Quantum Gravity and Quantum Curvature

DIAS-remote,
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Preview

The context of my talk is the search for ***quantum gravity*** as a nonperturbative, diffeomorphism-invariant quantum field theory of dynamical geometry in four spacetime dimensions, with a positive cosmological constant.

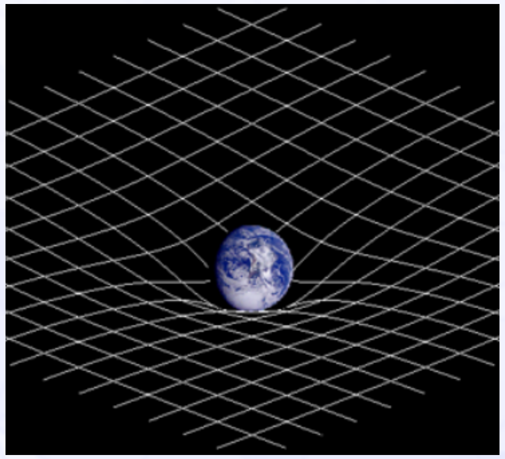
The key take-home messages today will be

1. we have (finally) understood how to correctly ***put gravity on a lattice***, without destroying diffeomorphism invariance
2. progress in quantum gravity will be achieved by studying ***observables***; the new kid on the block: ***quantum Ricci curvature***
3. **new result** (combining 1. and 2.): the geometry of the emergent quantum universe near the Planck scale is compatible with a ***constantly curved de Sitter space***

(N. Klitgaard and RL, Eur. Phys. J. C 80 (2020) 990, arXiv:2006.06263)

Life in the *Century of Gravity*

- **urgent**: complete our quantum gravity theories to make reliable predictions, minimizing free parameters and ad hoc assumptions
- **my route**: tackle quantum gravity and geometry *directly* in a non-perturbative, Planckian regime (no appeal to duality/dictionaries)
- the beauty of classical GR: "theory ***of*** spacetime", captured by its curvature properties
- given the central role of **curvature** classically, is it also true that



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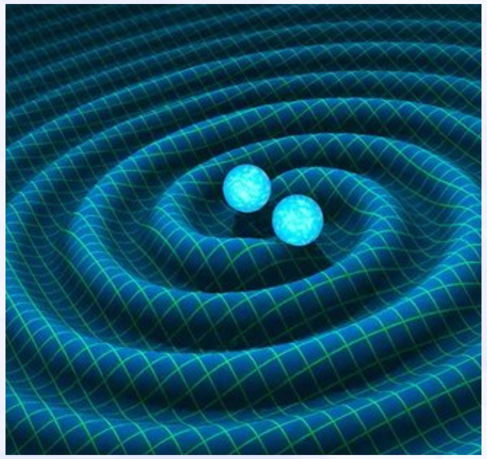


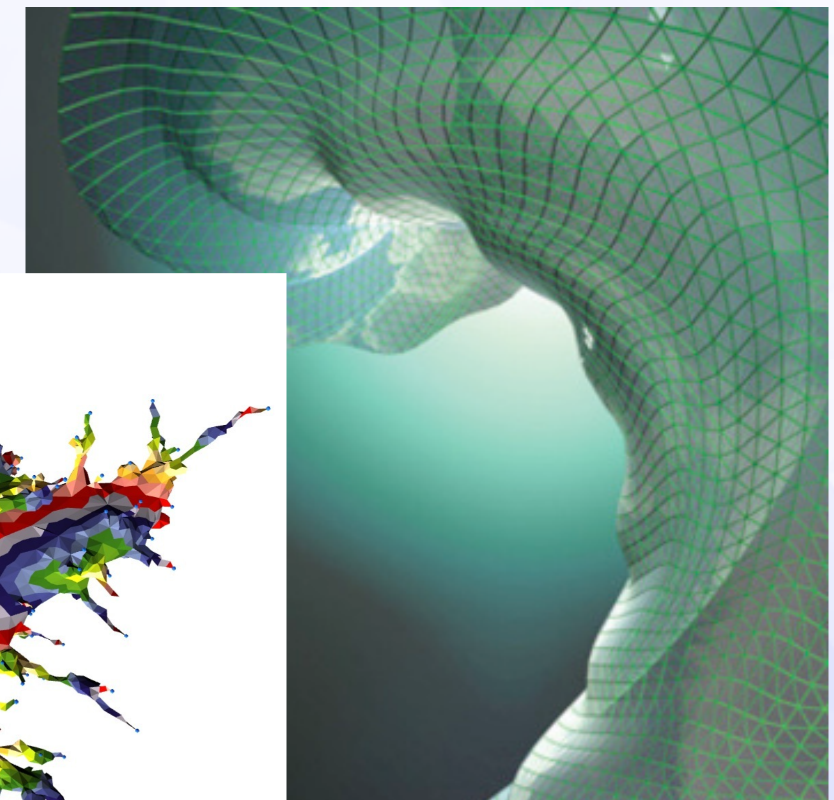
(©R. Hurt/Caltech-JPL/EPA)

nonperturb. quantum gravity = theory of quantum curvature?

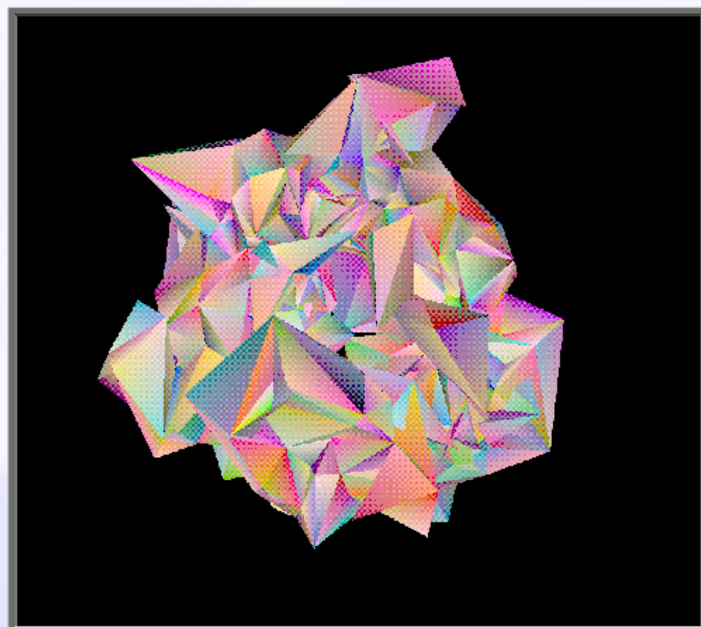
- So far, this proposition has remained largely unexplored. We have set up a new line of research into defining and measuring ***quantum Ricci curvature*** in quantum gravity, with intriguing results.

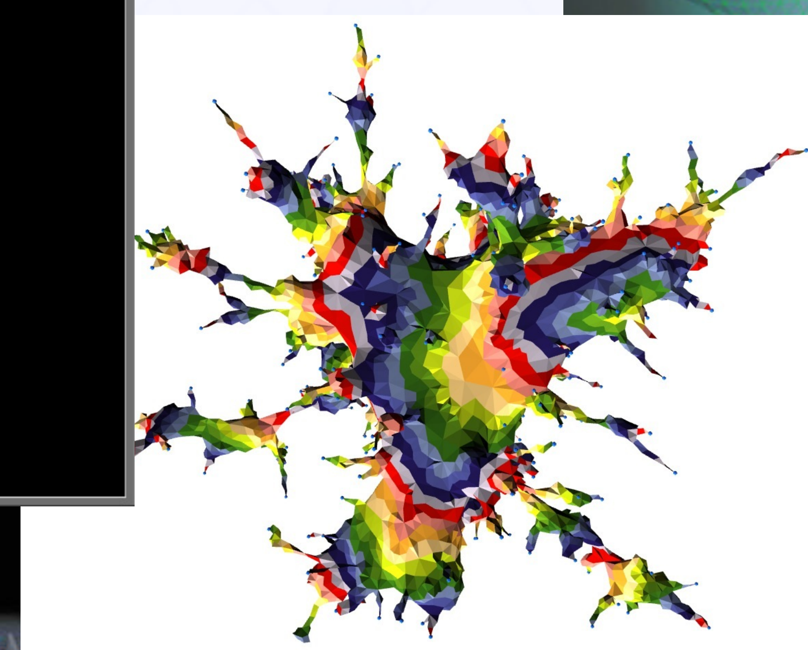
What *is* quantum spacetime?

