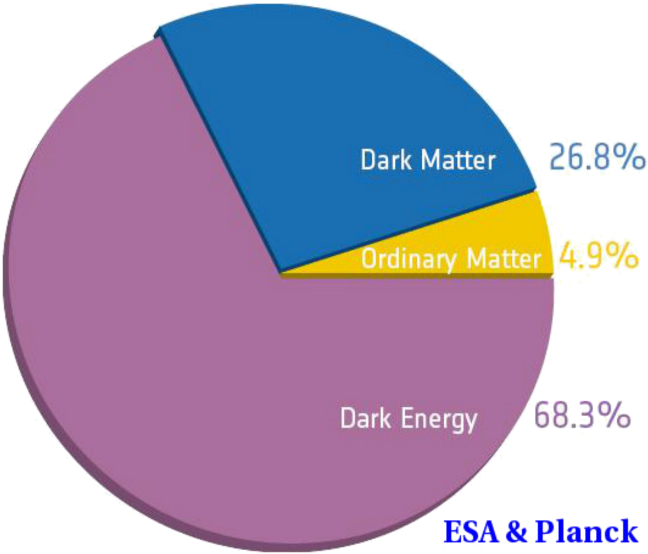
Composite dark matter and the role of lattice field theory

David Schaich (U. Liverpool)

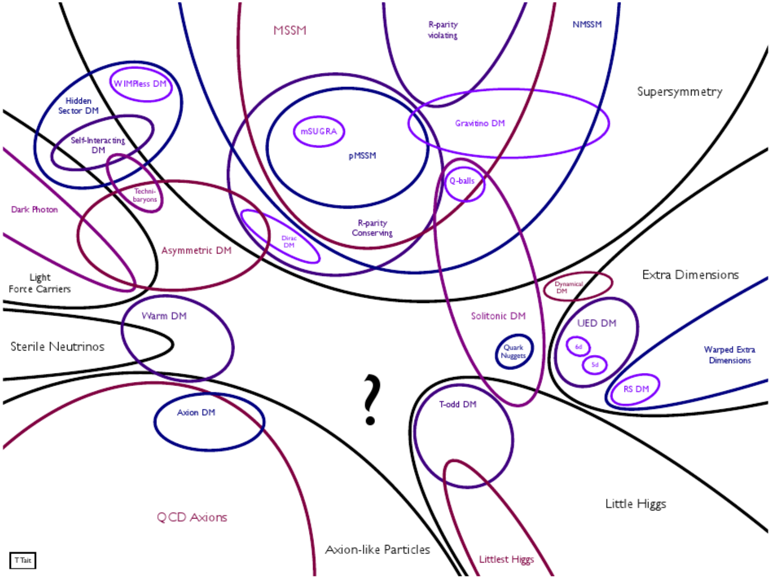
Theoretical Physics Seminar
Dublin Institute for Advanced Studies
17 November 2021

arXiv:2006.16429 and more to come
with the Lattice Strong Dynamics Collaboration

Dark matter — we observe it...



...we don't yet know what it is



Overview and plan

Composite dark matter is an attractive possibility

Lattice field theory is needed
to test models against experimental results

Why: Composite dark matter

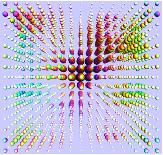
How: Lattice field theory

What: Recent, ongoing & planned work

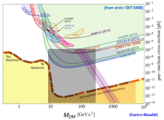
Direct detection experiments

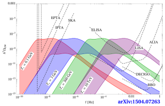
Gravitational-wave observatories

Collider experiments, galactic sub-structure, . . .



(Image credit: Claudio Rebbi)





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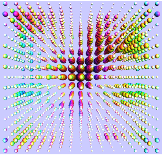

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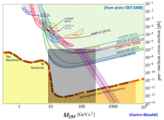
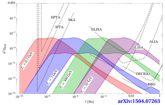


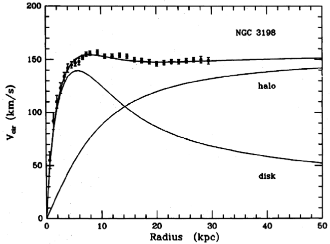


These slides: davidschaich.net/talks/2111Dublin.pdf

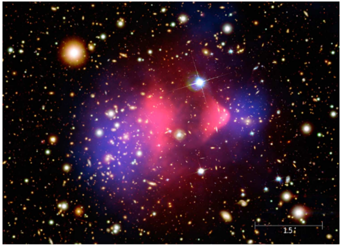
Interaction encouraged — complete coverage unnecessary

Gravitational evidence for dark matter

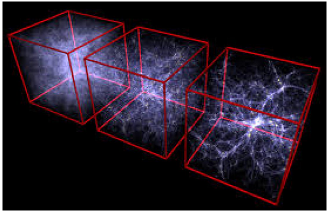
Rotation $\sim 10^3$–10^6 light-years



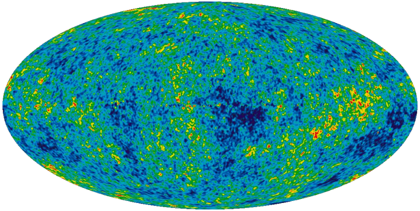
Lensing $\sim 10^6$ light-years



Structure $\sim 10^9$ light-years



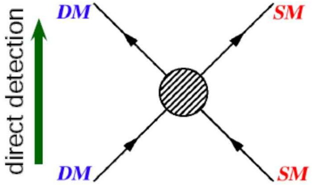
Cosmic background $\sim 10^{10}$ ly

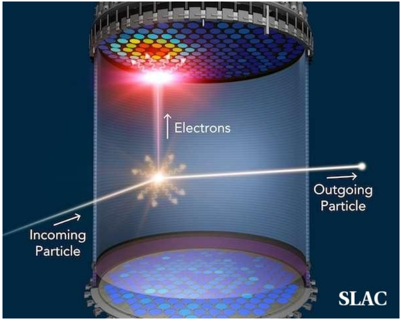


Non-gravitational dark matter interactions

Three search strategies

Direct scattering in underground detectors



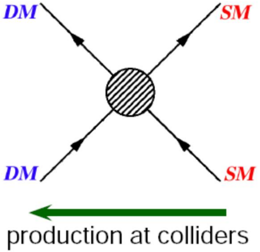


Non-gravitational dark matter interactions

Three search strategies

Direct scattering in underground detectors

Collider production at high energies



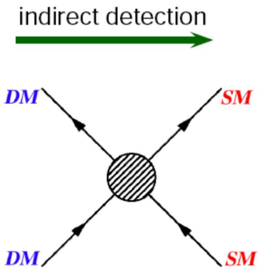
Non-gravitational dark matter interactions

