University of Southampton
School of Physics and Astronomy

elusives invisiblesPlus
neutrinos, dark matter & dark energy physics

Neutrino Physics

Steve King, 8th December 2021

DIAS
Institiúid Ard-Léinn Bhaile Átha Cliath | Dublin Institute for Advanced Studies

Neutrino Mass and Mixing

Reviews

F.Feruglio and A.Romanino, Rev.Mod.Phys.93(2021)1,015007[arXiv:1912.06028].

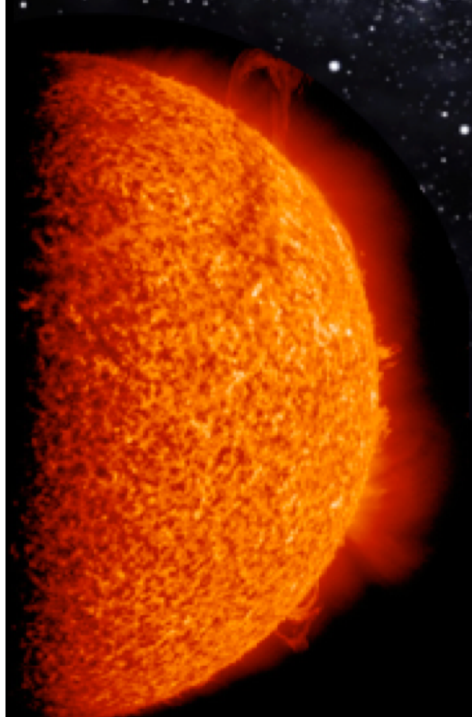
S.F.King, J.Phys.G 42(2015),123001 [arXiv:1510.02091].

S.F.King, A.Merle, S.Morisi, Y.Shimizu and M.Tanimoto,
New J.Phys.16(2014),045018 [arXiv:1402.4271].

S.F.King and C.Luhn, Rept.Prog.Phys.76(2013)056201 [arXiv:1301.1340].

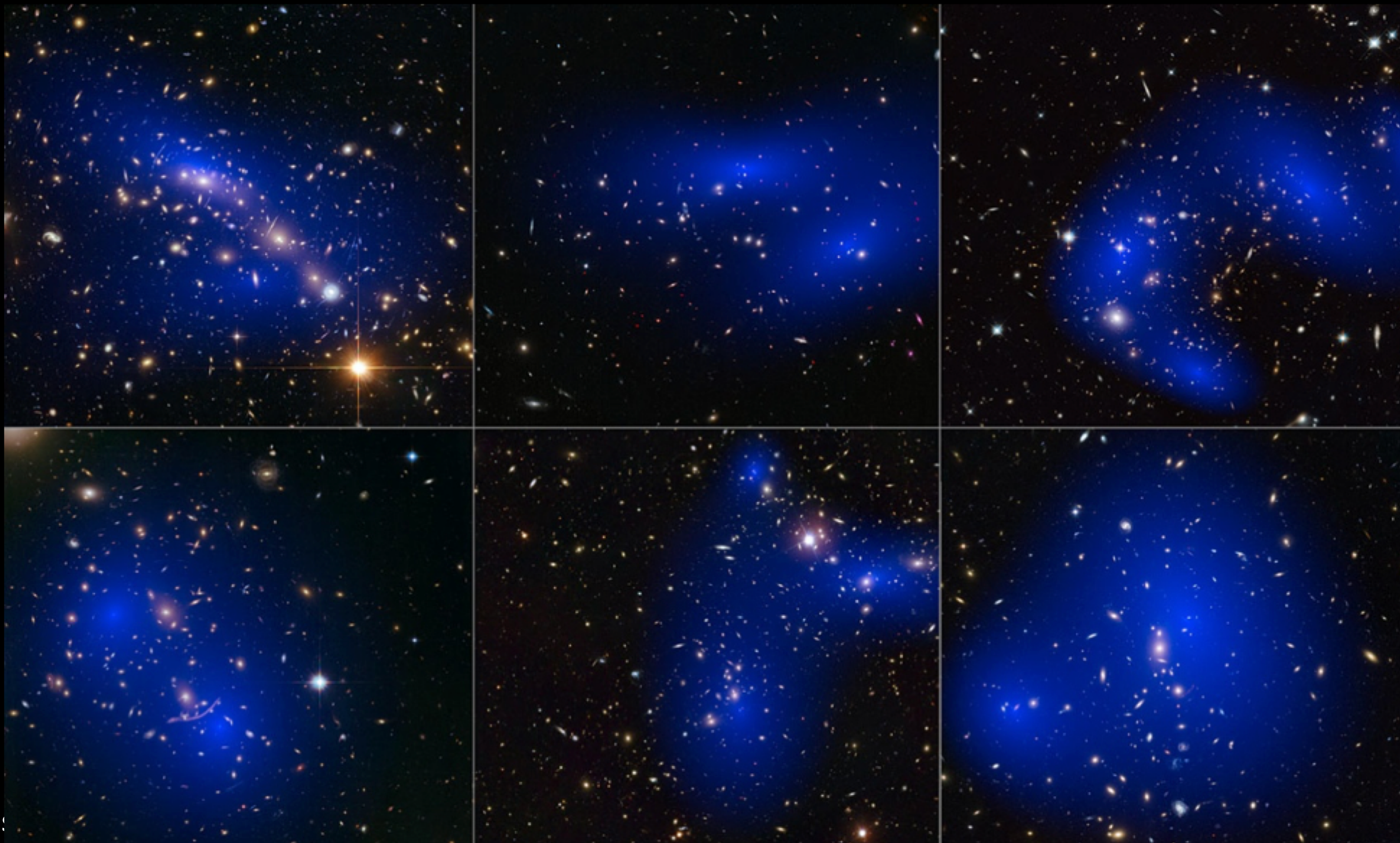
S.F.King, Rept.Prog.Phys.67(2004),107[arXiv:hep-ph/0310204].

Are neutrinos responsible for the matter-antimatter asymmetry?



$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = \frac{n_B}{n_\gamma} \approx 6 \times 10^{-10}$$

Dark Matter?



Dark Energy?

Steve King Southampton

Masterclass 2018

Implications for PP and Cosmology

- Neutrino mass and mixing

  See-saw mechanisms, flavour symmetry, Extra dimensions,...

- Unification of matter, forces and flavour

  SUSY, GUTs

Particle Physics

- Baryon asymmetry of the universe?

  Leptogenesis

- Dark Matter?

  Warm dark matter

- Inflation?