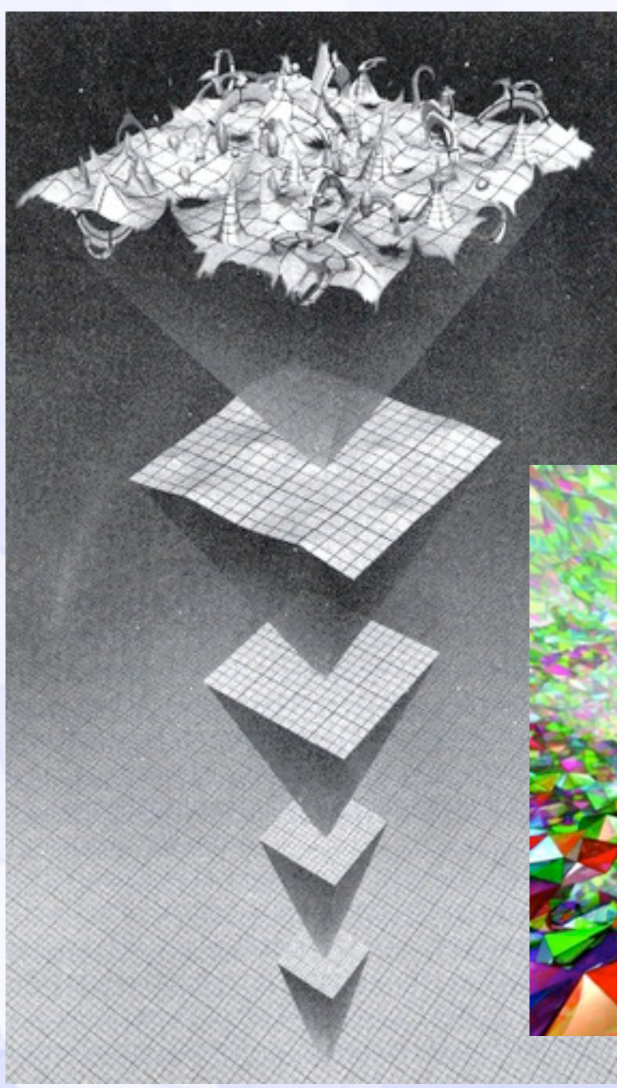
A “spacetime” with quantum properties near the Planck scale $\sim \ell_{\mathrm{Pl}}$, which in a suitable macroscopic limit can be approximated by a classical curved spacetime of General Relativity.

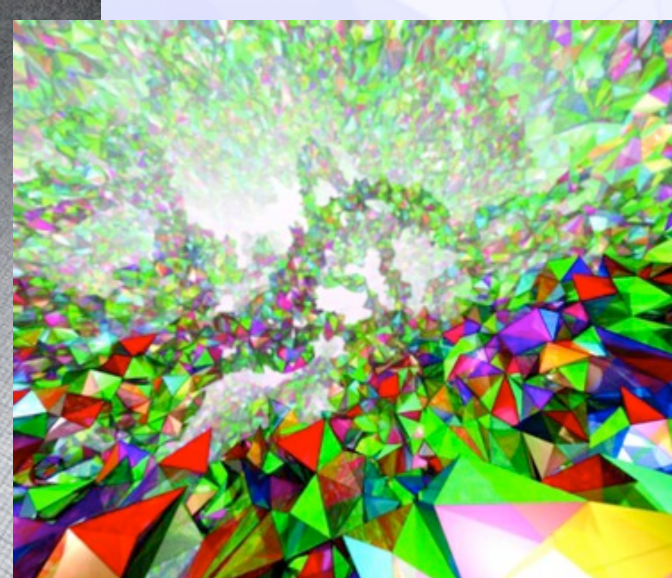


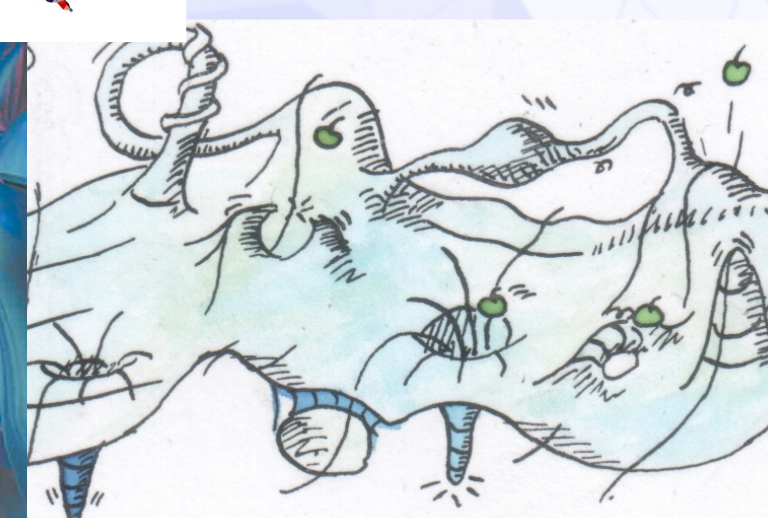
(artistic) impressions of “quantum foam”:



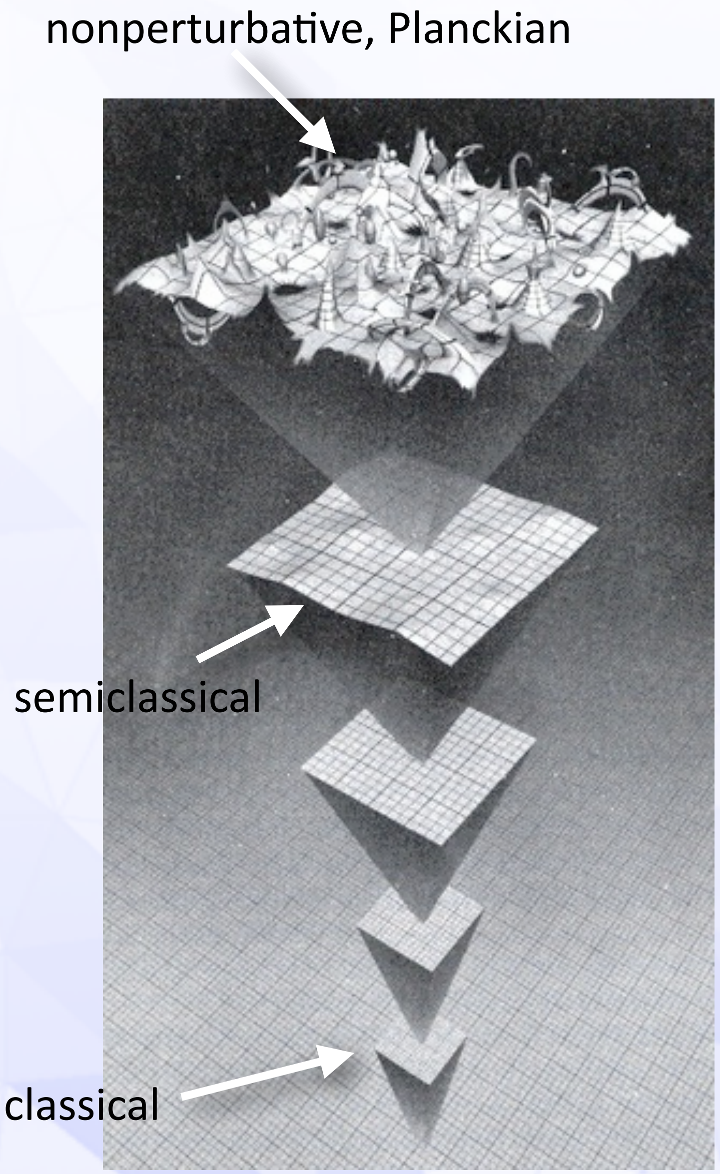








Going beyond classical and perturbative gravity



zooming in on a piece of empty spacetime

classical: $g_{\mu\nu}(x) \approx \eta_{\mu\nu}$, i.e. flat Minkowski metric on sufficiently small scales ($g_{\mu\nu}$ is a Lorentzian metric)

classical, linearized: $g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$, $|h_{\mu\nu}| \ll 1$, e.g. gravitational waves far away from source

perturbative quantum gravity: $\hat{g}_{\mu\nu}(x) = \eta_{\mu\nu} + \hat{h}_{\mu\nu}(x)$, but this theory is non-renormalizable, i.e. not useful near the Planck scale ℓ_{Pl}.