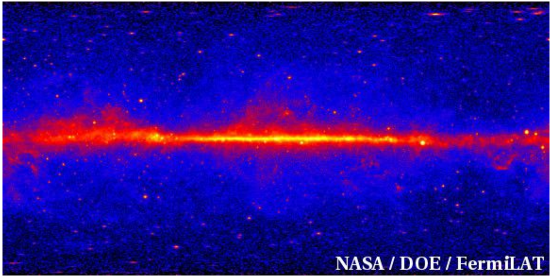
Three search strategies

Direct scattering in underground detectors

Collider production at high energies

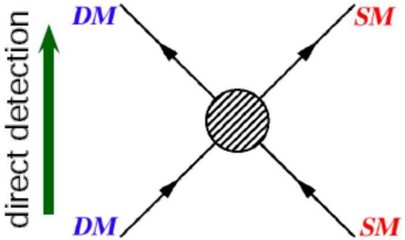
Indirect annihilation into cosmic rays

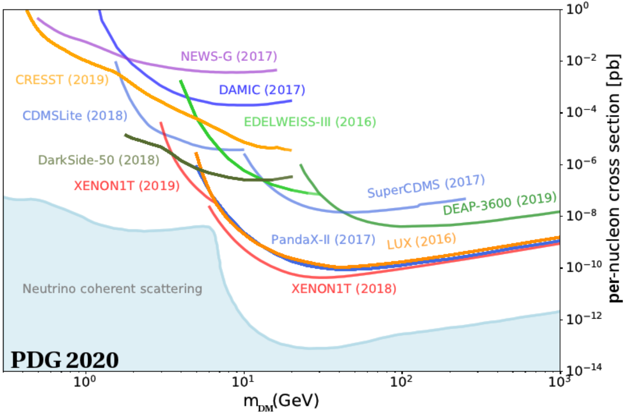




Non-gravitational dark matter interactions

No clear signals so far

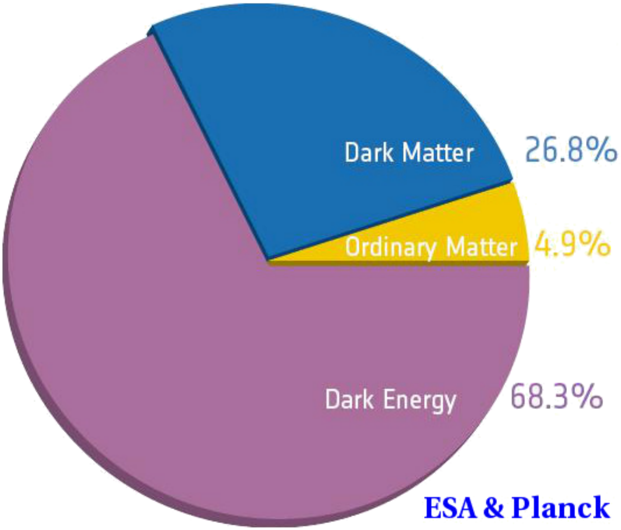




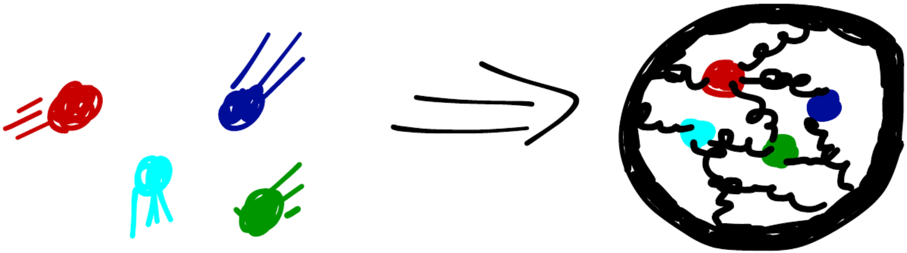
Why we expect non-gravitational interactions

$$\frac{\Omega_{\text{dark}}}{\Omega_{\text{ordinary}}} \approx 5 \quad \ldots \text{not } 10^5 \text{ or } 10^{-5}$$

Explained by non-gravitational interactions in the early universe



Composite dark matter



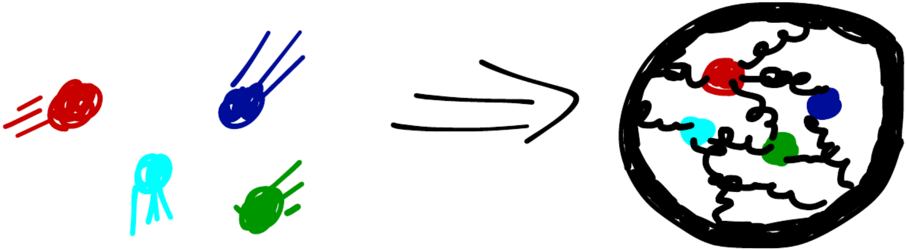
Early universe

Deconfined charged fermions $\longrightarrow$ explain relic density

Present day

Confined neutral 'dark baryons' $\longrightarrow$ no experimental detections

Composite dark matter



Present day

Confined neutral 'dark baryons' $\longrightarrow$ no experimental detections

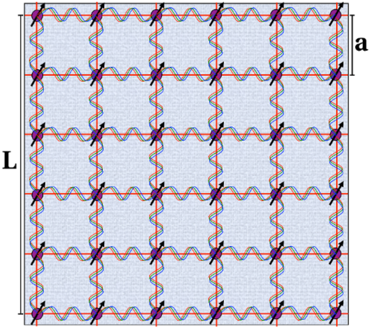
Interact via charged constituents

$\longrightarrow$ need **lattice calculations** for quantitative predictions

Lattice field theory in a nutshell

Formally $\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \;\; \mathcal{O}(\Phi) \;\; e^{-S[\Phi]}$

Regularize by formulating theory in finite, discrete, euclidean space-time
↖Gauge invariant, non-perturbative, 4-dimensional



P. Vranas LLNL

Spacing between lattice sites ("a")
$\longrightarrow$ UV cutoff scale $1/a$

Remove cutoff: $a \to 0$ $(L/a \to \infty)$

Hypercubic $\longrightarrow$ Poincaré symmetries ✓

Numerical lattice field theory calculations

High-performance computing $\longrightarrow$ evaluate up to $\sim$billion-dimensional integrals
(Dirac operator as $\sim 10^9 \times 10^9$ matrix)

Results to be shown, and work in progress, require state-of-the-art resources

Many thanks to national labs, USQCD–DOE, and computing centres!

Lassen @Livermore

USQCD @Fermilab

Barkla @Liverpool

Numerical lattice field theory calculations

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Importance sampling Monte Carlo

Algorithms sample field configurations with probability $\frac{1}{\mathcal{Z}} e^{-S[\Phi]}$

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \;\; \mathcal{O}(\Phi) \;\; e^{-S[\Phi]} \longrightarrow \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}(\Phi_i) \text{ with stat. uncertainty } \propto \frac{1}{\sqrt{N}}$$

Numerical lattice field theory calculations

Importance sampling Monte Carlo

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Lattice calculation requires specific theory $\longleftrightarrow$ lattice action $S[\Phi]$

Our strategy aims to gain generic insights into composite dark matter

Lattice Strong Dynamics Collaboration

Argonne	Xiao-Yong Jin, James Osborn
Bern	Andy Gasbarro
Boston	Venkitesh Ayyar, Rich Brower, Evan Owen, Claudio Rebbi
Colorado	Anna Hasenfratz, Ethan Neil, Curtis Peterson
UC Davis	Joseph Kiskis
Livermore	Dean Howarth, Pavlos Vranas
Liverpool	Chris Culver, DS
Michigan	Enrico Rinaldi
Nvidia	Evan Weinberg
Oregon	Graham Kribs
Siegen	Oliver Witzel
Trieste	James Ingoldby
Yale	Thomas Appelquist, Kimmy Cushman, George Fleming

Exploring the range of possible phenomena in strongly coupled field theories

Direct detection of composite dark matter

Charged constituents $\longrightarrow$ **form factors** $\longrightarrow$ experimental signals

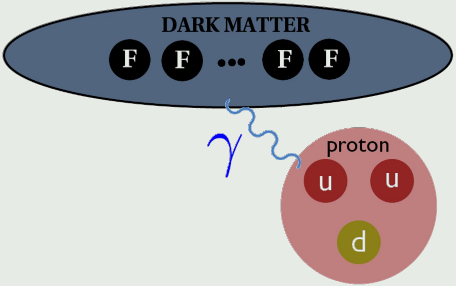
Photon exchange from electromagnetic form factors

Effective interactions suppressed by powers of dark matter mass

$$\text{Magnetic moment} \sim \frac{1}{M_{DM}}$$

$$\text{Charge radius} \sim \frac{1}{M_{DM}^2}$$

$$\text{Polarizability} \sim \frac{1}{M_{DM}^3}$$

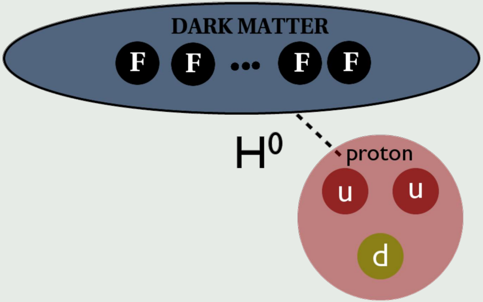


Direct detection of composite dark matter

Charged constituents $\longrightarrow$ **form factors** $\longrightarrow$ experimental signals

