Novel (Quantum) Computational Methods for Quantum Field Theories

Michael Spannowsky

IPPP, Durham University

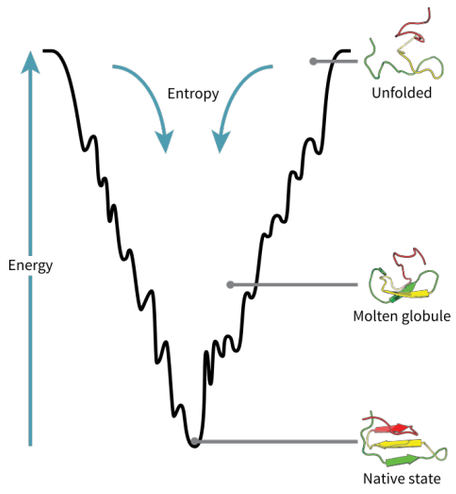
with S. Abel, N. Chancellor - 2003.07374

with S. Abel - 2006.06003

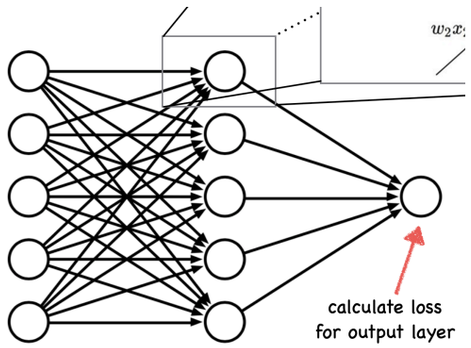
Outline of the talk

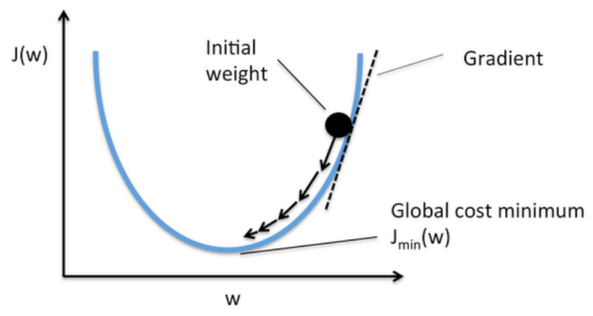
Prelude

2 observations

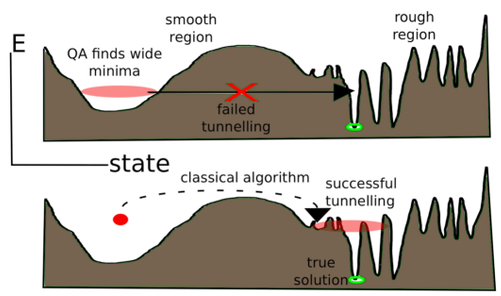


Classical





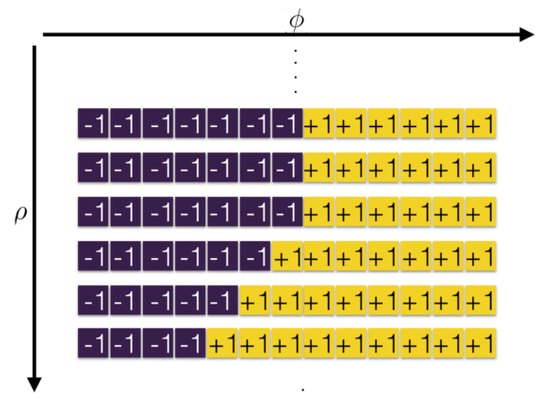
Hybrid



$$\mathcal{H}_{\mathrm{QA}}(t) = \sum_i \sum_j J_{ij}\sigma_i^Z\sigma_j^Z + \sum_i h_i\sigma_i^Z + \Delta(t)\sum_i \sigma_i^X$$

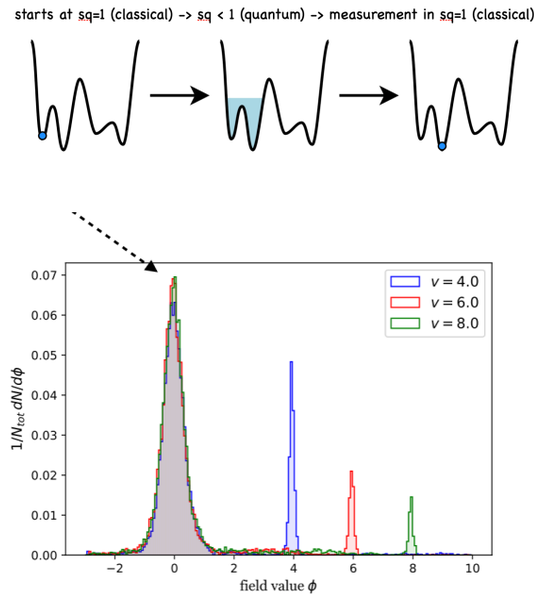
final Hamiltonian (encodes actual problem)

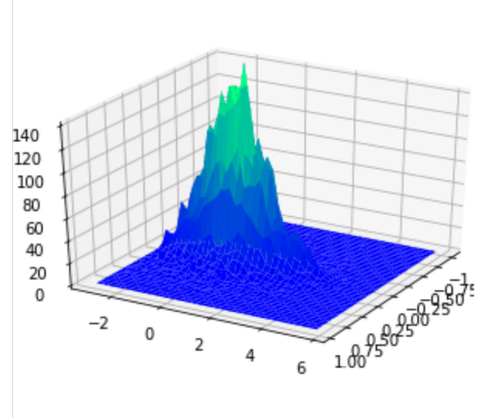
initial Hamiltonian (ground state = superposition of qubits with 0 and 1)



Quantum

starts at sq=1 (classical) -> sq < 1 (quantum) -> measurement in sq=1 (classical)





What is a calculation and how do we perform them?

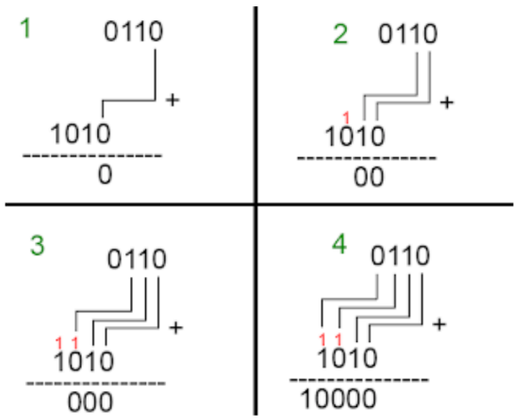
Example

convert into different representation

6+10 → Binary

```
  0110
+ 1010
```

→ define algebraic procedure on new representation



→ obtain result in new representation

10000 → convert back

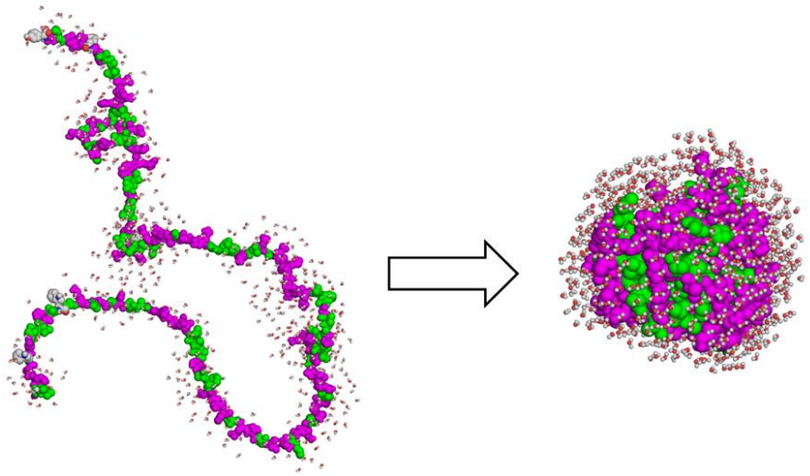
16

- Note, final solution manifold consists of 2^5 states

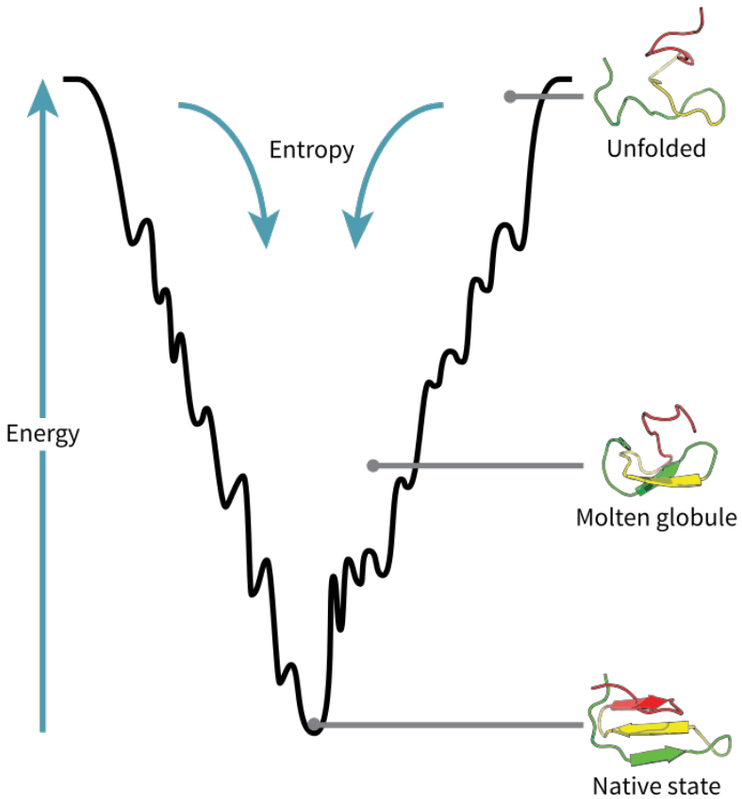
- However, convergence to 'correct' result very fast, as algorithm provides most direct path to solution state

- Puzzling: For this example, algebraic operation on original representation much simpler for human mind.

Protein-folding and Levinthal's Paradox



Unfolded Folded

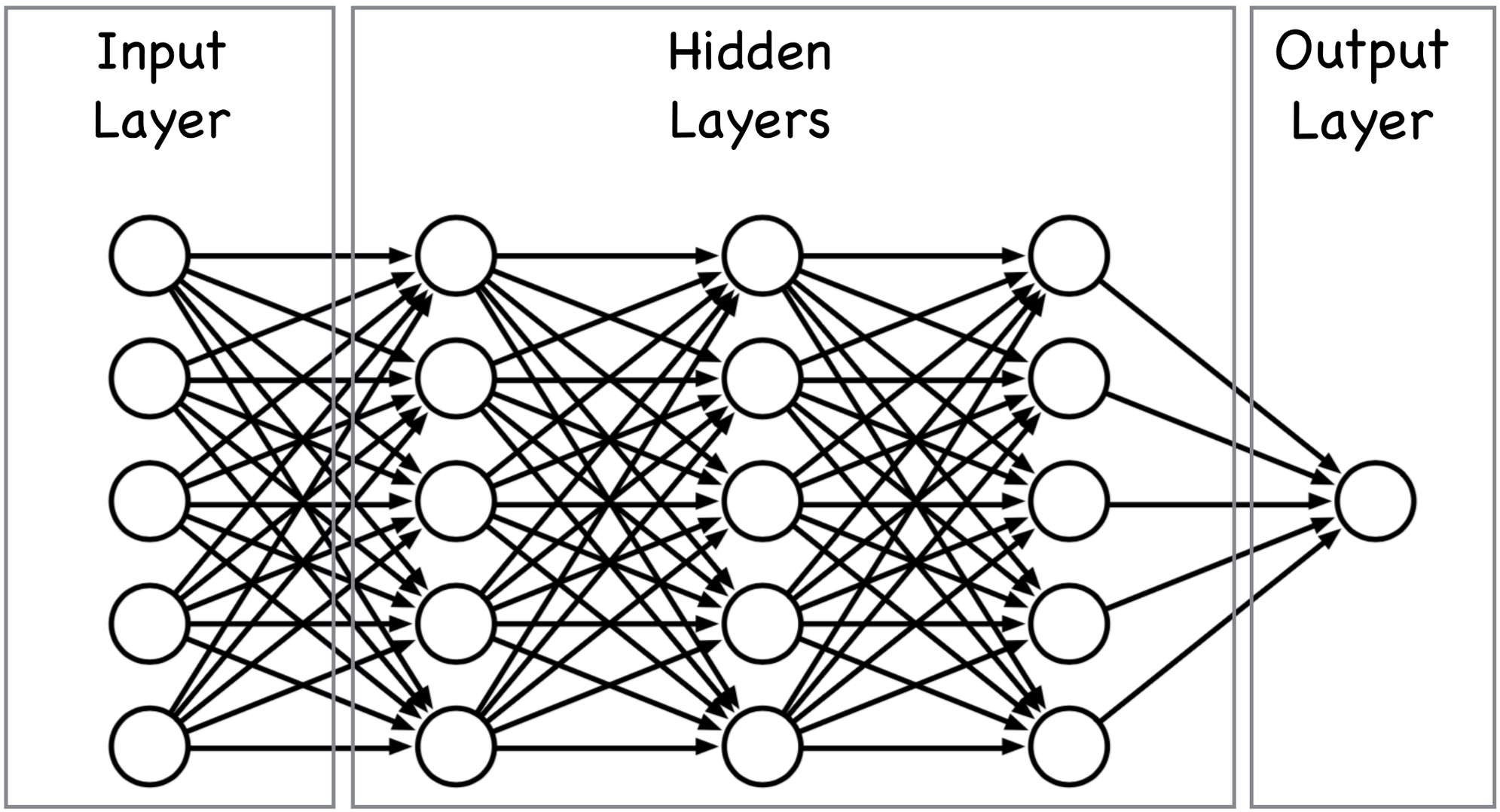


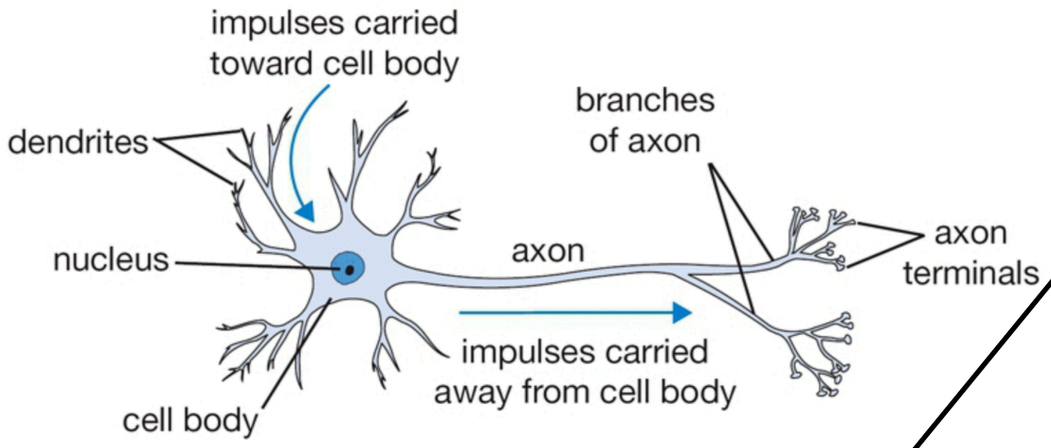
- Elongated proteins fold to same state within microseconds
- Some proteins have 3^{300} conformations
- Levinthal's Paradox (1969):
  Sequential sampling of states would take longer than lifetime of Universe (even if only nanoseconds per state spent)
- Solution: No sequential sampling, but rapid descend into the potential minimum. In proteins due to protein folding intermediates

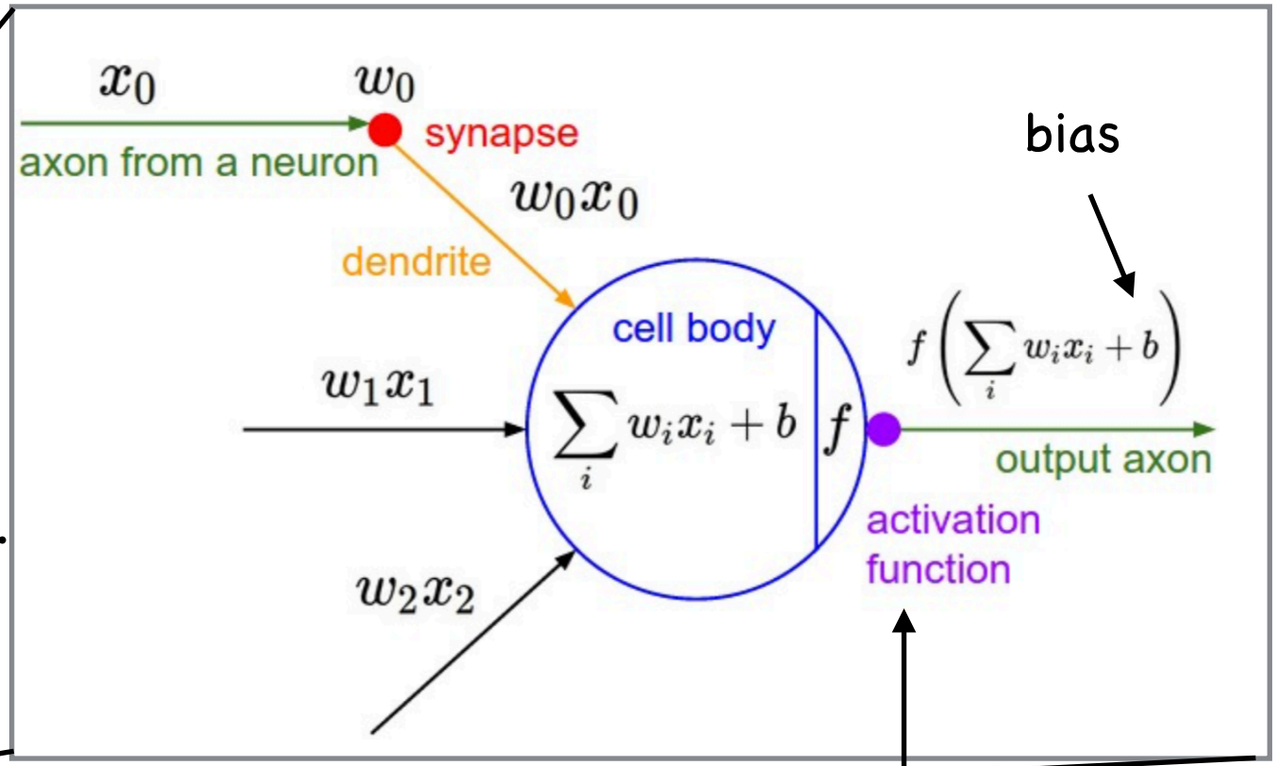
→ Solution of mathematical problem can be found quickly if encoded in ground state of complex system