  Sneutrino inflation

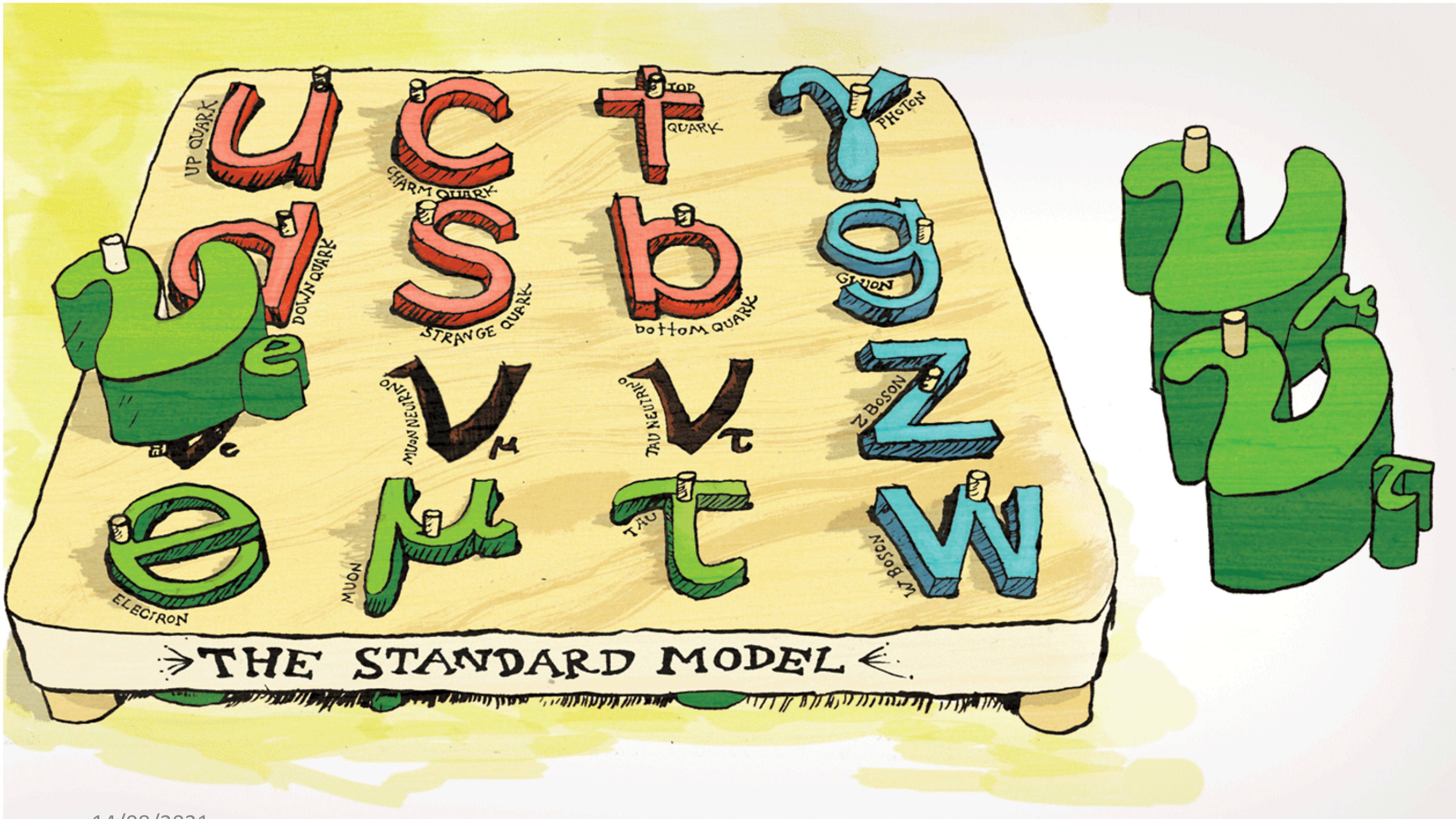
- Dark energy? $\Lambda \sim m_\nu^4$

Cosmology

Neutrino mass and mixing

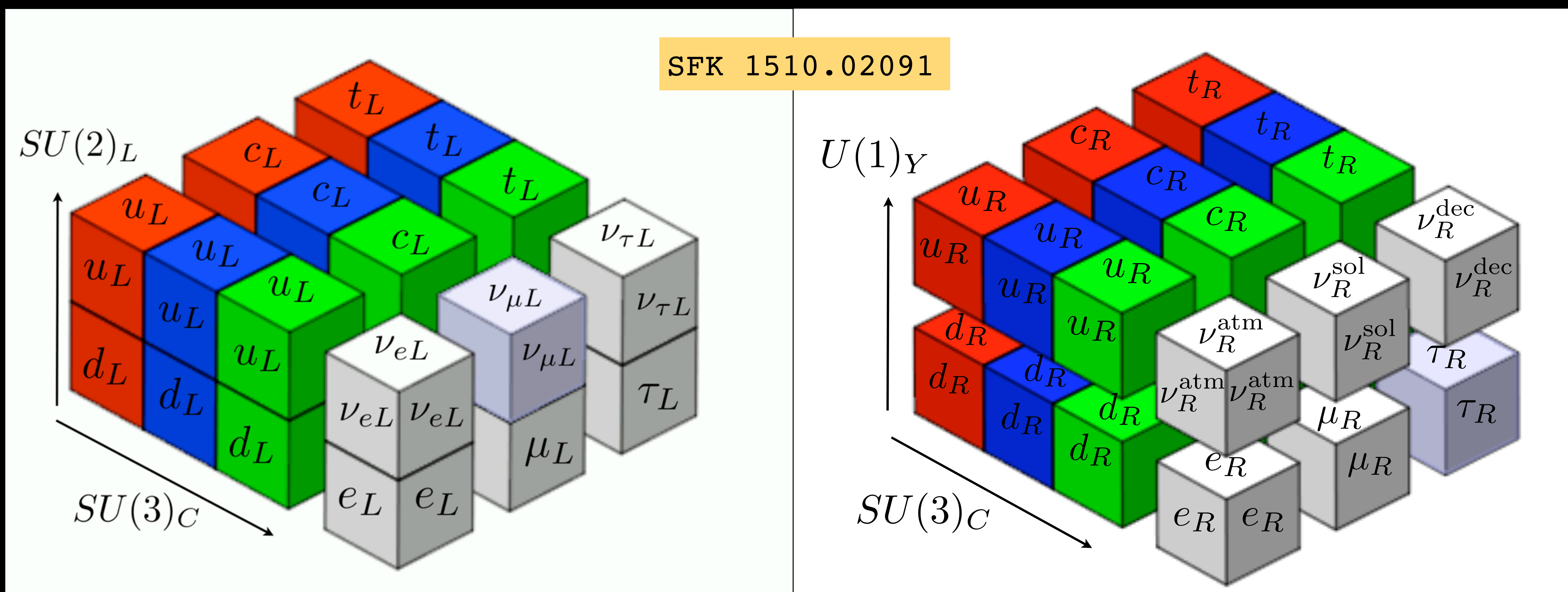
- ☐ Neutrinos have tiny masses (much less than electron)
- ☐ Neutrinos mix a lot (unlike the quarks)
- ☐ Up to 9 new params: 3 masses, 3 angles, 3 phases
- ☐ Origin of mass and mixing is unknown

How do the neutrinos fit into the Standard Model?