Higgs exchange from scalar form factor

Can dominate cross section. . . **if** F mass comes from Higgs

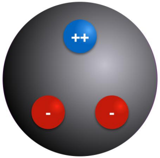


Direct detection of composite dark matter

Charged constituents $\longrightarrow$ **form factors** $\longrightarrow$ experimental signals

Simple first case: Dark matter as a "more-neutral neutron"

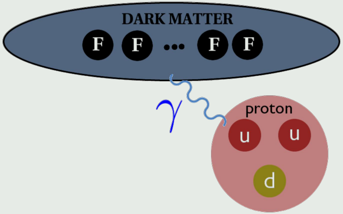
SU(3) with weak singlets $\longrightarrow$ no Higgs-exchange interaction



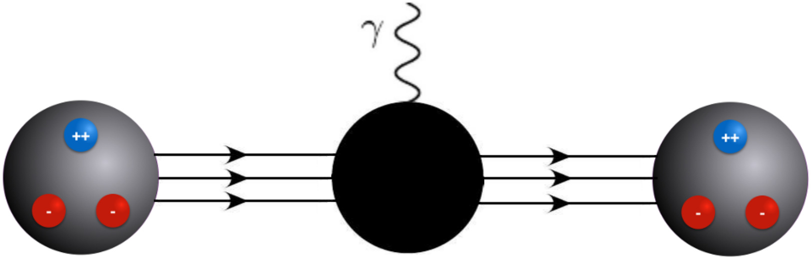
Investigate leading photon-exchange contributions

Magnetic moment $\sim \dfrac{1}{M_{DM}}$

Charge radius $\sim \dfrac{1}{M_{DM}^2}$



Magnetic moment and charge radius



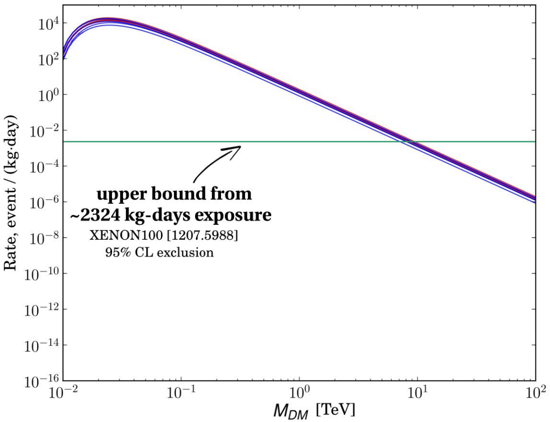
$$\langle DM(p') | \Gamma_\mu(q^2) | DM(p) \rangle \sim F_1(q^2)\, \gamma_\mu + F_2(q^2)\, \frac{i\sigma_{\mu\nu} q^\nu}{2M_{DM}}, \qquad q = p' - p$$

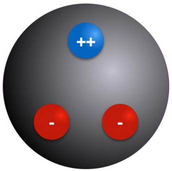
Electric charge: $F_1(0) = 0$ **Magnetic moment:** $F_2(0)$

Charge radius: $\langle r_E^2 \rangle = -6 \left. \frac{dF_1(q^2)}{dq^2} \right|_{q^2=0} + \frac{3F_2(0)}{2M_{DM}^2}$

Resulting direct detection constraints

Lattice calculations of magnetic moment and charge radius
$\longrightarrow$ event rate vs. dark matter mass





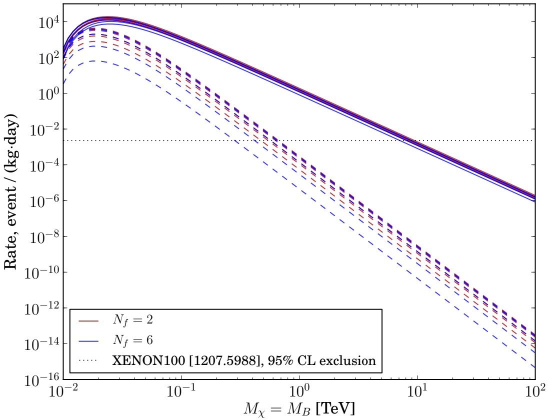
XENON100 $\longrightarrow$ $M_B \gtrsim 10$ TeV

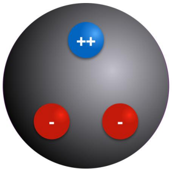
XENON1T $\longrightarrow$ $M_B \gtrsim 30$ TeV [1805.12562]

Little effect from varying model params

Magnetic moment dominates event rate

Dashed charge radius contributions suppressed $\sim 1/M_{DM}^2$





Can change symmetries to forbid both magnetic moment and charge radius

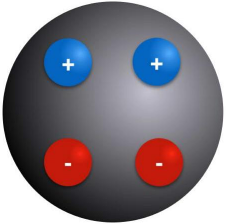
$\longrightarrow$ More interesting second case: 'Stealth Dark Matter'

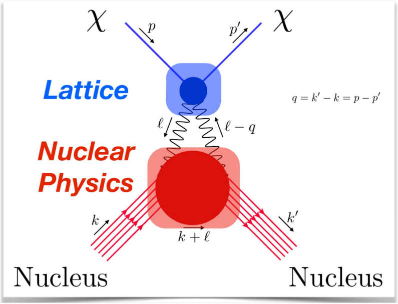
SU(4) Stealth Dark Matter

Fermions now include weak doublet & singlets

Scalar ‘baryon’ $\longrightarrow$ no magnetic moment ✓

+/- charge symmetry $\longrightarrow$ no charge radius ✓





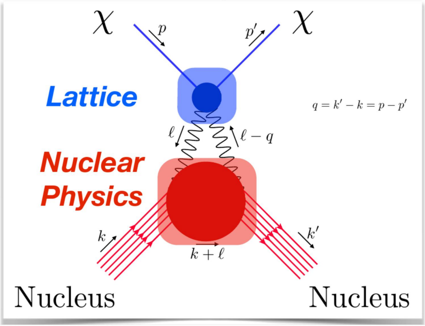
(Tiny) Coupling to Higgs needed for nucleosynthesis

Polarizability $\sim 1/M_{DM}^3$ dominates direct detection

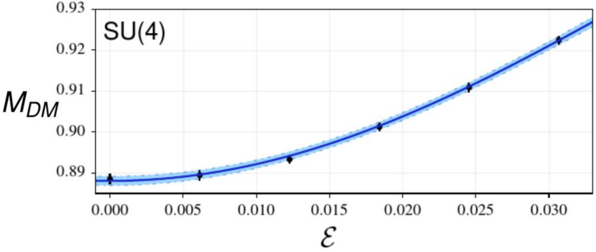
$\longrightarrow$ Unavoidable lower bound
on broad set of composite dark matter models

Polarizability of Stealth Dark Matter

Unavoidable lower bound
on broad set of composite dark matter models

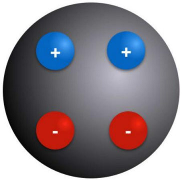


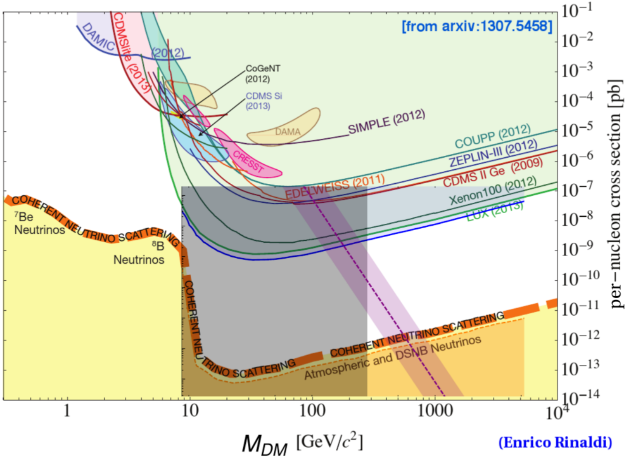
Nuclear physics very complicated
with large uncertanties