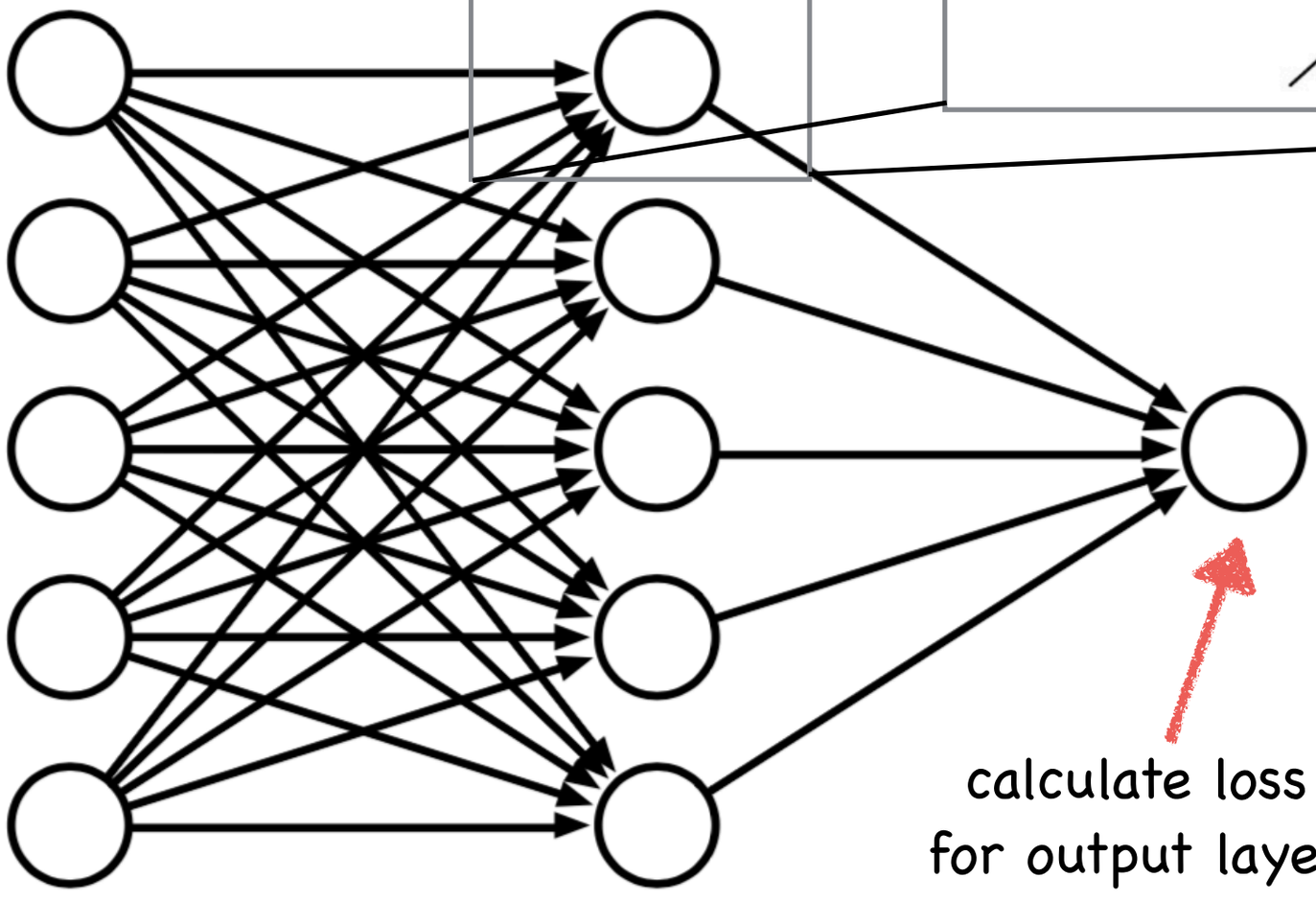
The classical (NN) approach

- Let's construct a complex system to optimise
- NN used in various ways. It is foremost a self-adaptive optimisation algorithm





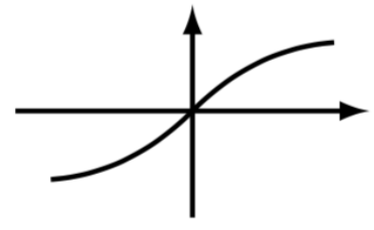




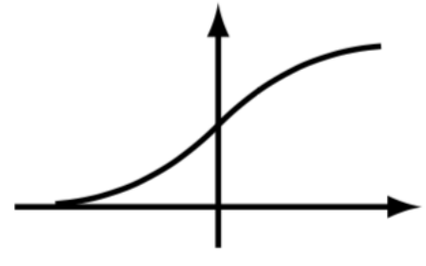
calculate loss
for output layer

Many options for
activation function f, e.g.

tanh



sigmoid



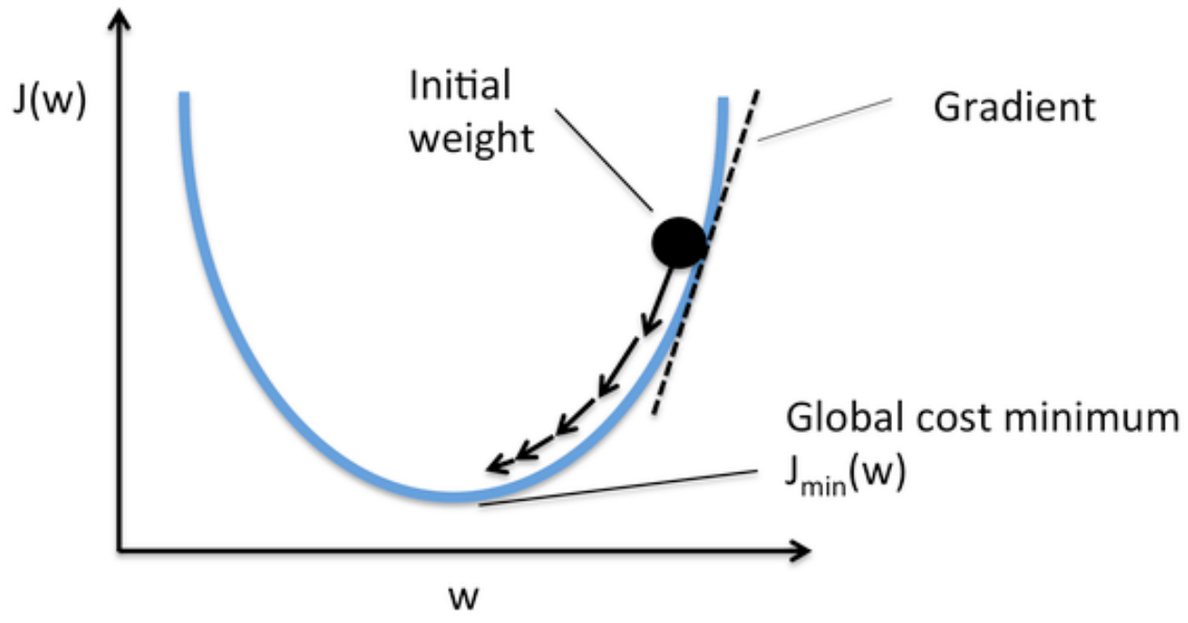
Gradient descent

- After forward propagation, ie. establishing the weights for all nodes (including the output node), we evaluate the **loss function**, to establish the error we are making.

  Loss function = difference between predicted and true function,

  e.g. $E(y, y') = \frac{1}{2}|y - y'|^2$

- Gradient descent: change network such, that you move towards the error minimum.
- Compute gradient -> get direction towards error minimum.



Universal approximation theorem: [Hornik et al '89] [Hornik '91]

feed-forward network with a single hidden layer containing a finite number of neurons can approximate continuous functions on compact subsets of R^n, under mild assumptions on the activation function.

Expression of the output of a neural net:

$$N_m(\vec{x}, \{w, \vec{b}\}) = \sum_{k,n} w^f_{mk} g(w^h_{kn} x_n + b^h_k) + b^f_m$$

final — hidden — m outputs — single hidden layer with k units — n inputs

[Piscopo, MS, Waite 1902.05563]

Our method can solve any equation that can be cast as an **optimisation problem**, i.e. can be brought into the form

$$\mathcal{F}_m(\vec{x}, \phi_m(\vec{x}), \nabla\phi_m(\vec{x}), \cdots, \nabla^j\phi_m(\vec{x})) = 0$$

Application to differential equations

(but used also for differo-integral equations)

Build the full function, here a DE into the loss function, incl boundary conditions

$$\mathcal{L}(\{w,\vec{b}\}) = \frac{1}{i_{\max}} \sum_{i,m} \hat{\mathcal{F}}_m(\vec{x}^i, \hat{\phi}_m(\vec{x}^i), \cdots, \nabla^j \hat{\phi}_m(\vec{x}^i))^2$$

$$+ \sum_{\text{B.C.}} (\nabla^p \hat{\phi}_m(\vec{x}_b) - K(\vec{x}_b))^2 \ ,$$

identify trial solution with network output $\hat{\phi}_m(\vec{x}) \equiv \breve{N}_m(\vec{x}, \{w,\vec{b}\})$

Build the full function, here a DE into the loss function, incl. boundary conditions

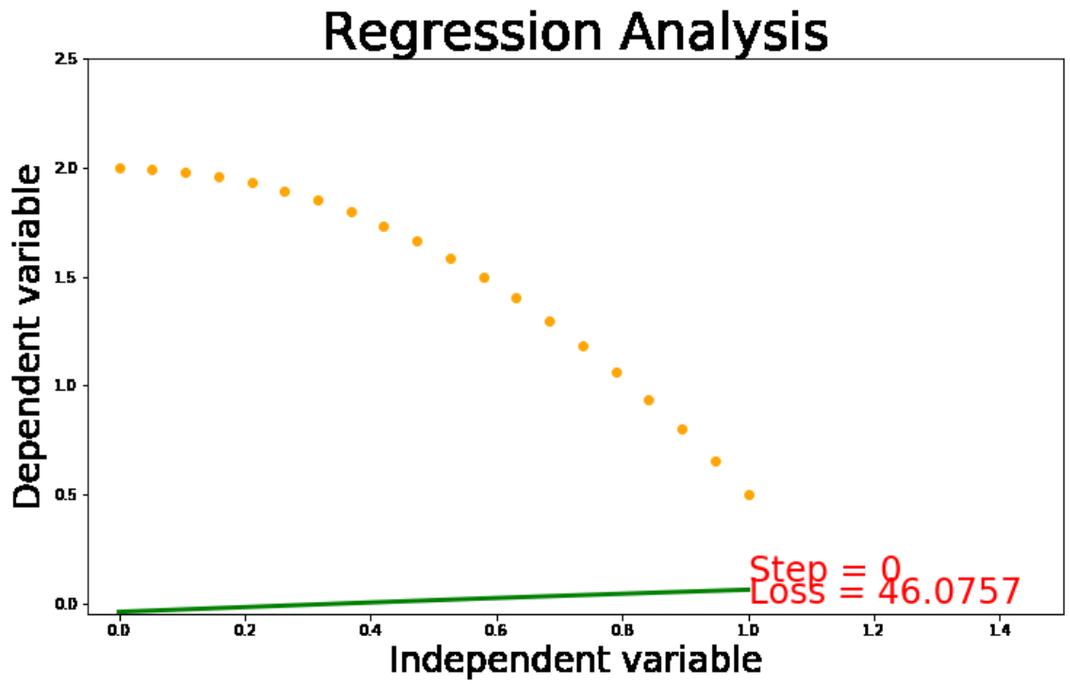
- Each part of the NN is differentiable (activation functions are chosen such)
- Hyperparameters can be optimised for speed of convergence and accuracy: activation function, nr hidden layers, width of NN, learning rate, epochs...

Applied to solution of simple differential equation

$$x\frac{d\phi(x)}{dx} = -3x^2$$

$$\phi(0) = 2$$

solution $\phi(x) = (4 - 3x^2)/2$



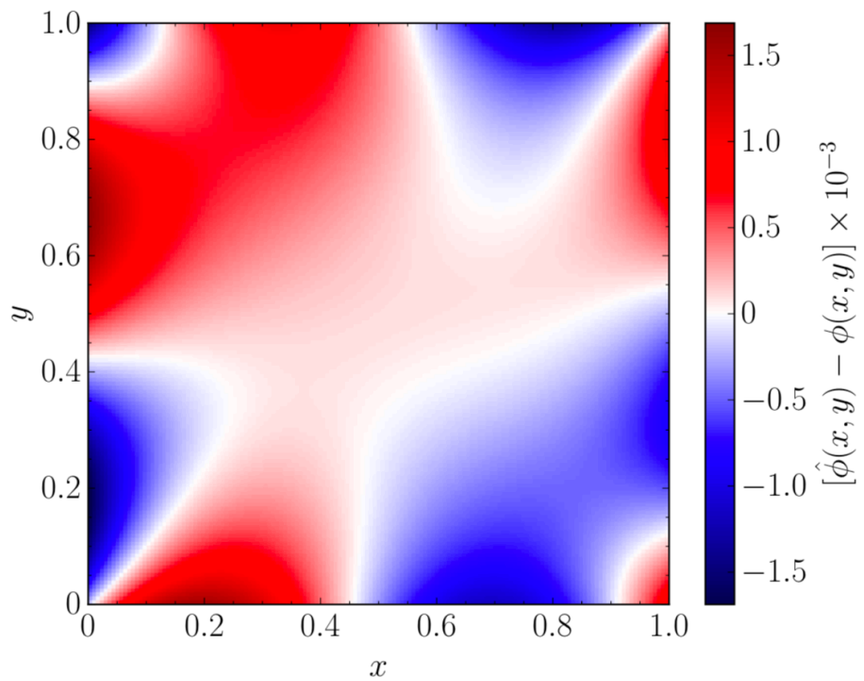
Differential equation:

$$\nabla^2\phi - e^{-x}(x - 2 + y^3 + 6y) = 0$$

boundary conditions

$$\phi(0, y) = y^3 \ , \quad \phi(1, y) = (1 + y^3)e^{-1} \ ,$$
$$\phi(x, 0) = xe^{-x} \ , \quad \phi(x, 1) = e^{-x}(x + 1) \ ,$$

domain $(x, y) \in [0, 1] \times [0, 1]$



Semiclassical calculations for bubbles and phase transitions

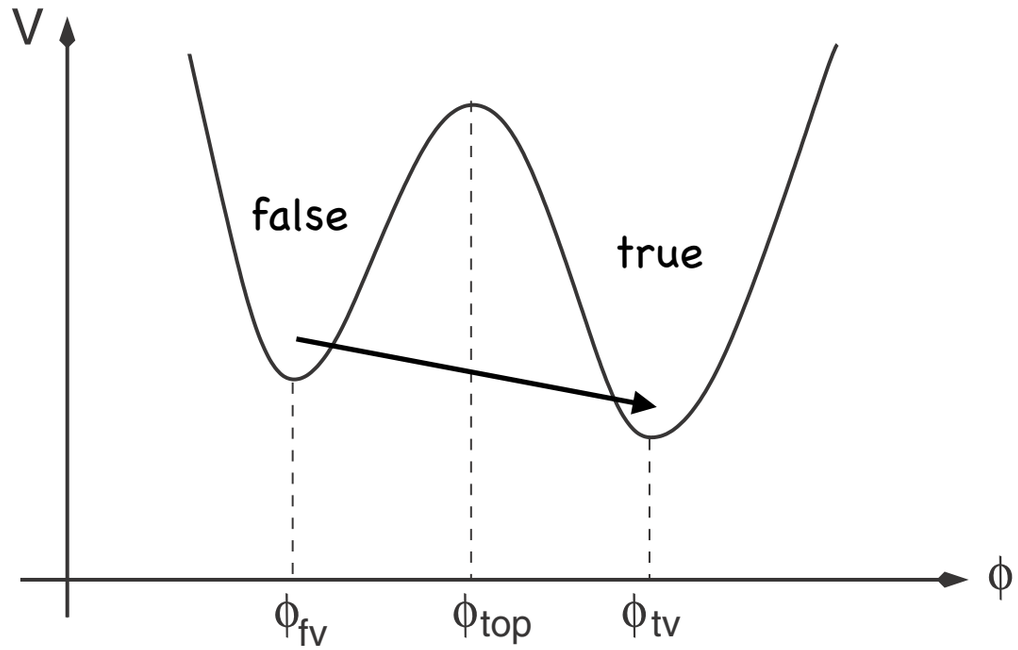
Need to find stationary points of Euclidean action:

Potential energy

$$S = \int d\tau\, d^3x \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial \tau} \right)^2 + \frac{1}{2} (\boldsymbol{\nabla} \phi)^2 + V(\phi) \right]$$

Transition from false to true via tunnelling. Bubble can nucleate anywhere, with nucleation rate per unit volume:

$$\frac{\Gamma}{\mathcal{V}} = A e^{-B} \qquad B = S(\phi_b) - S(\phi_{\rm fv})$$

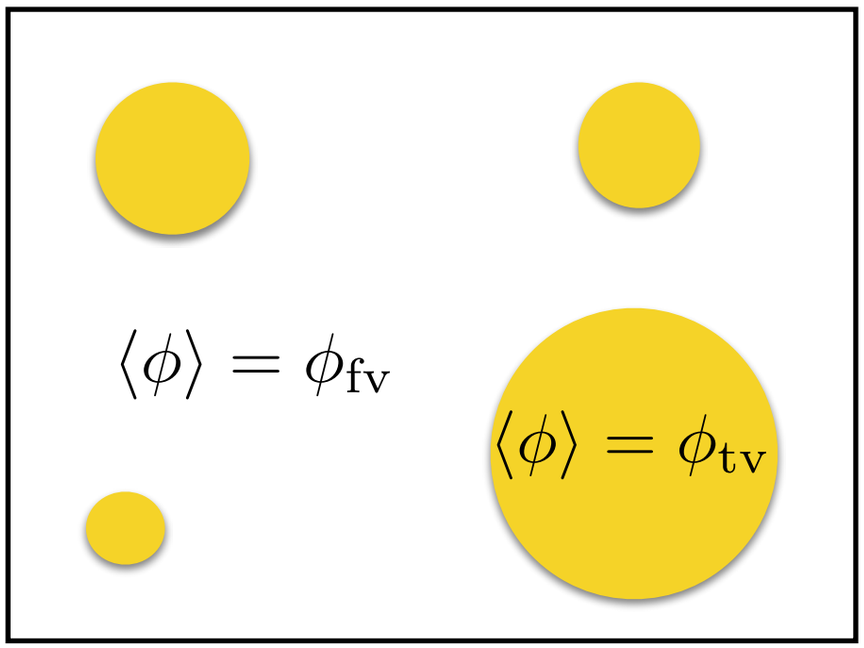


Growth of bubble via classical equation of motion

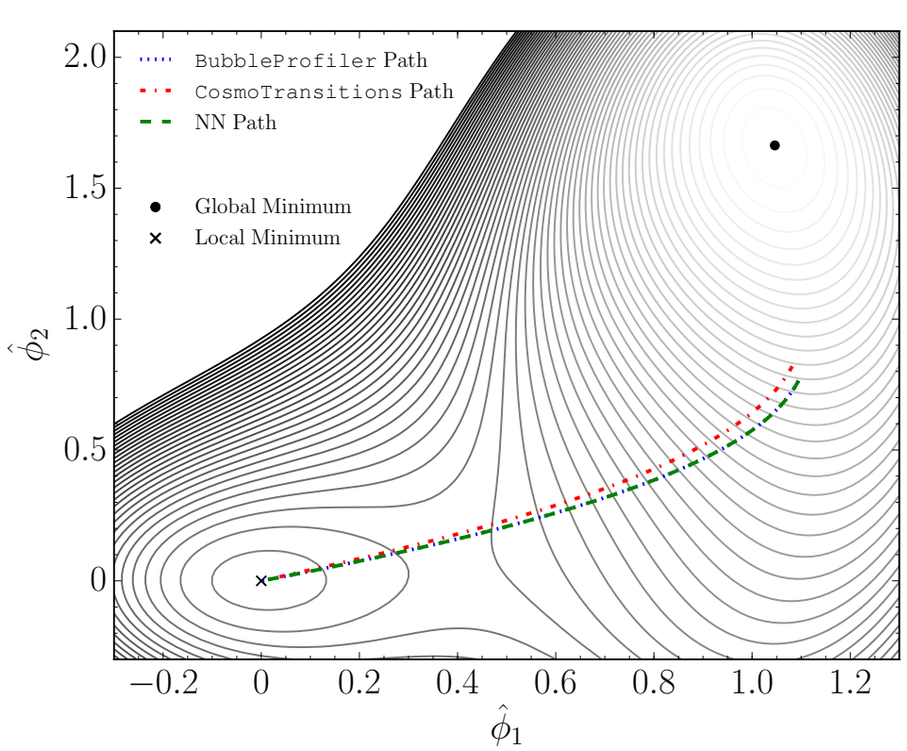
$$\left(-\frac{\partial^2}{\partial t^2} + \nabla^2 \right) \phi = \frac{\partial}{\partial \phi} V(\phi)$$