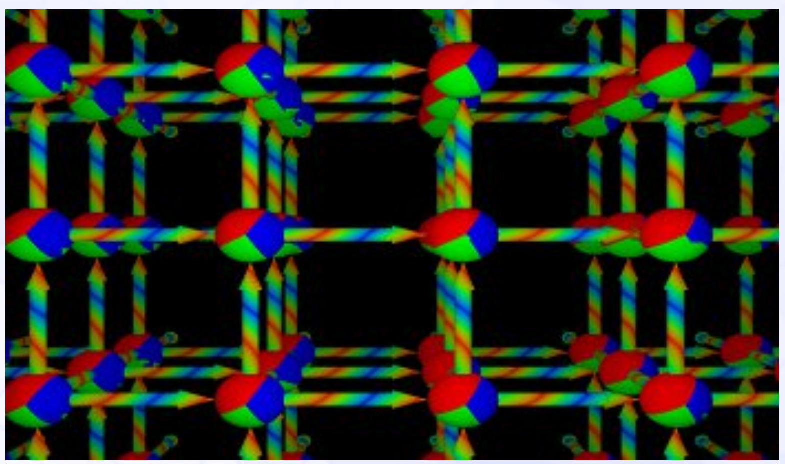
nonperturbative quantum gravity: what becomes of spacetime and the degrees of freedom of gravity at ℓ_{Pl} ?: quantum foam, wormholes? Unlike in $d=2$, nonperturbative systems of quantum geometry in higher dimensions are largely uncharted territory. Our classical geometric intuition is ***not*** a good guide. Unexpected things can and do happen!

How can we explore this extreme regime and obtain “quantum spacetime”?

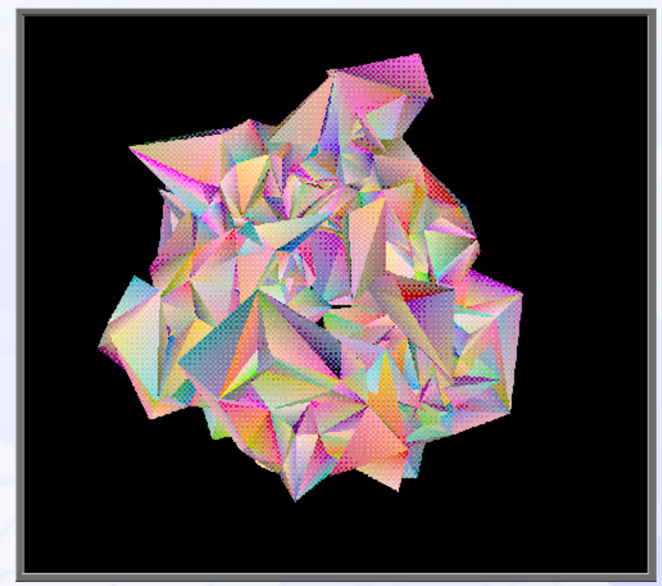
The setting

- following the extremely successful example of QCD, we explore the nonperturbative regime quantitatively by **“lattice quantum gravity”**
- lattice gauge field configurations à la Wilson (PRD 10 (1974) 2445) are replaced by piecewise flat geometries (triangulations) à la Regge (Nuovo Cim. 19 (1961) 558)

(© G. Bergner, Jena)



triangulated model of quantum space



- modern implementation: **Causal Dynamical Triangulations (CDT)**, a nonperturbative, background-independent, manifestly diffeo-morphism-invariant path integral, regularized on dynamical lattices
- N.B.: nontrivial scaling limit needed, no “fundamental discreteness”

Putting (quantum) gravity on a lattice …

- … presents some extra challenges, compared to non-abelian GFT
- early work: lattice versions of various first-order formulations of GR (vierbein $e_\mu{}^I$ + spin connection $\omega_\mu{}^{IJ}$) (Smolin (1979), Das, Kaku & Townsend (1979), Mannion & Taylor (1981), Kaku (1983), Tomboulis (1984), Caracciolo & Pelissetto (1984), Caselle, D'Adda & Magnea (1987), …)
- Monte Carlo simulations never found any interesting phase structure
- **issues:** measure (non-compact gauge groups)? status of compactified/ Euclideanized gravity? reflection positivity? metricity condition?

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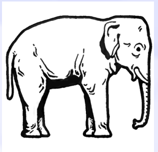
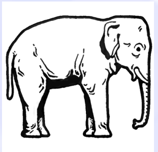
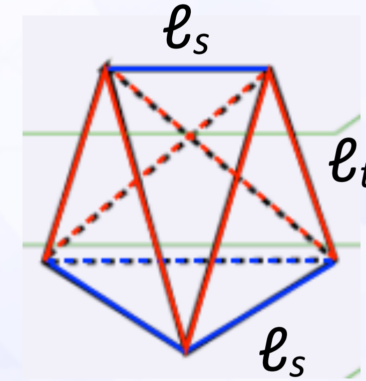
- … and a few   in the room:
  - what happened to diffeomorphism invariance? it's ***badly broken***!
  - unlike the YM action, the Einstein action in unbounded below; this is the ***potential killer*** of *any* 4D Euclidean gravitational path integral!

Crucial: lattice QG <u>without</u> diffeomorphisms

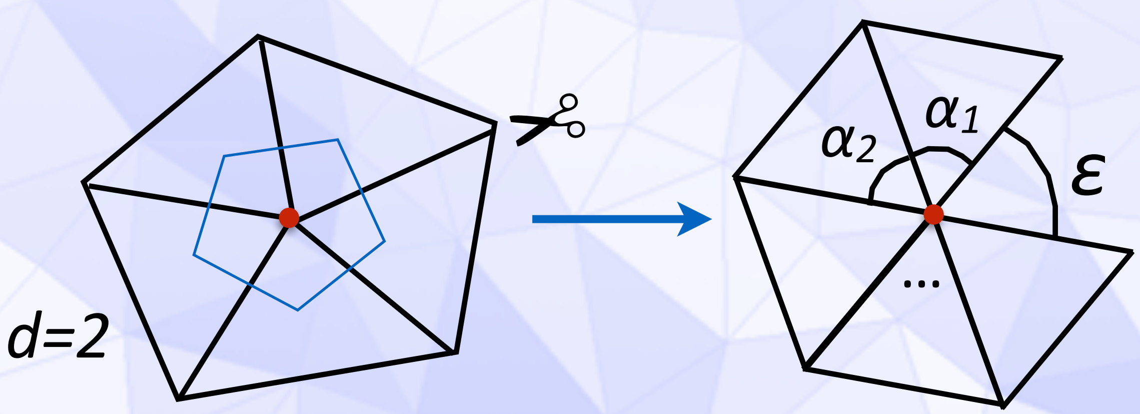
Strategy: approximate curved spacetimes by simplicial manifolds, following the profound, but underappreciated idea of "General Relativity without Coordinates" (Regge, 1961).



- 'piecewise flat' gluings of 4D triangular building blocks (four-simplices) describe intrinsically curved spacetimes

- Geometry is specified *uniquely* by the edge lengths ℓ of the simplices and how they are 'glued' together. **No coordinates are needed.**

- The full power of this idea is unleashed in the *quantum* theory, using a (C)DT path integral over dynamical, equilateral "lattices" ($\ell = a$ up to global time vs. space scaling, for a UV cut-off a).



- The CDT path integral has no coordinate redundancies. The MC simulations are relabeling invariant.

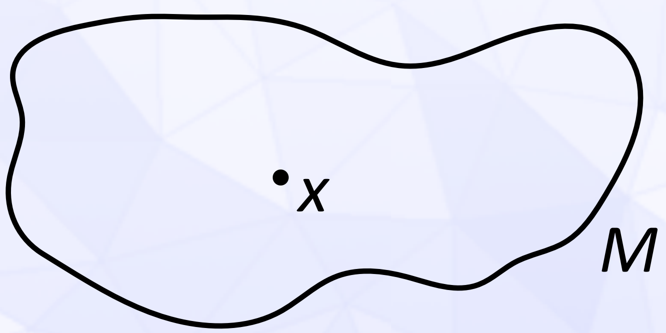
Gluing five equilateral triangles around a vertex generates a surface with Gaussian curvature (deficit angle ε) at the vertex.