Polarizability is dependence
of lattice M_{DM} on external field $\mathcal{E}$

Lower bound on direct detection



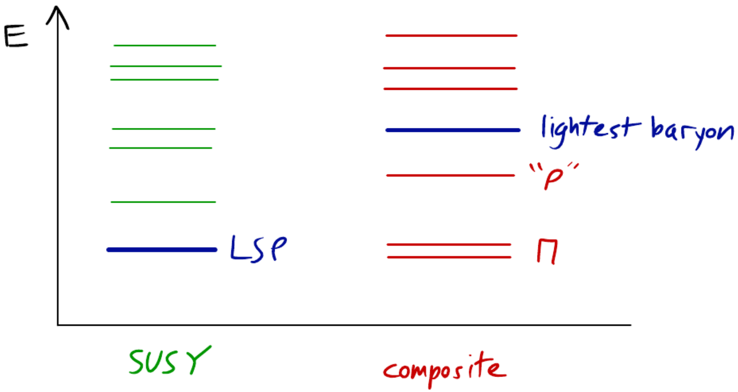


Results specific
to Xenon detectors

Uncertainty dominated
by Xenon nuclear physics

Shaded region is complementary constraint from particle colliders

Collider constraints



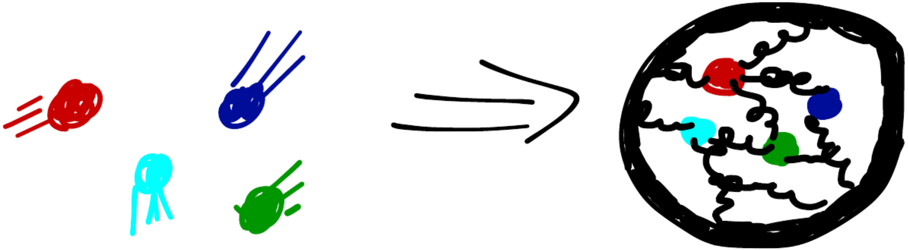
Dark baryon not lightest composite particle

'Missing energy' searches inefficient

Collider constraints from lighter **charged** 'Π' plus lattice calculation of M_{DM}/M_{Π}

Gravitational waves

Gravitational-wave observatories opening new window on cosmology



First-order confinement transition $\longrightarrow$ stochastic background of grav. waves

$\Longrightarrow$ Lattice studies of Stealth Dark Matter phase transition

Stealth Dark Matter phase diagram

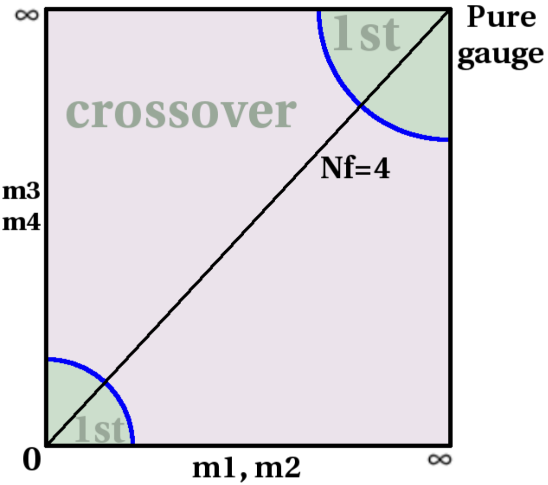
arXiv:2006.16429

Pure-gauge transition is first order

Becomes stronger as N increases

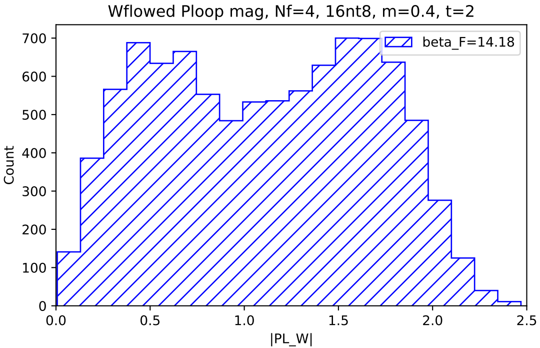
First-order transition persists
for sufficiently heavy fermions
$\longrightarrow M_P/M_V \gtrsim 0.9$

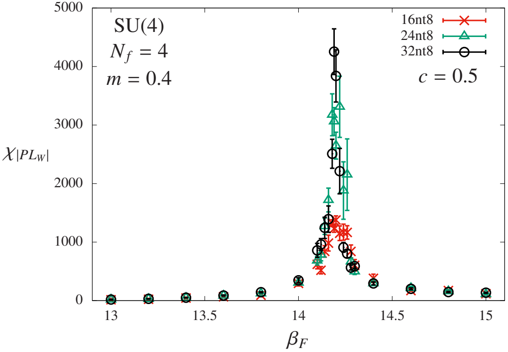
Form factor calculations considered
$0.55 \leq M_P/M_V \leq 0.77$



Determining order of thermal transition

arXiv:2006.16429





Left: Phase coexistence in Polyakov loop magnitude histogram

Right: Volume scaling of Polyakov loop susceptibility

From first-order transition to gravitational wave signal

First-order transition $\longrightarrow$ gravitational wave background will be produced

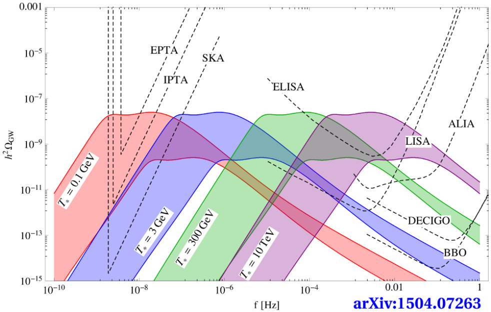
Four key parameters

Transition temperature $T_* \lesssim T_c$

Vacuum energy fraction from **latent heat**

Bubble nucleation rate (transition duration)

Bubble wall speed



arXiv:1504.07263

Low frequencies require space-based observatories or pulsar timing arrays

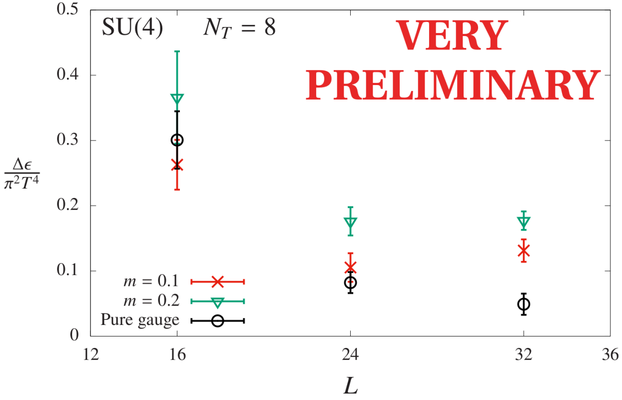
Work in progress: Latent heat $\Delta\epsilon$

First-order transition $\longrightarrow$ gravitational wave background will be produced

Vacuum energy fraction

$$\alpha \approx \frac{30}{4N(N^2-1)} \frac{\Delta\epsilon}{\pi^2 T_*^4}$$

Latent heat $\Delta\epsilon$
is change in energy density
at transition



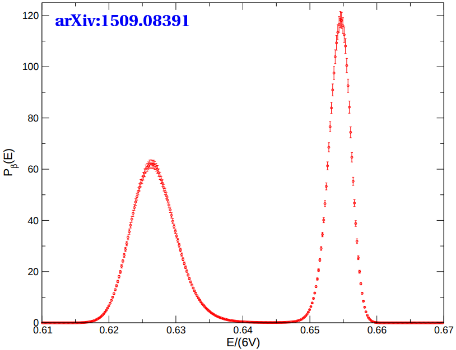
Work in progress: Density of states

Markov-chain importance sampling can struggle at first-order transition:
difficult to tunnel between coexisting phases

