Towards further insight into and new applications of gauge/gravity duality: Modular flow and new Dirac materials

Johanna Erdmenger

Julius-Maximilians-Universität Würzburg

Two recent papers of our research group:

1. J. E., Pascal Fries, Ignacio A. Reyes, Christian P. Simon,

   'Resolving modular flow: A toolkit for free fermions',
   arXiv/2008.07532 [hep-th].

2. D. Di Sante, J. E., M. Greiter, I. Matthaiakakis, R. Meyer, D. Rodriguez Fernandez, R. Thomale, E. van Loon, T. Wehling,

   'Turbulent hydrodynamics in strongly correlated Kagome materials',

   Nature Commun. 11 (2020) 1, 3997, arXiv/1911.06810 [cond-mat].

Modular Hamiltonian and Modular flow

Starting point: State given by density matrix ρ

Entangling region V

Entropy generalizes to entanglement entropy $S_V = -\mathrm{tr}(\rho_V \ln \rho_V)$

Hamiltonian generalizes to modular Hamiltonian K_V, defined implicitly via

$$\rho_V := \frac{e^{-K_V}}{\mathrm{tr}\left(e^{-K_V}\right)}$$

Generalized time evolution

Entanglement spectrum has many applications in many body physics and QFT

Topological order; relative entropy

AdS/CFT: Essential for gravity bulk reconstruction from QFT boundary data

Modular Hamiltonian and modular flow

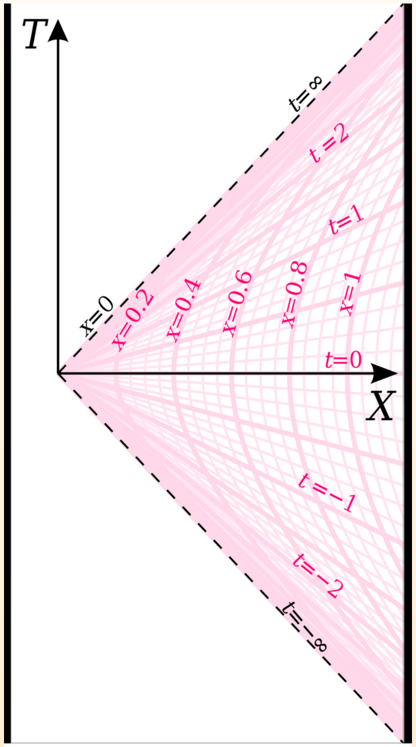
Modular Hamiltonian known explicitly only in a small number of cases

Universal and local result for QFT on Rindler spacetime:
(accelerated reference frame in Minkowski spacetime)

$$K - K_{\mathrm{vac}} = \frac{2\pi}{\hbar} \int_0^\infty dx\, x T_{tt}$$

(Bisognano-Wichmann theorem)

Further examples: CFT vacuum on a ball,
CFT_2 for single interval, vacuum on the cylinder or
thermal state on real line



(WIkipedia)

Modular Hamiltonian and modular flow

Modular flow generated by modular Hamiltonian:

Generalised time evolution with the density matrix:

$$\sigma_t(\mathcal{O}) := \rho^{it} \mathcal{O} \rho^{-it}$$

In general, modular flow is non-local

Result of 2008.07532 for free fermions in 1+1 dimensions:

For disjoint intervals $V = \bigcup_n [a_n, b_n]$:

$$\sigma_t\left(\psi^\dagger(y)\right) = \int_V \mathrm{d}x\, \psi^\dagger(x)\Sigma_t(x,y)\,,$$

$$\Sigma_t = \left(\frac{1 - G|_V}{G|_V}\right)^{it}.$$

Modular flow expressed in terms of reduced propagator $G|_V$

Modular flow

A few facts from Tomita-Takesaki modular theory: (see S. Hollands, 1904.08201)

Tomita conjugation:

$$S\mathcal{O}|\Omega\rangle := \mathcal{O}^\dagger|\Omega\rangle$$

for operator $\mathcal{O}$ in von Neumann algebra $\mathcal{R}$

S may be decomposed into $J\Delta^{1/2}$, J antiunitary and Δ positive

Tomita theorem: $J\mathcal{R}J^\dagger = \mathcal{R}'$, $\Delta^{it}\mathcal{R}\Delta^{-it} = \mathcal{R}$

Modular flow: $\sigma_t(\mathcal{O}) = \Delta^{it}\mathcal{O}\Delta^{-it}$

Modular Hamiltonian: $e^{-itK} := \Delta^{it}$

Two operators satisfy the KMS (Kubo-Martin-Schwinger) condition

$$\langle\Omega|\mathcal{O}_1\sigma_t(\mathcal{O}_2)|\Omega\rangle = \langle\Omega|\sigma_{t+i}(\mathcal{O}_2)\mathcal{O}_1|\Omega\rangle$$

by analogy to time evolution at finite temperature

Modular two-point function for free fermions

Modular two-point function

$$G_{\text{mod}}(x,y;t) := \begin{cases} -\langle\Omega|\sigma_t(\psi^\dagger(y))\psi(x)|\Omega\rangle & \text{for } 0 < \mathrm{Im}(t) < 1 \\ +\langle\Omega|\psi(x)\sigma_t(\psi^\dagger(y))|\Omega\rangle & \text{for } -1 < \mathrm{Im}(t) < 0. \end{cases}$$

Introduce Σ_t as test or smearing function

$$\sigma_t\big(\psi^\dagger(y)\big) = \int_V \mathrm{d}x\, \psi^\dagger(x)\Sigma_t(x,y)$$

From fermion anticommutator it follows that

$$G_{\text{mod}}(x,y;t-i0^+) - G_{\text{mod}}(x,y;t+i0^+) = \Sigma_t(x,y)$$

Modular flow for free fermions

Modular Hamiltonian, space region V

$$K = \int_V \mathrm{d}^d x \int_V \mathrm{d}^d y \, \psi^\dagger(x) k(x,y) \psi(y) + \int_{V^c} \mathrm{d}^d x \int_{V^c} \mathrm{d}^d y \, \psi^\dagger(x) k^c(x,y) \psi(y)$$

$$G(x,y) := \langle \Omega | \psi(x) \psi^\dagger(y) | \Omega \rangle$$

$$\mathrm{e}^{-k} = \frac{1 - G|_V}{G|_V}$$

$$\sigma_t(\psi^\dagger(y)) = \int_V \mathrm{d}^d x \, \psi^\dagger(x) \Sigma_t(x,y) \quad \text{with} \quad \Sigma_t = \left[\frac{1 - G|_V}{G|_V} \right]^{\mathrm{i}t}$$