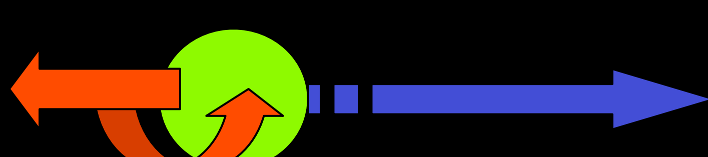


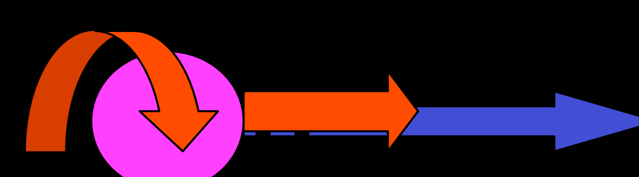
The Standard Model (plus RHNs)

Left-handed | Right-handed

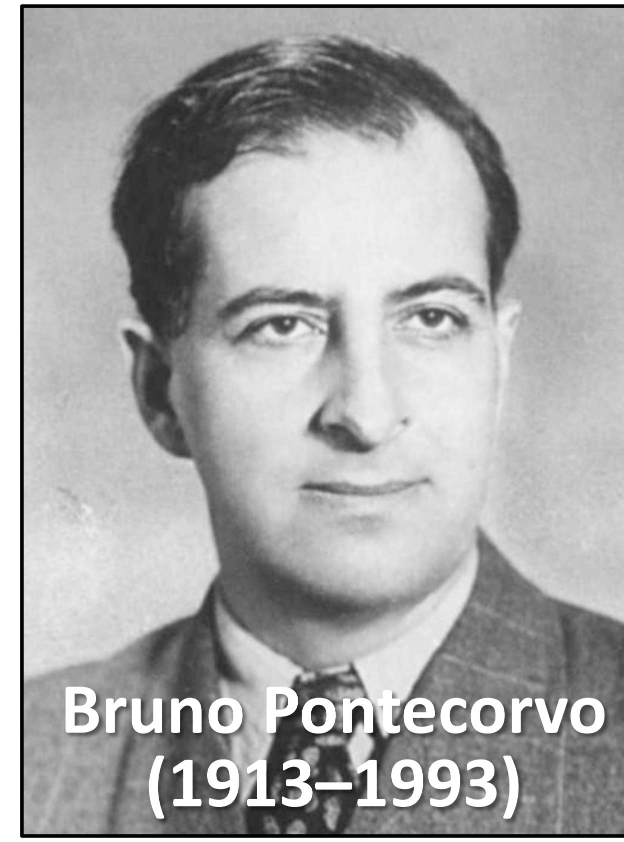


$$\nu_L \qquad \nu_R$$





Neutrino-Oscillations



Only possible if neutrinos have mass

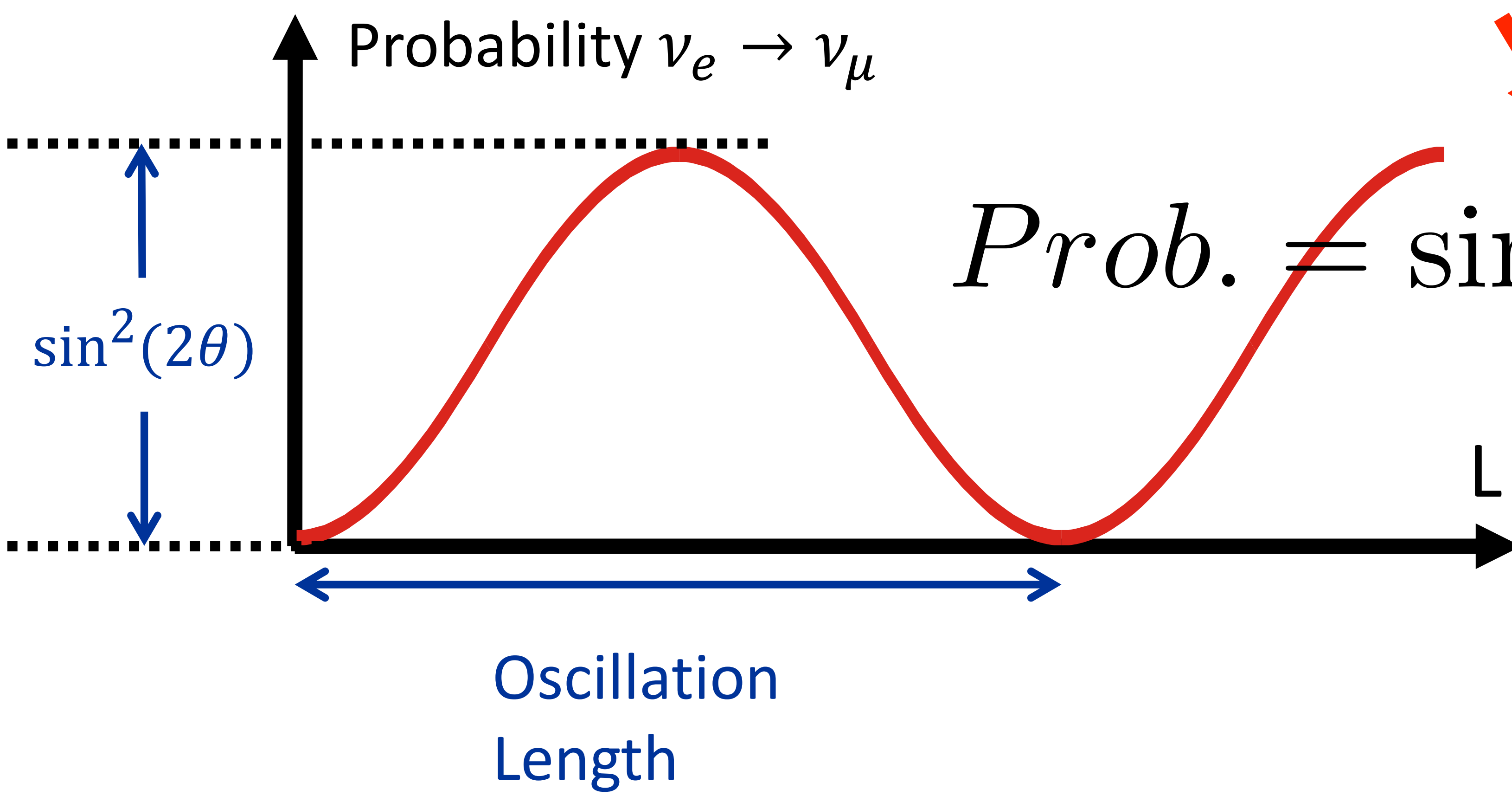
Pontecorvo & Gribov (1968 „ Solar neutrino problem“)

- Neutrinos are quantum superpositions of mass states

$$\nu_e = +\cos\Theta\ \nu_1 + \sin\Theta\ \nu_2$$
$$\nu_\mu = -\sin\Theta\ \nu_1 + \cos\Theta\ \nu_2$$