'LLR' generalization of Landau–Wang algorithm
$\longrightarrow$ continuous density of states $\rho(E)$ with exponential error suppression

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \;\; \mathcal{O}(\Phi) \;\; e^{-S[\Phi]}$$

$$\longrightarrow \frac{1}{\mathcal{Z}} \int dE \;\; \mathcal{O}(E) \;\; \rho(E) \;\; e^{-E}$$



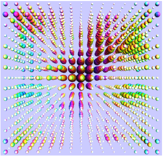
Work by Felix Springer

SU(4) code developed, analyses underway

Recapitulation and outlook

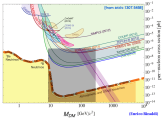
Composite dark matter is an attractive possibility

Lattice field theory is needed
to test models against experimental results

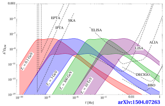


(Image credit: Claudio Rebbi)

Form factors for direct detection
$\longrightarrow$ Stealth Dark Matter setting lower bound



First-order early-universe transition
$\longrightarrow$ gravitational waves depending on latent heat etc.



And more: Collider experiments; galactic sub-structure;
indirect detection; relic abundance; . . .

Thank you!

Lattice Strong Dynamics Collaboration & Felix Springer

Funding and computing resources

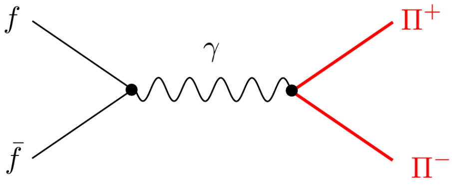
UK Research and Innovation

USQCD

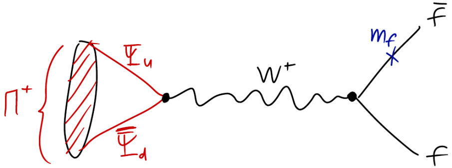
Supplement: Stealth Dark Matter at colliders

arXiv:1809.10184

Production



Decay



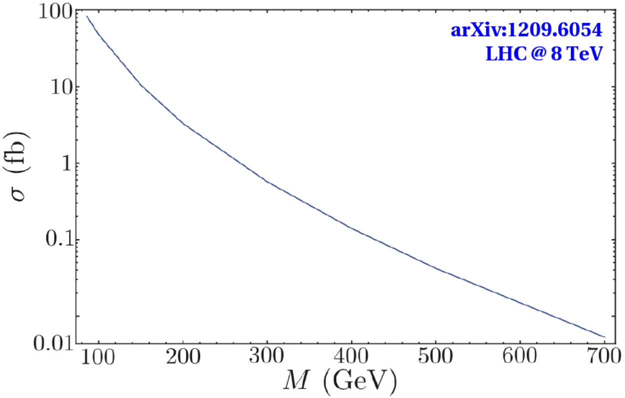
"Particularly tricky" at the LHC

Published bounds $M_\Pi \gtrsim 130$ GeV similar to $M_\Pi \gtrsim 100$ GeV from LEP

[ATLAS-CONF-2020-051 reports $M_\Pi \gtrsim 340$ GeV for lifetimes $\sim$0.1 ns]

More form factors to compute: $F_1(4M_\Pi^2)$ for Π and decay constant F_Π

Form factors for collider searches



Π pair production cross section

Integrate over proton parton dist.,
here setting $F_1(4M_\Pi^2) = 1$

For $M_\Pi \gtrsim 200$ GeV, LHC can search for $\Pi^+\Pi^- \longrightarrow t\bar{b} + \bar{t}b$
in addition to $\tau^+\tau^- + \not{E}_T$

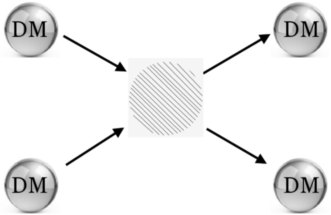
Supplement: Self-interactions and ‘small-scale’ structure

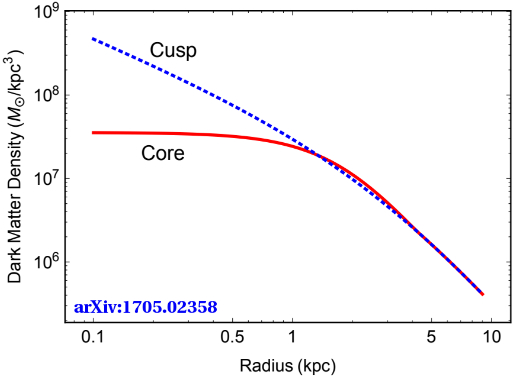
Astrophysical observations vs. collisionless dark matter

Persistent discrepancies on galactic scales

[“core vs. cusp”; “too big to fail”; “missing satellites”; “diversity” — Review: arXiv:1705.02358]

Can be addressed by
dark matter self-interactions



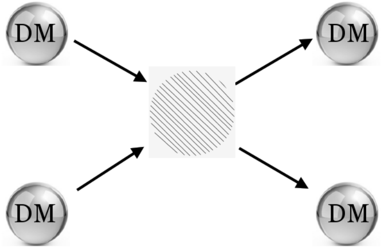


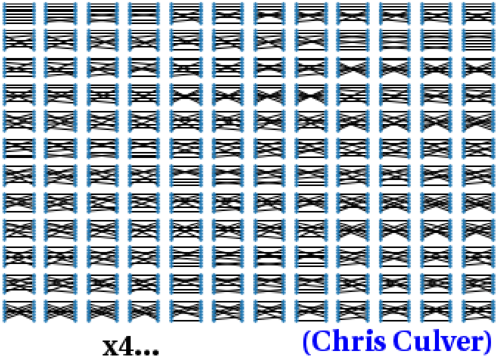
Baryon–baryon scattering work in progress

2×4fermions $\times$ SU(4) gauge group $\longrightarrow$ proliferation of contractions

[comparable to QCD triton or He nucleus]

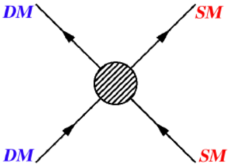
Work in progress to apply state-of-the-art stochastic LapH methods





Backup: Thermal freeze-out for relic density

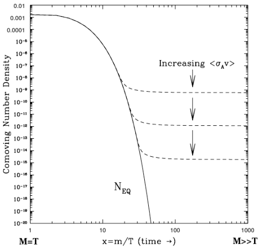
Requires non-gravitational interactions with known particles



DM $\longleftrightarrow$ SM for $T \gtrsim M_{DM}$

DM $\longrightarrow$ SM for $T \lesssim M_{DM}$
$\Longrightarrow$ rapid depletion of Ω_{DM}

Hubble expansion
$\Longrightarrow$ dilution $\longrightarrow$ freeze-out



$2 \to 2$ scattering relates coupling and mass, $200\alpha \sim \dfrac{M_{DM}}{100\text{ GeV}}$

Strong $\alpha \sim 16$ $\longrightarrow$ 'natural' mass scale $M_{DM} \sim 300$ TeV

Smaller $M_{DM} \gtrsim 1$ TeV possible from $2 \to n$ scattering or asymmetry

Backup: Two roads to natural asymmetric dark matter

Idea: Dark matter relic density related to baryon asymmetry

$$\Omega_D \approx 5\Omega_B$$
$$\implies M_D n_D \approx 5 M_B n_B$$

$n_D \sim n_B \quad \implies \quad M_D \sim 5M_B \approx 5$ GeV

High-dim. interactions relate baryon# and DM# violation

$M_D \gg M_B \quad \implies \quad n_B \gg n_D \sim \exp\left[-M_D/T_s\right] \qquad T_s \sim 200$ GeV