Problem reduced to computing functions of the restricted propagator $G|_V$

For reduced density matrices: Araki 1971, Peschel 2003

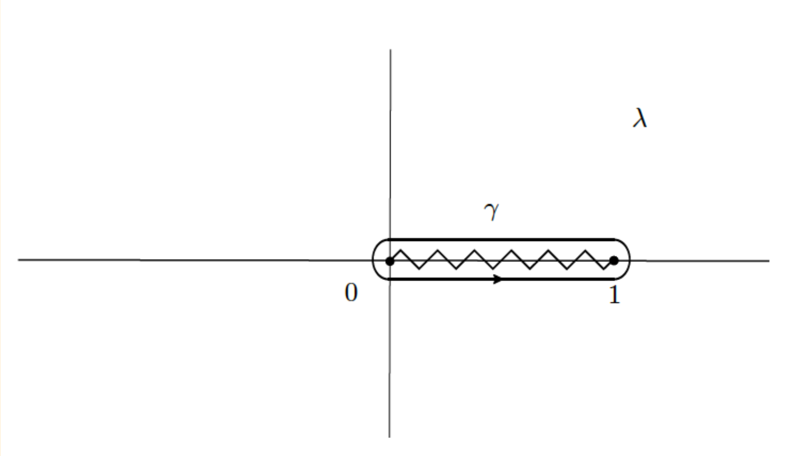
Modular flow for free fermions: Resolvent

$$\sigma_t(\psi^\dagger(y)) = \int_V \mathrm{d}^d x \, \psi^\dagger(x) \Sigma_t(x,y) \quad \text{with} \quad \Sigma_t = \left[\frac{1-G|_V}{G|_V}\right]^{it}$$

Introduce resolvent as shown for function f:

$$f(G|_V) = \frac{1}{2\pi \mathrm{i}} \oint_\gamma \mathrm{d}\lambda \, f(\lambda) \frac{1}{\lambda - G|_V}$$

Contour



Ansatz for the resolvent:

$$\frac{1}{\lambda - G|_V} = \frac{1}{\lambda} + \frac{F_\lambda}{\lambda^2}$$

$(\lambda - G_V) \times 1/(\lambda - G_V) = 1$ leads to an integral equation.

Modular flow then obtained from

$$\Sigma_t = \left(\frac{1 - G|_V}{G|_V}\right)^{it}$$

$$\Sigma_t = \frac{1}{2\pi \mathrm{i}} \oint_\gamma \mathrm{d}\lambda \left(\frac{1-\lambda}{\lambda}\right)^{it} \left[\frac{1}{\lambda} + \frac{F_\lambda}{\lambda^2}\right]$$

Locality properties

We compute the free fermion modular flow for a number of examples:

plane, cylinder (Ramond and Neveu-Schwarz sectors), torus

Locality:

Non-local: Kernel $\Sigma_t(x,y)$ is a smooth function on all of the region V

Bi-local: $\Sigma_t(x,y) \sim \delta(f(x,y))$. Discrete set of contributions. Couples pairs of distinct points since $x \neq y$ at $t = 0$.

Local: As bi-local but with $x = y$ at $t = 0$

Locality properties depend on boundary conditions and temperature

Reflected in structure of poles and cuts in modular correlator

Part II: New materials for holographic hydrodynamics

- Collaboration between string theorists and condensed matter theorists
- Proposing new materials to test predictions from gauge/gravity duality

Motivation and Overview

- Turbulent hydrodynamics in strongly correlated Kagome metals

  Domenico Di Sante, J. E., Martin Greiter, Ioannis Matthaiakakis, René Meyer, David Rodriguez Fernandez, Ronny Thomale, Erik van Loon, Tim Wehling

  arXiv:1911.06810 [cond-mat], Nat. Comm.

- Proposal for a new Dirac material with stronger electronic coupling than in graphene: Scandium-Herbertsmithite

- in view of enhanced hydrodynamic behaviour of the electrons

  Reaching smaller η/s (ratio of shear viscosity over entropy density)

When phonon and impurity interactions are suppressed,

Electron-electron interactions may lead to a hydrodynamic electron flow

(Small parameter window)

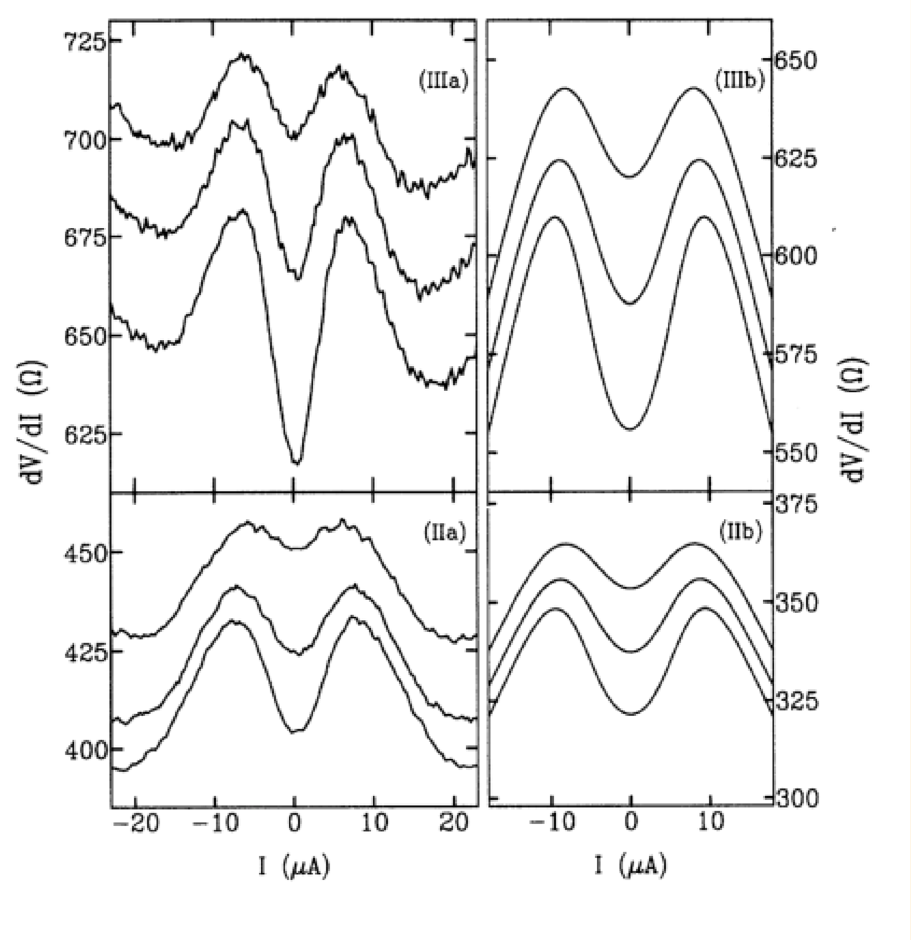
Some Implications:

- Decrease of differential resistance dV/dI with increasing current I

Weak coupling: High mobility wires

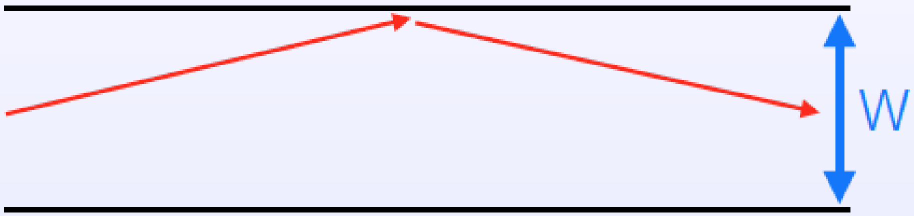
Transition: Knudsen flow $\Rightarrow$ Poiseuille flow   Gurzhi effect

Molenkamp, de Jong Phys. Rev. B 51 (1995) 13389 for GaAs in 2+1 dimensions



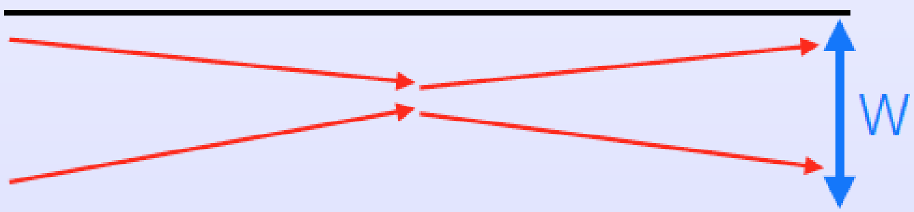
- 2D Electrons in (Al)GaAs Heterostructures

Knudsen flow regime

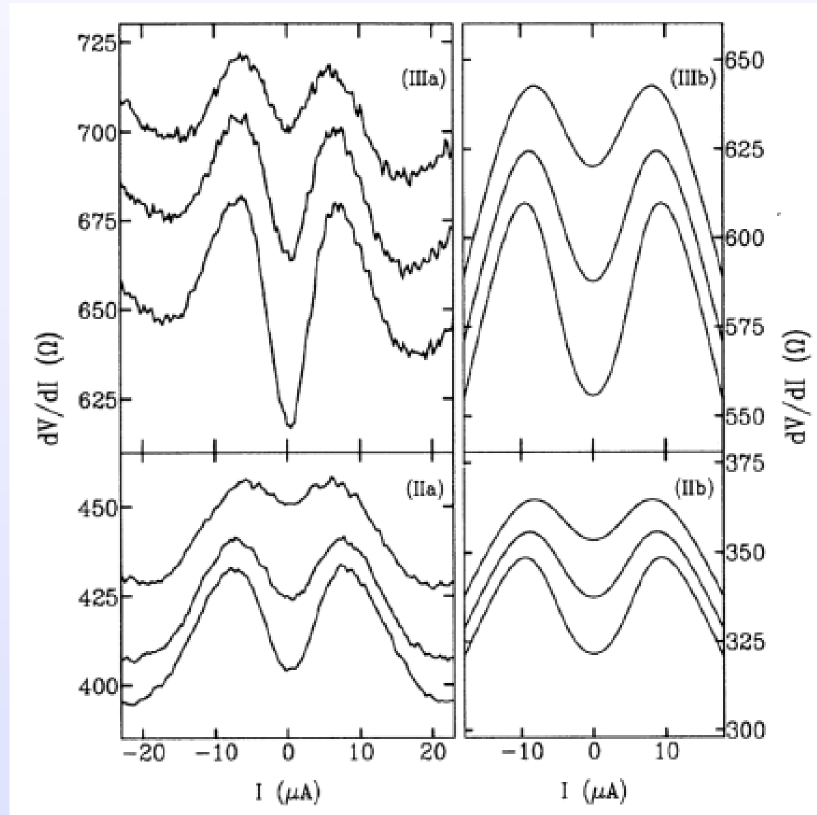


$$\ell_{ee}(T) > W$$

Poiseuille flow regime



$$\ell_{ee}(T) < W$$



[Gurzhi 1968]

