Cliath
School of Cosmic Physics, Dublin Institute for Advanced Studies
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THE GRAVITY BASE STATIONS FOR IRELAND

Abstract

A grid of 82 gravity base stations has been set up in Ireland. 37 of these form a network of 69 connections and 45 are intermediate. The network has been adjusted and values for each station are given as a difference from the principal base, at Dublin, which in turn is connected to the pendulum station at Dunsink. The measurements were carried out in 1954-5 with a standard Worden gravimeter. This meter has been compared with four other Worden gravimeters at Macclesfield, England, one of which has been compared with pendulum observations in Great Britain.

Introduction

The earlier gravity work in Ireland, *Memoir* No. 2, *Parts* 2, 3 and 4, was carried out with a small Graf gravimeter. A network of stations was set up a few miles apart which, as well as providing a general picture of the gravitational field in Ireland, was hoped to serve as base stations for subsequent more detailed surveys. The more modern instruments now available enable the base stations to be positioned further apart; the adjustment becomes easier and the overall accuracy increases.

In 1954 the School of Cosmic Physics obtained a standard Worden gravimeter No. 211 and after some preliminary trial surveys it was decided to set up a grid of base stations incorporating earlier sites of stations if these were convenient and to take measurements within a short period of time and adjust them in a single operation.

The Sites for the Stations

Measurements of gravity in Ireland are, because of adverse weather conditions mainly high winds, usually carried out inside a vehicle and so for convenience the base stations were chosen on roadways. The choice of the actual site for the station was usually controlled by the accessibility, the ease of manœuvring and parking the vehicle, and conveniency to the centre of a town. The bases were chosen about 40 km apart. In some cases when the distances between two towns was greater than this an additional base was inserted. With only a few exceptions the sites for the base stations were within 30 metres of a bench mark, very often within 3 metres. The positions are marked on two sets of maps, scale 6″ to one mile, one set stored at the Ordnance Survey, Phœnix Park, Dublin and the other at the School of Cosmic Physics. Sketches of the sites are kept, as well, at the latter address and copies can be obtained on request.

The Measurement of the Connections between Bases

It would not be feasible to link each base with every adjacent one and in several cases it would not be even possible within a reasonable time because of the lack of direct road connections. The network of connections was mainly determined by travelling facilities so that in some cases rather large loops had to be utilised (*Fig.* 1).

The measurements were carried out with a single gravimeter, Worden No. 211, a standard model without the large dial. The greatest gravity difference between two bases was 63 mgals while the mean difference was 18 mgals. No readings were taken within 12 hours after resetting the zero with the coarse spring and the meter was protected as far as possible from extremes of temperature. Throughout the measurements the meter had

a small positive drift of approximately 0·02 mgal per hour. The drift was not constant over a day being greater during working hours. No jumps in the zero were observed or suspected. It was found that the meter is very sensitive to vibration so that readings can only be taken under very quiet conditions. It is also microphonic but this is not of much importance in Ireland outside of the cities. The effect of vibration and sound not only broadens the index line but displaces it to one side usually so that the reading is higher than it should be. This can be serious if it passes unnoticed.

The method employed for making the measurements was the " Forward Looping " method (NETTLETON 1940, p. 38) with the interval between successive readings of about one hour. It was discovered that this method has a few drawbacks and the following investigation will point these out.

THE FORWARD LOOPING METHOD

In the " Forward Looping " method let the readings at two stations be

r_1 at time t_1 at station A
r_2 at time t_2 at station B
r_3 at time t_3 at station A
r_4 at time t_4 at station B

In practice, the intervals (t_2-t_1), (t_3-t_2) and (t_4-t_3) are very nearly equal, within a few per cent, and to simplify the argument we will assume these intervals to be equal to t.