- Different propagation speeds gives neutrino oscillations

Remember this formula



$$Prob. = \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{E}$$

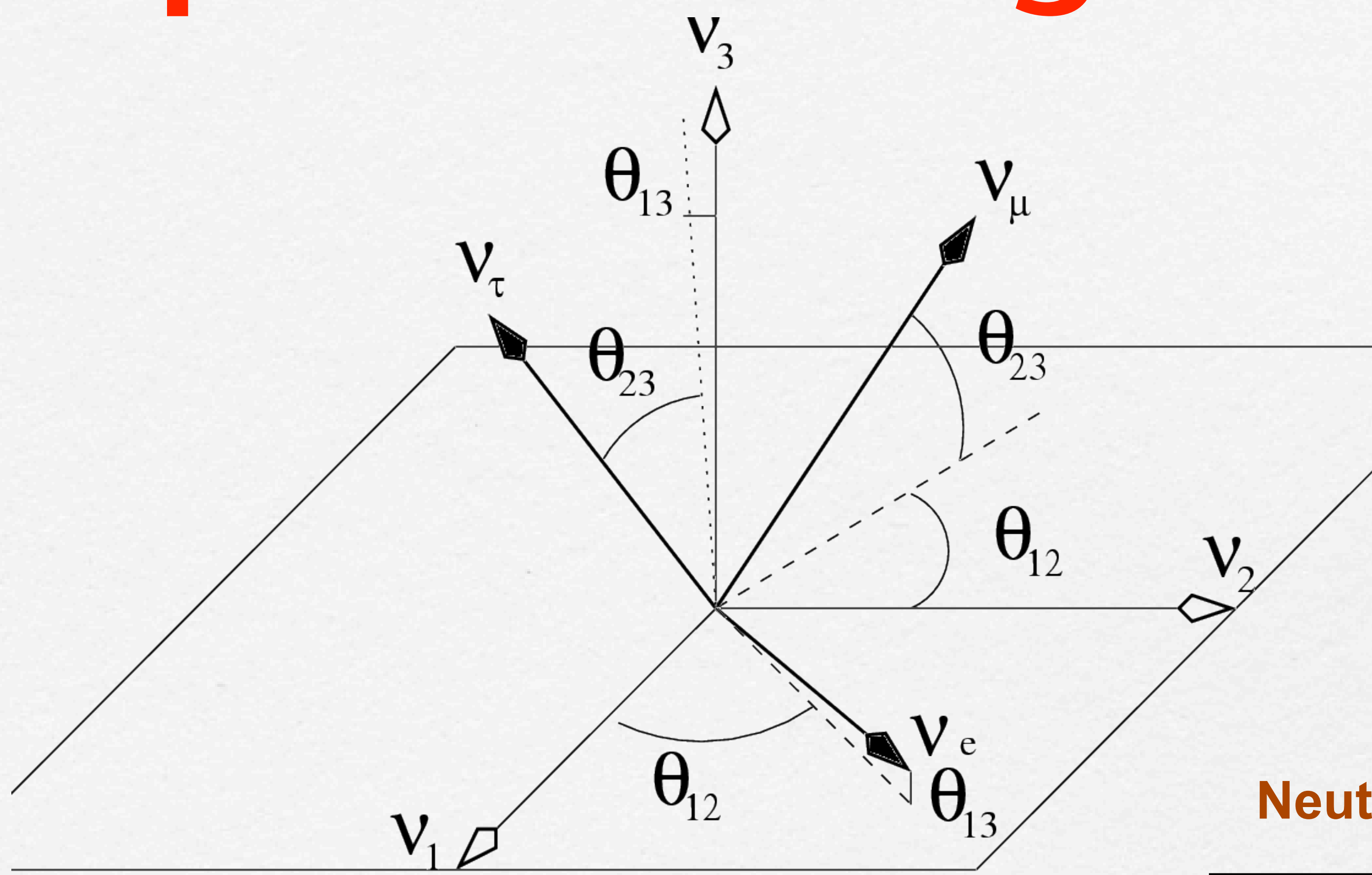
L is distance travelled

E is energy of neutrino

$$\Delta m^2 = m_2^2 - m_1^2$$

PMNS Lepton mixing matrix

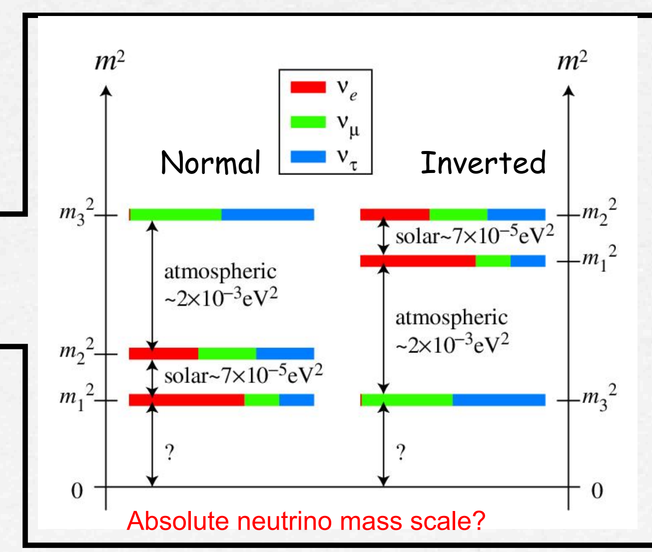
Pontecorvo
Maki
Nakagawa
Sakata



Standard Model states

$$\begin{pmatrix}\nu_e \\ e^-\end{pmatrix}_L \quad \begin{pmatrix}\nu_\mu \\ \mu^-\end{pmatrix}_L \quad \begin{pmatrix}\nu_\tau \\ \tau^-\end{pmatrix}_L$$

Neutrino mass states



$$\begin{pmatrix}\nu_e \\ \nu_\mu \\ \nu_\tau\end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3}\end{pmatrix} \begin{pmatrix}\nu_1 \\ \nu_2 \\ \nu_3\end{pmatrix}$$

PMNS Lepton mixing matrix

$$U_{PMNS}=\begin{pmatrix}1&0&0\\0&c_{23}&s_{23}\\0&-s_{23}&c_{23}\end{pmatrix}\begin{pmatrix}c_{13}&0&s_{13}e^{-i\delta}\\0&1&0\\-s_{13}e^{i\delta}&0&c_{13}\end{pmatrix}\begin{pmatrix}c_{12}&s_{12}&0\\-s_{12}&c_{12}&0\\0&0&1\end{pmatrix}\begin{pmatrix}1&0&0\\0&e^{i\frac{\alpha_{21}}{2}}&0\\0&0&e^{i\frac{\alpha_{31}}{2}}\end{pmatrix}$$