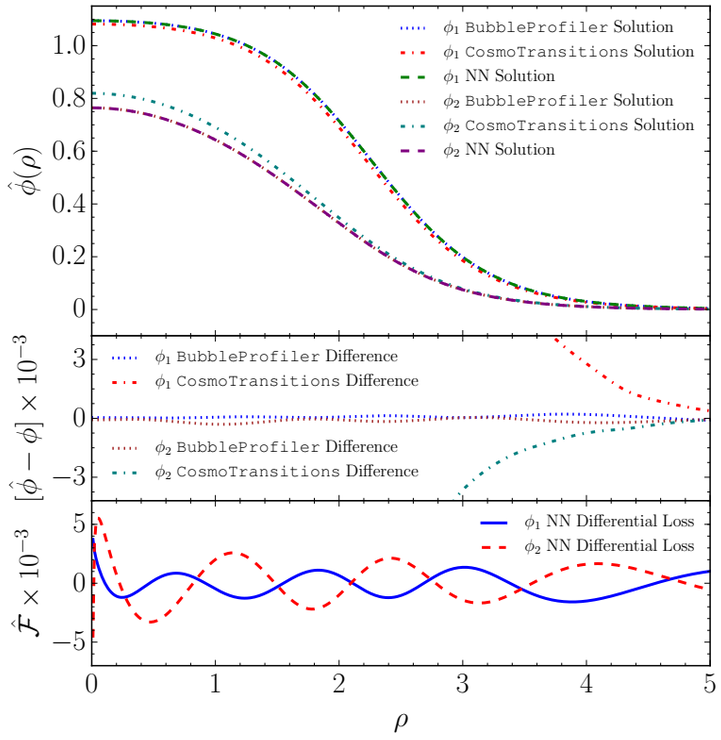
Methods to calculate bubble nucleation:

- Thin-wall approximation
- Polygon approximation [Guada, Maiezza, Nemevsek '18]
- over/undershoot method
- **Neural-Net approach** [Piscopo, MS, Waite '19]



Can be used for various other examples, flexible and powerful





[Piscopo, MS, Waite '19]

- NN-approach often more reliable and more precise than dedicated PT solver, e.g. CosmoTransitions, BubbleProfiler
- $\hat{\mathcal{F}}$ is measure for quality of the solution
- Many applications for phase transitions and beyond

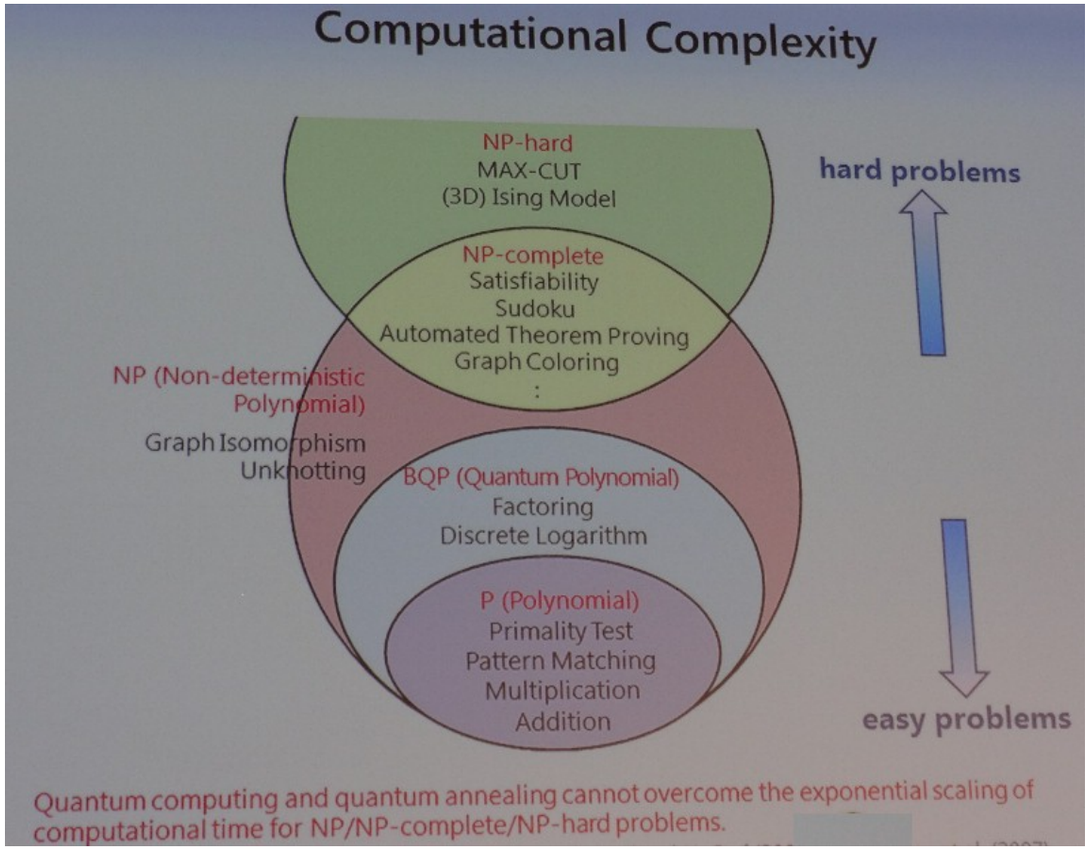
see [Balaji, MS, Tamarit 2010.08013]

The Quantum Computing approach

Why quantum computing for computations?

- We know that in theory quantum computers can perform certain tasks much faster than classical computers/algorithms,
  e.g. search of unstructured database with N entries in a time prop. to sqrt(N)

  → classically only random guess or exhaust search



P - solvable, deterministically in polynomial time

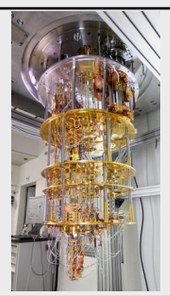
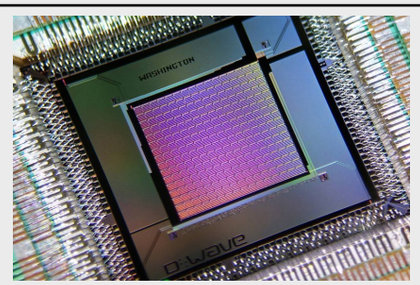
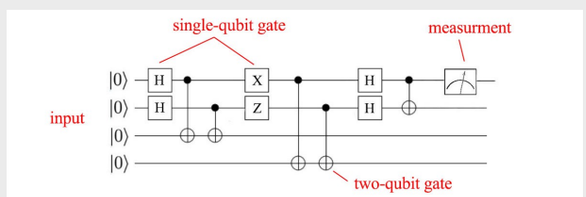
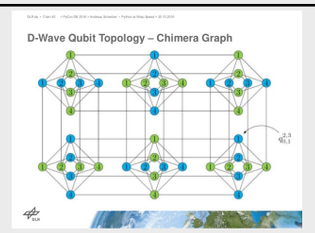
NP - non-deterministic polynomial.
Solutions verifiable in polynomial time

→ Not the defining question for us
We want use QC as quantum lab for FT

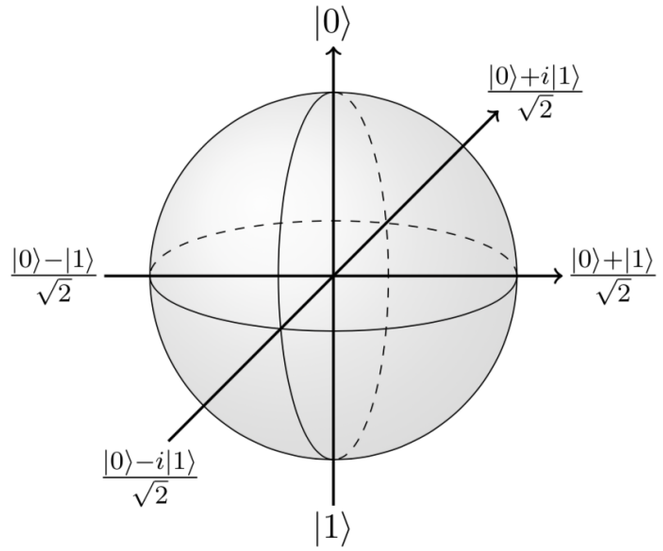
- But how do we use real imperfect, quantum machines to solve problems particle physicists care about?

The Quantum Computing approach

Quantum computing has long and distinguished history but is only now becoming practicable.

Type	Discrete Gate	Quantum Annealer
Property	Universal (any quantum algorithm can be expressed)	Not universal — certain quantum systems
How?	IBM - Qiskit ~ 50 Qubits	DWave - LEAP ~5000 Qubits
What?		
		

- Both QC systems operate on the Bloch sphere:



  measuring $\sigma_i^Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

  where $(\sigma^Z|0\rangle = |0\rangle,\ \sigma^Z|1\rangle = -|1\rangle)$ are the possible eigenvector equations.

- each i represents a single qubit

- A discrete quantum gate system is good for looking at things like entanglement, crypotgraphy, Shor's, Grover's algorithms, etc.

- Quantum annealer is good for network problems, but it is also more natural tool to think about field theory. It is based on the general transverse field Ising model:

[Kadowaki, Nishimori]

$$\mathcal{H}_{\mathrm{QA}}(t) = \sum_i \sum_j J_{ij}\sigma_i^Z\sigma_j^Z + \sum_i h_i\sigma_i^Z + \Delta(t)\sum_i \sigma_i^X$$

- What does the "anneal" mean?

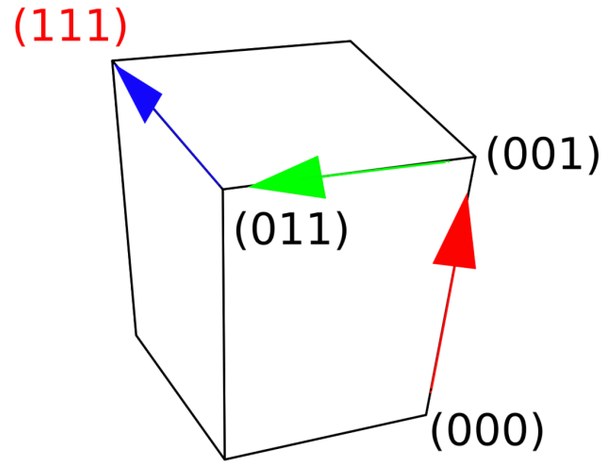
$$\mathcal{H}_{\mathrm{QA}}(t) = \sum_i \sum_j J_{ij}\sigma_i^Z\sigma_j^Z + \sum_i h_i\sigma_i^Z + \Delta(t)\sum_i \sigma_i^X$$

final Hamiltonian (encodes actual problem)

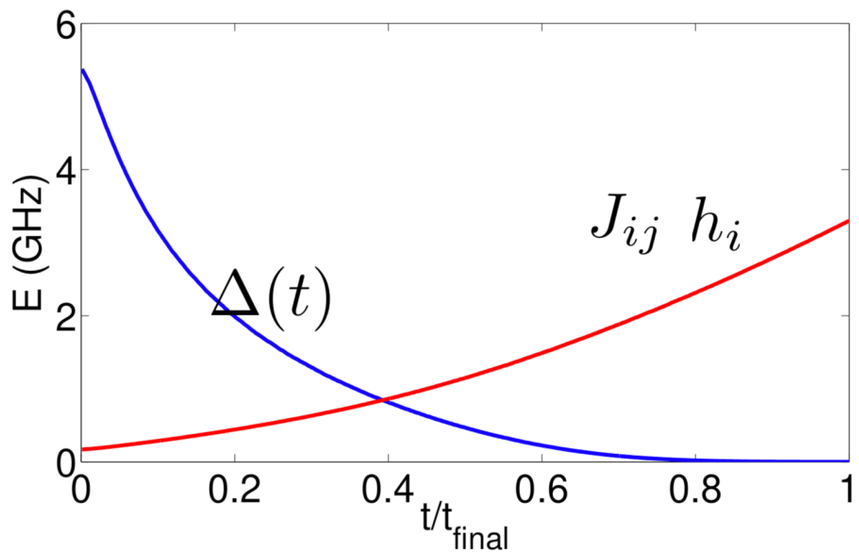
initial Hamiltonian (ground state = superposition of qubits with 0 and 1)

$\Delta(t)$ induces bit-hopping in the Hamming/Hilbert space

- Anneal idea: transition from ground state of initial Hamiltonian into ground state of problem Hamiltonian



- The idea is to dial this parameter to land in the global minimum (i.e. the solution) of some "problem space" described by J, h:

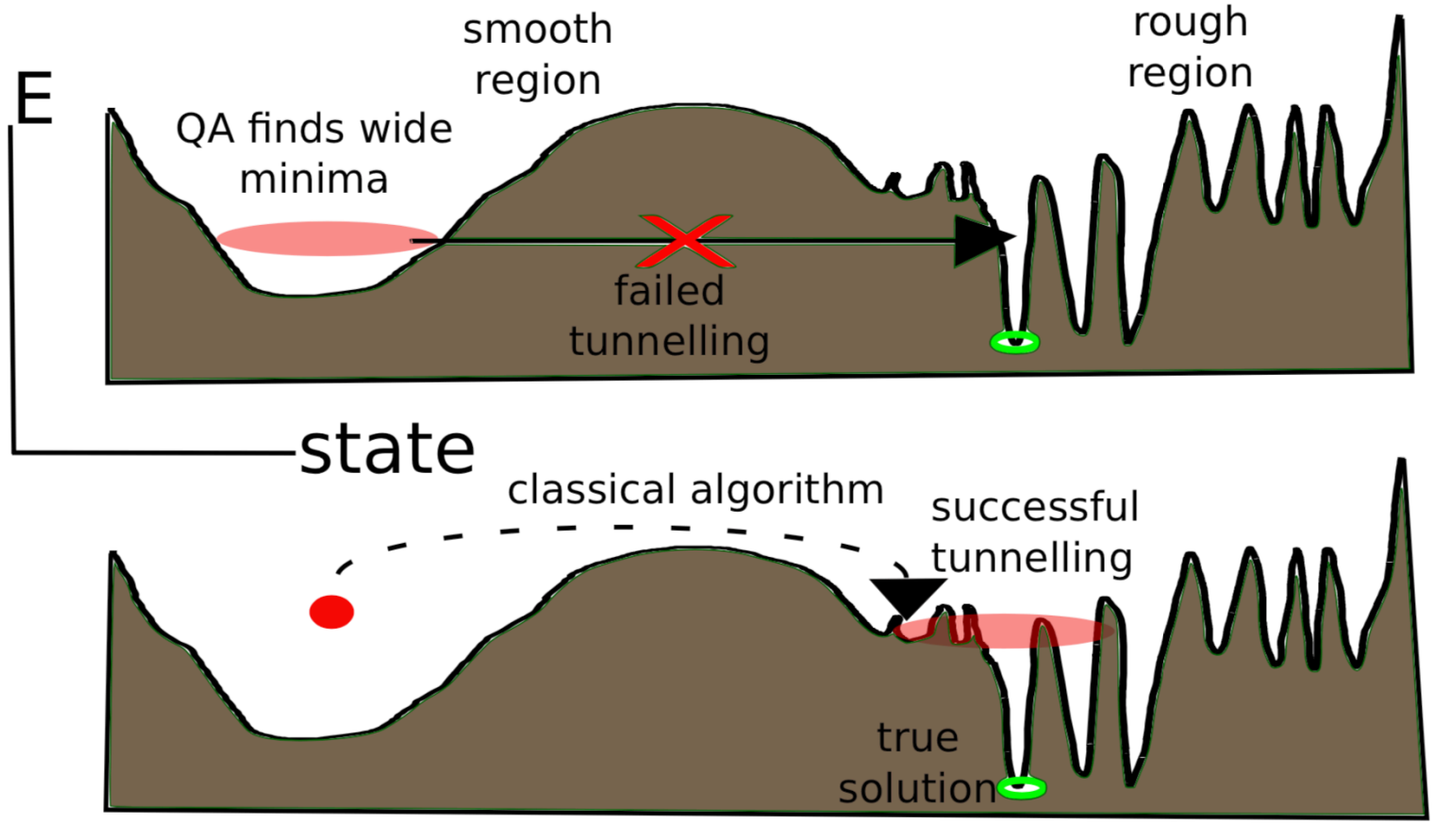


Thermal (classical) and Quantum Annealing are complementary:

- Thermal tunnelling is fast over broad shallow potentials
  (Quantum "tunnelling" is exponentially slow)

- Quantum tunnelling is fast through tall thin potentials
  (Thermal "tunnelling" is exponentially slow - Boltzmann suppression)