Observables are key

Regardless of microscopic degrees of freedom and dynamical principle governing QG at the Planck scale, observables are needed to

- understand the gauge-invariant content of a given candidate theory,
- compare to other models, before developing genuine phenomenology,
- establish the existence of a classical limit consistent with GR.

Classical gravitational observables are diffeomorphism-invariant and usually *nonlocal* quantities, unlike in YM.
For example, $g_{\mu\nu}(x)$ and $R(x)$ are **not** observables while $\int_M d^4x\sqrt{g}\,R(x)$ is.



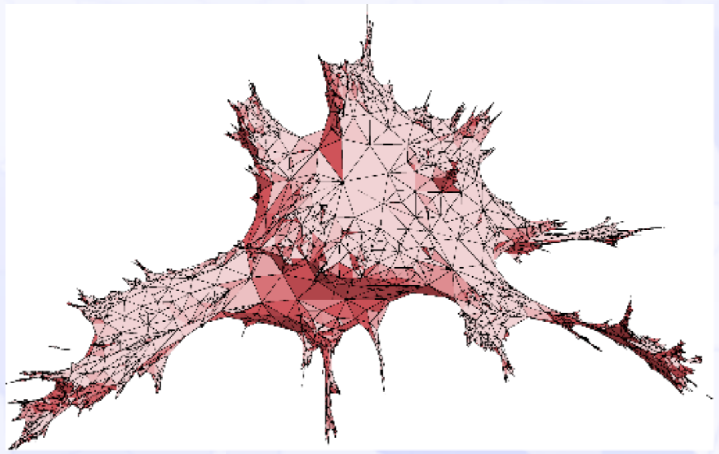
Nonperturbative QG and "quantum spacetime" are unlikely to be described by smooth metric fields $g_{\mu\nu}(x)$, but there ought to be notions of *distance* and *volume* which can be used to construct (pre-geometric) **quantum observables** $\hat{\mathcal{O}}$ with suitable invariance properties.

Implementation is another matter

$$\langle\hat{\mathcal{O}}\rangle = \frac{1}{Z}\int \mathcal{D}g\, \mathcal{O}[g]\, \mathrm{e}^{-S^{\mathrm{EH}}[g]} \quad ??$$

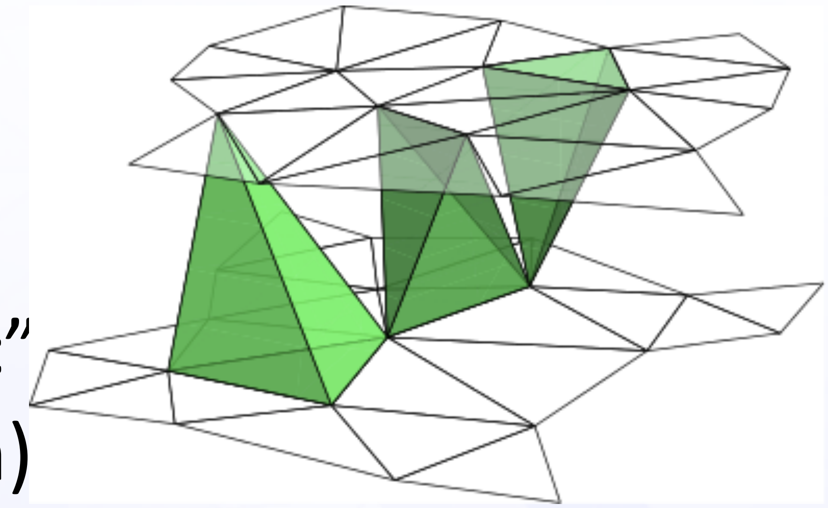
The longstanding problem of nonperturbative quantum gravity was that we had no idea *what* observables $\hat{\mathcal{O}}$ to calculate and *how*.

This is no longer true, thanks to a significant body of results on "dynamical triangulations (DT)" since the mid-1980s. One assembles curved manifolds from identical flat building blocks and investigates their ensemble behaviour in suitable limits, analytically in 2D (David (1985), Ambjørn, Durhuus & Fröhlich (1985), Kazakov (1985), …), and numerically in 4D (Agishtein & Migdal (1992), Ambjørn & Jurkiewicz (1992), …), amounting to nonperturbative, manifestly coordinate-independent implementations of the (Euclidean) gravitational path integral.



2D DT path integral history (T. Budd)

CDT Quantum Gravity

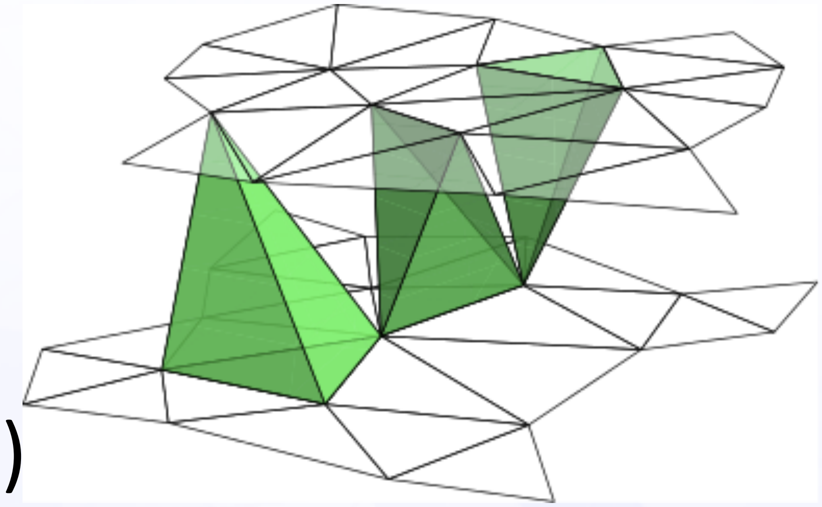


part of a (piecewise flat) causal triangulation

DT QG provides a concrete lattice framework to define and ***quantitatively evaluate*** "pre-geometric" observables (like spectral and Hausdorff dimension) ***in a regime far away from classicality***, and addresses several important issues (measure, unboundedness problem, uniqueness, invariance, cosmological constant $\Lambda^{ren} > 0$).

$$Z^{\mathrm{eu}}(G_{\mathrm{N}},\Lambda) = \int_{g\in\frac{Riem(M)}{Diff(M)}} \mathcal{D}g\ \mathrm{e}^{-S^{\mathrm{EH}}[g]} \rightarrow Z^{\mathrm{DT}}(G_{\mathrm{N}},\Lambda) = \lim_{a\to 0} \sum_{\substack{inequiv.\\ triang.\,T}} \frac{1}{C(T)}\,\mathrm{e}^{-S^{\mathrm{Regge}}_{\mathrm{eu}}[T]}$$

CDT Quantum Gravity



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$$Z(G_{\rm N},\Lambda) = \int_{g\in\frac{Lor(M)}{Diff(M)}} \mathcal{D}g\, {\rm e}^{iS^{\rm EH}[g]} \to Z^{\rm CDT}(G_{\rm N},\Lambda) = \lim_{a\to 0} \sum_{\substack{inequiv.\\ causal\\ triang.\, T}} \frac{1}{C(T)}\, {\rm e}^{iS^{\rm Regge}[T]}$$

To find extended 4D spacetime in a large-scale limit, realize reflection positivity and obtain second-order phase transitions, one seems to need a causal, Lorentzian version (with Wick rotation) of this set-up, ***Causal Dynamical Triangulations***. (Ambjørn & RL (1998), Ambjørn, Jurkiewicz & RL (2004), …)