Electroweak sphaleron processes above T_s distribute asymmetries

Both require non-gravitational interactions with known particles

Backup: More details about form factors

Photon exchange via electromagnetic form factors

Interactions suppressed by powers of confinement scale $\Lambda \sim M_{DM}$

Dimension 5: Magnetic moment $\longrightarrow$ $(\overline{X}\sigma_{\mu\nu}X)\, F^{\mu\nu}/\Lambda$

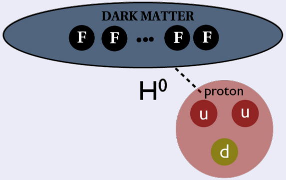
Dimension 6: Charge radius $\longrightarrow$ $(\overline{X}X)\, v_\mu \partial_\nu F^{\mu\nu}/\Lambda^2$

Dimension 7: Polarizability $\longrightarrow$ $(\overline{X}X)\, v_\mu v_\nu F^{\mu\alpha} F_\alpha^{\ \nu}/\Lambda^3$

Higgs exchange via scalar form factors

Higgs couples through σ terms $\langle B | m_\psi \overline{\psi}\psi | B \rangle$

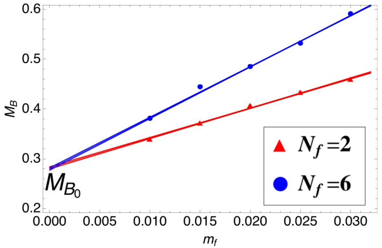
Produces rapid charged 'Π' decay needed for Big Bang nucleosynthesis



Backup: More details about SU(3) composite dark matter model

Same SU(3) gauge group as QCD

Re-analyze existing data sets:
$32^3 \times 64$ lattices, domain wall fermions



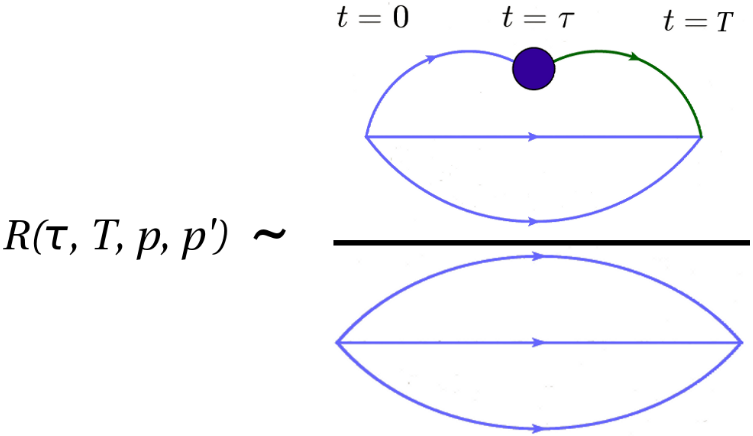
Scan relatively heavy fermion masses $m_F \longrightarrow 0.55 \lesssim M_\Pi / M_V \lesssim 0.75$

Compare $N_F = 2$ or 6 degenerate flavors with same $M_{B_0} \equiv \lim_{m_F \to 0} M_B$

Unlike QCD, fermions are all SU(2)$_L$ singlets $\longrightarrow Q = Y$

Setting $Q_\mathrm{P} = 2/3$ and $Q_\mathrm{M} = -1/3$,
dark matter candidate is singlet "dark baryon" $B = \mathrm{PMM}$

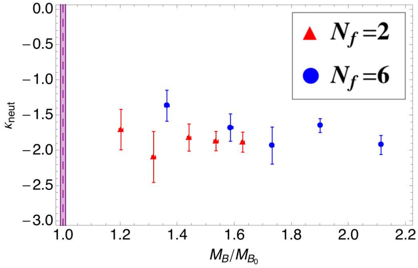
Backup: Form factor calculations on the lattice



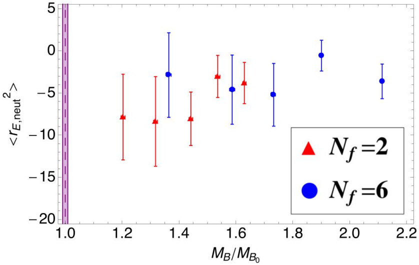
$$R_\Gamma(\tau, T, p, p') \longrightarrow \langle DM(p') | \Gamma_\mu(q^2) | DM(p) \rangle + \mathcal{O}\left(e^{-\Delta\tau},\ e^{-\Delta T},\ e^{-\Delta(T-\tau)}\right)$$

Backup: Electromagnetic form factor results

Magnetic moment κ



Charge radius $\langle r^2 \rangle$



Little dependence on N_F or on $m_F \sim M_B/M_{B_0}$

κ comparable to neutron's $\kappa_N = -1.91$

$\langle r^2 \rangle$ smaller than neutron's $\langle r^2 \rangle_N \approx -38$ (related to larger M_Π/M_V)

Insert into standard event rate formulas. . .

Backup: Event rate formulas and lattice input

$$\text{Rate} = \frac{M_{detector}}{M_T} \frac{\rho_{DM}}{M_{DM}} \int_{E_{min}}^{E_{max}} dE_R\, \mathcal{A}cc(E_R) \left\langle v_{DM} \frac{d\sigma}{dE_R} \right\rangle_f$$

$$\frac{d\sigma}{dE_R} = \frac{\overline{|\mathcal{M}_{SI}|^2} + \overline{|\mathcal{M}_{SD}|^2}}{16\pi \left(M_{DM} + M_T\right)^2 E_R^{max}} \qquad E_R^{max} = \frac{2 M_{DM}^2 M_T v_{col}^2}{\left(M_{DM} + M_T\right)^2}$$

From **magnetic moment** κ and **charge radius** $\langle r^2 \rangle$

$$\frac{\overline{|\mathcal{M}_{SI}|^2}}{e^4 \left[Z F_c(Q)\right]^2} = \left(\frac{M_T}{M_{DM}}\right)^2 \left[\frac{4}{9} M_{DM}^4 \left\langle r^2 \right\rangle^2 + \frac{\kappa^2 \left(M_T + M_{DM}\right)^2 \left(E_R^{max} - E_R\right)}{M_T^2 E_R}\right]$$

$$\overline{|\mathcal{M}_{SD}|^2} = e^4 \frac{2}{3} \left(\frac{J+1}{J}\right) \left[\left(A \frac{\mu_T}{\mu_n}\right) F_s(Q)\right]^2 \kappa^2$$

Backup: Event rate formulas and lattice input

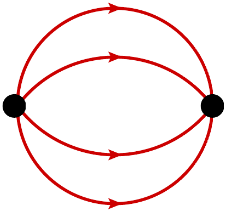
$$\text{Rate} = \frac{M_{detector}}{M_T} \frac{\rho_{DM}}{M_{DM}} \int_{E_{min}}^{E_{max}} dE_R \, \mathcal{A}cc(E_R) \left\langle v_{DM} \frac{d\sigma}{dE_R} \right\rangle_f$$

$$\frac{d\sigma}{dE_R} = \frac{\overline{|\mathcal{M}_{SI}|^2} + \overline{|\mathcal{M}_{SD}|^2}}{16\pi \left(M_{DM} + M_T\right)^2 E_R^{max}} \qquad E_R^{max} = \frac{2 M_{DM}^2 M_T v_{col}^2}{\left(M_{DM} + M_T\right)^2}$$