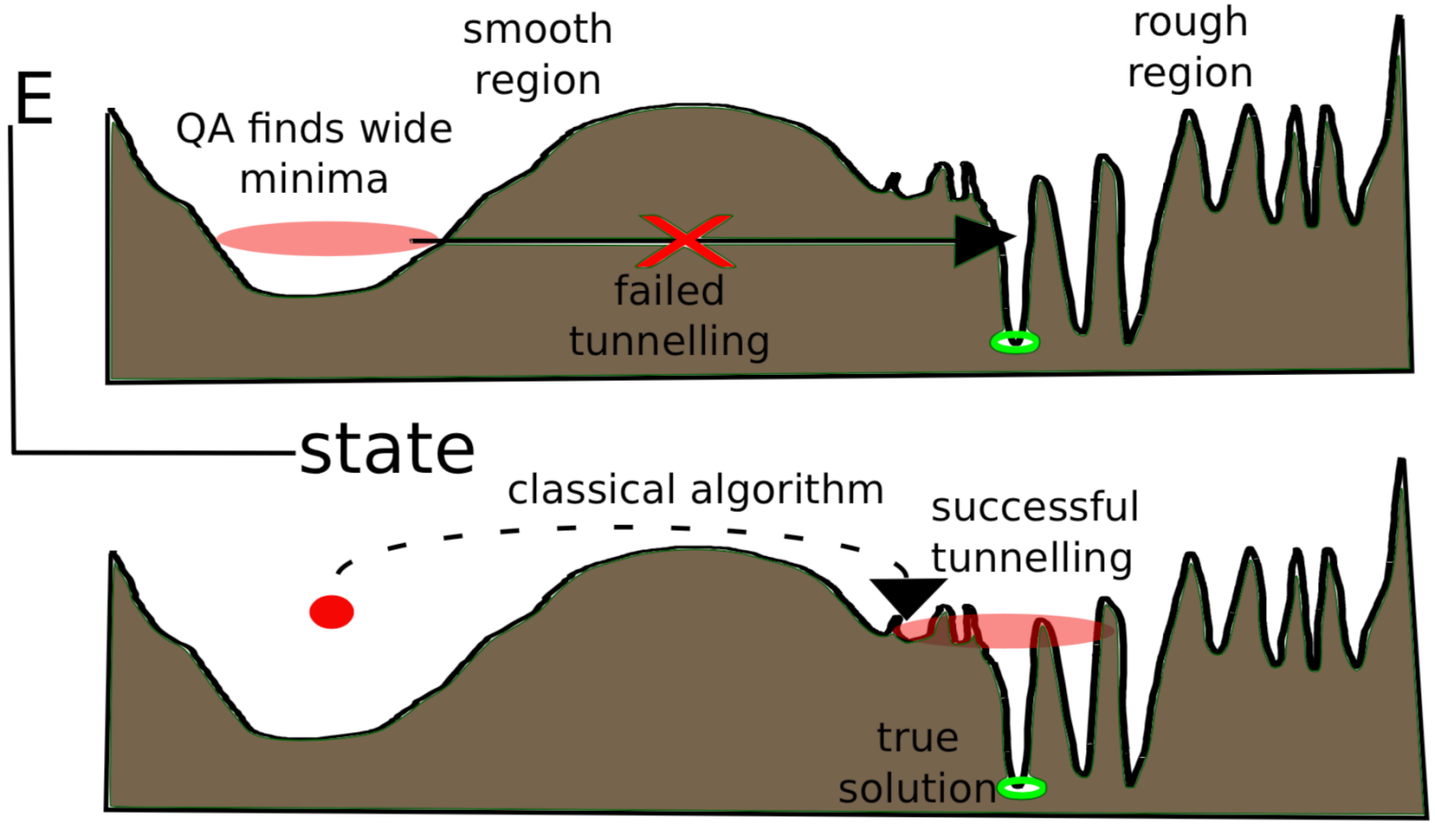
- Hybrid approach can be useful depending on solution landscape



- More specifically - thermal annealing uses Metropolis algorithm: accept random σ_i^Z flips with probability

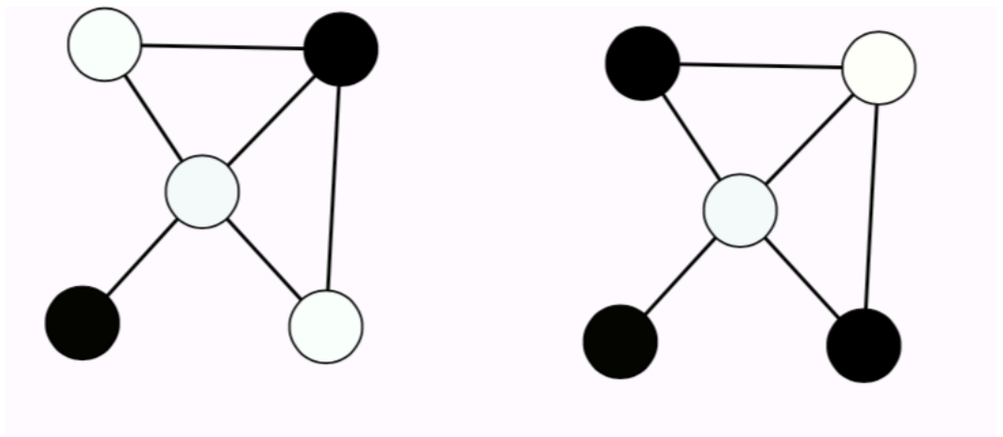
$$P = \begin{cases} 1 & \Delta H \leq 0 \\ e^{-\Delta H/KT} & \Delta H > 0 \end{cases}$$

- Quantum tunnelling in QFT happens with probability $P \sim e^{-w\sqrt{2m\Delta H}/\hbar}$ so by contrast it can be operative for tall barriers if they are thin



Simple example of Ising encoding

Example 1: how many vertices on a graph can we colour so that none touch?



NP problem

Let non-coloured vertices have $\sigma_i^Z = -1$ and coloured ones have $\sigma_i^Z = +1$

Add a reward for every coloured vertex, and for each link between vertices i,j we add a penalty if there are two +1 eigenvalues:

$$\mathcal{H} = -\Lambda \sum_i \sigma_i^Z + \sum_{\text{linked pairs } \{i,j\}} \left[\sigma_i^Z + \sigma_j^Z + \sigma_i^Z \sigma_j^Z\right]$$

Example 2: N^2 students are to sit an exam in a square room with NxN desks 1.5m apart. Half the students (A) have a virus while half of them (B) do not.
How can they be arranged to minimise the number of infections due to <2m social distancing?

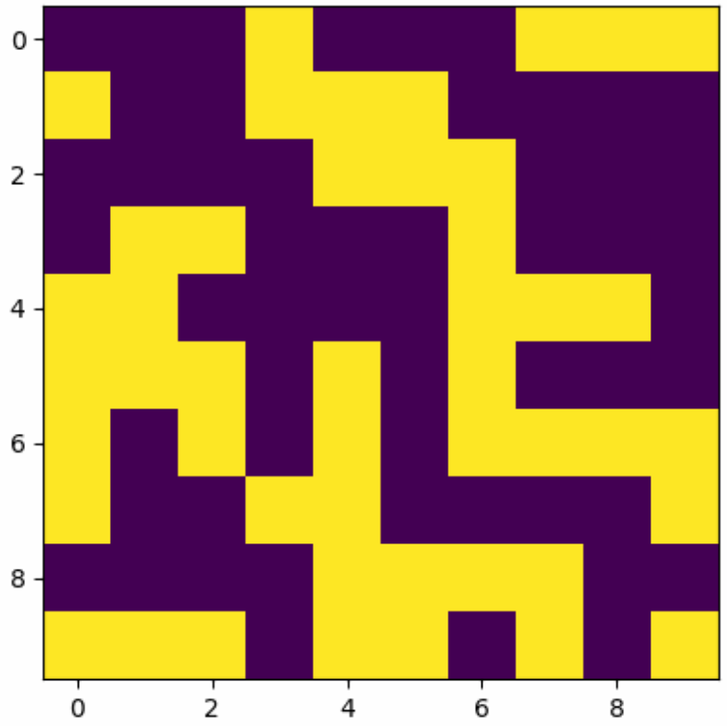
There are N^2 spins σ^Z_{lN+j} arranged in rows and columns. We do not care if A>=<A or B>=<B, but if A>=<B then we put a penalty of 2+ on the Hamiltonian (ferromagnetic coupling)

$$\mathcal{H} = \sum_{\ell m=1}^{N} \sum_{ij=1}^{N} \left(\delta_{\ell m}(\delta_{(i+1)j} + \delta_{(i-1)j}) + \delta_{ij}(\delta_{(\ell+1)m} + \delta_{(\ell-1)m})\right) \left[1 - \sigma^Z_{\ell N+i}\sigma^Z_{mN+j}\right]$$

Finally we need to apply constraint that #A=#B:

$$\mathcal{H}^{(\mathrm{constr})} = \Lambda\,(\#A - \#B)^2 \quad = \Lambda \left(\sum_{\ell,i}^{N} \sigma^Z_{\ell N+i}\right)^2 \quad = \Lambda \sum_{\ell m=1}^{N} \sum_{ij=1}^{N} \sigma^Z_{\ell N+i}\sigma^Z_{mN+j}$$

- Example 2 done with classical thermal annealing using the Metropolis algorithm. Note this represents a search over solution space of 2^100 configurations



- Importantly the constraint hamiltonian cannot be too big otherwise the hills are too high and it freezes too early. This makes the process require a (polynomial sized) bit of "thermal tuning".

- Could be done more easily on quantum annealers as constraints could be high and it would still work, e.g. D-Wave quantum annealer. However, architecture (connectivity of J, h) is limited.

Instantons on spin-lattice [Schenk, MS in prep]

Action with DM term

$$S = \int \mathrm{d}\tau \mathrm{d}x \left[\frac{1}{2} \left(\partial_i n^a\right)^2 + \kappa \left(n^1 \partial_x n^2 - n^2 \partial_x n^1 \right) + \frac{\mu}{2} \left(1 - \left(n^3\right)^2 \right) + B n^3 \right]$$

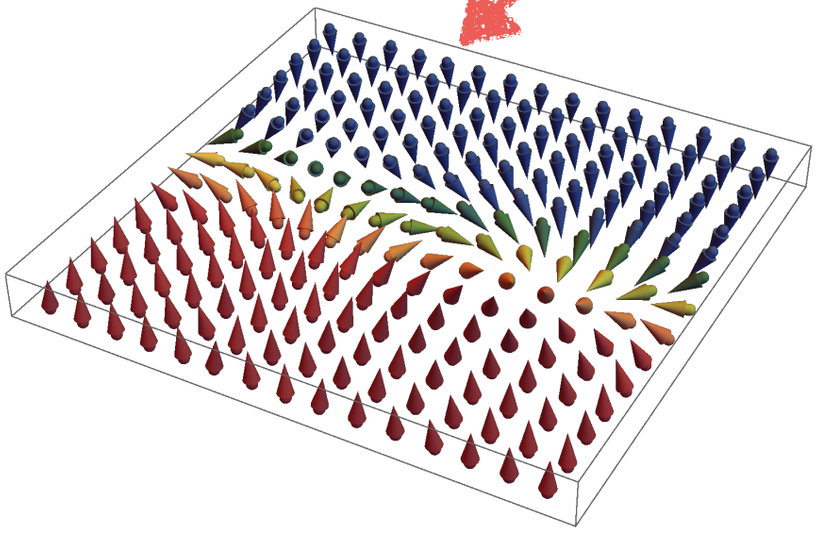
Example 1+1, but extension to higher dim in paper

latticised

$$S_{\mathrm{lat}} = -\sum_{\langle ij \rangle} \hat{n}_i^a \hat{n}_j^a + \sum_i a^2 \left[B \hat{n}_i^3 + \frac{\mu - \kappa^2}{2} \left(1 - \left(\hat{n}_i^3\right)^2 \right) \right]$$

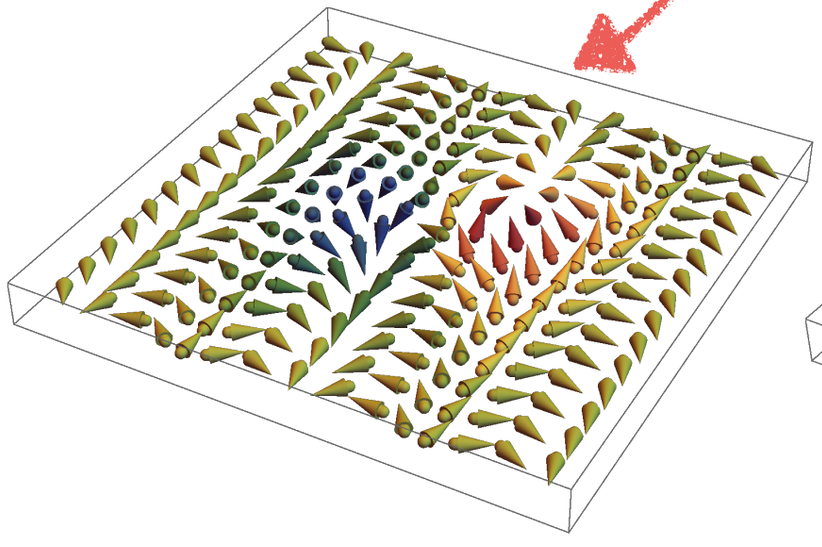
Metropolis algorithm + Thermal annealing

Domain wall



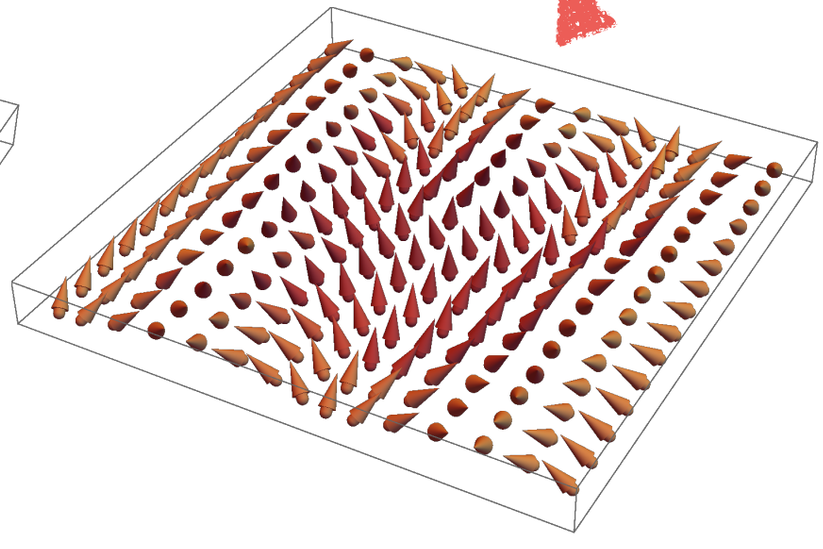
$Q = 1 \quad \mu > \kappa^2 \quad B = 0$

Instanton



$\mu = \kappa^2 \quad B = 0$
$Q = 1$

Merons



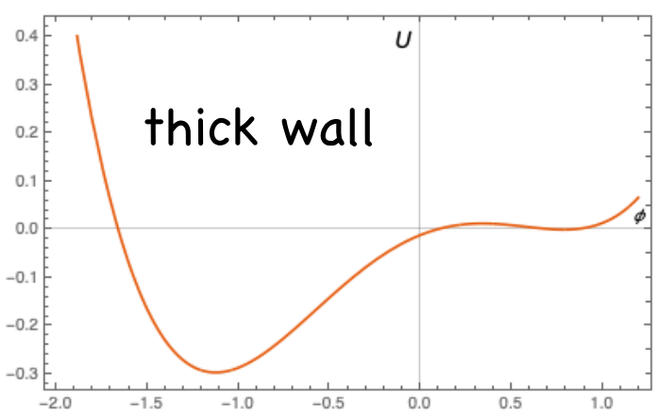
$\mu - \kappa^2 = -1 \quad B = 1/2$
$Q = 1/2$

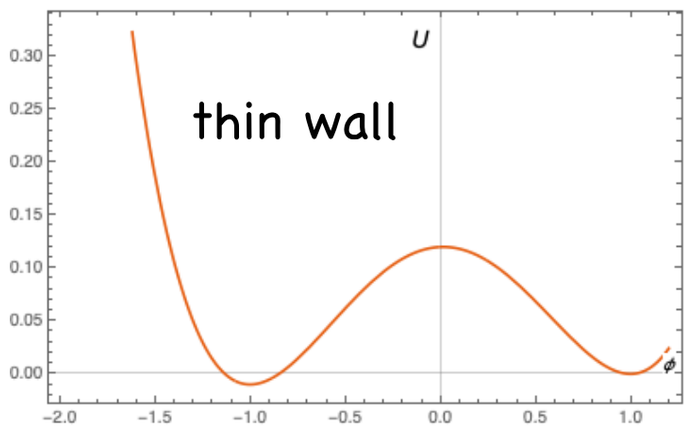
A field theory problem: Tunnelling in QFT

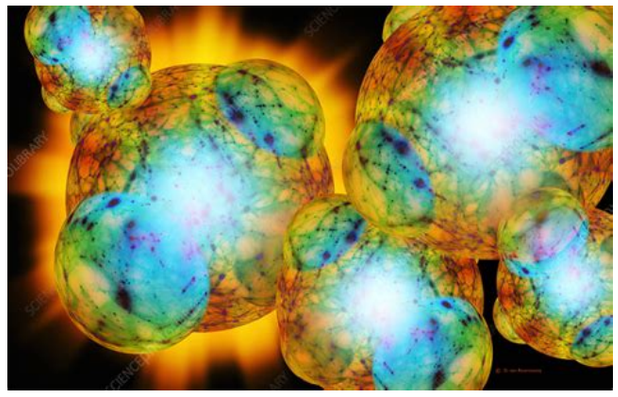
- think of Ising model as a "universal QFT computer"
- Simple problem to demonstrate encoding QFT - quantum tunnelling in a scalar theory.
- Advantage 1: easy to prepare initial state (this non-pert. process easier to prepare than scattering states)
- Advantage 2: on a quantum annealer we could observe genuine tunnelling, rather than just simulate it

$$V(\phi) = \frac{\lambda}{8}(\phi^2 - a^2)^2 + \frac{\epsilon}{2a}(\phi - a)$$

$$U(\phi) = V(\phi) - V(\phi_+)$$







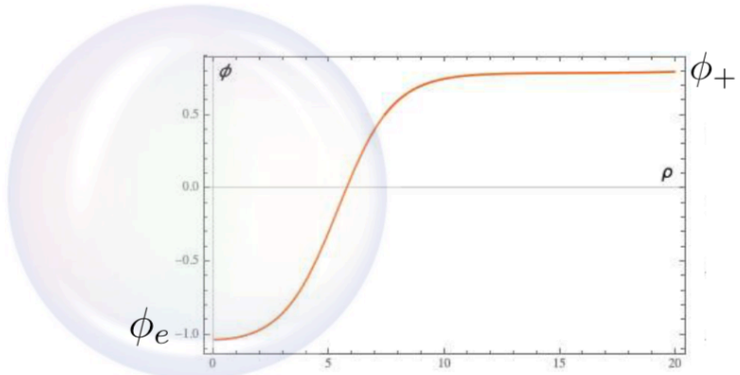
- Example has know analytic solution, see work by Callan, Coleman, de Luccia and Linde

- Decay rate per unit volume is given by the Euclidean actions of the O(4) and O(3) symmetric "bounce" solution (for instanton or thermal respectively)

$$\Gamma_4 \;=\; A_4\, e^{-S_4[\phi]} \qquad\qquad S_{c+1} = \int_0^\infty d\rho \rho^c \left(\frac{\dot{\phi}^2}{2} + U(\phi) \right)$$

- Normally, solution found by solving Euler-Lagrange equations with boundary conditions:

$$\frac{d^2\phi}{d\rho^2} + \frac{c}{\rho}\frac{d\phi}{d\rho} \;=\; U' \qquad \rho \;\to\; 0, \infty \qquad d\phi/d\rho \;=\; 0$$



- Solution for thin-wall approximation: action written in terms of c=0 action

finding the extremum of the action is a quasi-convex problem (convex in a finite box)

$$S_4 = \frac{27\pi^2 S_1^4}{2\epsilon^3} \quad \text{with} \quad \boxed{S_1 \;=\; \int_0^\infty d\rho \; \frac{1}{2}\dot{\phi}^2 + U(\phi)}$$

[Chancellor '19] [Abel, Chancellor, MS '20]

Consider encoding a continuous filed value $\phi(\rho)$ at some point, and discretise into N

$$\phi(\rho_l) \;=\; \phi_0 + \alpha_l \xi \;=\; \phi_0 + \xi \ldots \phi_0 + N\xi$$

Wish to represent it as a point on a spin chain == domain wall encoding:

We translate this to a field value using $\phi(\rho_\ell) = \frac{1}{2}\sum_{j=1}^{N-1}(\phi_0 + j\xi)\,\langle \sigma^Z_{\ell N+j+1} - \sigma^Z_{\ell N+j}\rangle$

receiving only contribution from frustration at $j = \alpha_\ell$

For this domain wall encoding to work we have to avoid mult. frustrations e.g.

