Non-Relativistic Supergeometry in the Moore-Read Fractional Quantum Hall State

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Outline

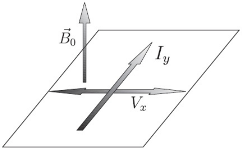
- Abelian and non-Abelian Fractional Quantum Hall States
- Non-Relativistic Geometry of the FQHE
- Supergeometry and CS theory in the Moore-Read state
- Boundary super-CFT and Majorana fermion
- Goldstino and topological response of the Moore-Read state

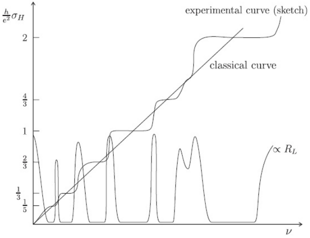
A Standard Model for the FQHE

An effective quantum field theory in the low-energy (IR) regime

- It needs to be based on phenomenological and numerical evidences.
- It cannot be derived from any high-energy quantum field theory (quantum gravity, etc.).
- It has to incorporate topological and non-topological (geometric) features of the FQHE.
- It has to take into account not only topological order, but also nematic, ferromagnetic, superconducting (etc.) phases.
- It has to be a predictive (and not just a descriptive) theory.

Chern-Simons theory in the Integer Quantum Hall Effect





U(1) Chern-Simons theory in the IQHE

$$S_{CS}[A] = \int d^3x \left(\frac{\nu}{4\pi} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda + j^\mu A_\mu \right),$$

Hall current ($e^2/\hbar \equiv 1$):

$$J^i = \frac{\nu}{2\pi} \epsilon^{ij} E_j.$$

- Quantized Hall conductivity with filling factor $\nu \in \mathbb{Z}$
- Bulk-edge correspondence: CS_{2+1}/CFT_{1+1}

Chern-Simons theory in the Abelian Laughlin states

Coulomb interactions give rise to the fractionalization of the electric charge, $\nu = \frac{1}{p}$, with p a positive odd number.

$$S_{CS}[a, A] = \int d^3x \left(\frac{1}{2\pi} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu a_\lambda - \frac{p}{4\pi} \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda + \ldots \right) .$$

By integrating out the hydro-dynamical field a_μ we obtain the correct fractional Hall conductivity

$$S_{CS}[A] = \int d^3x \left(\frac{1}{4\pi p} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda \right) .$$

On a disk geometry, the edge theory is given by a chiral boson action in polar coordinates (t, ϕ)

$$S_\varphi = \frac{1}{2} \int d\phi \, dt \, \left[\partial_t \varphi \partial_\phi \varphi + v_b (\partial_\phi \varphi)^2 \right] ,$$

namely a chiral CFT with $c = 1$.

FQHE on curved space and Hall viscosity

The Hall viscosity comes from the response of the system to shear or strain. From a suitable effective topological action $S[a, A, \omega]$, with ω the $SO(2)$ spin connection, we integrate out a and obtain

$$S[\omega, A] = \int \left[\left(\frac{\nu \bar{s}^2}{4\pi} - \frac{c}{48\pi} \right) \omega d\omega + \frac{\nu}{4\pi} AdA + \frac{\nu \bar{s}}{2\pi} Ad\omega \right],$$

$$\rho = \frac{\nu}{2\pi} B + \frac{\nu \bar{s}}{4\pi} R, \qquad J^i = \frac{\nu}{2\pi} \epsilon^{ij} E_j + \frac{\nu \bar{s}}{2\pi} \epsilon^{ij} \mathcal{E}_j,$$

where $\bar{s} = p/2$ is the average orbital spin (shift).

$$s^\mu = \frac{\nu \bar{s}}{2\pi} \epsilon^{\mu\nu\lambda} \partial_\nu A_\lambda + \left(\frac{\nu \bar{s}^2}{2\pi} - \frac{c}{24\pi} \right) \epsilon^{\mu\nu\lambda} \partial_\nu \omega_\lambda,$$

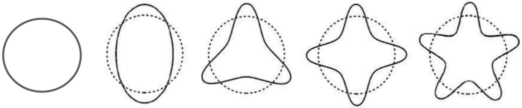
where

$$\frac{\nu \bar{s}}{2\pi} \epsilon^{\mu\nu\lambda} \partial_\nu A_\lambda = \eta_H u^\mu,$$

with u^μ the covariant drift velocity and $\eta_H = \nu \bar{s} B / 4\pi$ the Hall viscosity.