Back to quantum observables

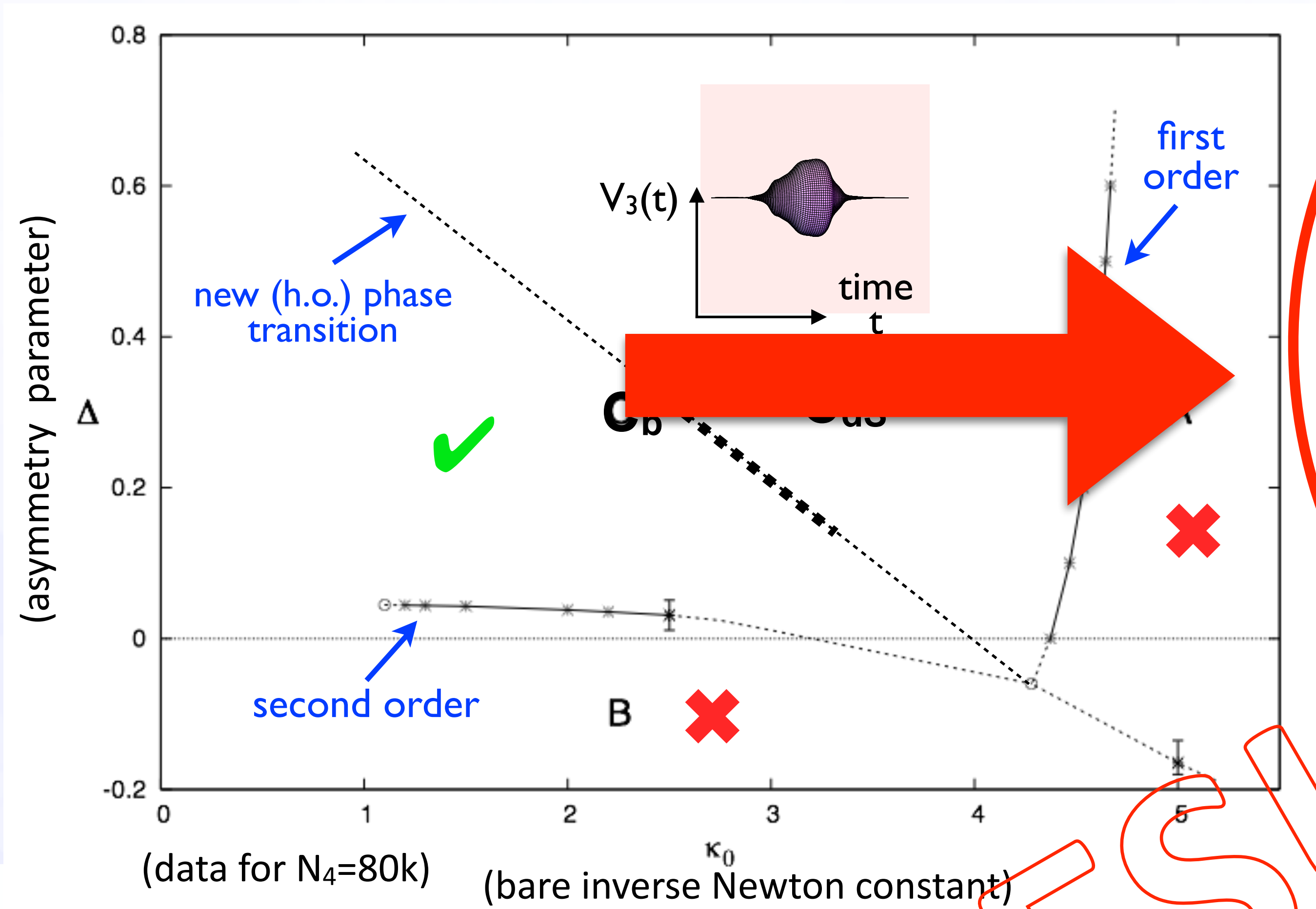
New results from and about CDT QG (on phase structure, critical behaviour, RG trajectories, properties of quantum spacetime) have come from evaluating a few **nonperturbative quantum observables**, operationally defined in terms of *distance and volume measurements*.

(Pre-)geometric observables display a significant degree of universality and can signal generic pathologies (aka 'those d… branched polymers').

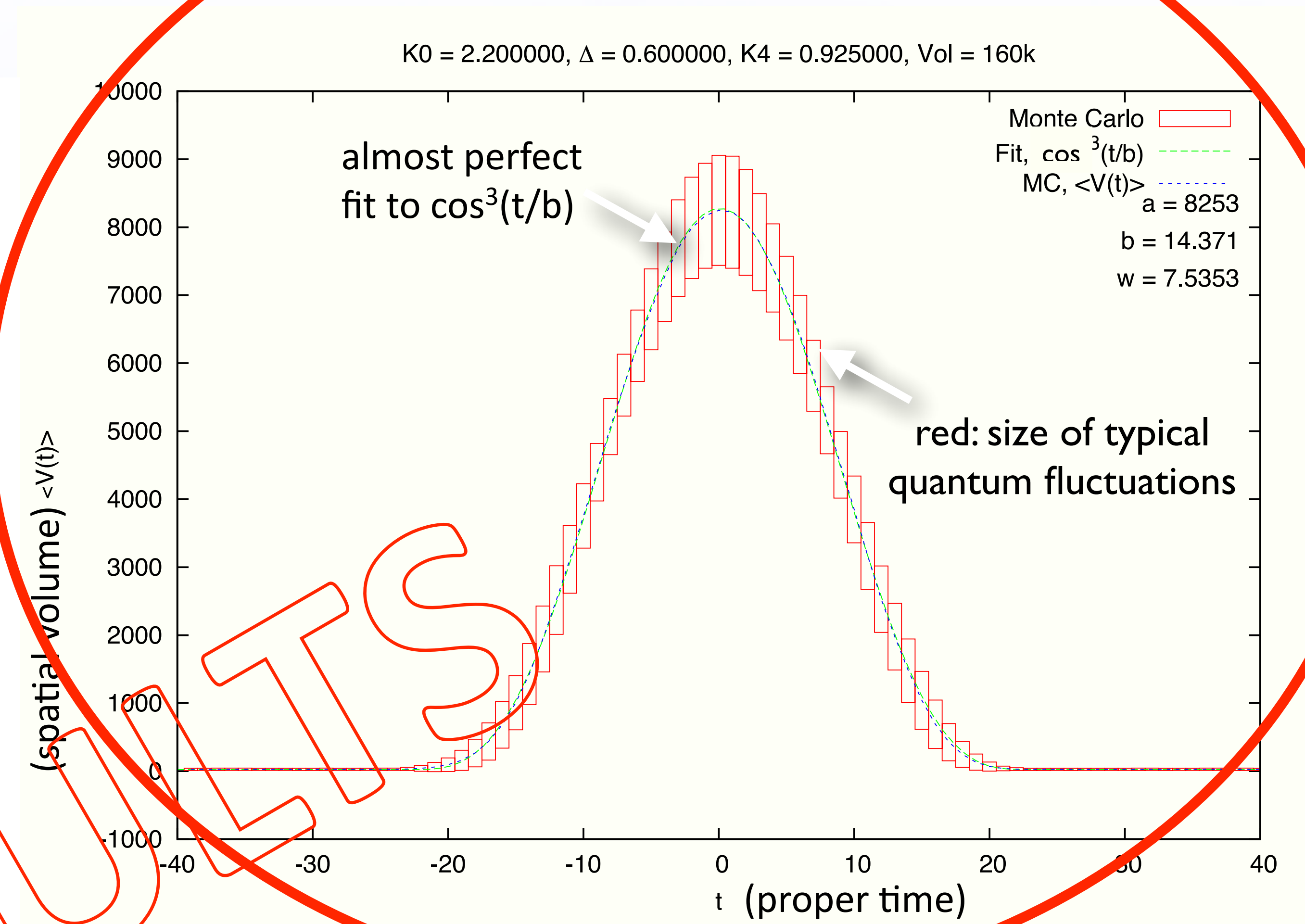
Impact across approaches of the ***spectral dimension***: extracting a dimension from the behaviour of random walkers on quantum spacetime one finds a scale-dependent "dynamical dimensional reduction" 4 → 2 near ℓ_{Pl} (Ambjørn, Jurkiewicz & RL (2005), Lauscher & Reuter (2005), …); perhaps a universal feature of QG? (Carlip (2017))

Going beyond "dimension", one has measured the global shape $\langle V(t) \rangle$ (=spatial volume as a function of proper time) of the quantum universe emerging in CDT QG, and it matches that of a ***de Sitter universe***! (Ambjørn, Görlich, Jurkiewicz & RL (2008))