→ include $\mathcal{H}^{(\text{chain})}_\ell = -\Lambda\left(\sum_{j=1}^{N-1}\sigma^Z_{\ell N+j}\sigma^Z_{\ell N+j+1} - \sigma^Z_{\ell N+1} + \sigma^Z_{\ell N+N}\right)$

enforces domain walls if Lambda large

$$\mathcal{H}_\ell^{(\text{chain})} = -\Lambda \left(\sum_{j=1}^{N-1} \sigma^Z_{\ell N+j} \sigma^Z_{\ell N+j+1} - \sigma^Z_{\ell N+1} + \sigma^Z_{\ell N+N} \right)$$

penalty for different adjacent spins

pins end at opposite values

$$J_{ij}^{(\text{chain})} = -\frac{\Lambda}{2} \begin{pmatrix} 0 & 1 & & & & \\ 1 & 0 & 1 & & & \\ & 1 & 0 & & & \\ & & & \ddots & & \\ & & & & 0 & 1 \\ & & & & 1 & 0 \end{pmatrix}_{ij}$$

$$h_j^{(\text{chain})} = \Lambda \left(\delta_{j1} - \delta_{jN} \right)$$

To add a potential we can add a contribution to the linear h couplings

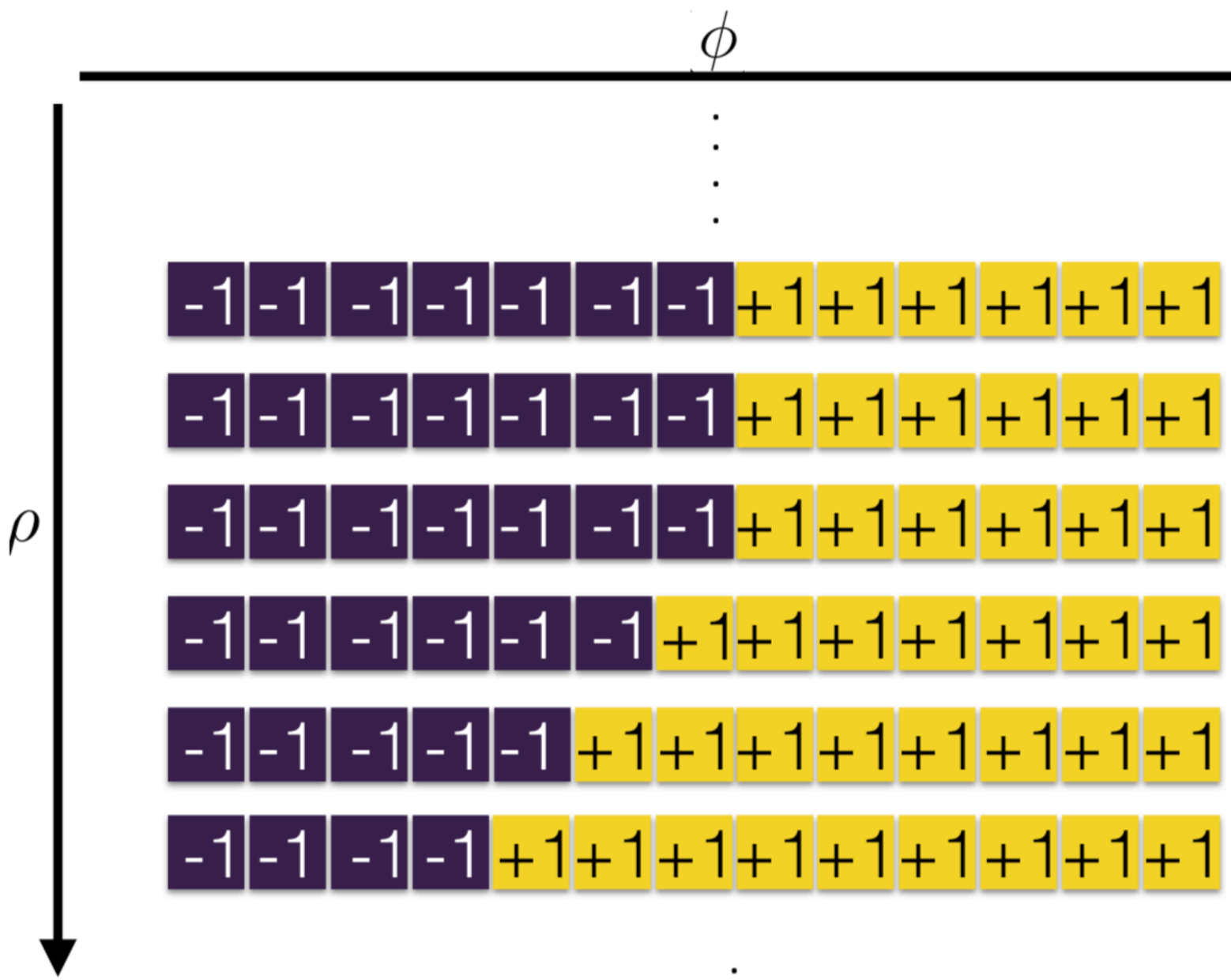
only frustrated link contributes

$$U(\phi(\rho_\ell)) = \frac{1}{2} \sum_{j=1}^{N-1} U(\phi_0 + j\xi) \left\langle \sigma^Z_{\ell N+j+1} - \sigma^Z_{\ell N+j} \right\rangle \rightarrow h_{N\ell+j}^{(\text{QFT})} = \begin{cases} \frac{\nu}{2} \left(U(\phi_0 + (j-1)\xi) - U(\phi_0 + j\xi) \right) ; & j < N \\ \frac{\nu}{2} U(\phi_0 + (N-1)\xi) ; & j = N \end{cases}$$

Pictorial representation of a solution

spacetime discretised

$$\rho_\ell = \ell\nu$$
$$= \nu \ldots M\nu$$



$$\phi(\rho_\ell) = \phi_0 + \frac{N\xi}{2} - \frac{\xi}{2}\sum_{j=1}^{N}\langle\sigma^Z_{\ell N+j}\rangle$$

The kinetic term is included as follows:

$$S_{KE} \equiv \int_0^{\Delta\rho} d\rho \frac{1}{2}\dot{\phi}^2 = \lim_{M\to\infty} \sum_{\ell=1}^{M} \frac{1}{2\nu} \left(\phi(\rho_{\ell+1}) - \phi(\rho_\ell)\right)^2 \qquad \nu = \Delta\rho/M$$

$$= \sum_{\ell=1}^{M-1} \sum_{ij}^{N-1} \frac{\xi^2}{8\nu} \left[\sigma^Z_{(\ell+1)N+i} - \sigma^Z_{\ell N+i}\right] \left[\sigma^Z_{(\ell+1)N+j} - \sigma^Z_{\ell N+j}\right]$$

encoded in Ising model $J^{(\mathrm{QFT})}_{\ell N+i,mN+j} = \frac{\xi^2}{8\nu} \left(2\delta_{\ell m} - \delta_{\ell(m+1)} - \delta_{(\ell+1)m}\right)$

For the boundary conditions: $\mathcal{H}^{(BC)} = \frac{\Lambda'}{2}(\phi(0)+v)^2 + \frac{\Lambda'}{2}(\phi(\rho_M)-v)^2$

Can think of these as boundary mass-term potentials in U:

$$h^{(\mathrm{BC})}_{N\ell+j} = \begin{cases} -\Lambda'(\phi_0 + j\xi + v)\ ; & \ell = 1, \forall j \\ -\Lambda'(\phi_0 + j\xi - v)\ ; & \ell = M-1, \forall j \end{cases}$$

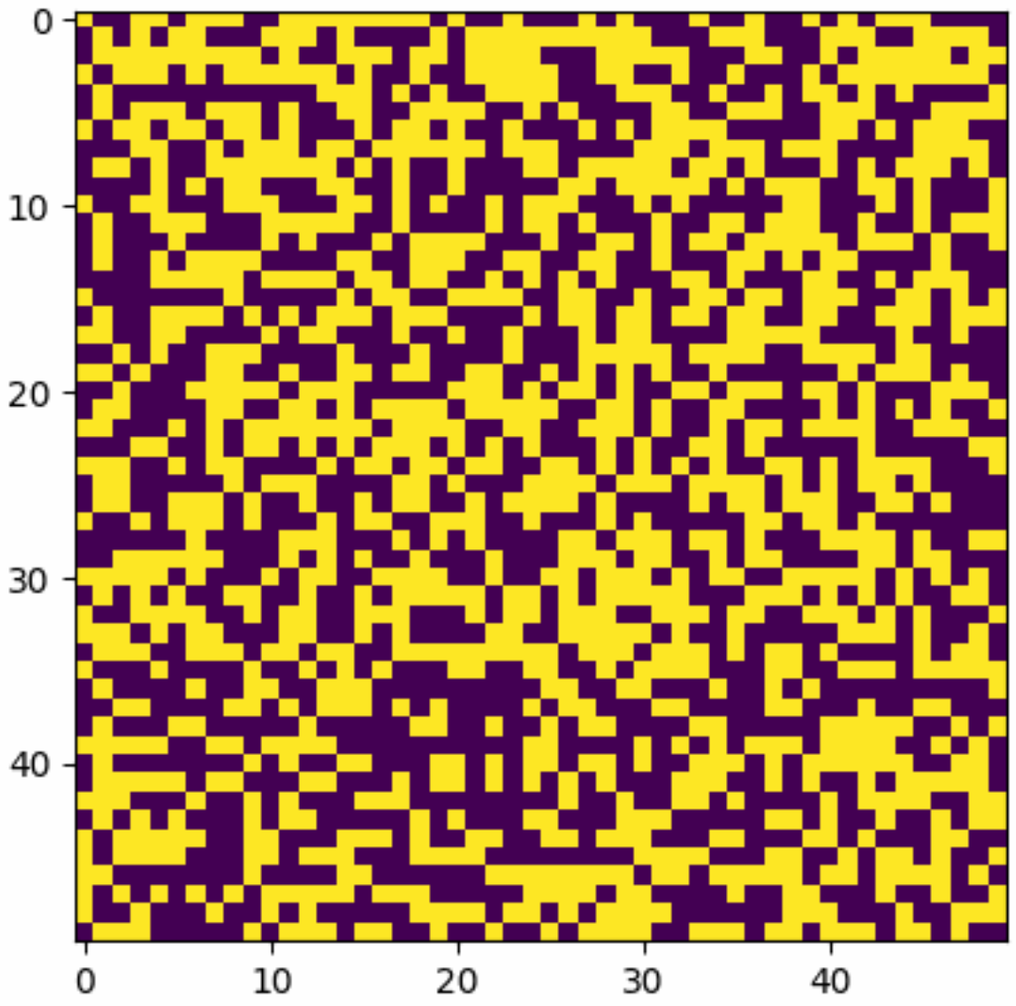
Finally add everything together $\mathcal{H} = \mathcal{H}^{(\mathrm{chain})} + \mathcal{H}^{(\mathrm{QFT})} + \mathcal{H}^{(\mathrm{BC})}.$

Result for thin-wall limit

Can solve classical simulated annealing with Metropolis algorithm.

Result depends on hyperparameters, e.g. temperature etc.

Too hot



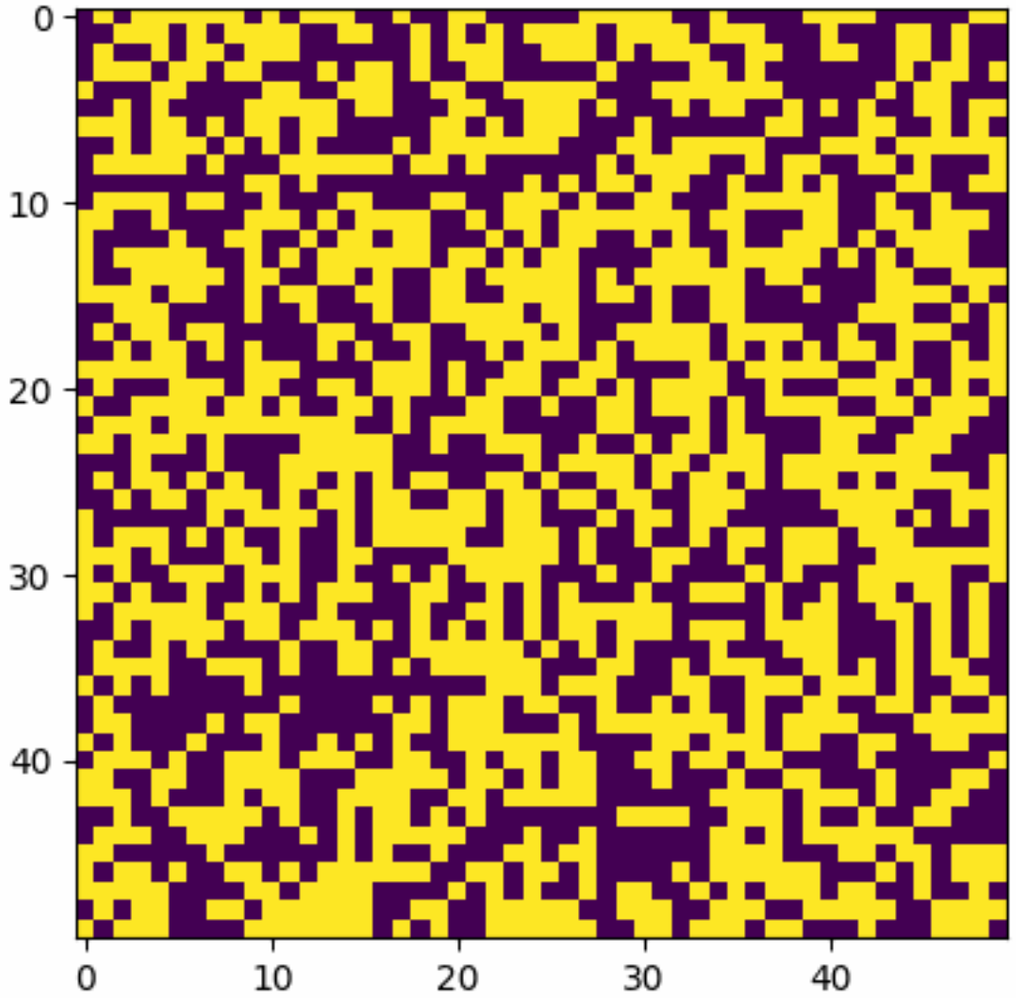
Result for thin-wall limit

Can solve classical simulated annealing with Metropolis algorithm.

Result depends on hyperparameters, e.g. temperature etc.

Too cold

(shock freezing)

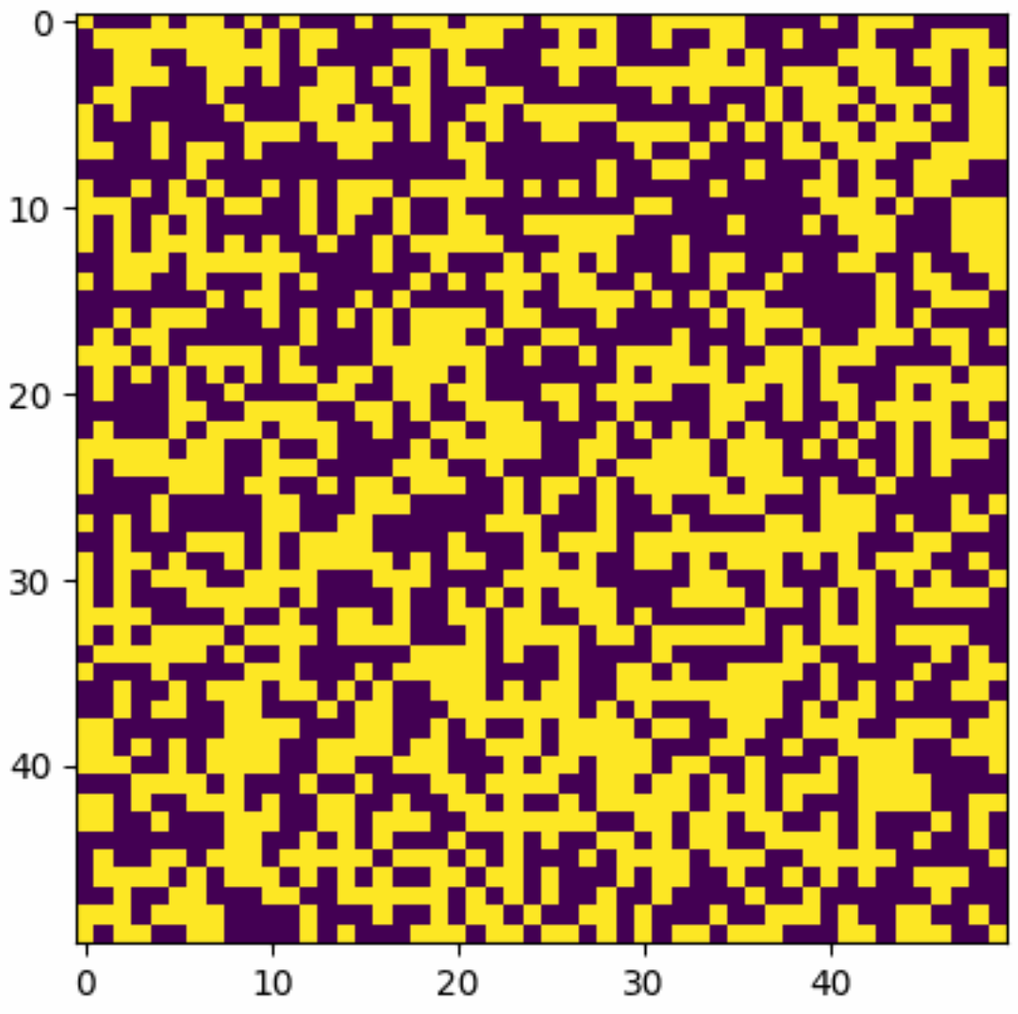


Result for thin-wall limit

Can solve classical simulated annealing with Metropolis algorithm.