We will assume first that the zero drift is linear throughout the measurements and equal to b mgals per hour. When the time interval t is of the order of an hour the effect of the earth tides will be included in b. If the difference in gravity between stations A and B, which we wish to ascertain, is written as δ_{AB}

then
$$r_2=r_1+bt+\delta_{AB} \tag{1}$$
$$r_3=r_1+2bt \tag{2}$$
$$r_4=r_1+3bt+\delta_{AB} \tag{3}$$

From (1) and (2) we can obtain a value for δ_{AB}
$$\delta'_{AB}=r_2-\tfrac{1}{2}(r_3+r_1) \tag{4}$$
and substituting r_1 from *Equ.* (1) in *Equ.* (2) and (3)
$$\delta''_{AB}=-r_3+\tfrac{1}{2}(r_2+r_4) \tag{5}$$
The two values δ'_{AB} and δ''_{AB} are then combined to give a mean
$$\delta_{AB}=\tfrac{1}{2}(\delta'_{AB}+\delta''_{AB})=\tfrac{1}{4}\{(r_4-r_1)+3(r_2-r_3)\} \tag{6}$$

It is customary to take the difference $\delta'_{AB}-\delta''_{AB}$ as a measure of the accuracy of the measurements of δ_{AB} and to use it in the adjustment of a network (BULLERWELL, 1952, p. 308).

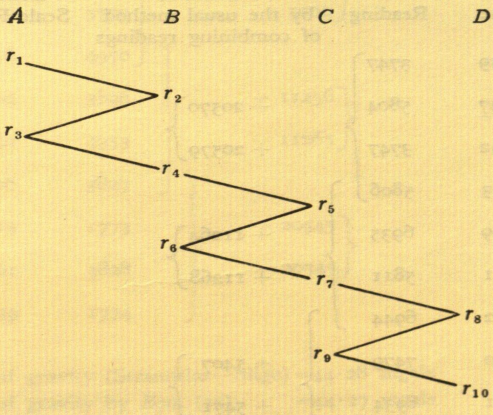
Proceeding further, if the drift is assumed to be not linear but parabolic then equations corresponding to (1), (2) and (3) can be written thus
$$r_2=r_1+bt+ct^2+\delta_{AB} \tag{7}$$
$$r_3=r_1+2bt+4ct^2 \tag{8}$$
$$r_4=r_1+3bt+9ct^2+\delta_{AB} \tag{9}$$

Solving *Equ.* (7), (8) and (9)
$$\delta_{AB}=\tfrac{1}{4}\{(r_4-r_1)+3(r_2-r_3)\} \tag{10}$$

Thus the two results given by *Equ.* (6) and (10) are identical. In other words whether we assume the drift is linear or parabolic we arrive at the same numerical value for δ_{AB} from the same initial readings. The difference $\delta'_{AB}-\delta''_{AB}$ mentioned above can only be assessed as a measure of accuracy of δ_{AB} if it can be shown that the drift is linear.

The *Equ.* (6) and (10) have important properties in the continuation of the " Forward Looping " method. The time interval does not enter and if the drift is assumed parabolic the earth tide effect will be completely allowed for. Let us take two more links and if, as before, we assume the time intervals within each link are equal they may be omitted. The readings can be represented thus :—

Stations



$$\delta_{AB}=\tfrac{1}{4}\{(r_4-r_1)+3(r_2-r_3)\}$$

$$\delta_{BC}=\tfrac{1}{4}\{(r_7-r_4)+3(r_5-r_6)\}$$

$$\delta_{CD}=\tfrac{1}{4}\{(r_{10}-r_7)+3(r_8-r_9)\}$$

Adding we get

$$\delta_{AD}=\tfrac{1}{4}\{(r_{10}-r_1)+3(r_2-r_3)+3(r_5-r_6)+3(r_8-r_9)\} \qquad (11)$$

Equ. (11) does not contain r_4 and r_7 ; and r_1 and r_{10} have one-third the weight of each of the others. Hence in a line of links the $(3n+1)^{th}$ reading has either zero or one-third the weight of each of the others when the overall difference of gravity only is required.

Fig. 1. Sketch map of Ireland showing the locations of the Gravity Base Stations together with their names and serial numbers. The connections between the stations are the measured gravity differences in mgals and the closing error, measured clockwise, is given in the centre of each loop.

the two values δ'_{AB} and δ''_{AB} differing by only 0·01 mgal. When the network was completed the closing errors of the two loops on either side of this connection were large and of opposite signs. The connection was remeasured on October 7, 1955 and the difference was 34·28 mgals with the two values differing again by only 0·01 mgal. If one accepts the difference of the two values, δ'_{AB} and δ''_{AB}, as a measure of the error in the measurement then the differences of gravity as measured on the two days are equally good but they differ by 0·11 mgal which is well outside the limits of error of the instrument.