Phase diagram of CDT QG

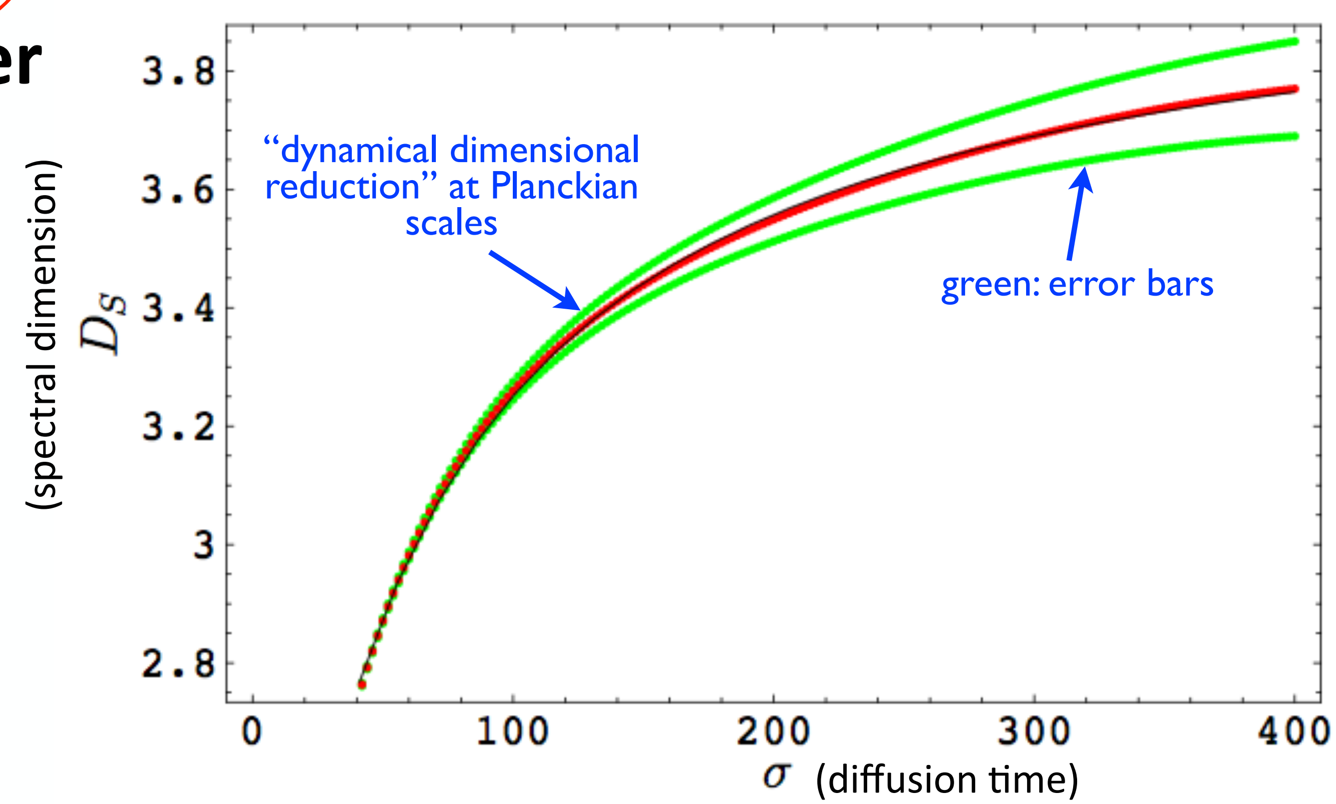


The universe is de Sitter-shaped

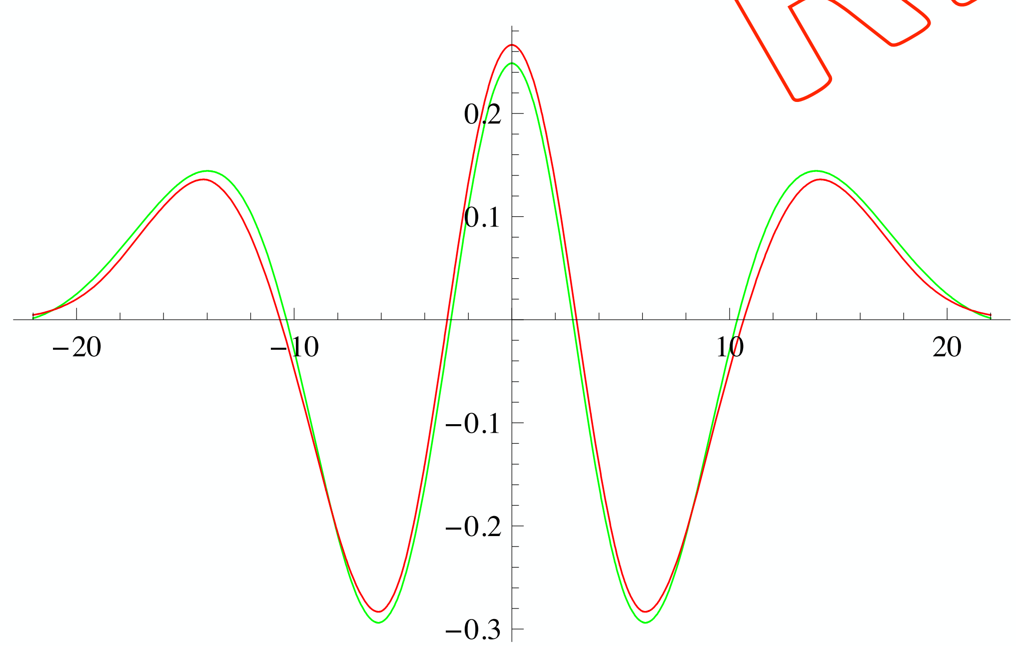


RESULTS

Spectral dimension of the universe

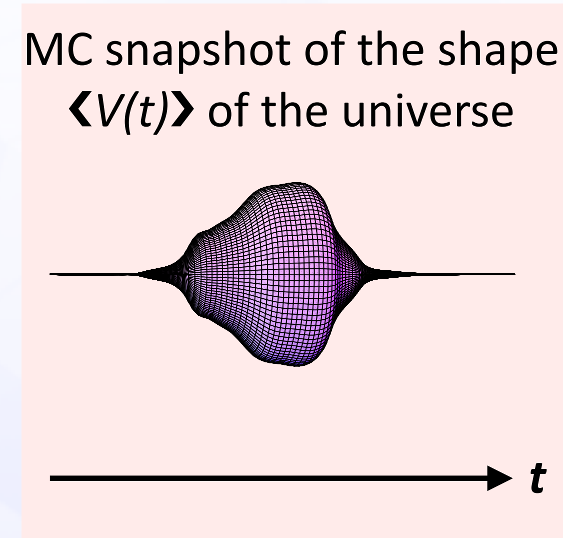


Volume fluctuations around de Sitter



(low-lying eigenmode matches with semiclassical expectation)

The challenge of "*quantum curvature*"

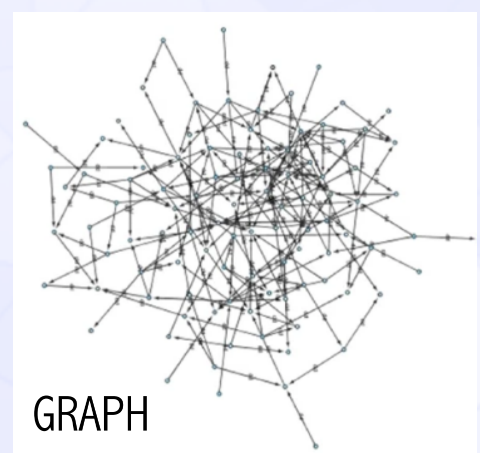


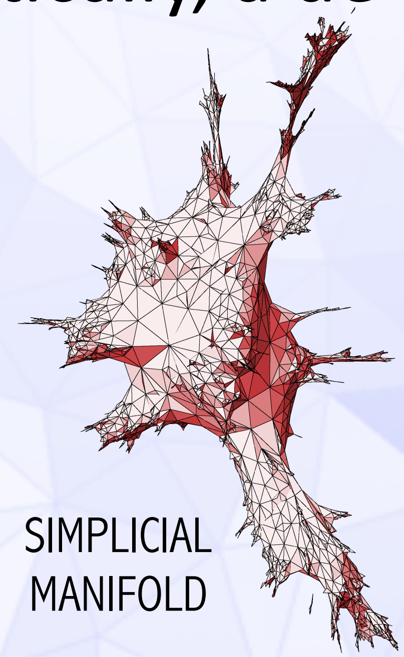
Even if the quantum universe is de Sitter-shaped, it does ***not*** mean it's a (Euclidean) de Sitter space S^4,