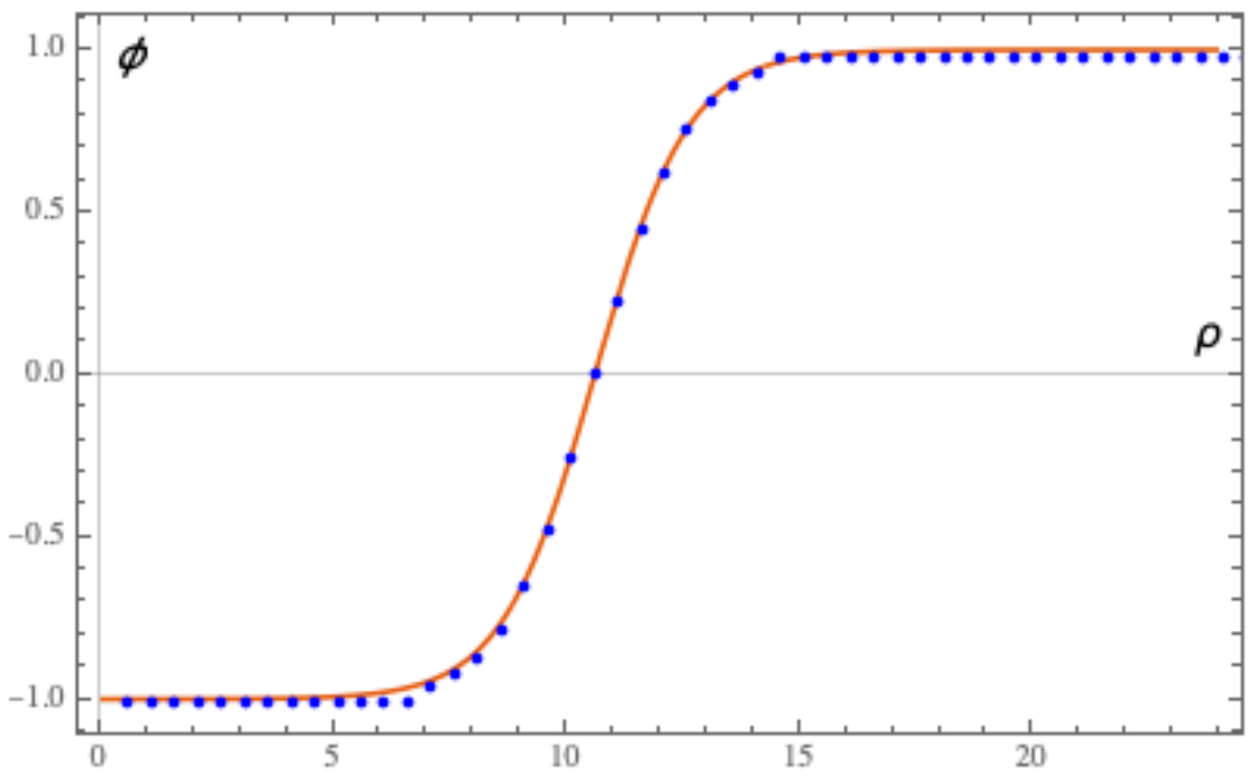
Result depends on hyperparameters, e.g. temperature etc.

Just right



Same result on D-Wave using hybrid quantum/classical Kerberos annealer (Finds best samples of parallelised tabu search + simulated annealing + D-Wave subproblem sampling)

blue dots

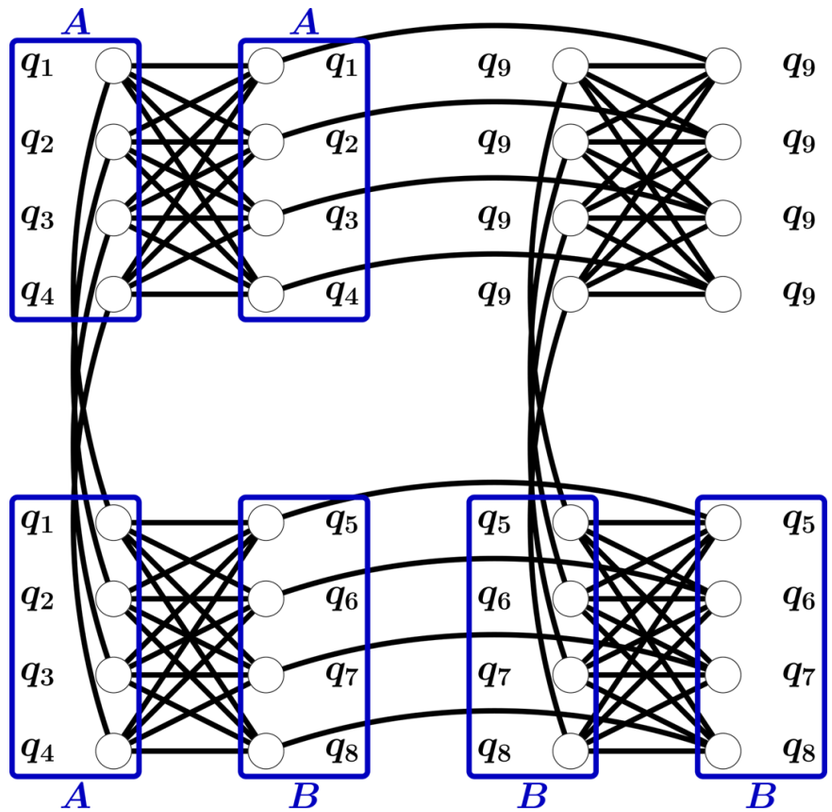


Kerberos hybrid algo is much more robust than pure simulated (thermal) annealing

Why not pure Quantum annealer on D-Wave system?

The connectivity is not general enough for this problem (in particular encoding the kinetic terms) - due to Chimera structure

Embedding of the problem difficult, as mapping of spacetime and field discretisation challenging/ impossible if solution knows about relation between different spacetime values



But principle has been proven: we can encode a pure field theory potential chimera structure, so we an experiment with QFT tunnelling

A quantum laboratory for QFTs

- going beyond the reach of classical computers -

- Using the spin-chain approach for field theories discussed before, we can encode a QFT on a quantum annealer and study its dynamics directly.

[Abel, MS 2006.06003]

- To show that the system is a true and genuine quantum system we investigate if the state can tunnel from a meta-stable vacuum into a the true vacuum.

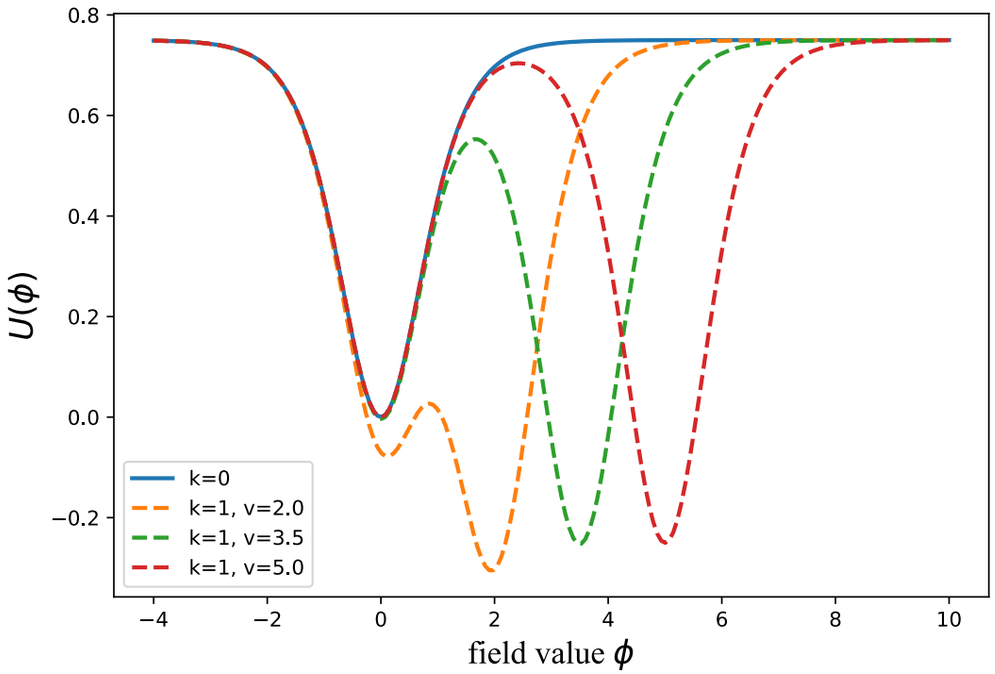
- Choose a potential of interest:

$$U(\phi) = \frac{3}{4}\tanh^2\phi - k(t)\,\mathrm{sech}^2\left(c(\phi - v)\right)$$

where $\phi = \eta/\eta_0$

time dependent

$\phi(t)$ is the field and c, v are dimless constants



The tunnelling probability in a QFT is calculated by evaluating the path-integral in Euclidean space around the action's critical points using the steepest gradient-descent method

$$\langle \eta_i | \eta_f \rangle_E = \int \mathcal{D}\delta\eta \, e^{-\hbar^{-1} \int dt \left(\frac{m(\dot{\eta}_{cl}+\delta\dot{\eta})^2}{2} + U(\eta_{cl}+\delta\eta) - E_0 \right)} = A e^{-\hbar^{-1} S_{E,cl}}$$

quantum annealer

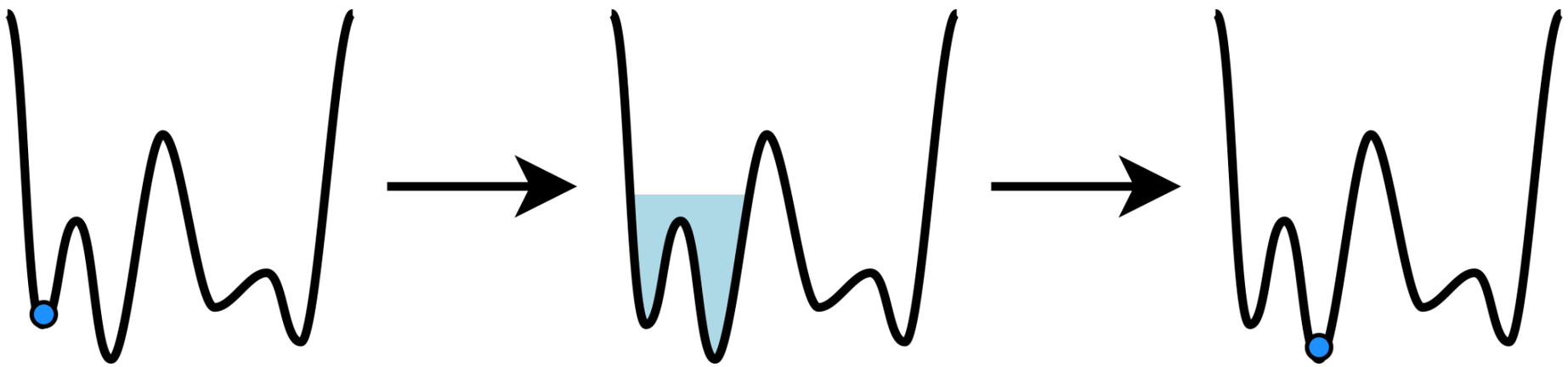
For the tunnelling rate $\Gamma = |\langle \eta_i | \eta_f \rangle_E|^2 \approx e^{-2\hbar^{-1} S_{E,cl}}$ with $S_{E,cl} = \int_{\eta_+}^{\eta_e} d\eta \sqrt{2m(U - E_0)}$

Exponent is object of interest: $\hbar^{-1} S_E \approx \gamma^{-\frac{1}{2}} \int_{\phi_+}^{\phi_e} \sqrt{\frac{3}{4} \tanh^2 \phi - \operatorname{sech}^2(\phi - v)} \, d\phi$ with $\gamma \stackrel{\text{def}}{=} \hbar^2 / 2m\eta_0^2$

$$\log \Gamma \approx -2\hbar^{-1} S_E \approx \sqrt{\frac{3}{\gamma}} \left(\frac{5}{3} - v \right)$$

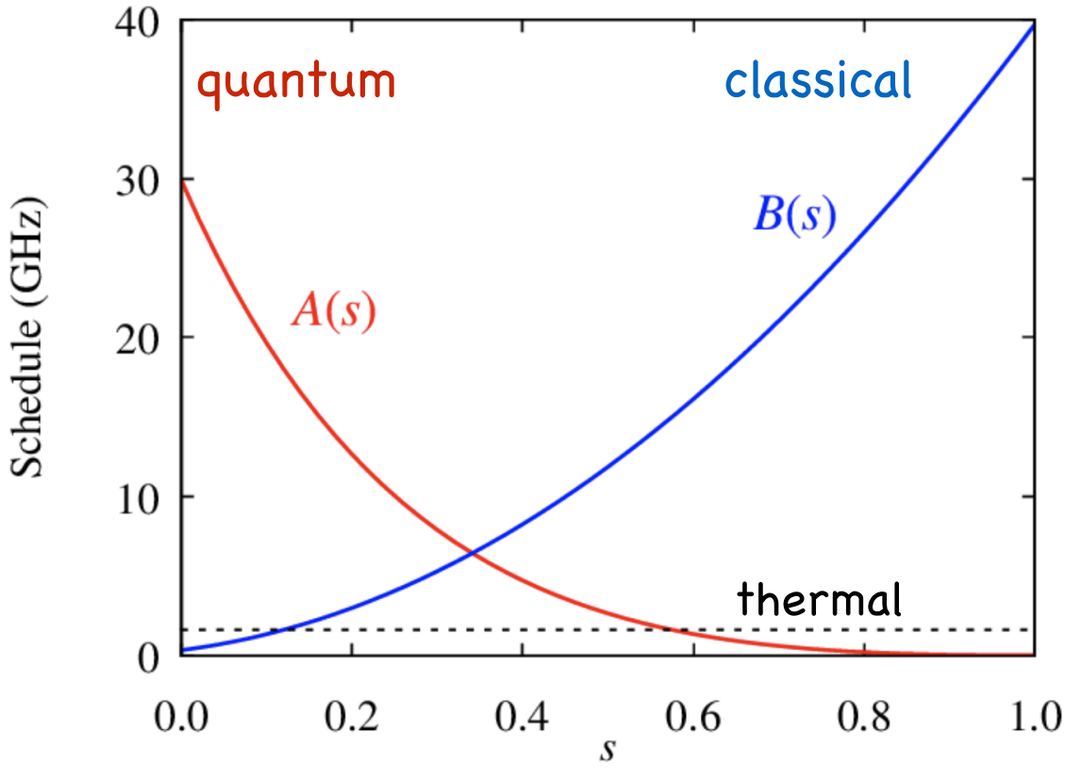
D-Wave reverse annealing

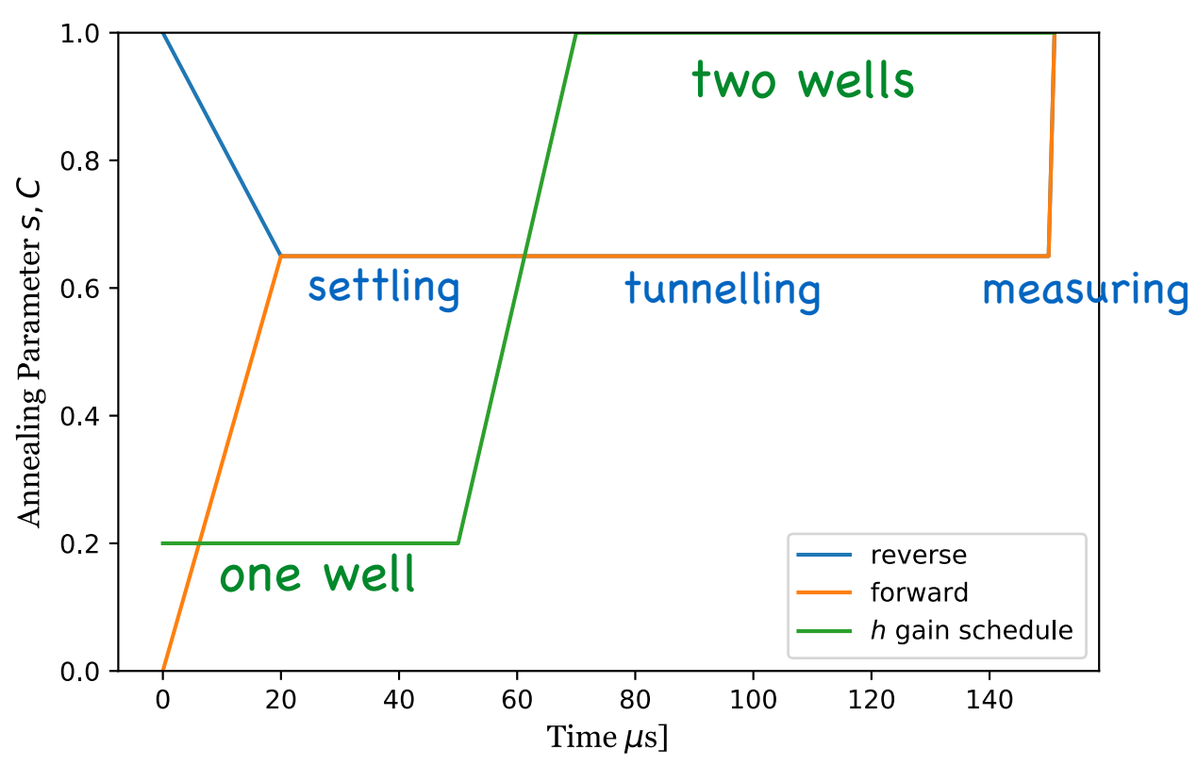
starts at sq=1 (classical) -> sq < 1 (quantum) -> measurement in sq=1 (classical)



To show that the system behaves like a quantum system, we do reverse annealing as follows:

A. begin with system in a classical state, with just a single well potential, i.e. k=0
B. bring it to a quantum state and wait 50 microseconds for it to become stable
C. change the potential to introduce the second well
D. wait t microseconds and bring it back to a classical state to measure the sigmas
E. rinse and repeat 10k times
F. work out the tunnelling fraction