Atmospheric — Reactor — Solar — Majorana

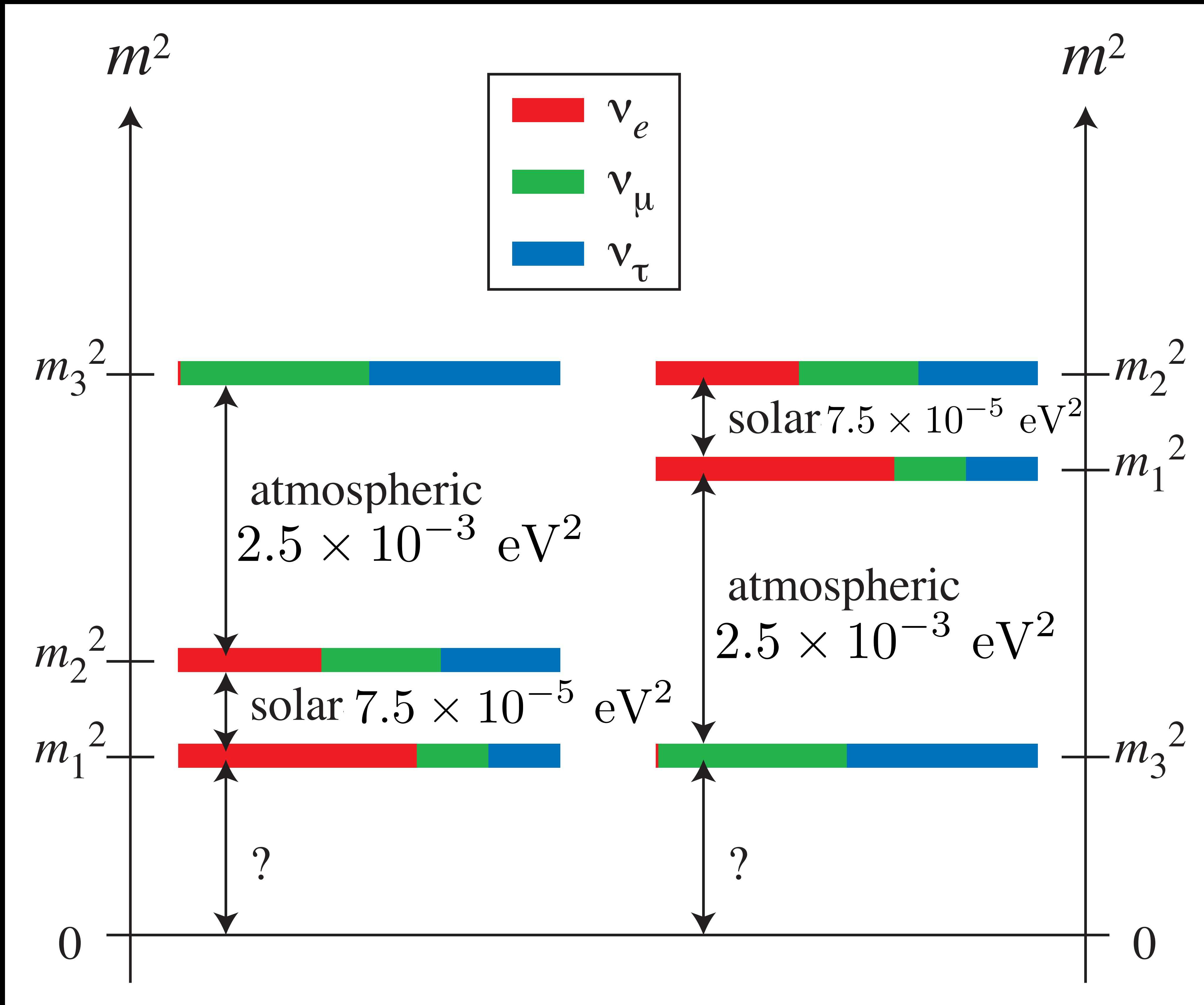
$$=\begin{pmatrix}c_{12}c_{13}&s_{12}c_{13}&s_{13}e^{-i\delta}\\-s_{12}c_{23}-c_{12}s_{13}s_{23}e^{i\delta}&c_{12}c_{23}-s_{12}s_{13}s_{23}e^{i\delta}&c_{13}s_{23}\\s_{12}s_{23}-c_{12}s_{13}c_{23}e^{i\delta}&-c_{12}s_{23}-s_{12}s_{13}c_{23}e^{i\delta}&c_{13}c_{23}\end{pmatrix}$$

$$\times\,\mathrm{diag}(1,e^{i\alpha_{21}/2},e^{i\alpha_{31}/2})$$

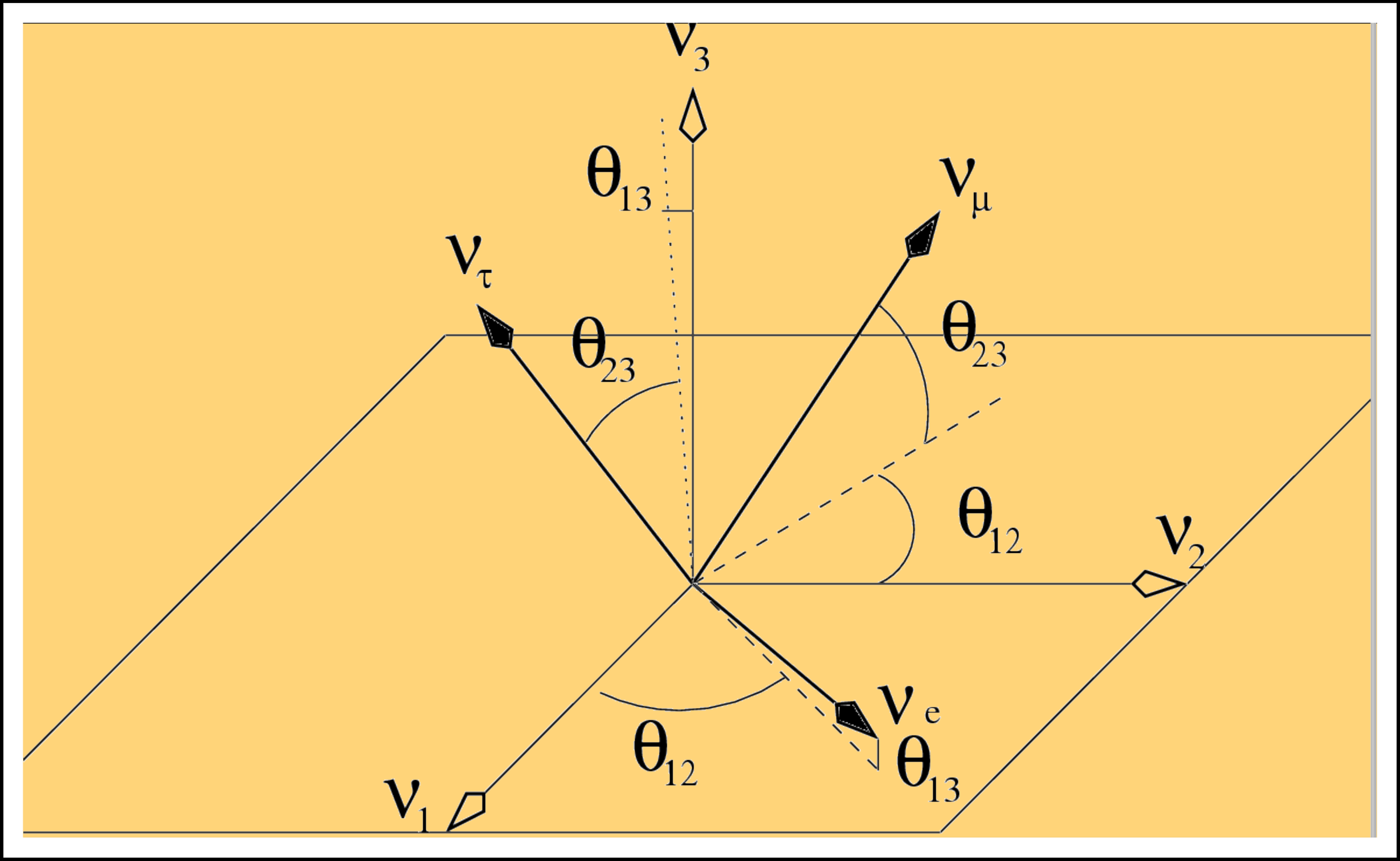
The 6 parameters measurable in neutrino oscillations (assuming 3 active neutrinos):