At first it seems as if the earlier readings must be in error and should be omitted but this is not permissible because these readings are used in the first and second links taken that day, namely Sligo—Tubbercurry and Ballyhaunis—Tuam, and these are quite good as can be seen later in *Fig.* 1. Here again it is not possible, without additional readings, to find out which readings were in error.

To sum up then, it can be said that from theoretical reasoning and from practical examples the individual readings taken in the method of " Forward Looping " are weighted disproportionally and can give rise to a false estimate of accuracy.

As already pointed out, the readings for the network of bases were made by this method of " Forward Looping " before the above details had been worked out and hence the computations for the gravity differences had to be made using these readings. No other method of combination could be found which would be better than the usual one i.e. *Equ.* (4) and (5). As 70% of the differences $\delta'_{AB} - \delta''_{AB}$ was less than 0·02 mgal it was felt that calculations assuming a parabolic drift would not be justified and allowance for earth tides was found by experiment to be unnecessary as the time intervals were too small.

Adjustment of the Network

The bases and the various links and connections are given in *Fig.* 1. The sum for each loop is given in the centre. 70% of these sums lie in the range $\pm$ 0·04 mgal. The highest is 0·11 mgal in a loop of 5 links. These values are quite satisfactory and within the limits expected.

Fig. 2 was constructed from *Fig.* 1 by simple addition when a connection consisted of more than one link. Before adjusting the resulting network, weights had to be assigned to each connection. As mentioned earlier it is not possible to ascertain the error in any one link and hence it was decided to treat them all as of equal weight. Thus each connection was given a weight inversely proportional to the square root of the number of links constituting it, in an analogous manner to a levelling network. The weights are written in italics in *Fig.* 2. There were 69 connections and hence 69 equations of conditions. With 37 unknowns a like number of normal equations was obtained from the equations of conditions. The 37 simultaneous equations were solved by Messrs. Ferranti of Portland Place, London on their electronic calculator " Pegasus ". The data punching took 5¼ hours while the actual computation was accomplished in 7 minutes.

The values of gravity were then deduced for each base in the network and from these *Fig.* 3 was constructed. As a check, the sum of each loop is now zero. Comparing *Figs.* 2 and 3 it can be seen that the changes to the connections are, in general, very small. The largest changes are concerned with Stations *1* and *83*. These stations are alongside each other, *83* being used while the pillar for Station *1* was being constructed. The discrepancies here are undoubtedly connected with the vibration always experienced at these stations and of which mention has already been made.

During the fieldwork this theory was worked out and *Equ.* (11) was used to compute gravity differences for each day's work without the more rigorous method making allowance for the fact that the time intervals are not exactly the same. When the final computations were done later and the results corresponding to *Equ.* (4) and (5) worked out it was found that discrepancies between the two methods of calculation only occurred when the time intervals were much different.

Some time later an opportunity arose to test out the implications of *Equ.* (11). A connection consisting of three links was repeated because on the first day the weather turned adverse to good readings and one reading in particular was suspected to be in error. The results were as follows :—

October 5, 1955

Station	Time	Reading	Difference by the usual method of combining readings	Difference in mgals Scale Factor $1 \cdot 187 \times 10^{-3}$
Sligo ..	08·59	3747		
Bundoran ..	09·47	5804	+ 20570	24·42
Sligo ..	10·32	3747	+ 20579	
Bundoran ..	11·23	5806		
Donegal ..	12·09	6935	+ 11264	13·37
Bundoran ..	12·51	5811	+ 11268	
Donegal ..	14·22	6944		
Stranorlar ..	16·30	7479	+ 5407	6·46
Donegal ..	16·21	6934	+ 5471	
Stranorlar ..	17·09	7483		

Difference of gravity (Stranorlar—Sligo) = 44·25 mgals (12)

Difference of gravity by *Equ.* (11) = 44·26 mgals (13)

The reading which was questioned was taken at Donegal at 14·22 thus

Station Donegal

Time	Reading
12·09	6935
14·22	6944
16·21	6934

This reading was not used at all in computing the difference of gravity given by *Equ.* (13) and the latter is in excellent agreement with *Equ.* (12) which was deduced by the more rigorous method.

On the following day the next set of readings were taken.