Calibrating the system with a simple harmonic oscillator

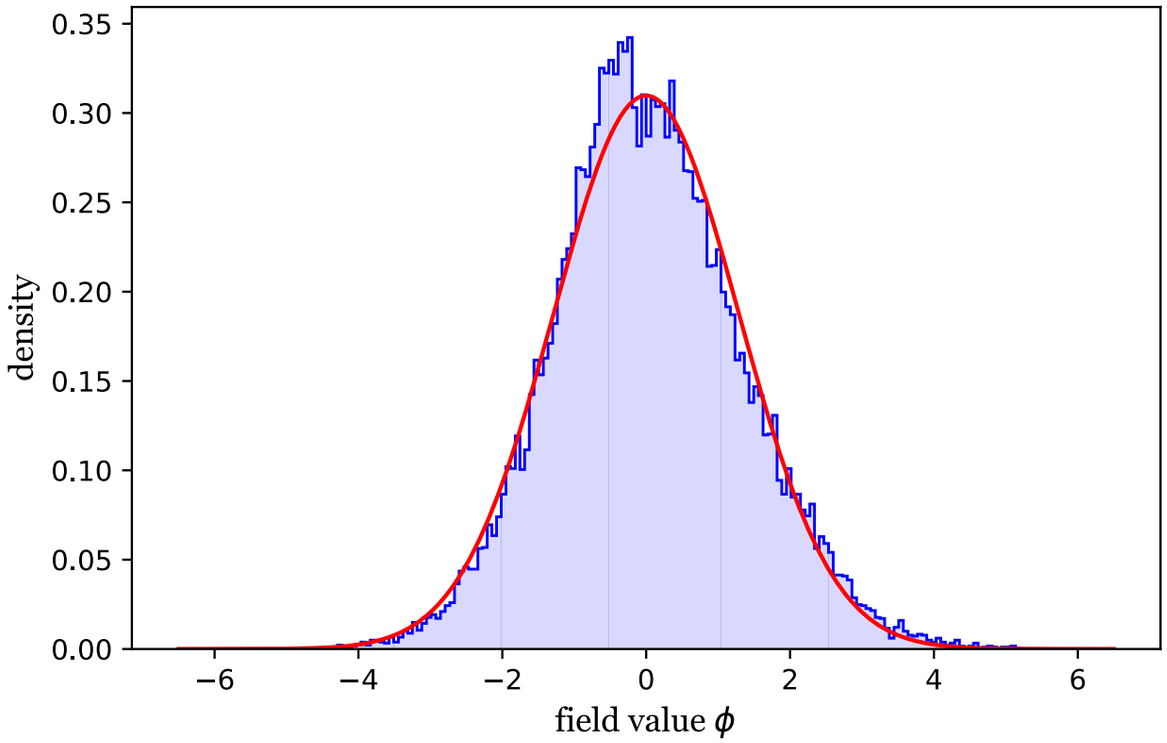
we assume for the potential $U_0(\phi) = \frac{\kappa}{2}\phi^2$ with $\kappa = 0.06$

we initialise classically at $\phi = 0$ and let it settle for $75\ \mu\mathrm{s}$ with $s_q = 0.7$

Fitted with Gaussian

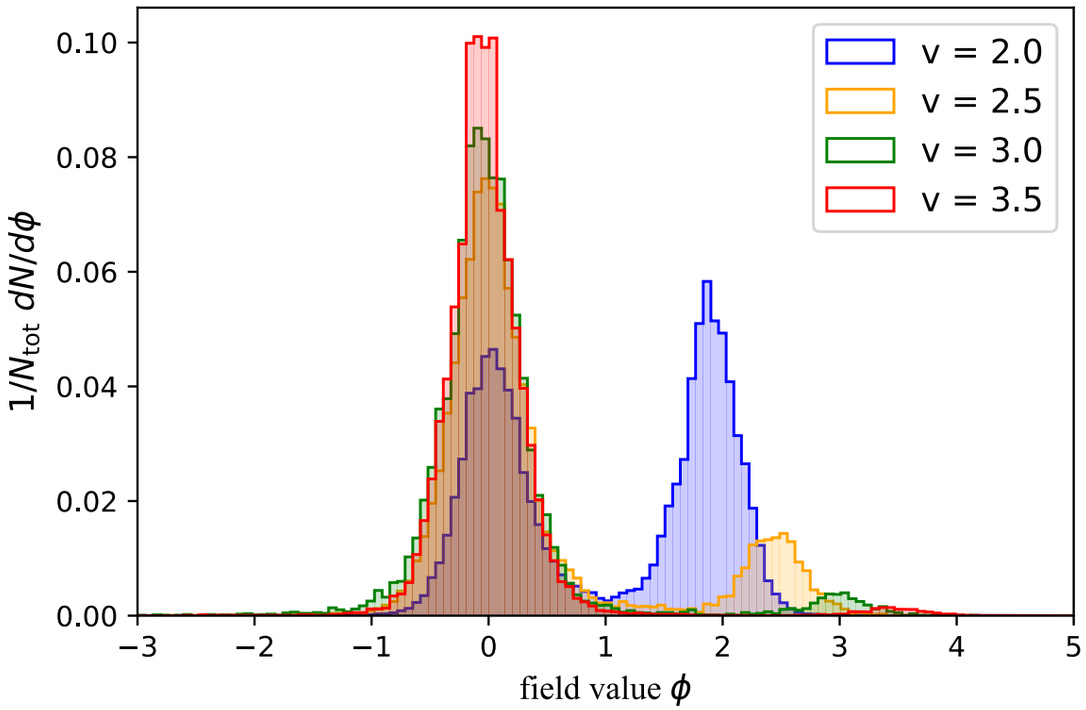
$$|\psi|^2 = \frac{(\kappa/2\gamma)^{\frac{1}{4}}}{\pi^{\frac{1}{2}}} e^{-\sqrt{\kappa/2\gamma}\,\phi^2} \longrightarrow \gamma = 0.33$$

No dynamics was induced by hand on annealer. Thus, constitutes a genuine measurement of the ground state wave function of a quantum mechanical system



30k shots on D-Wave machine

Results: it appears to decay with v as expected

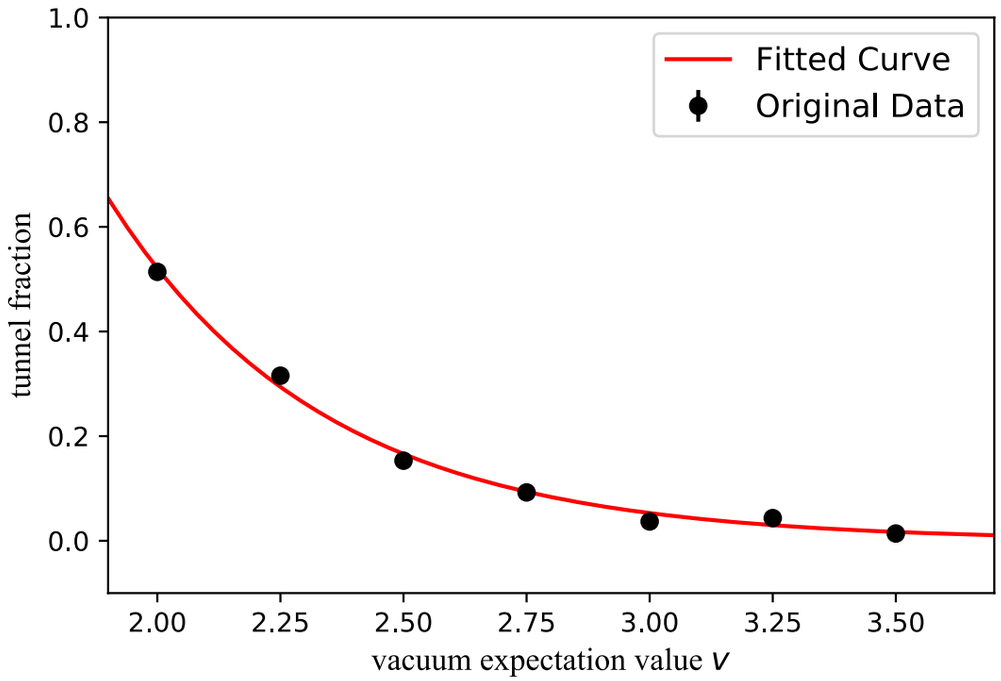


Perform tunnelling for

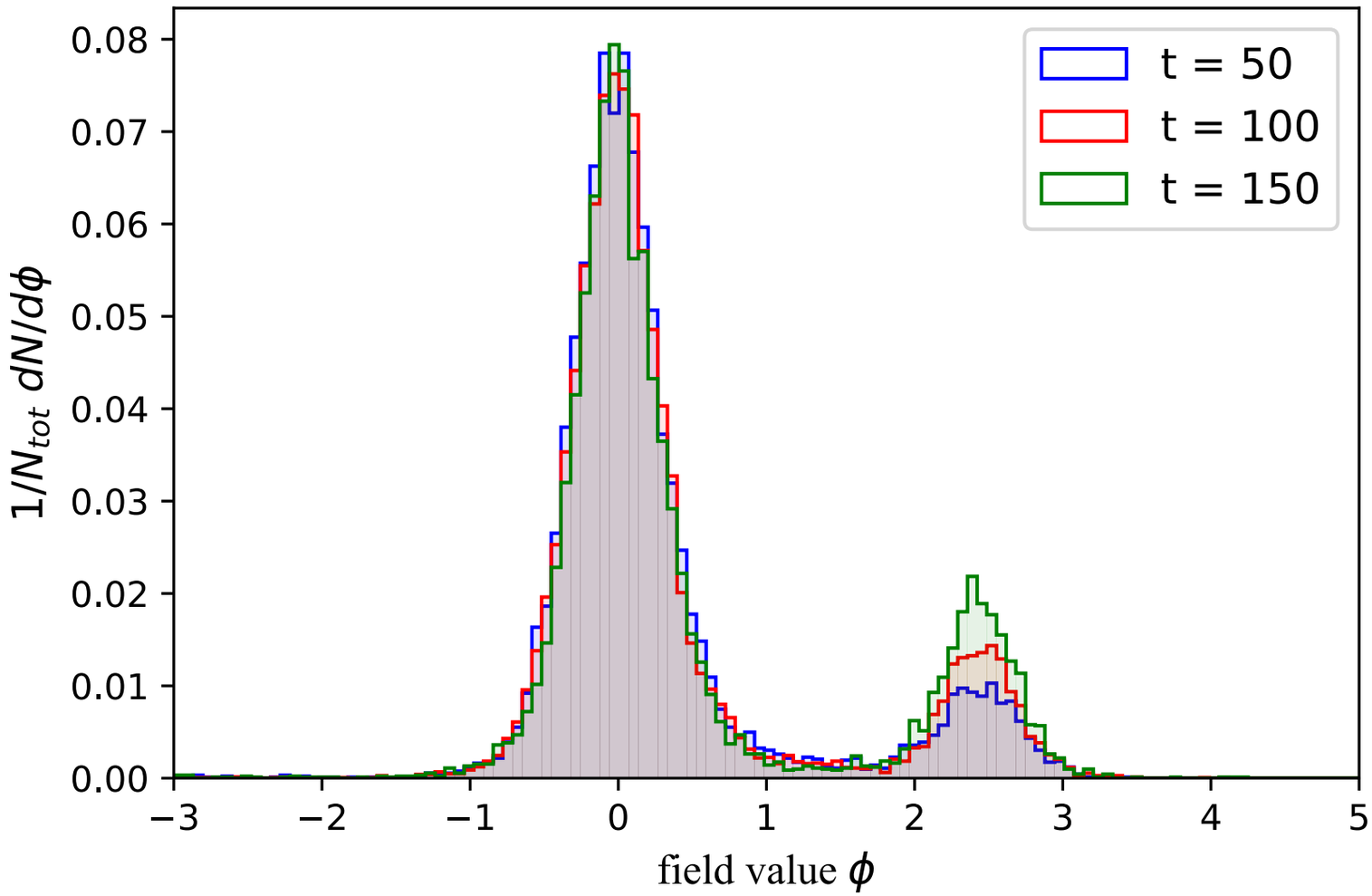
$t_{\text{tunnel}} = 100\mu s$ at $s_q = 0.7$

Theory: $\log \Gamma = 3.0 \times (1.66 - v)$

Exp: $\log \Gamma = 2.29 \times (1.71 - v)$



Results: we see tunnelling, e.g. at v=2.5



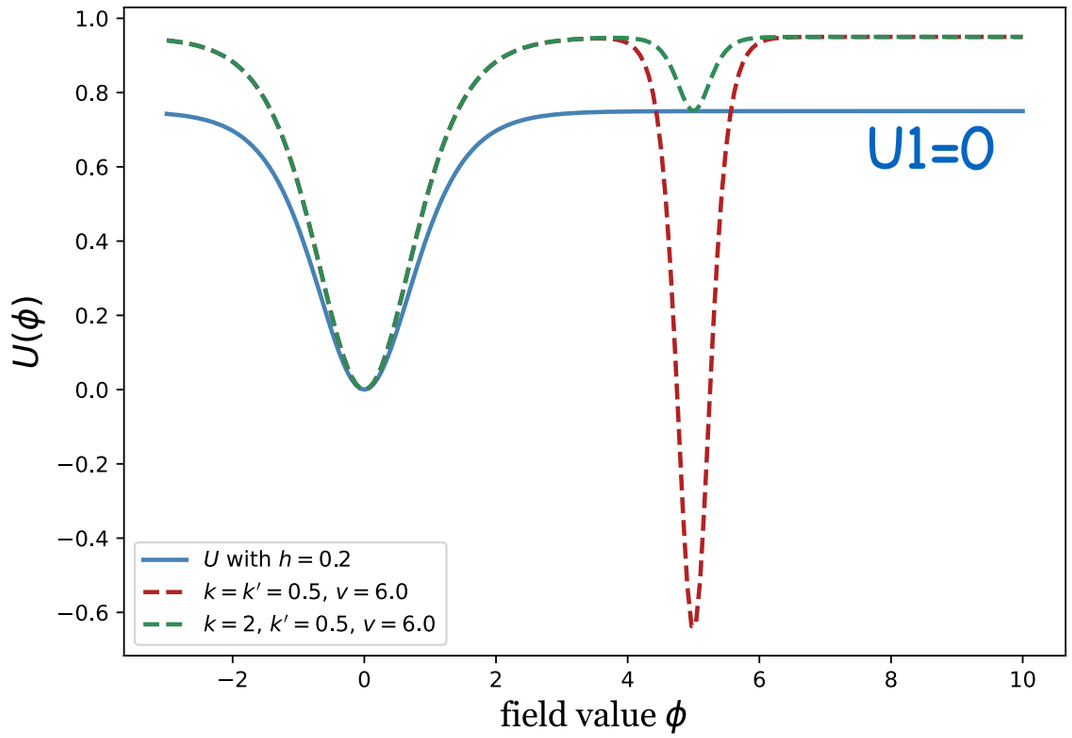
where $N_{\text{tunnelled}} = N_{\text{tot}} \left(1 - \exp^{-t\Gamma}\right)$

Quantum vs Thermal

To test if process over or through barrier, we compare time-dependent potential, starting at U0 and going either to k=k'=0.5 or k=2,k'=0.5

$$U_0 = \frac{3}{4}\tanh^2\phi - C_0\,U_1 \quad \text{where} \quad C_0 = 0.2$$

$$U_1 = k'\,\tanh^2\phi - k\,\mathrm{sech}^2 c(\phi - v)$$

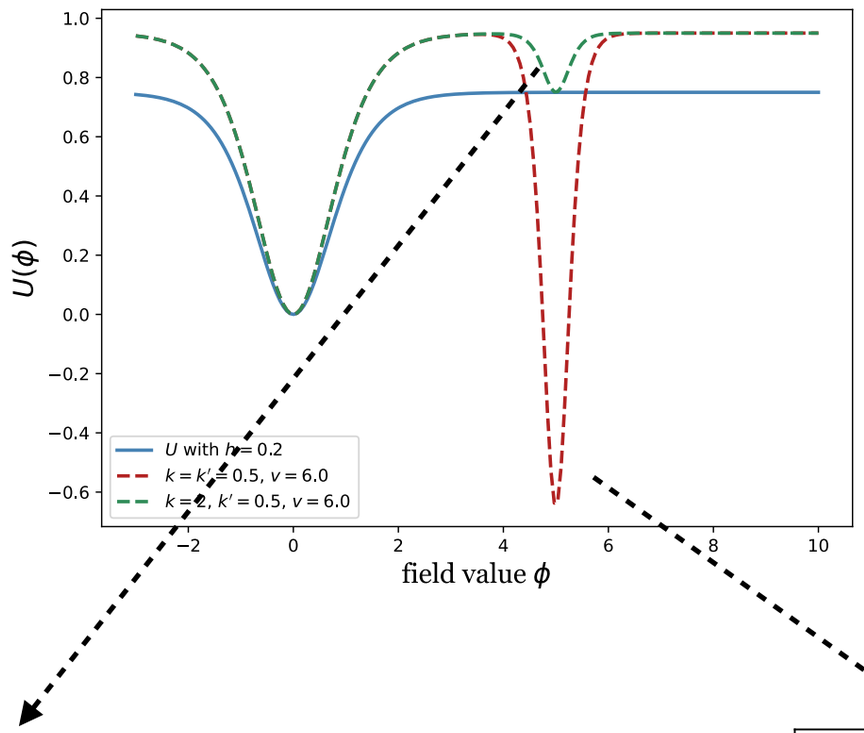


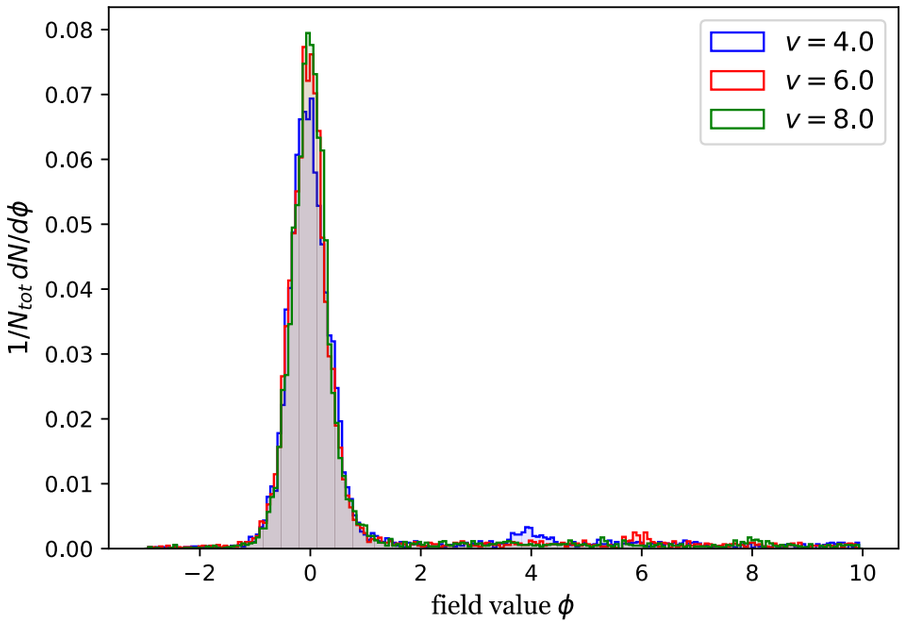
Note, only depth of well different, width of well is the same