- ✳ The atmospheric mass squared difference Δm^2_{31}
- ✳ The solar mass squared difference $\Delta m^2_{21} = m^2_2 - m^2_1$
- ✳ The atmospheric angle θ_{23}
- ✳ The solar angle θ_{12}
- ✳ The reactor angle θ_{13}
- ✳ The CP violating phase δ

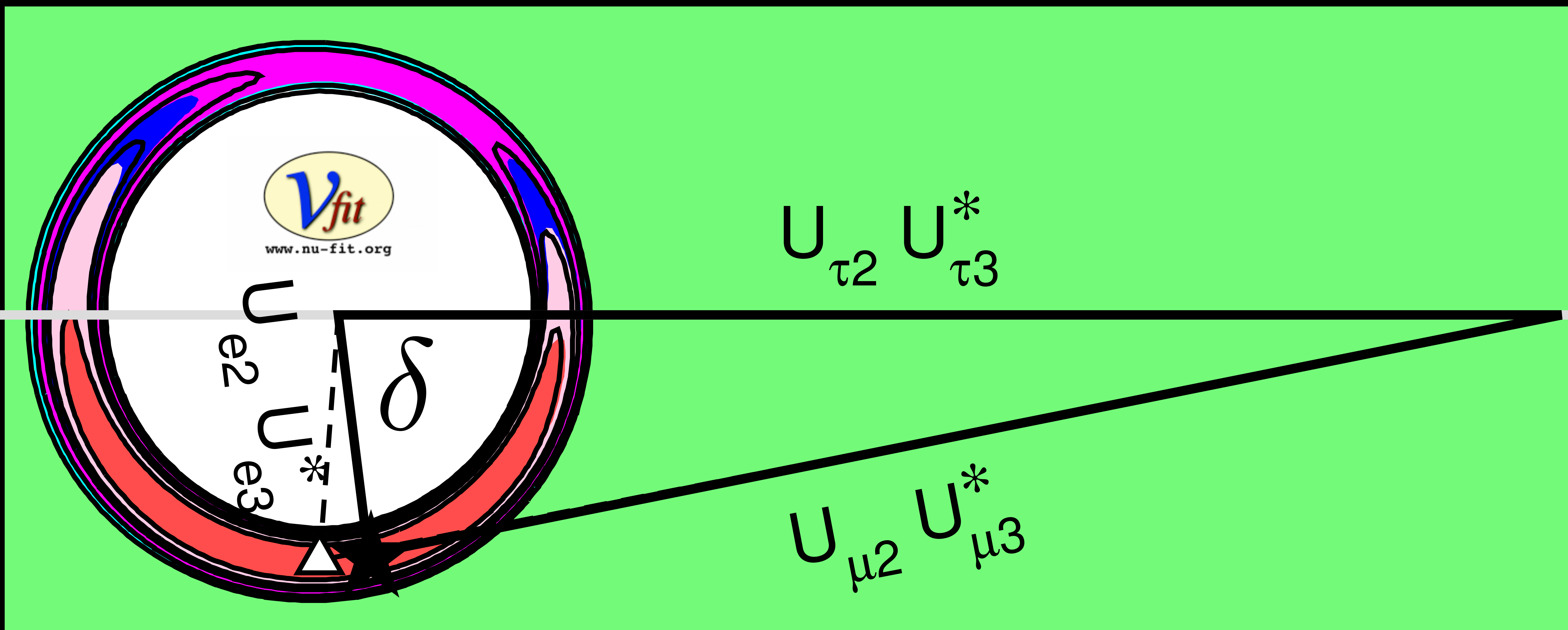
2 Mass Squared Differences