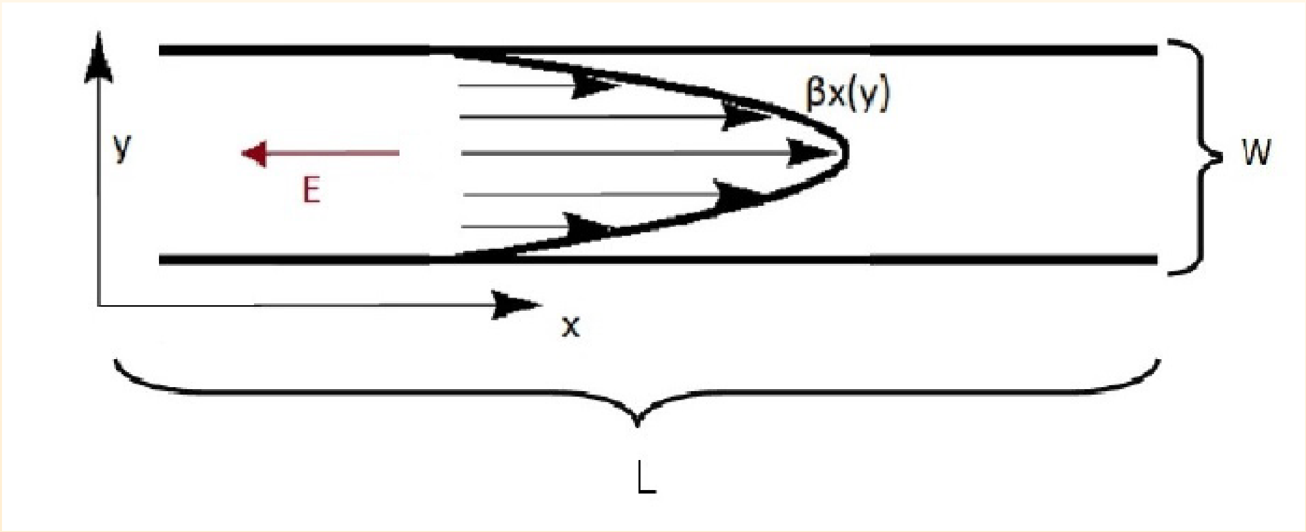
[Molenkamp+de Jong 1994,95]

Conditions for hydrodynamic behaviour of electrons in solids

$$\ell_{ee} < \ell_{\rm imp}, \ell_{\rm phonon}, W$$

ℓ_{ee}: Typical scale for electron-electron scattering

Flow profile in wire



Effective electron-electron coupling strength

$$\alpha_{\text{eff}} = \frac{e^2}{\epsilon_0 \epsilon_r \hbar v_F}$$

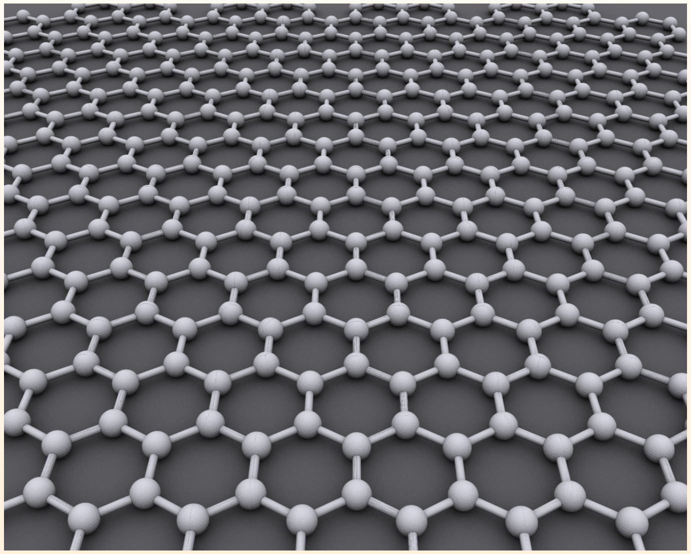
Electron-electron scattering length:

$$\ell_{\text{ee}} \propto \frac{1}{{\alpha_{\text{eff}}}^2}$$

Larger electronic coupling $\Rightarrow$ More robust hydrodynamic behaviour

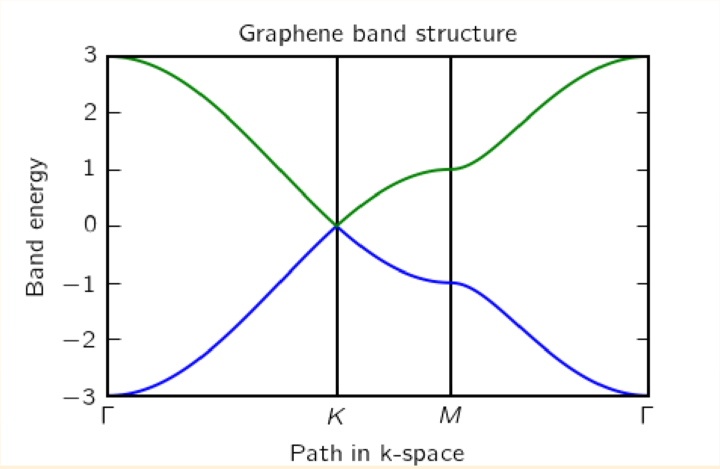
Hydrodynamics in Dirac materials: Graphene

Hexagonal carbon lattice



Source: Wikipedia

Dirac material:
Linear dispersion relation



Considerable theoretical and experimental effort for viscous fluids

Review: Polini + Geim, arXiv:1909.10615

Relativistic hydrodynamics: Expansion in four-velocity derivatives

$$T_{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P\eta^{\mu\nu} - \sigma^{\mu\nu} + \ldots$$

$$\sigma^{\mu\nu} = P^{\mu\alpha}P^{\nu\beta}\left(\eta(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha - \frac{2}{3}\nabla_\gamma u^\gamma \eta_{\alpha\beta}) \; + \; \zeta\nabla_\gamma u^\gamma \eta_{\alpha\beta}\right)$$

Shear viscosity η, bulk viscosity ζ

$P^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu$

Relativistic hydrodynamics: Expansion in four-velocity derivatives

$$T_{\mu\nu} = (\epsilon + P) u^\mu u^\nu + P\eta^{\mu\nu} - \sigma^{\mu\nu} + \dots$$

$$\sigma^{\mu\nu} = P^{\mu\alpha} P^{\nu\beta} \left(\eta (\nabla_\alpha u_\beta + \nabla_\beta u_\alpha - \frac{2}{3} \nabla_\gamma u^\gamma \eta_{\alpha\beta}) \; + \; \zeta \nabla_\gamma u^\gamma \eta_{\alpha\beta} \right)$$