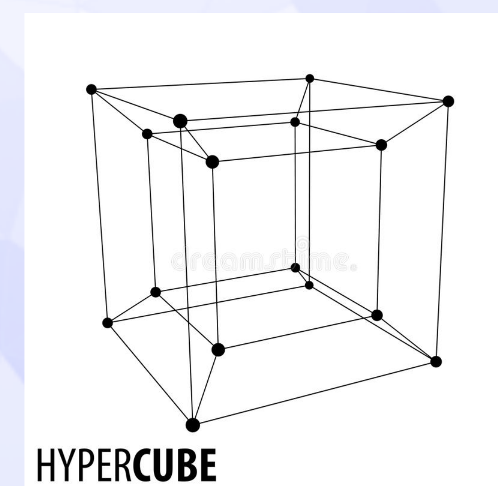
$$ds^2 = dt^2 + c^2 \cos^2(t/c) d\Omega^2_{(3)}.$$

The shape of the universe (= the spatial volume *V(t)* as a function of proper time *t*) is a single geometric variable, but there are many more. Characteristically, a de Sitter space has *constant positive curvature*.

Challenge for afficionados of "quantum bits": ***what is the curvature of a non-smooth metric space?*** $R^{\kappa}{}_{\lambda\mu\nu}(x) = ?$







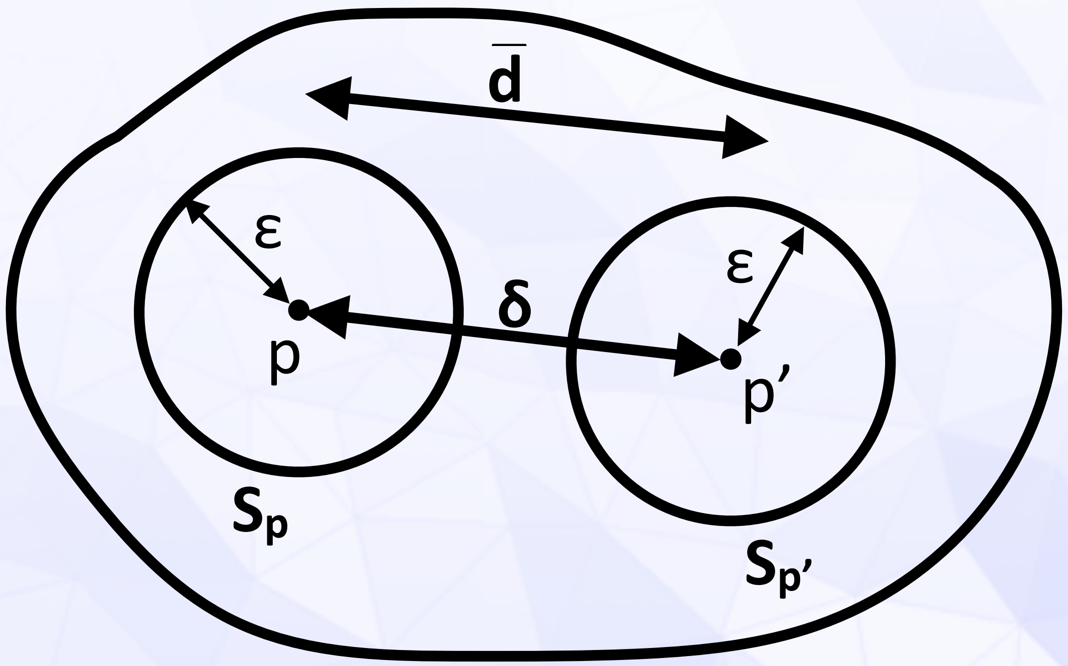
We have successfully defined and tested ***quantum Ricci curvature.***

(N. Klitgaard & RL, PRD 97 (2018) no.4, 0460008 and no.10, 106017)

Introducing quantum Ricci curvature

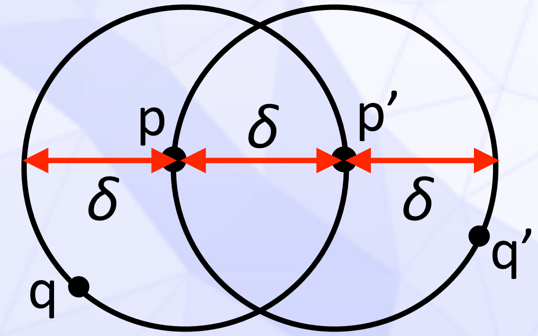
In D dimensions, the key idea is to compare the distance $\bar{d}$ between two $(D\text{-}1)$-spheres with the distance δ between their centres.

The sphere-distance criterion:
"On a metric space with positive (negative) Ricci curvature, the distance $\bar{d}$ of two nearby spheres S_p and $S_{p'}$ is smaller (bigger) than the distance δ of their centres."



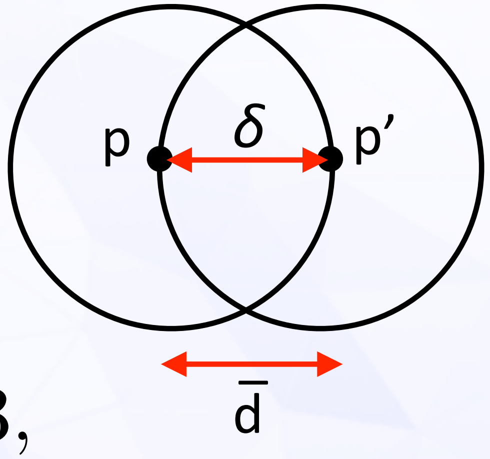
(c.f. Y. Ollivier, J. Funct. Anal. 256 (2009) 810)

Our variant uses the ***average sphere distance*** of two spheres of radius δ whose centres are a distance δ apart,



$$\bar{d}(S_p^\delta, S_{p'}^\delta) := \frac{1}{vol(S_p^\delta)} \frac{1}{vol(S_{p'}^\delta)} \int_{S_p^\delta} d^{D-1}q\,\sqrt{h} \int_{S_{p'}^\delta} d^{D-1}q'\,\sqrt{h'}\; d(q, q'),$$

Implementing quantum Ricci curvature



From the quotient of sphere distance and centre distance we extract the **"quantum Ricci curvature K_q at scale δ"**,

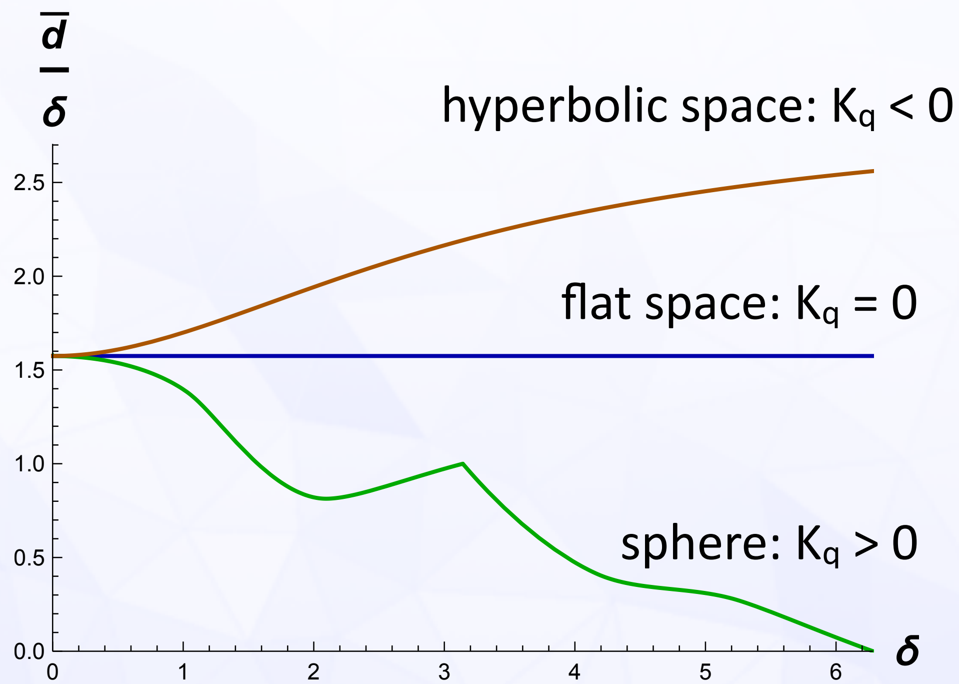
$$\frac{\bar{d}(S_p^\delta, S_{p'}^\delta)}{\delta} = c_q(1 - K_q(p,p')), \quad \delta = d(p,p'), \quad 0 < c_q < 3,$$

where c_q is a non-universal constant depending on the type and the dimension D of the space. For smooth Riemannian manifolds and $\delta \ll 1$:

$$\frac{\bar{d}}{\delta} = \begin{cases} 1.5746 + \delta^2\left(-0.1440\, Ric(v,v) + \mathcal{O}(\delta)\right), & D=2, \\ 1.6250 + \delta^2\left(-0.0612\, Ric(v,v) - 0.0122\, R + \mathcal{O}(\delta)\right), & D=3, \\ 1.6524 + \delta^2\left(-0.0469\, Ric(v,v) - 0.0067\, R + \mathcal{O}(\delta)\right), & D=4, \end{cases}$$