Note also that energy eigenstate of tanh^2 potential (U1=0) is not at 0, but lowest energy eigenstate is ~0.41

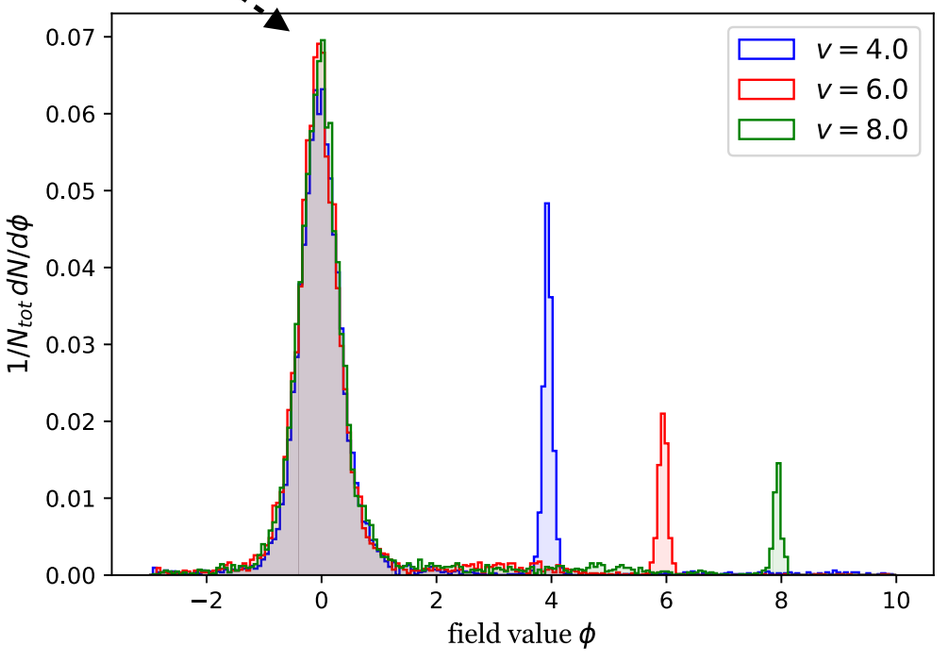
→ Better not to have only 2 differently deep second potential wells, but to also raise the barrier to suppress thermal transitions

Result: clear quantum behaviour





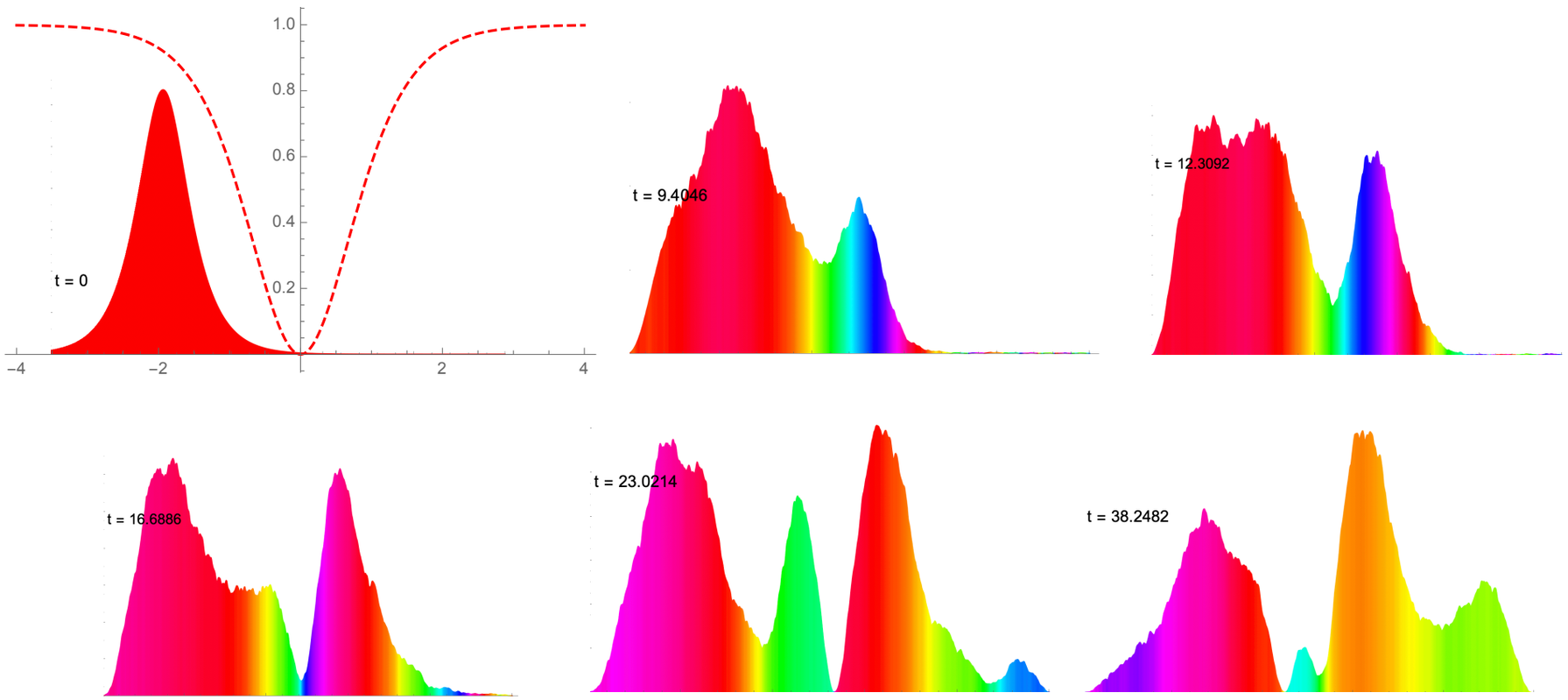
with varying v



→ Clearly dominant part of transition through barrier and not over barrier

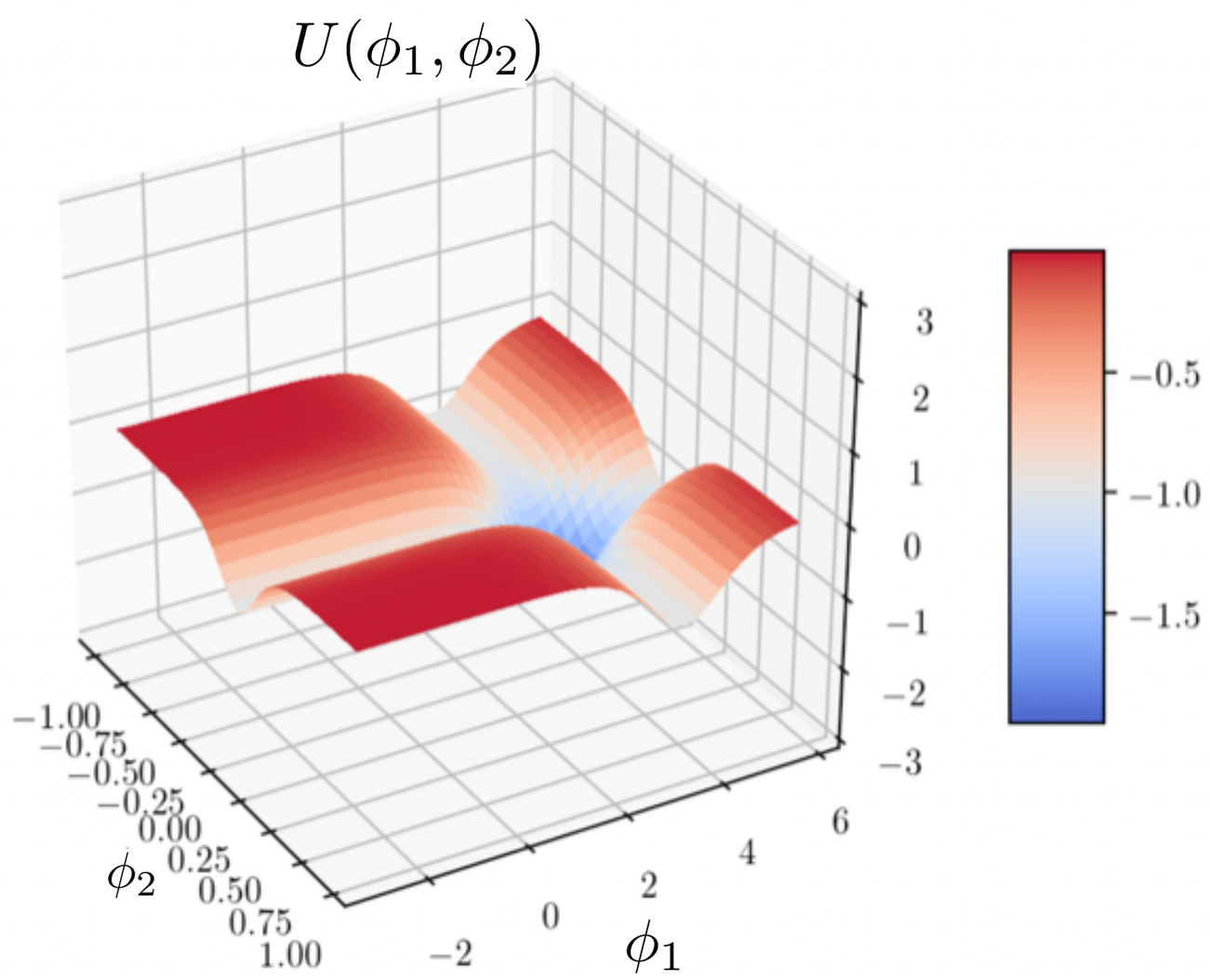
Also dynamics has characteristic behaviour. For example it still "tunnels" to the bottom of a potential even if there is no barrier: i.e. the wave function leaks across, rather than rolling as a lump —

Numerically solving S.E. we find (this takes an hour!)



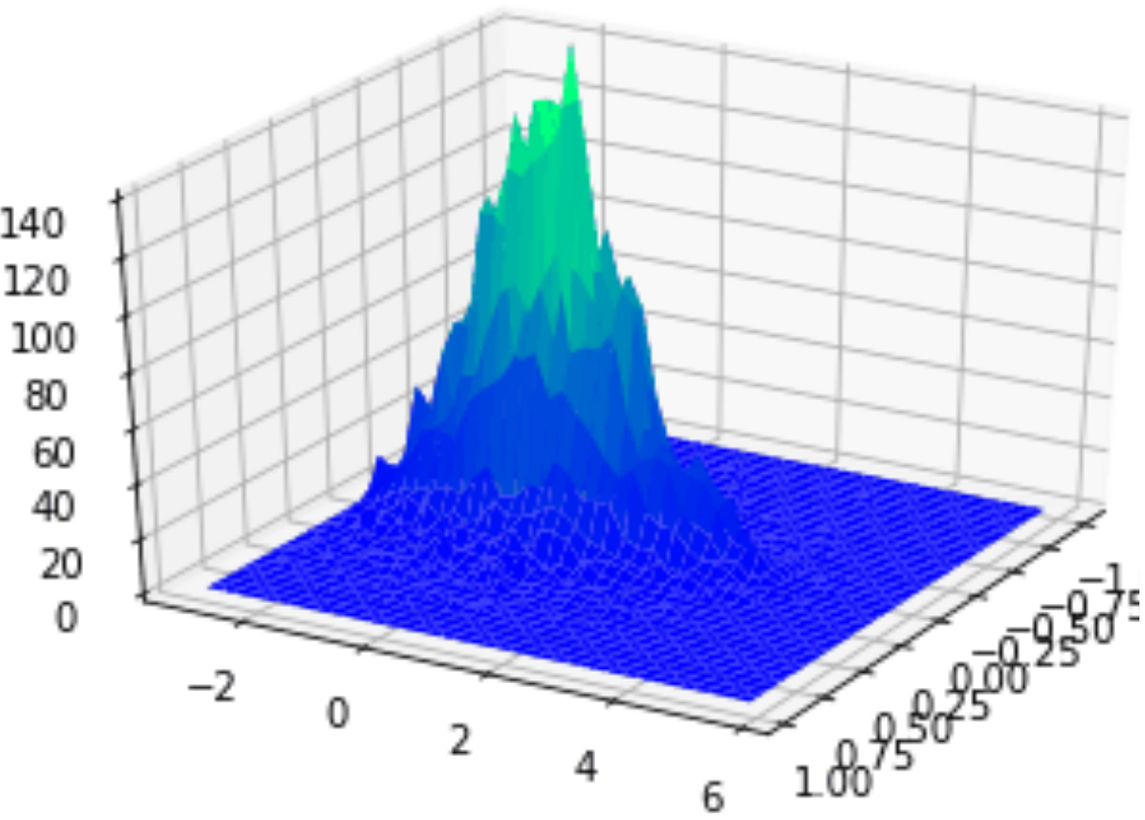
Also dynamics has characteristic behaviour. For example it still “tunnels” to the bottom of a potential even if there is no barrier: i.e. the wave function leaks across, rather than rolling as a lump —

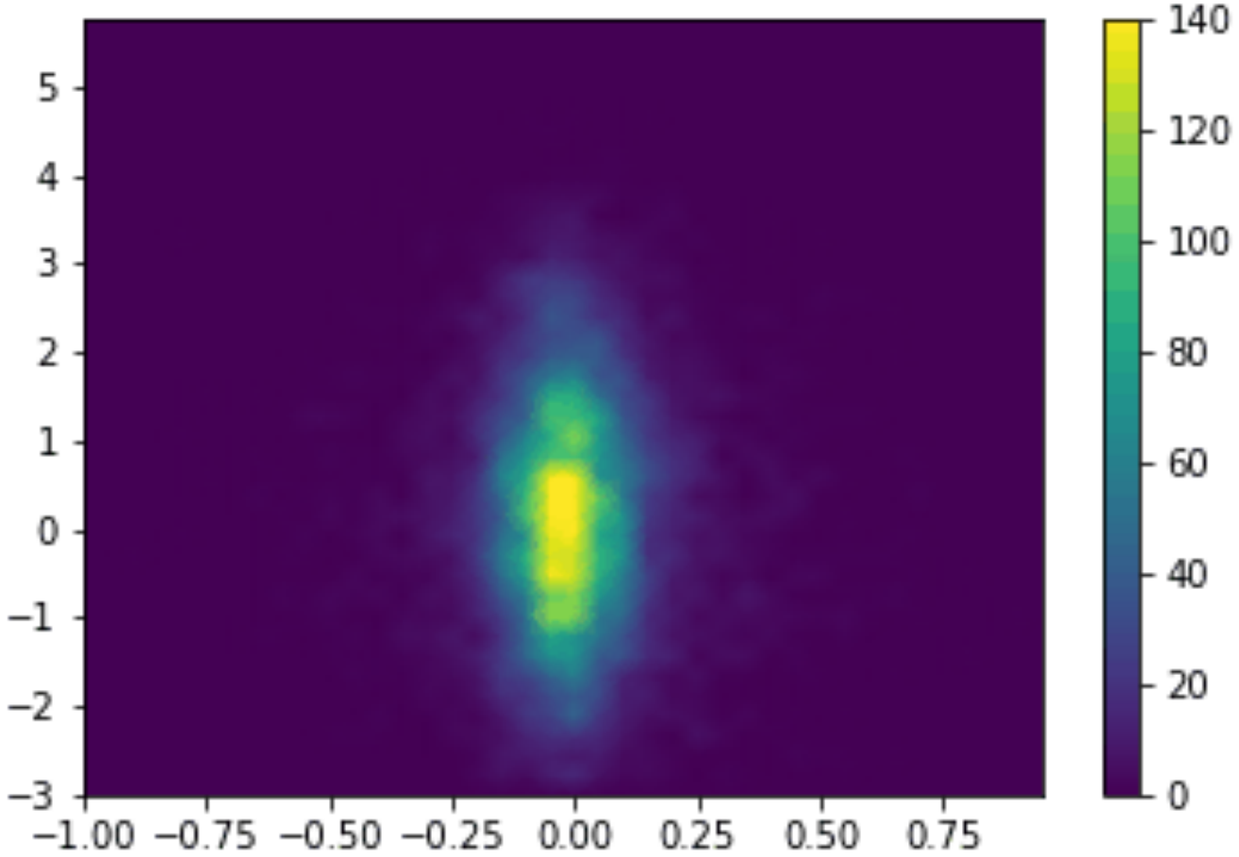
Multiple measurements on the quantum annealer:

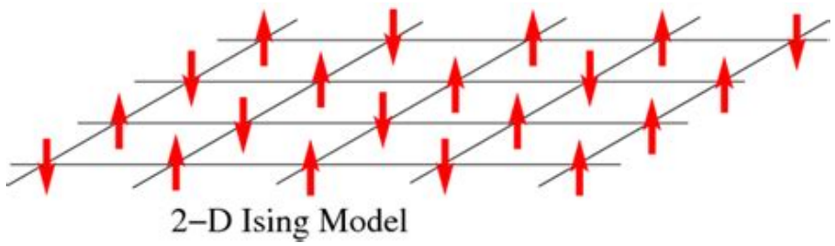


Also dynamics has characteristic behaviour. For example it still “tunnels” to the bottom of a potential even if there is no barrier: i.e. the wave function leaks across, rather than rolling as a lump —

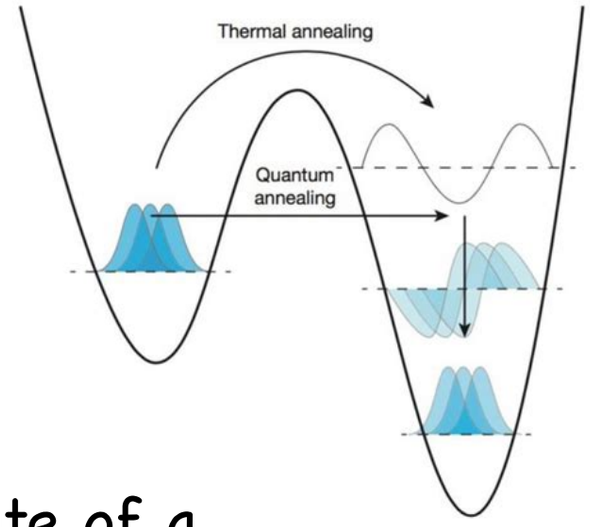
Multiple measurements on the quantum annealer:







Summary and Future directions



- Encoding the solution of a problem into the ground state of a complex system can be a fast and reliable way to calculate highly non-trivial problems.

- Quantum computers are near-to-midterm future experiments that can be used to encode and solve questions in field theory

- We have achieved so far:
  - ➡ First instance of being able to build a QFT by hand in quantum lab
  - ➡ First observation of instantons for freely chosen QFT (d=1)

- Future research directions include:
  - ➡ Study strongly-coupled systems
  - ➡ Extend to more dimensions, gauge theories, condensed matter, etc etc