Shear viscosity η, bulk viscosity ζ

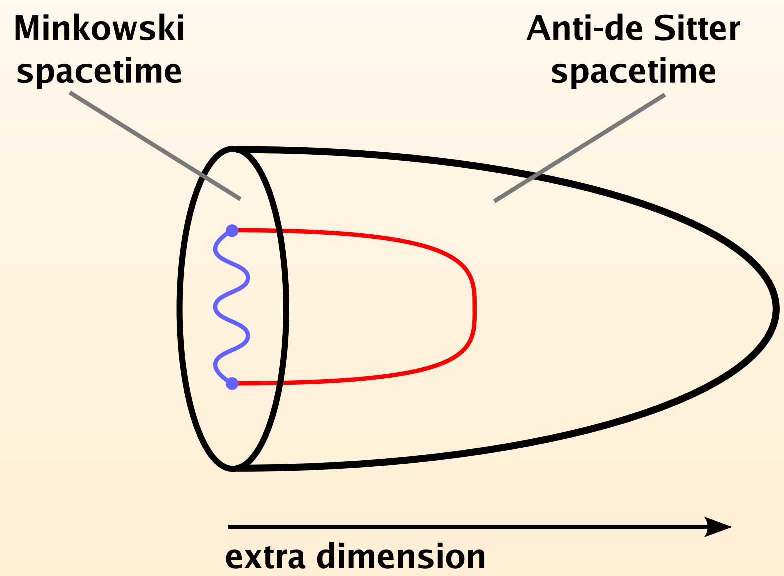
$P^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu$

Shear viscosity for strongly correlated systems may be calculated from gauge/gravity duality!

Gauge/Gravity Duality: Bulk-boundary correspondence

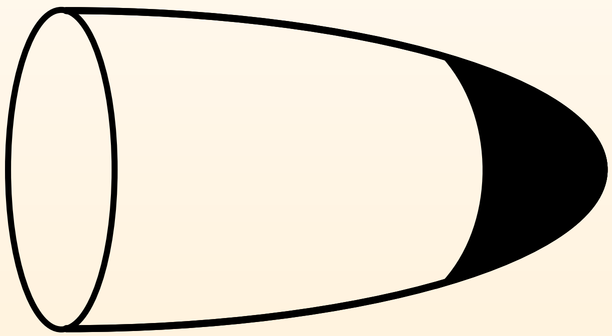
Quantum observables at the boundary of the curved space

may be calculated from propagation through curved space



Quantum theory at finite temperature:

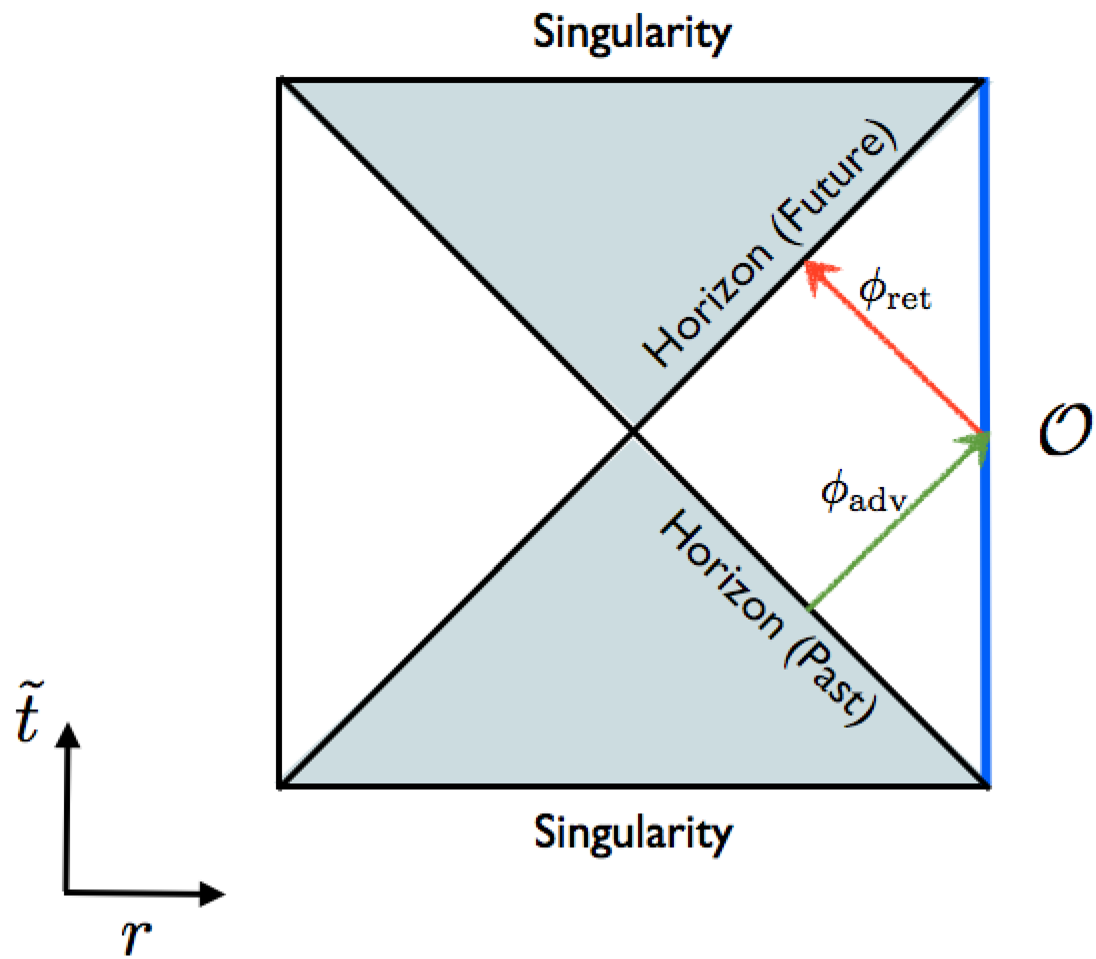
Dual to gravity theory with black hole (in Anti-de Sitter space)