From **polarizability** C_F

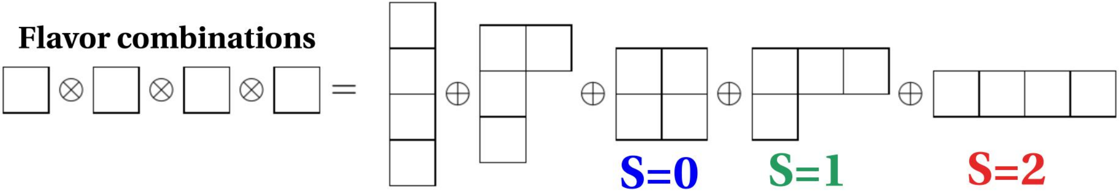
$$\sigma_{SI} = \frac{Z^4}{A^2} \frac{144\pi \alpha_{em}^4 \widetilde{M}_{n,DM}^2}{M_{DM}^6 R^2} C_F^2 \propto \frac{Z^4}{A^2} \quad \text{per nucleon}$$

Backup: More details about SU(4) Stealth Dark Matter



Quenched SU(4) lattice ensembles

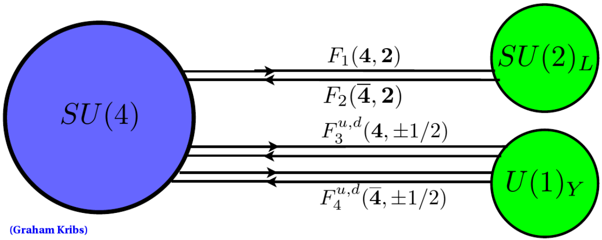
Lattice volumes up to $64^3 \times 128$,
several lattice spacings to check systematic effects



Dark matter candidate is spin-zero baryon $\longrightarrow$ no magnetic moment

Need at least two flavors to anti-symmetrize $\longrightarrow$ no charge radius

Backup: Even more details about SU(4) Stealth Dark Matter



(Graham Kribs)

Field	$SU(N_D)$	$(SU(2)_L,\ Y)$	Q
$F_1 = \begin{pmatrix} F_1^u \\ F_1^d \end{pmatrix}$	$\mathbf{N}$	$(\mathbf{2},\ 0)$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$
$F_2 = \begin{pmatrix} F_2^u \\ F_2^d \end{pmatrix}$	$\bar{\mathbf{N}}$	$(\mathbf{2},\ 0)$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$
F_3^u	$\mathbf{N}$	$(\mathbf{1}, +1/2)$	$+1/2$
F_3^d	$\mathbf{N}$	$(\mathbf{1}, -1/2)$	$-1/2$
F_4^u	$\bar{\mathbf{N}}$	$(\mathbf{1}, +1/2)$	$+1/2$
F_4^d	$\bar{\mathbf{N}}$	$(\mathbf{1}, -1/2)$	$-1/2$

Mass terms $m_V\left(F_1F_2 + F_3F_4\right) + y\left(F_1 \cdot HF_4 + F_2 \cdot H^\dagger F_3\right) + \text{h.c.}$

Vector-like masses evade Higgs-exchange direct detection bounds

Higgs couplings $\longrightarrow$ charged meson decay before Big Bang nucleosynthesis

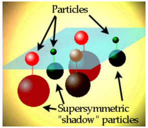
Both required $\longrightarrow$ four flavors

Backup: ‘Stealth’ composites from conspicuous constituents

Direct detection cross section (pb)

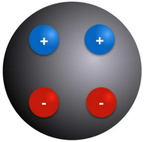
Neutrino

$\sigma \sim 10^{-2}$



SUSY neutralino

$10^{-6} \lesssim \sigma \lesssim 10^{-5}$



Stealth Dark Matter

$$\sigma \sim \left(\frac{200\ \text{GeV}}{M_{DM}}\right)^{6} \times 10^{-9}$$

Radar cross section (m^2)

747

$\sigma \sim 10^{2}$

Falcon

$\sigma \sim 10^{-2}$

Stealth F-22

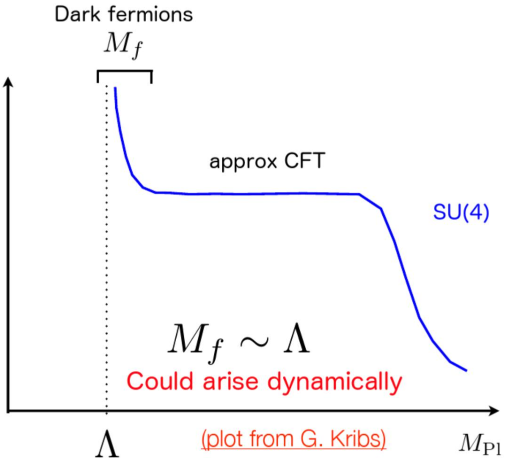
$\sigma < 10^{-3}$

Backup: Stealth Dark Matter mass scales

Lattice studies focus on $m_\psi \simeq \Lambda_{DM}$ where effective theories least reliable

$m_\psi \simeq \Lambda_{DM}$ could arise dynamically

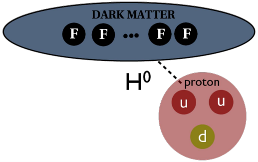
Collider constraints on M_{DM} become stronger as m_ψ decreases



Backup: Effective Higgs interaction

$M_H = 125$ GeV $\longrightarrow$ Higgs exchange can dominate direct detection

$$\sigma_H^{(SI)} \propto \left| \frac{\widetilde{M}_{DM,N}}{M_H^2} \; y_\psi \langle DM \left| \overline{\psi}\psi \right| DM \rangle \;\; y_q \langle N \left| \overline{q}q \right| N \rangle \right|^2$$



Quark $\; y_q = \dfrac{m_q}{v}$

Dark $\; y_\psi = \alpha \dfrac{m_\psi}{v}$ suppressed by $\alpha \equiv \dfrac{v}{m_\psi} \dfrac{\partial m_\psi(h)}{\partial h} \Big|_{h=v} = \dfrac{yv}{yv + m_V}$

Determine using Feynman–Hellmann theorem $\quad \langle DM \left| \overline{\psi}\psi \right| DM \rangle = \dfrac{\partial M_{DM}}{\partial m_\psi}$

Backup: Feynman–Hellmann theorem

$m_\psi \overline{\psi}\psi$ is the only term in the hamiltonian that depends on m_ψ

$$\Longrightarrow \left\langle B \left| \frac{\partial \widehat{H}}{\partial m_\psi} \right| B \right\rangle = \langle B \left| \overline{\psi}\psi \right| B \rangle$$

Since $\widehat{H}\,|B\rangle = M_B\,|B\rangle$ and $\langle B|\,\widehat{H} = \langle B|\,M_B$ we have

$$\frac{\partial}{\partial m_\psi} M_B = \frac{\partial}{\partial m_\psi} \left\langle B \left| \widehat{H} \right| B \right\rangle = \left\langle \frac{\partial B}{\partial m_\psi} \left| \widehat{H} \right| B \right\rangle + \left\langle B \left| \widehat{H} \right| \frac{\partial B}{\partial m_\psi} \right\rangle + \left\langle B \left| \frac{\partial \widehat{H}}{\partial m_\psi} \right| B \right\rangle$$

$$= M_B \langle \frac{\partial B}{\partial m_\psi} | B \rangle + M_B \langle B | \frac{\partial B}{\partial m_\psi} \rangle + \langle B \left| \overline{\psi}\psi \right| B \rangle$$

$$= M_B \frac{\partial}{\partial m_\psi} \langle B | B \rangle + \langle B \left| \overline{\psi}\psi \right| B \rangle = \langle B \left| \overline{\psi}\psi \right| B \rangle \qquad \square$$

Backup: Lattice results for Higgs exchange constrain α

$$\sigma_H^{(SI)} \propto \left| y_\psi \left\langle DM \left| \overline{\psi}\psi \right| DM \right\rangle \right|^2$$