Incompressibility and area-preserving diffeomorphisms

The incompressibility of the FQH fluid is due to an underlying quantum area-preserving diffs induced by interactions.



$$H_V = \int d^2q\, V_q \bar{\rho}_{-q} \bar{\rho}_q,$$

$$[\bar{\rho}_{\tilde{q}}, \bar{\rho}_q] = f(\tilde{q}, q)\bar{\rho}_{\tilde{q}+q}, \quad f(\tilde{q}, q) = 2i\, e^{\tilde{q}\cdot q} \sin\left(\frac{\tilde{q}\times q}{2}\right).$$

The projected density operators $\bar{\rho}_q$ on the lowest Landau level satisfy the Girvin-MacDonald-Platzman (GMP)/ W_∞ algebra.

GMP mode: $|q\rangle = \bar{\rho}_q|0\rangle$ ($|0\rangle$ is the exact ground state of H_V).

It is possible to show that the GMP mode is a massive spin-2 propagating mode (non-relativistic graviton!).

From the GMP mode to emergent geometry

Emergent geometry in the FQHE induced by interactions (Haldane, PRL (2011)).

- Being topological, Chern-Simons theories cannot encode dynamical massive gravitons.
- TQFTs have to be embedded in a more general framework that should be able to encode both topological and geometric features of the FQHE.
- Non-relativistic bimetric theory: $(g_{\mu\nu}, \hat{g}_{\mu\nu})$ (Gromov and Son, PRX (2017)).
- The GMP algebra give rise to higher-spin modes (S. Golkar, D. X. Nguyen, M. M. Roberts, D. T. Son, PRL (2016), Randellini and Cappelli, JHEP (2016)).

Non-Abelian Moore-Read State

For the filling factor $\nu = 1/2$, Moore-Read state describes composite fermions which pair up and condense by giving rise to a p-wave superconductor (Moore and Read, NPB, (1991)).

- Quasi-particles have fractional charge $e/4$.
- Non-Abelian Ising anyons in the bulk.
- A massless chiral Majorana mode appears together with the chiral boson on the boundary.
- Effective field theory:
  $SU(2)_2$ Chern-Simons theory in the bulk and $SU(2)_2$ chiral WZW on the boundary, with $c = 3/2 = 1 + 1/2$.

Alternative picture on the boundary: SCFT

It has been recently conjectured that the chiral Majorana fermion ψ and chiral boson φ are superpartners and described by a $\mathcal{N} = (1, 0)$ supersymmetric conformal field theory (K.W. Ma, R. Wang, and K. Yang, PRL, (2021)).

$$S[\varphi, \psi] = \frac{1}{2} \int d\phi \, dt \, \left[\partial_t \varphi \partial_\phi \varphi + v_b (\partial_\phi \varphi)^2 + i\psi(\partial_t + v_f \partial_\phi)\psi \right] ,$$

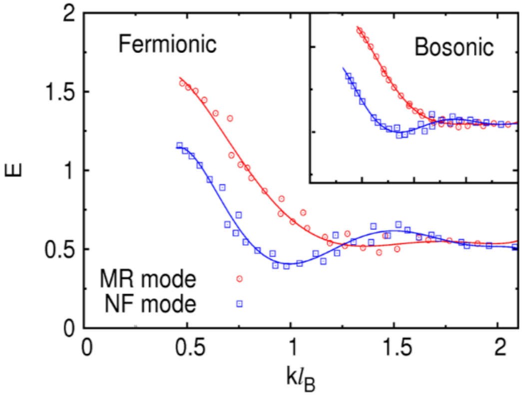
SUSY is broken when $v_b \neq v_f$ and a goldstino emerges.

There exists a previous proposal with $\mathcal{N} = 2$ SCFT for the Moore-Read state (E. Sagi and R. A. Santos, PRB (2017)), which is compatible with the fermionization of the $SU(2)_2$ WZW model (J.-B. Bae and S. Lee, arXiv:2105.02148).

Neutral spin-3/2 collective mode

B. Yang, Z.-X. Hu, Z. Papic, and F. D. M. Haldane, PRL (2012).

Magneto-Rotor (MR)=GMP mode, NF=neutral fermion mode.



These two modes can be seen as superpartners of each other (Gromov, Martinec and Ryu, PRL (2020)).