Hawking temperature identified with temperature in the dual field theory

Retarded Green's Functions in Strongly Coupled Systems

Anti-de Sitter
black hole



Retarded Green's function: $$G^R_{\mathcal{O}_A \mathcal{O}_B} = \left.\frac{\delta \langle \mathcal{O}_A \rangle}{\delta \phi_{B(0)}}\right|_{\delta\phi=0} = \frac{\delta \phi_{A(1)}}{\delta \phi_{B(0)}}$$

subject to infalling boundary condition at horizon

Holographic calculation of shear viscosity Kovtun, Son, Starinets 2004

- Energy-momentum tensor $T_{\mu\nu}$ dual to graviton $g^{\mu\nu}$
- Calculate correlation function $\langle T_{xy}(x_1) T_{xy}(x_2) \rangle$ from propagation through black hole space
- Shear viscosity is obtained from Kubo formula:

$$\eta = -\lim \frac{1}{\omega} \operatorname{Im} G^R_{xy,xy}(\omega)$$

- Shear viscosity $\eta = \pi N^2 T^3/8$, entropy density $s = \pi^2 N^2 T^3/2$

$$\frac{\eta}{s} = \frac{1}{4\pi} \frac{\hbar}{k_B}$$

(Note: Quantum critical system: $\tau = \hbar/(k_B T)$)

Holographic hydrodynamics

Holography: From propagation of graviton in dual gravity subject to

$$S_{E-H} = \int d^{d+1}x\sqrt{-g}\,(R - 2\Lambda)$$

For $SU(N)$ gauge theory at infinite coupling, $N \to \infty$, $\lambda = g^2 N \to \infty$:

$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$

Holography: From propagation of graviton in dual gravity subject to

$$S_{E-H} = \int d^{d+1}x \sqrt{-g}\,(R - 2\Lambda)$$

For $SU(N)$ gauge theory at infinite coupling, $N \to \infty$, $\lambda = g^2 N \to \infty$:

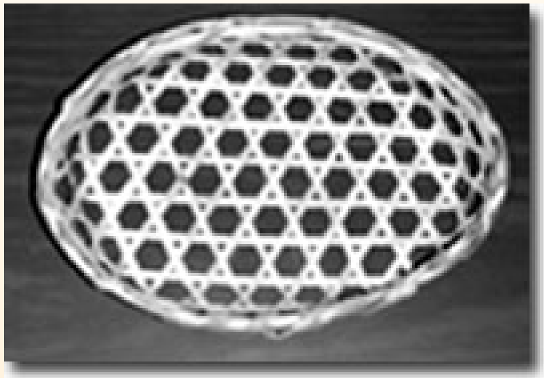
$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$