Matrix element $\propto \dfrac{\partial M_{DM}}{\partial m_\psi}$

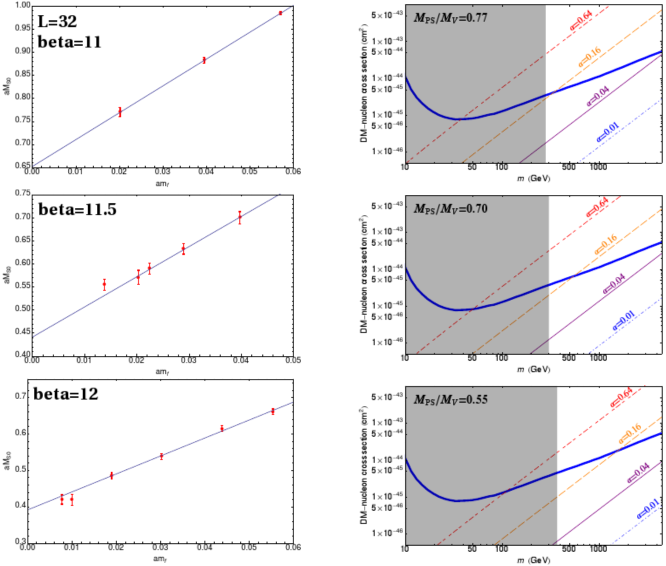
(Feynman–Hellmann)

Stealth Dark Matter:

$$0.15 \lesssim \frac{m_\psi}{M_{DM}} \frac{\partial M_{DM}}{\partial m_\psi} \lesssim 0.34$$

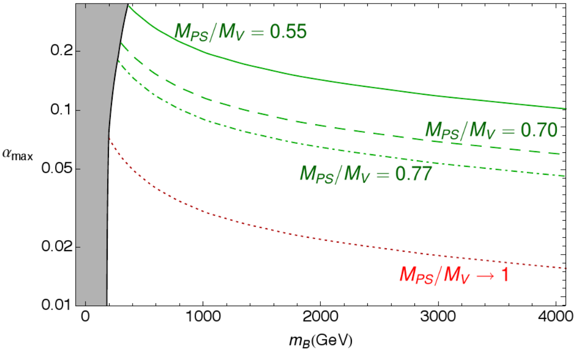
Larger than QCD

$$0.04 \lesssim \frac{m_q}{M_N} \frac{\partial M_N}{\partial m_q} \lesssim 0.08$$



Backup: Bounds on effective Higgs coupling

Higgs-exchange cross section $\longrightarrow$ maximum α allowed by LUX [1310.8214]



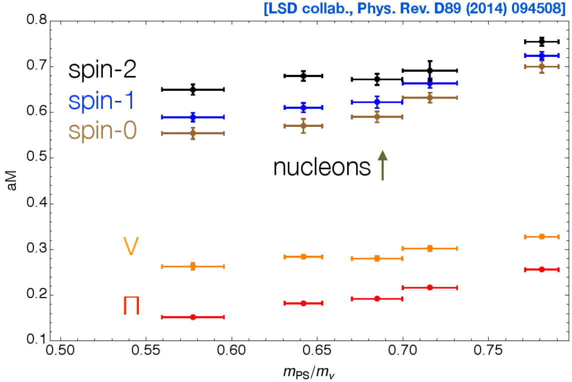
Maximum α depends on M_Π/M_V and dark matter mass

Smaller $M_\Pi/M_V \longleftrightarrow m_F$
$\longrightarrow$ stronger constraints from colliders

Effective Higgs interaction tightly constrained

$\alpha \lesssim 0.3$ for $M_\Pi/M_V \gtrsim 0.55$ $\longrightarrow$ fermion masses must be mainly vector-like

Backup: Indirect detection



Lattice results for composite spectrum

Predict γ-rays from splitting between baryons with spin $S = 0$, 1 and 2

Much more challenging future work

DM–$\overline{\text{DM}}$ annihilation into (many) lighter Π that then decay

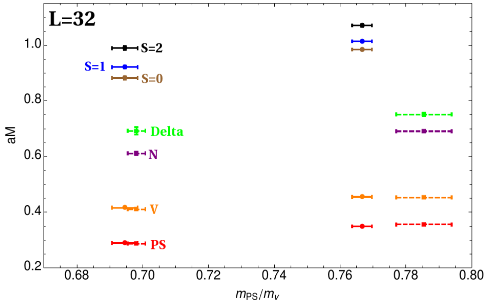
Backup: Large-N predictions for SU(4) baryons

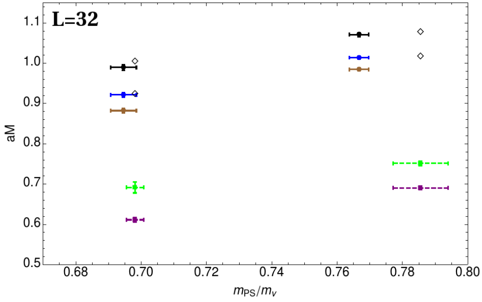
Tune (β, m_F) to match SU(3) M_Π and M_V (dashed)

Rotor spectrum for spin-J baryons: $M(N,J) = NM_0 + C + B\frac{J(J+1)}{N} + \mathcal{O}\left(\frac{1}{N^2}\right)$

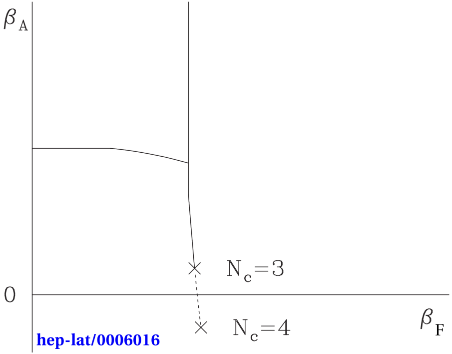
Fit M_0, C and B with nucleon, Δ and spin-0 baryon masses

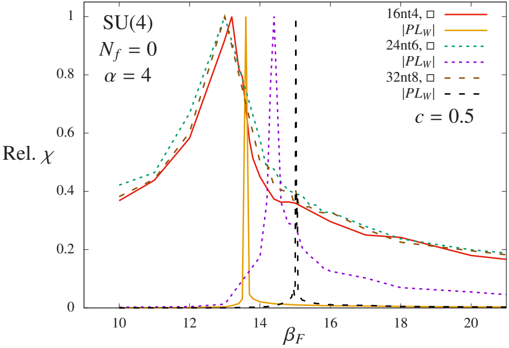
$\longrightarrow$ predictions for $S = 1, 2$ baryons (diamonds)





Backup: Thermal transition vs. bulk transition

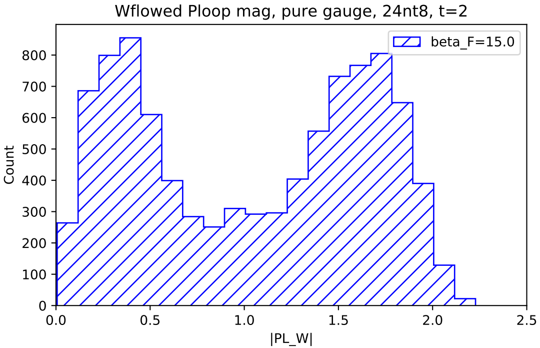


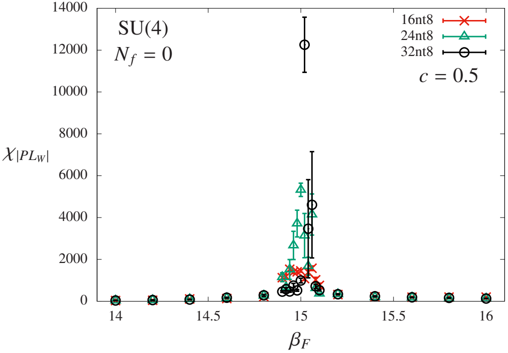


Try to avoid bulk transition for small $L^3 \times N_T$ volumes $\longrightarrow$ use $\beta_A = -\beta_F/4$

Still need $N_T > 4$ for clear separation between bulk & thermal transitions

Backup: Compare with known first-order pure-gauge transition





Signals are stronger but qualitatively same as for $M_P/M_V \approx 0.96$

No clear hysteresis even in pure-gauge case