October 6, 1955

Station	Time	Reading	Difference by the usual method of combining readings	Difference in mgals Scale Factor $1 \cdot 187 \times 10^{-3}$
Stranorlar ..	11·32	5496		
Donegal ..	12·13	4950	— 5481	— 6·52
Stranorlar ..	12·52	5500	— 5500	
Donegal ..	14·16	4950		
Bundoran ..	15·02	3826	— 11256	— 13·37
Donegal ..	15·42	4953	— 11265	
Bundoran ..	16·26	3827		
Sligo ..	17·13	1773	— 20545	— 24·39
Bundoran ..	18·01	3828	— 20545	
Sligo ..	18·59	1774		

Difference of gravity (Stranorlar—Sligo) = 44·28 mgals
Difference of gravity by *Equ.* (11) = 44·27 mgals

Thus the gravity difference Stranorlar—Sligo as measured on the two days are 44·25 and 44·28 mgals and are within the limits of error of the instrument which does not measure better than 0·01 mgal. This apparent good agreement must be due to the fortuitous elimination or diminution of weight of the doubtful reading or readings shown by the analysis of *Equ.* (11). It might be thought that the questionable reading mentioned before would show up in the computation of the individual links but here this particular reading is either "r_4" in one link or "r_1" in the next link and hence has only the weight of one-third the others. Thus without additional readings it is not possible to assess the error in the series of readings given for Donegal above.

This is a case in which repeat readings are in agreement but on the other hand if the $(3n+1)^{th}$ readings have less error than the remaining ones there can be quite a large error in the final answer e.g. outside what is expected from the probable error. Such a case was suspected in the survey undertaken on the connection Tubbercurry—Ballyhaunis. On November 10, 1954 this difference, the middle link of three, was measured as 34·17 mgals

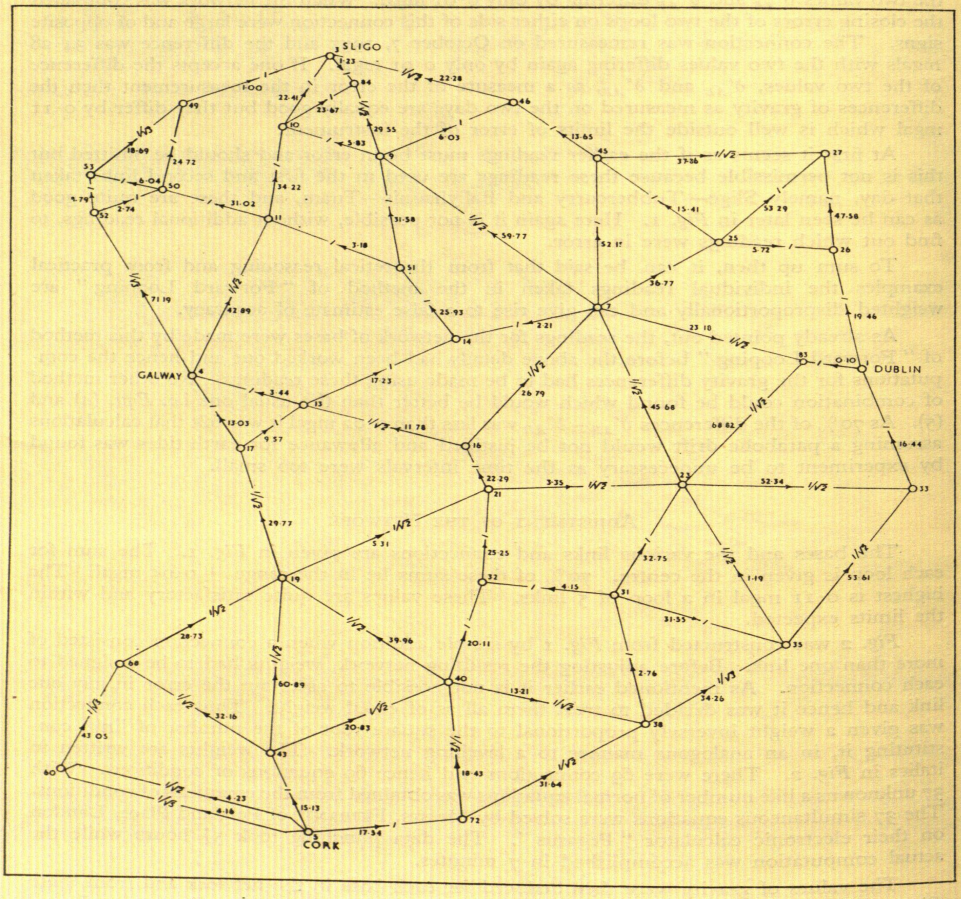


Fig. 2. The Network of gravity readings prepared for adjustment. The number alongside each circle is the serial number of the Base Station. The number along each connection at the arrow is the measured difference of gravity between two Base Stations, the arrow pointing in the direction of increasing gravity ; the figure, in italics, is the weight attached to each connection.