Leading correction in the inverse 't Hooft coupling $\propto \lambda^{-3/2}$

From R^4 terms contributing to the gravity action

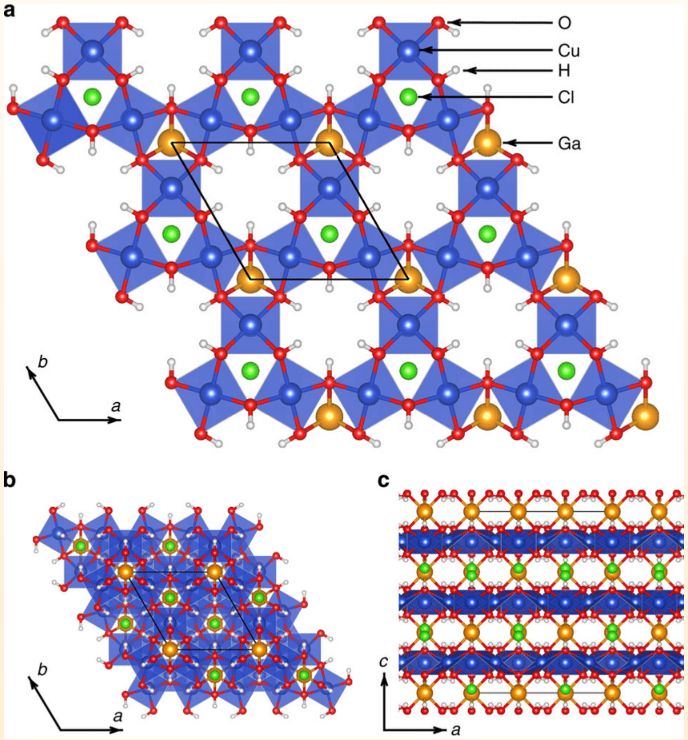
Kagome materials

Kagome: Japanese basket weaving pattern



Source: Wikipedia

Hexagonal lattice



Source: Nature

Herbertsmithite: $\mathrm{ZnCu_3(OH)_6Cl_2}$

Source: Wikipedia

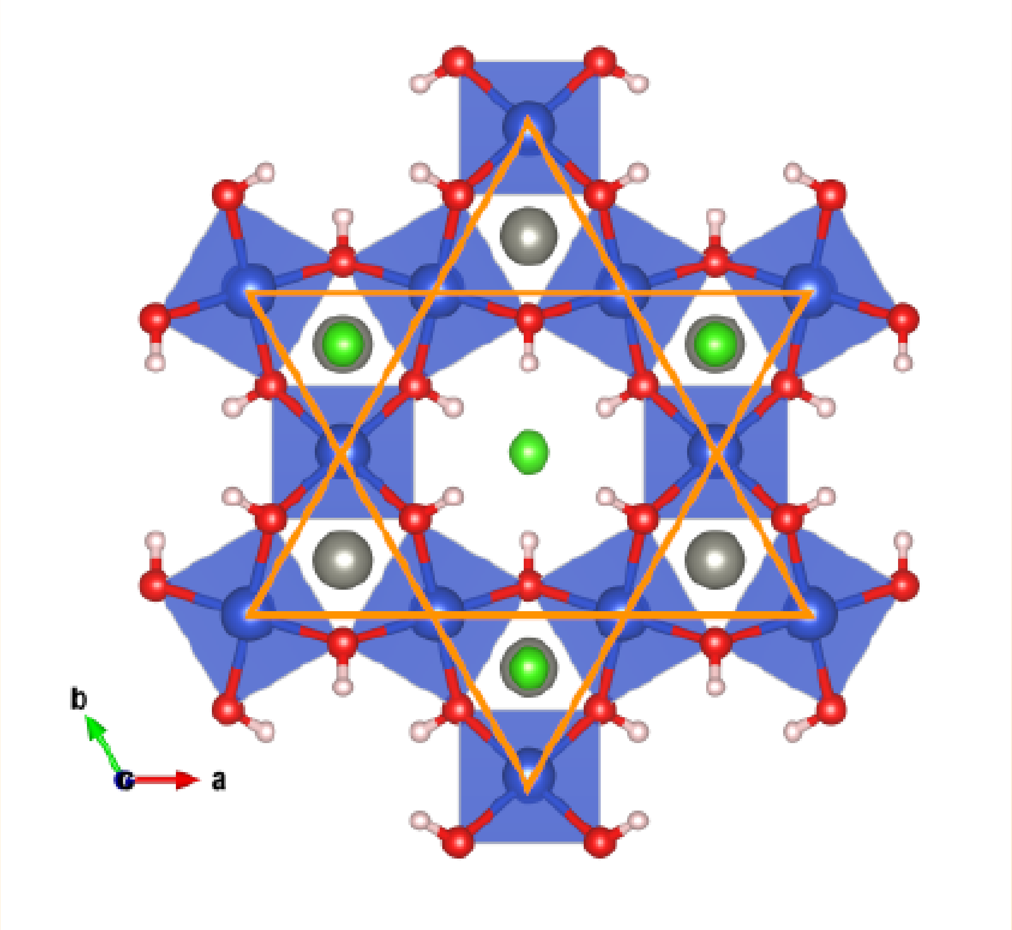
Original Herbertsmithite has Zn^{2+}

Fermi surface below Dirac point

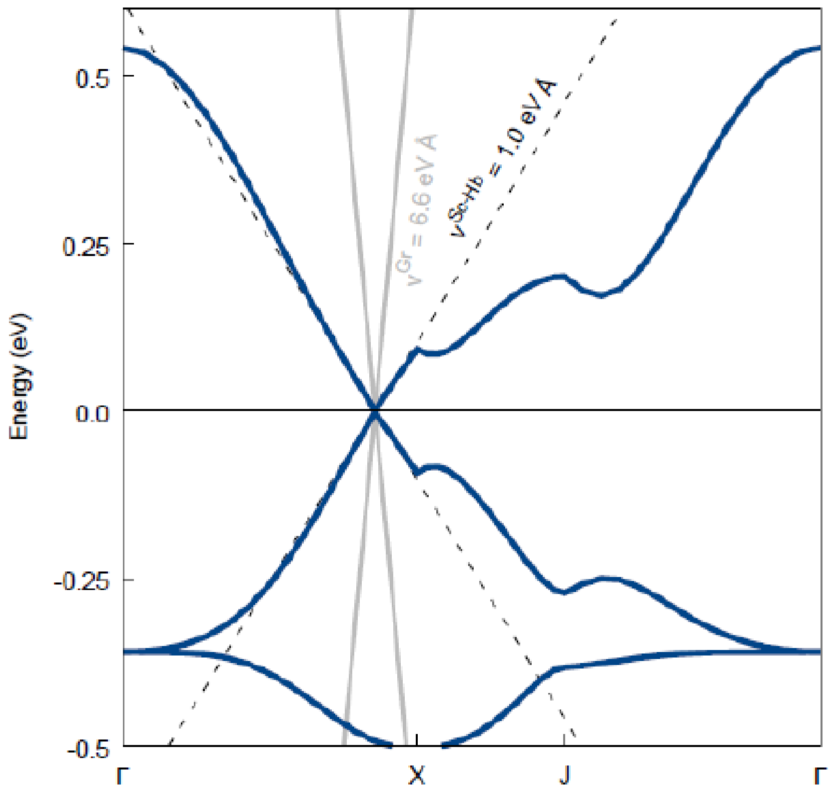
Idea: Replace Zinc by Scandium, Sc^{3+}

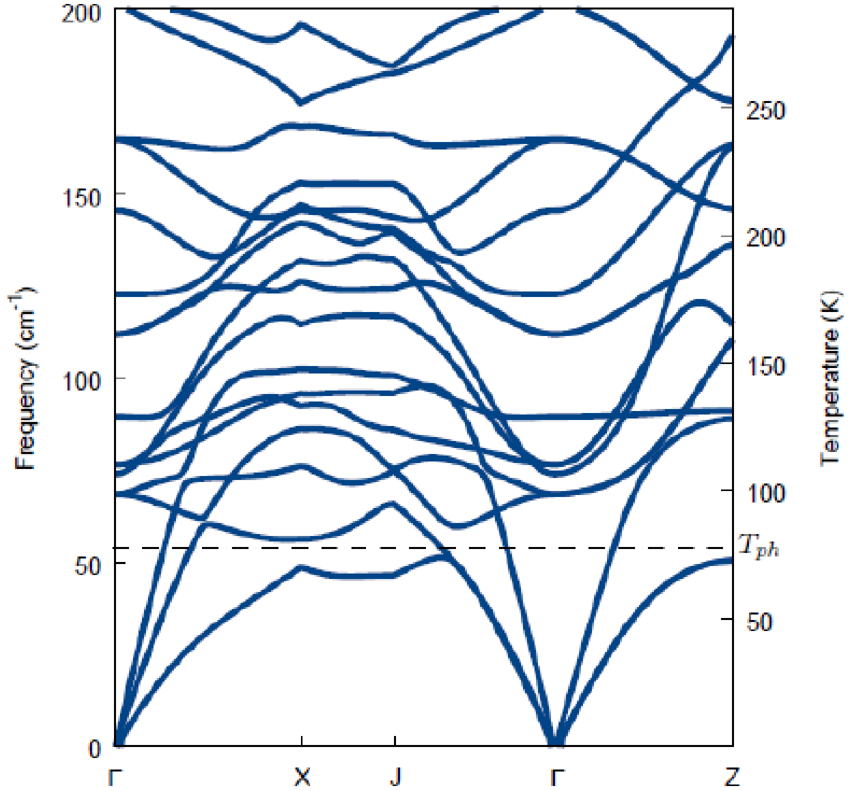
Places Fermi surface exactly at Dirac point

Scandium-Herbertsmithite



Scandium-Herbertsmithite





Band structure

Phonon dispersion

Scandium-Herbertsmithite

- CuO_4 plaquettes form Kagome lattice
- Low-energy physics captured by $d_{x^2-y^2}$ orbital at each Cu site
- Fermi level is at Dirac point (filling fraction $n = 4/3$)
- Orbital hybridization allows for larger Coulomb interaction (confirmed by cRPA calculation)
- Prediction: $\alpha^{\mathrm{Sc-Hb}} = 2.9$ versus $\alpha^{\mathrm{Graphene}} = 0.9$
- Optical phonons are thermally activated only for temperatures above $T = 80$K
- Enhanced hydrodynamic behaviour: $\ell_{ee}^{\mathrm{Sc-Hb}} = \frac{1}{6}\ell_{ee}^{\mathrm{graphene}}$
- Candidate to test universal predictions from holography