Global symmetries and features in the Moore-Read state

- Magnetic translations (broken time-reversal symmetry)
- Galilei rotational invariance
- Supersymmetry
- TQFT at ground state

Superalgebra and first-order formalism in (super-)geometry

$$[\mathcal{J},\mathcal{P}_a]=\epsilon_a{}^b\mathcal{P}_b\,,\ \ [\mathcal{P}_a,\mathcal{P}_b]=-\epsilon_{ab}\mathcal{T}\,,$$
$$[\mathcal{J},\mathcal{Q}_\alpha]=\frac{1}{2}\left(\gamma_0\right)^\beta{}_\alpha\,\mathcal{Q}_\beta\,,\ \ \{\mathcal{Q}_\alpha,\mathcal{Q}_\alpha\}=\frac{1}{2}\left(C\gamma_0\right)_{\alpha\beta}\mathcal{T}\,,$$
$$[\mathcal{Z},\mathcal{P}_a]=\epsilon_a{}^b\mathcal{P}_b\,,\ \ [\mathcal{Z},\mathcal{Q}_\alpha]=\frac{1}{2}\left(\gamma_0\right)^\beta{}_\alpha\,\mathcal{Q}_\beta\,,$$

Q_α are Majorana supercharges, $C_{\alpha\beta}=\epsilon_{\alpha\beta}$ is the charge conjugation matrix. The Nappi-Witten algebra is a subalgebra.

$$\langle\mathcal{J},\mathcal{J}\rangle=\mu_0\,,\quad\langle\mathcal{P}_a,\mathcal{P}_b\rangle=\mu_1\delta_{ab}\,,\quad\langle\mathcal{J},\mathcal{T}\rangle=-\mu_1\,,$$
$$\langle\mathcal{Q}_\alpha,\mathcal{Q}_\beta\rangle=C_{\alpha\beta}\mu_1,\quad\langle\mathcal{Z},\mathcal{J}\rangle=\mu_1\,,\quad\langle\mathcal{Z},\mathcal{T}\rangle=-\mu_1\,,$$
$$\langle\mathcal{Z},\mathcal{Z}\rangle=\mu_2,$$

$$\mathbb{A}=\omega\mathcal{J}+a\mathcal{Z}+e^a\mathcal{P}_a+A\mathcal{T}+\mathcal{Q}_\alpha\Psi^\alpha.$$

Chern-Simons theory and non-relativistic supergeometry

$$S_{\mathrm{CS}} = \frac{1}{4\pi}\int \left\langle \mathbb{A} d\mathbb{A} + \frac{2}{3}\mathbb{A}\wedge\mathbb{A}\wedge\mathbb{A} \right\rangle,$$

$$\mathbb{F} = d\mathbb{A} + (1/2)[\mathbb{A},\mathbb{A}] = R\mathcal{J} + f\mathcal{Z} + R^a\mathcal{P}_a + F\mathcal{T} + \mathcal{Q}_\alpha D\Psi^\alpha .$$

$$R = d\omega\,, \qquad f = da,$$

$$R^a = de^a + \epsilon^a{}_b e^b\left(\omega + a\right)\,,$$

$$F = dA - \frac{1}{2}\epsilon_{ab}e^a e^b - \frac{1}{4}\bar{\Psi}_\alpha\left(\gamma_0\right)^\alpha{}_\beta\,\Psi^\beta,$$

$$D\Psi^\alpha = d\Psi^\alpha + \frac{1}{2}\left(\omega + a\right)\left(\gamma_0\right)^\alpha{}_\beta\,\Psi^\beta.$$

$$S_{\mathrm{CS}} = \frac{1}{4\pi}\int\Big[\mu_0\,\omega\,d\omega + 2\mu_1\,ad\omega + \mu_2\,ada + \mu_1\,e^a R_a - 2\mu_1\,Ad\left(\omega + a\right) - \mu_1\,\bar{\Psi}_\alpha D\Psi^\alpha\Big].$$