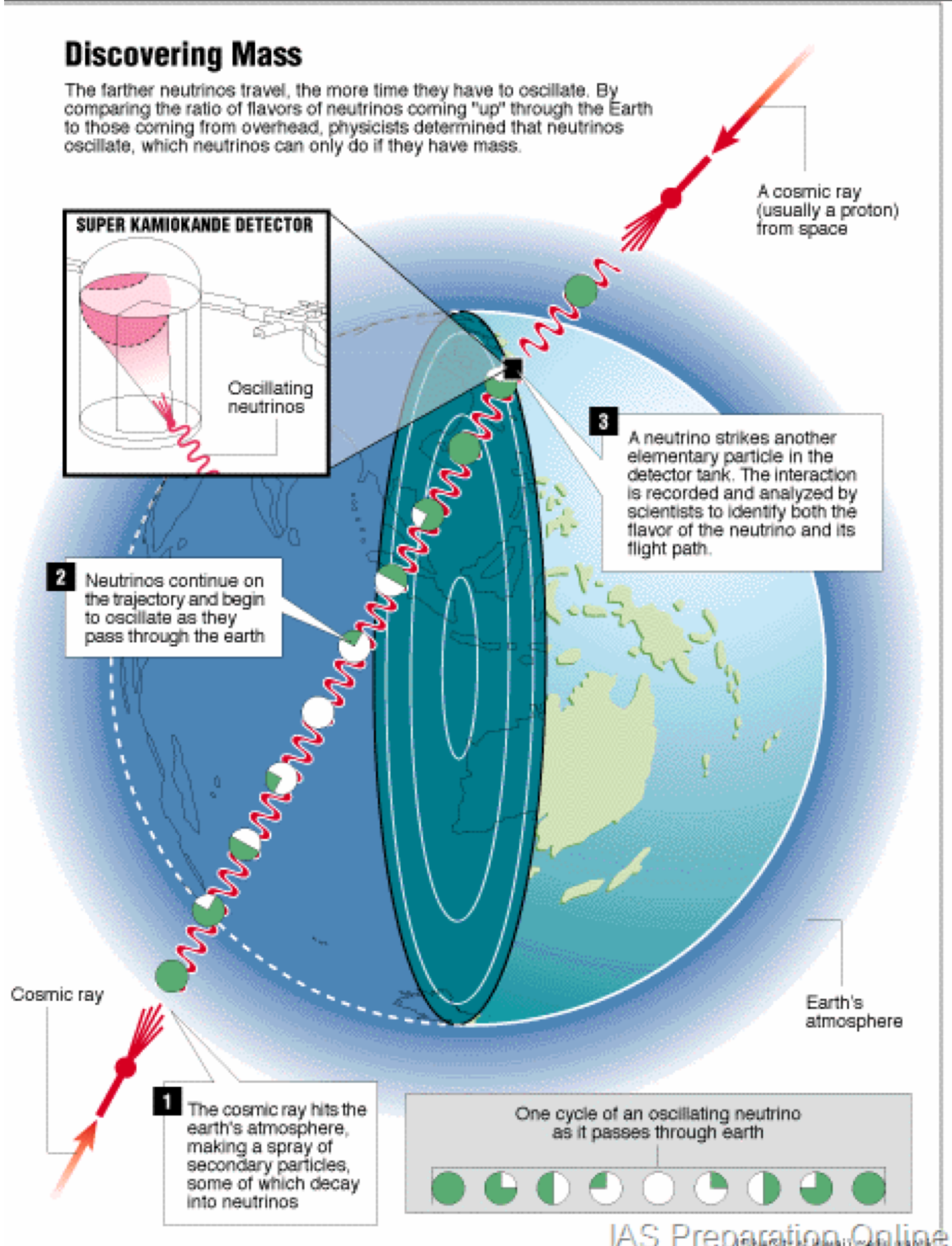
The 3 Lepton Mixing Angles



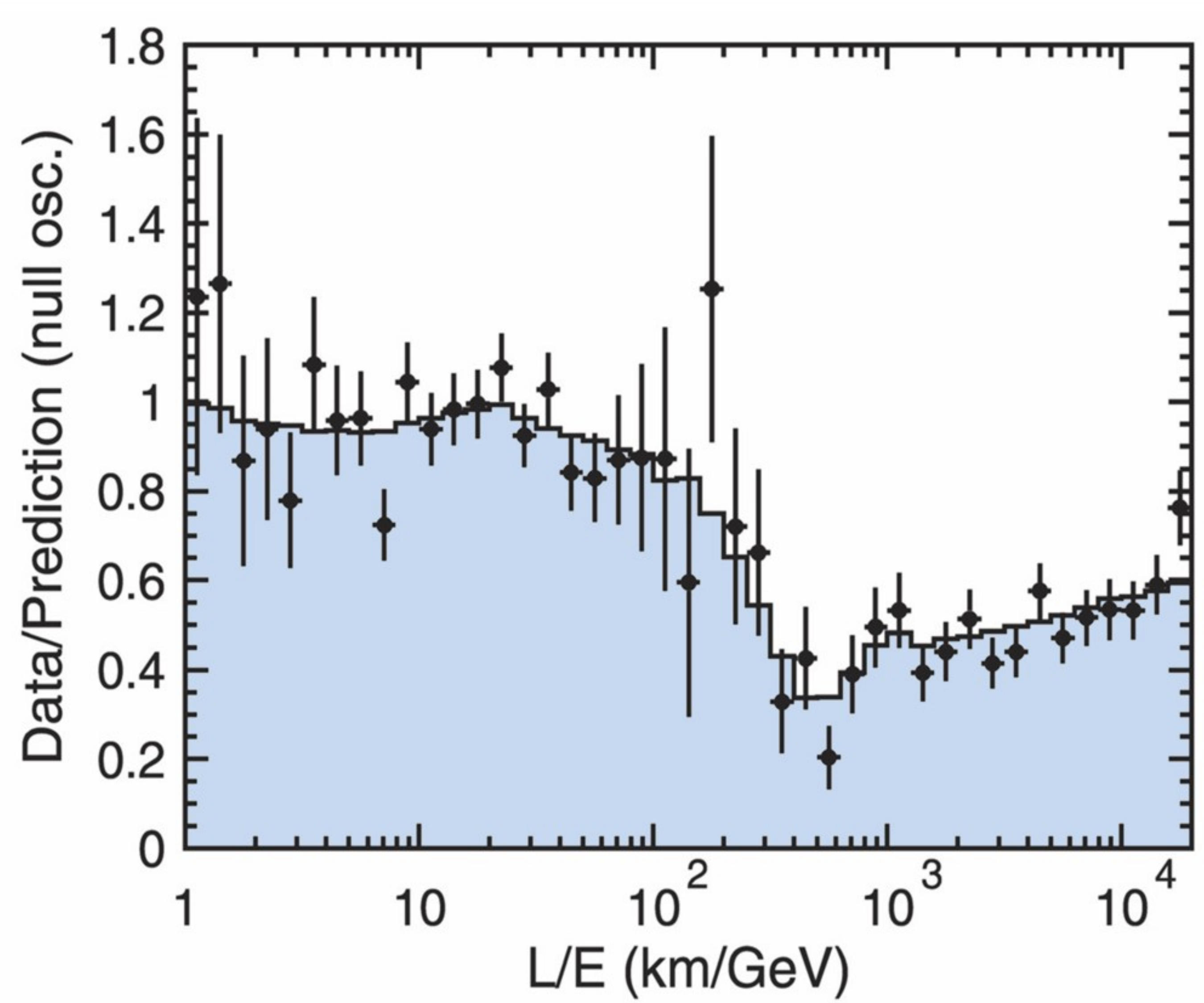
The one oscillation CP Violating Phase



Atmospheric Neutrino Oscillations (1998)



Proof that neutrinos have mass



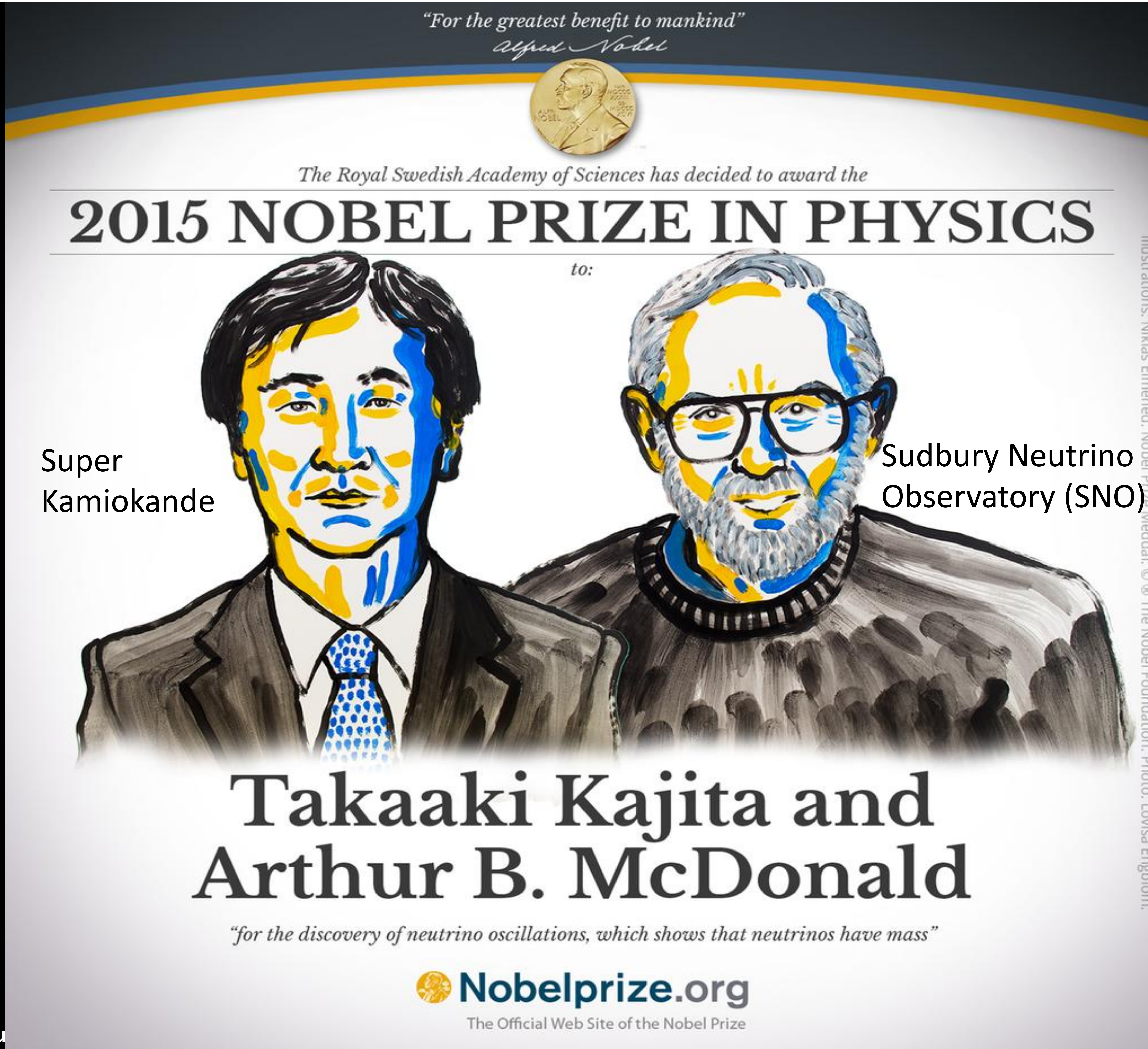
$$Prob. = \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{E}$$