- involves only *distance and volume measurements*
- the directional/tensorial character is captured by the "double sphere", coarse-graining by the variable scale δ
- simplest observable (average Ricci scalar): average first over p′, then p

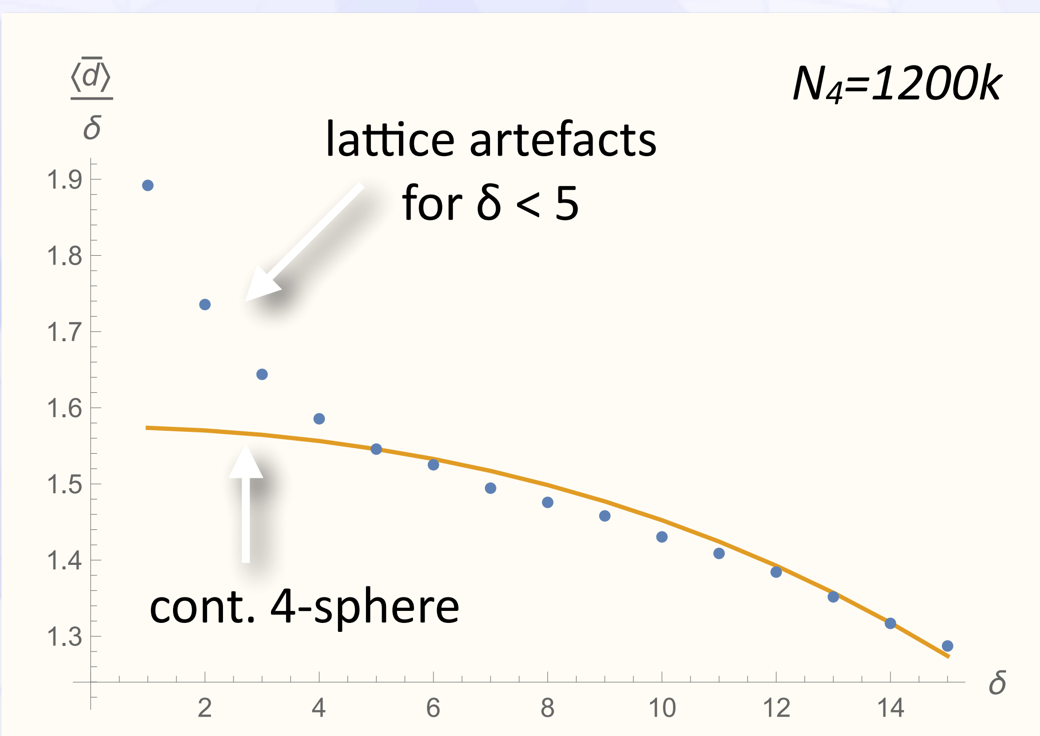
Quantum curvature of the de Sitter universe

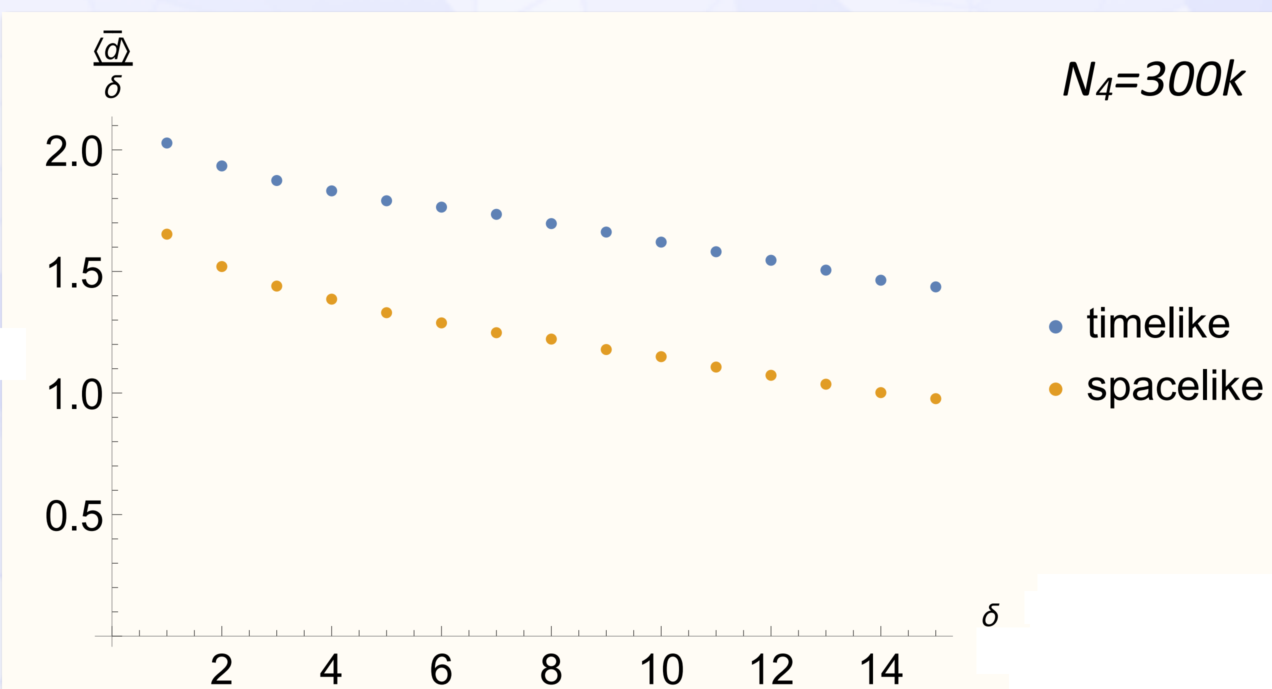


K_q on classical, constantly curved spaces in D=2 (curvature radius 1)

To interpret quantum results, we are building a reference library of *curvature profiles $K_q(\delta)$* on various classical spaces. (J. Brunekreef & RL, arXiv:2011.10168)

Measurements in 4D CDT QG on $S^3 \times S^1$ at volumes $N_4 \leq 1.2 \times 10^6$ clearly show that $\langle K_q \rangle > 0$, with a good fit to S^4! Same *Ric* in time- and spacelike directions!





$\langle K_q \rangle$ of the dynamically generated dS universe

(N. Klitgaard & RL, Eur. Phys. J. C 80 (2020))

Summary

Nonperturbative quantum gravity can be studied in a **lattice** setting, in close analogy with lattice QCD, but taking into account the dynamical nature of geometry. Quantum observables are crucial.

The full power of Regge's idea of describing geometry without coordinates unfolds in nonperturbative QG in terms of Causal Dynamical Triangulations, yielding a **truly geometric path integral**.

Despite the absence of smoothness, one can define a notion of **Ricci curvature** that appears to be well-defined all the way to the Planck scale. Remarkably, we have found good evidence that the emergent quantum de Sitter universe at $\sim$10 ℓ_{Pl} is compatible with a round S^4.

CDT REVIEWS: J. Ambjørn, A. Görlich, J. Jurkiewicz & RL, Phys. Rep. 519 (2012) 127, arXiv: 1203.3591; RL, CQG 37 (2020) 013002, arXiv:1905.08669


Thank you!