Boundary theory: chiral boson and chiral Majorana mode

$$S_{\mathrm{WZW}}=\frac{1}{4\pi}\int dtd\phi\,\left\langle g^{-1}\partial_{+}gg^{-1}g'\right\rangle-\frac{1}{12\pi}\int_{M_3}\left\langle\left(g^{-1}\mathrm{d}g\right)^{3}\right\rangle,$$

with $\partial_\pm=\partial_t\pm v\partial_\phi$ and $g'\equiv\partial_t g$, which is obtained after replacing the local solution of the CS field equations $\mathbb{F}=0$, given by $\mathbb{A}=g^{-1}dg$ with the boundary condition $\mathbb{A}_t+v\mathbb{A}_\phi=0$ back in the action. The left-invariant Maurer-Cartan form

$$\Omega=g^{-1}dg=\Omega_{\mathcal{J}}\mathcal{J}+\Omega_{\mathcal{Z}}\mathcal{Z}+\Omega^{a}_{\mathcal{P}}\mathcal{P}_a+\Omega_{\mathcal{T}}\mathcal{T}+\mathcal{Q}_\alpha\Omega^{\alpha}_{\mathcal{Q}},$$

satisfies the Maurer-Cartan equation

$$d\Omega+\Omega\wedge\Omega=0.$$

$$S_{\mathrm{WZW}}=\frac{1}{4\pi}\int dtd\phi\left(\tilde{\mu}\,\partial_{+}\varphi\varphi'+\mu_1\psi_\alpha\partial_{+}\psi^{\alpha}\right),$$

with $\tilde{\mu}=(\mu_2-2\mu_1+\mu_0)$.

Broken supersymmetry and goldstino in the bulk

$$S_\eta = \frac{i\,\mu_1}{4\pi}\int D\bar{\eta}\wedge\hat{\gamma}\wedge\, D\eta,$$

where η is a goldstino spinor field describing a neutral fermion and $\bar{\eta} = \eta^\dagger\gamma_0$ its conjugate, D is the covariant derivative and $\hat{\gamma} = e^a\gamma_a$. It looks a bit different from the standard 2+1-D Volkov-Akulov goldstino (Bansal and Sorokin, JHEP (2018)).

Goldstino-gravitino minimal coupling:

$$D\eta \to D\eta - \sqrt{m}\Psi, \quad D\bar{\eta} \to D\bar{\eta} - \sqrt{m}\bar{\Psi}.$$

$$S_m = \frac{i\,\mu_1\,m}{4\pi}\int \bar{\Psi}\wedge\hat{\gamma}\wedge\Psi,$$

This is the standard 2+1-D Rarita-Schwinger mass term that breaks SUSY (Deser, 1984).

Integrating out the massive gravitino

We assume that m is large and positive such that we can neglect the terms that couple η and Ψ (they are proportional to $\sqrt{m}$, which is small compared to m).

By integrating out the massive gravitino Ψ, we finally obtain a CS term

$$(\mu_1/4\pi)(\omega + a)d(\omega + a).$$

Moreover, in order to have a unique geometric response from the background geometry, we vary the action with respect to the spatial dreibein e^a to obtain the field equation $R_a = 0$, which in turn yields the following equation for torsion

$$T^a \equiv de^a + \epsilon^a{}_b e^b \omega = -\beta \epsilon^a{}_b e^b a.$$

Topological response

$$S[A,\omega] = \frac{1}{4\pi}\int \left[\hat{c}\,\omega d\omega + \nu\, AdA + \nu\mathcal{S}\, Ad\omega\right].$$

Here, for the Moore-Read state, $\hat{c} = (\nu - c)/12$, where $\nu = 1/2$, $\mathcal{S} = 3$, $c = 3/2$.

The exact values of these physical coefficients are derived by taking $\mu_0 = \hat{c} - \nu\mathcal{S}/2$, $\mu_1 = -\nu\mathcal{S}/2$ and $\mu_2 = -\nu\left(\mathcal{S}/2 + (\mathcal{S}/2)^2\right)$ in our theory.

The first and third terms in the above action are known as gravitational Chern-Simons and Wen-Zee term, respectively. The former is associated to the gravitational anomaly, while the latter is related to the Hall viscosity η_H.

$$\rho = \frac{\nu}{2\pi}B + \frac{\nu\mathcal{S}}{8\pi}R, \quad J^i = \frac{\nu}{2\pi}\epsilon^{ij}E_j + \frac{\nu\mathcal{S}}{4\pi}\epsilon^{ij}\mathcal{E}_j.$$

Conclusions and outlook

- I have presented a novel non-relativistic supergeometric theory to describe the topological response of the Moore-Read state in the low-energy regime.
- This theory gives rise to a massive gravitino in the bulk and a chiral Majorana a chiral boson modes on the boundary in agreement with previous works.
- It is crucial to generalize our theory by including also the dynamical GMP mode (massive graviton) and the further hypothetical higher-spin modes.
- It would be very interesting to generalize our approach to the hierarchies of FQH states in the second Landau level build from the Moore-Read state (Bonderson-Slingerland states).