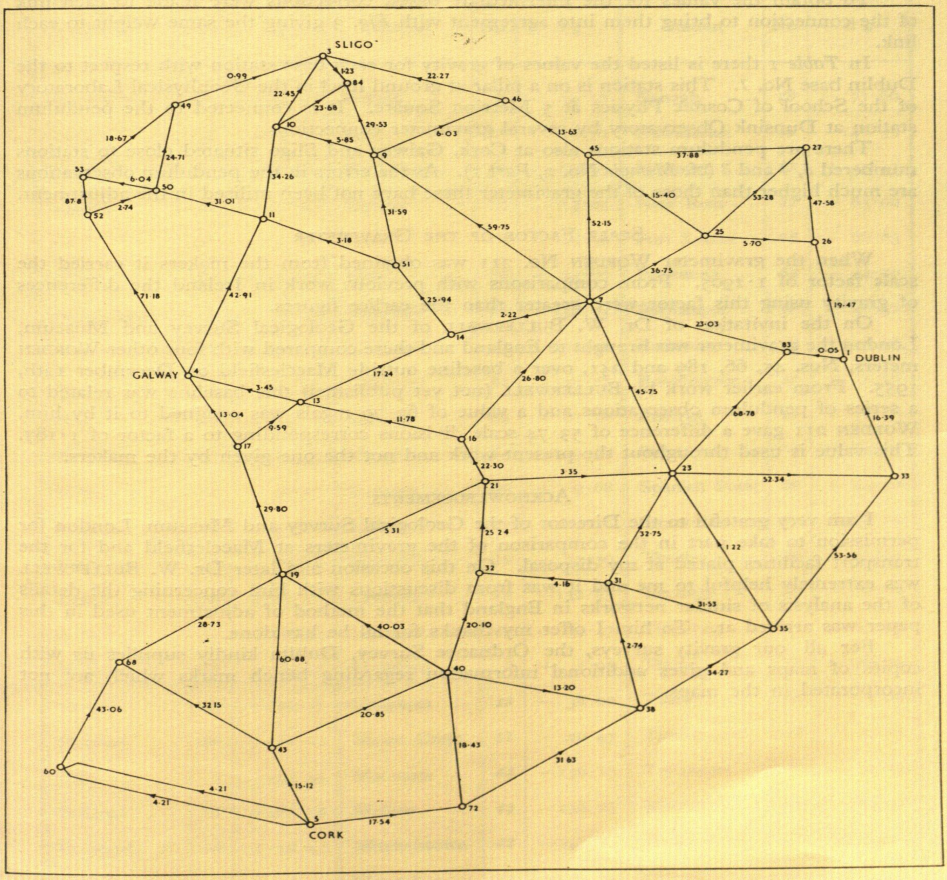


Fig. 3. The network of gravity connections showing the differences of gravity between two Base Stations after adjustment.

To obtain the values for the intermediate bases, corrections were made to each link of the connection to bring them into agreement with *Fig.* 3 giving the same weight to each link.

In *Table* 1 there is listed the values of gravity for each base station with respect to the Dublin base No. *1*. This station is on a pillar at ground level in the Geophysical Laboratory of the School of Cosmic Physics at 5 Merrion Square. It is connected to the pendulum station at Dunsink Observatory by several gravimeter connections.

There are pendulum stations also at Cork, Galway and Sligo situated close to stations numbered *5*, *4* and *3* (cf. *Memoir* No. 2, *Part* 1). As the errors in the pendulum observations are much higher than those of the gravimeter these have not been utilised in the adjustment.

Scale Factor of the Gravimeter

When the gravimeter Worden No. 211 was obtained from the makers it carried the scale factor of 1·1905. From comparisons with previous work in Ireland the differences of gravity using this factor were greater than the earlier figures.