Weak coupling : Kinetic theory

$$\frac{\eta}{s} \propto \frac{1}{\alpha^2}$$

Strong coupling: Holography

Take correction

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}\left(1 + \frac{\mathcal{C}}{\alpha^{3/2}}\right)$$

Vary $\mathcal{C}$ from 0.0005 to 2

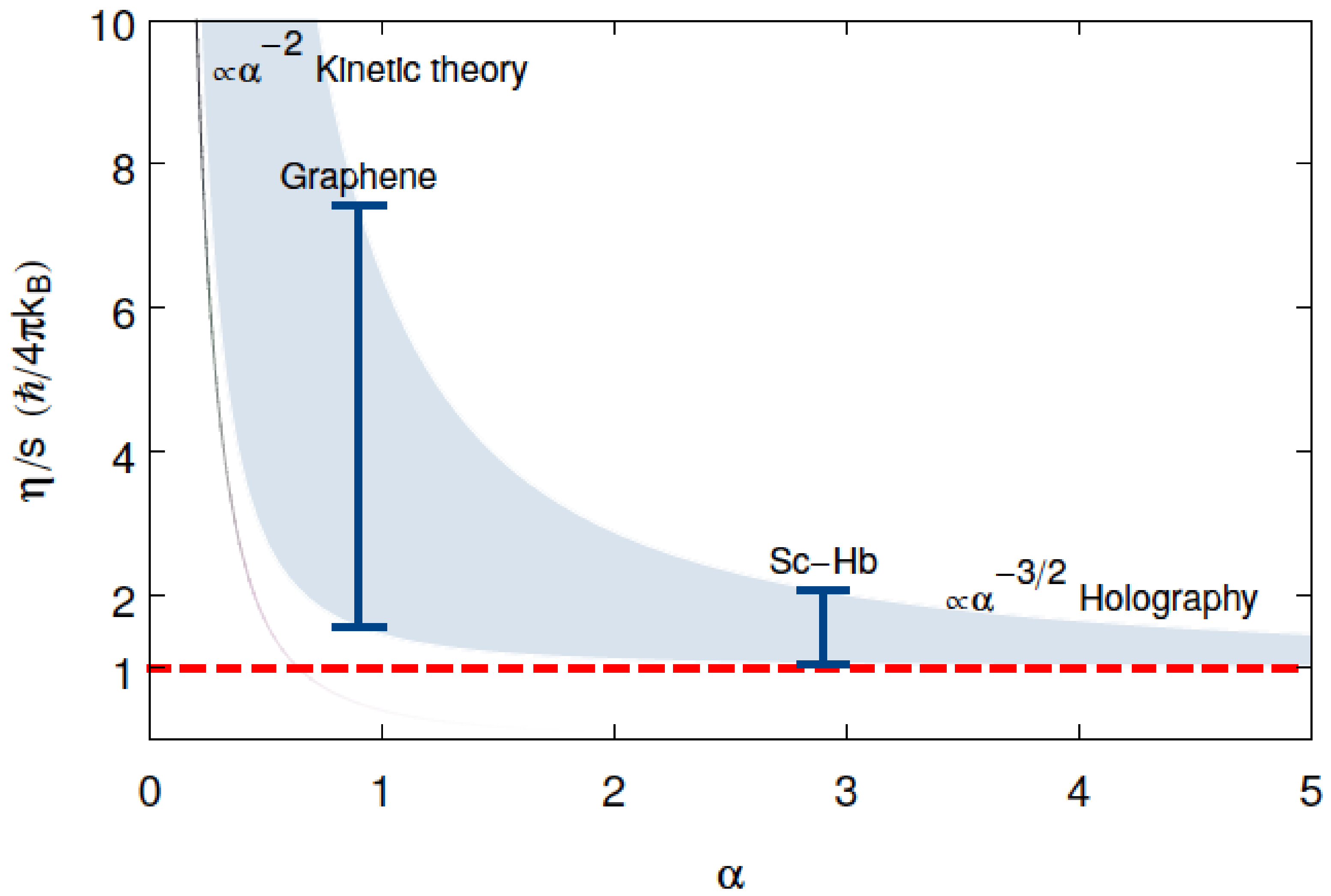
Estimate of the Shear viscosity

AdS gravity computation: Corrections of higher order in the curvature

$$S = S_{E-H} + \int \sqrt{-g}\,\left(\gamma_2 R^2 + \gamma_3 R^3 + \gamma_4 R^4 + \ldots\right)$$

- R^2 term is topological for bulk theory in $d = 4$
- R^3 terms absent in type II supergravity parent theories
- R^4 term: Coefficient $\mathcal{O}(\lambda^{-3/2})$
- $$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}\left(1 + \frac{\mathcal{C}}{\alpha^{3/2}}\right)$$
- R^4 correction is model-dependent.
  We parametrize this by varying the coefficient $\mathcal{C}$

Estimate of the Shear viscosity



Estimate of the Reynolds number

$$\mathrm{Re} = \left(\frac{\eta}{s}\frac{k_B}{\hbar}\right)^{-1} \frac{k_B T}{\hbar v_F} \frac{u_{\mathrm{typ}}(\eta/s)}{v_F} W$$

u_{typ} typical velocity, enhanced at strong coupling

Navier-Stokes equation:

$$\frac{d\bar{v}}{dt} = -\nabla P + \frac{1}{\mathrm{Re}}\nabla^2 \bar{v} + f$$

Turbulence: Reynolds number must be $\mathcal{O}(1000)$

In Sc-Hb, factor 100 larger than in graphene

Conclusion and outlook

- Explicit expressions for modular flow of free fermion theories
- Non-locality explicitly realized
- Scandium-substituted Herbertsmithite has predicted coupling $\alpha_{\text{eff}} = 2.9$
- Factor 3.2 larger than Graphene
- May reach region of robust hydrodynamics in solids
- Smaller ratio of η/s - parameter region where gauge/gravity duality applies