On the invitation of Dr. W. Bullerwell of the Geological Survey and Museum, London the gravimeter was brought to England and there compared with four other Worden meters, Nos. 45, 66, 189 and 241, over a baseline outside Macclesfield on December 12th, 1955. From earlier work by Bullerwell (not yet publishcd) this baseline was related to a series of pendulum observations and a value of 63·79 mgals was assigned to it by him. Worden 211 gave a difference of 53·74 scale divisions corresponding to a factor of 1·187. This value is used throughout the present work and not the one given by the makers.

Acknowledgements

I am very grateful to the Director of the Geological Survey and Museum, London for permission to take part in the comparison of the gravimeters at Macclesfield and for the transport facilities placed at my disposal. On this occasion and later Dr. W. Bullerwell was extremely helpful to me and it was from discussions with him concerning the details of the analysis of similar networks in England that the method of adjustment used in this paper was arrived at. To him I offer my thanks for all he has done.

For all our gravity surveys, the Ordnance Survey, Dublin kindly supplies us with copies of maps and gives additional information regarding bench marks which are not incorporated in the maps.

Table 1

THE GRAVITY BASE STATIONS WITH THEIR GRAVITY DIFFERENCES FROM THE PRINCIPAL BASE (DUBLIN)

Station	No.	Δg	Station	No.	Δg	Station	No.	Δg
Abbeyfeale	68	−106·12	Enfield	6	− 22·57	Nenagh	20	− 71·66
Adare	70	− 79·42	Ennis	18	− 49·72	Newmarket	69	−140·95
Arklow	34	− 36·08	Enniscorthy	35	− 69·95	Newport	53	+ 45·41
Athlone	14	− 20·76	Galway	4	− 34·55	New Ross	37	− 83·00
Athy	23	− 68·73	Gort	17	− 47·59	Port Laoise	22	− 66·83
Ballina	49	+ 64·08	Kells	25	+ 13·77	Portumna	58	− 44·09
Ballyhaunis	11	+ 8·36	Kenmare	61	−158·94	Rathangan	59	− 46·32
Bangor	55	+ 57·36	Kilkenny	31	−101·48	Rathluirc	74	−125·00
Bantry	62	−154·42	Killarney	60	−149·18	Roscommon	51	+ 5·18
Birr	16	− 49·78	Kilrush	29	− 62·10	Roscrea	21	− 72·08
Blacklion	48	+ 60·55	Kinsale	64	−149·51	Sligo	3	+ 65·07
Boyle	9	+ 36·77	Leenane	56	+ 9·68	Spanish Point	30	− 44·78
Brackley	46	+ 42·80	Lifford	79	+120·18	Stranorlar	78	+109·64
Bundoran	76	+ 89·78	Limerick	19	− 77·39	Thurles	32	− 97·32
Cahir	40	−117·42	Lismore	71	−132·63	Tinahely	75	− 80·15
Carlow	44	− 98·51	Listowel	67	− 94·47	Tipperary	41	−115·16
Castlebar	50	+ 39·37	Longford	8	+ 4·95	Tralee	66	−106·11
Cavan	45	+ 29·17	Loughrea	13	− 38·00	Tuam	12	+ 3·09
Clonmel	39	−112·00	Maam Cross	57	− 20·47	Tubbercurry	10	+ 42·62
Cork	5	−153·39	Macroom	65	−156·17	Tullamore	15	− 37·15
Donegal	77	+103·15	Mallow	43	−138·27	Waterford	38	−104·22
Drogheda	26	+ 19·47	Mitchelstown	42	−143·34	Westport	52	+ 36·63
Dublin	1	0·0	Monaghan	47	+ 62·01	Wexford	36	− 75·31
Dundalk	27	+ 67·05	Mullingar	7	− 22·98	Wicklow	33	− 16·39
Dungarvan	73	−122·71	Mulranny	54	+ 39·65	Youghal	72	−135·85
Dunmanway	63	−169·26	Naas	24	− 43·21			

g(Dublin) − g(Dunsink)	9·60 mgals	from gravimeter observations
g(Dunsink) − g(Pendulum House, Cambridge)	121·4 mgals	from pendulum observations
g(Pendulum House, Cambridge)	981265·0 mgals	

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AUGUST 24, 1956.