That formula again

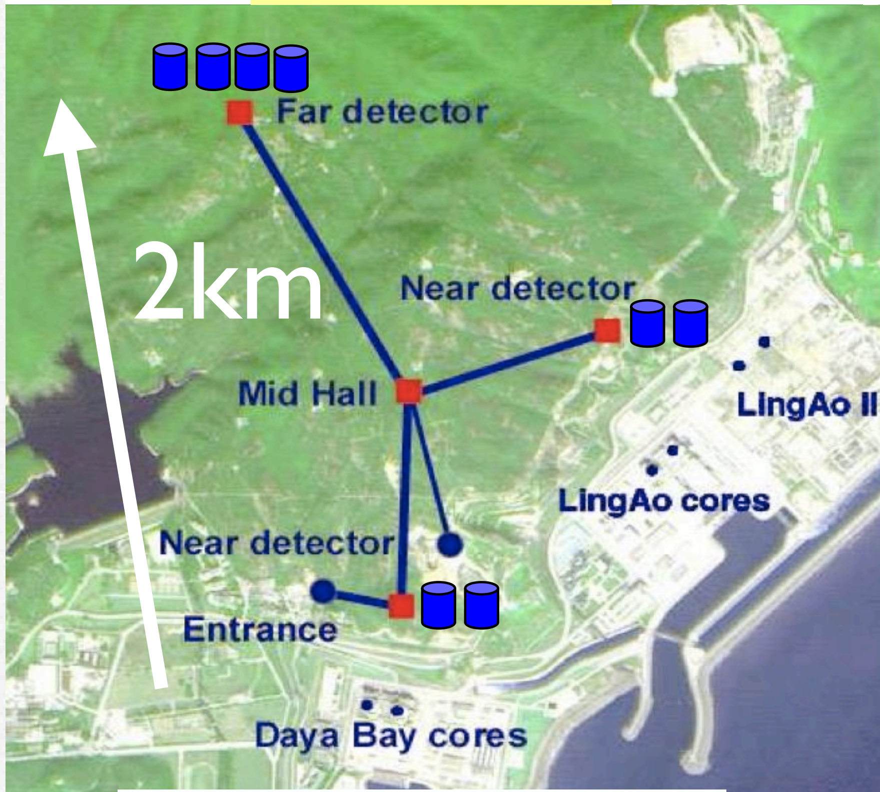
Atmospheric neutrino oscillations show characteristic L/E variation

Brief History of Neutrino Physics post 1998

- ☑ Atmospheric ν_μ disappear, large θ_{23} (1998)  SK
- ☑ Solar ν_e disappear, large θ_{12} (2002)  SK, SNO
- ☑ Solar ν_e are converted to $\nu_\mu+\nu_\tau$ (2002) SNO
- ☑ Reactor anti-ν_e disappear/reappear (2004) Kamland
- ☑ Accelerator ν_μ disappear (2006) MINOS
- ☑ Accelerator ν_μ converted to ν_τ (2010) OPERA
- ☑ Accelerator ν_μ converted to ν_e , θ_{13} hint (2011) T2K
- ☑ Reactor anti-ν_e disapp θ_{13} meas.(2012) DB, Reno,DC

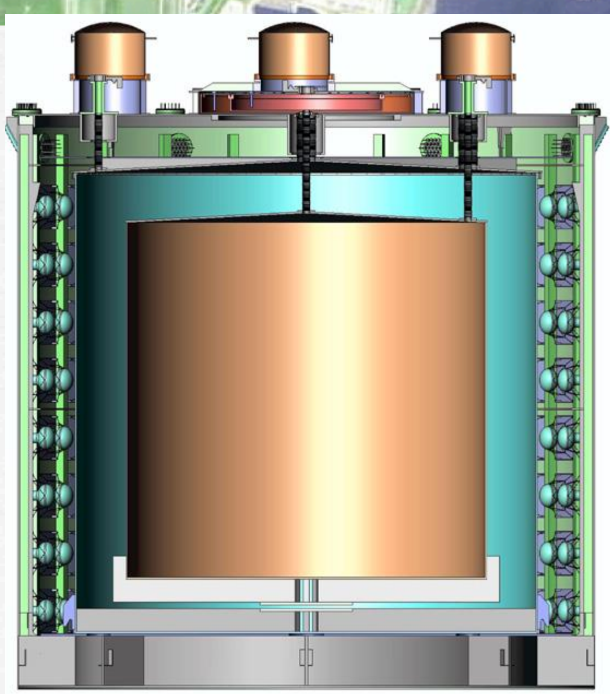

"For the greatest benefit to mankind"
Alfred Nobel
The Royal Swedish Academy of Sciences has decided to award the
2015 NOBEL PRIZE IN PHYSICS
to:
Super
Kamiokande
Sudbury Neutrino
Observatory (SNO)
Takaaki Kajita and
Arthur B. McDonald
"for the discovery of neutrino oscillations, which shows that neutrinos have mass"
Nobelprize.org
The Official Web Site of the Nobel Prize
Illustrations: Niklas Elmehed. Nobel Prize Meddal: © ® The Nobel Foundation. Photo: Lovisa Engblom.

Daya Bay

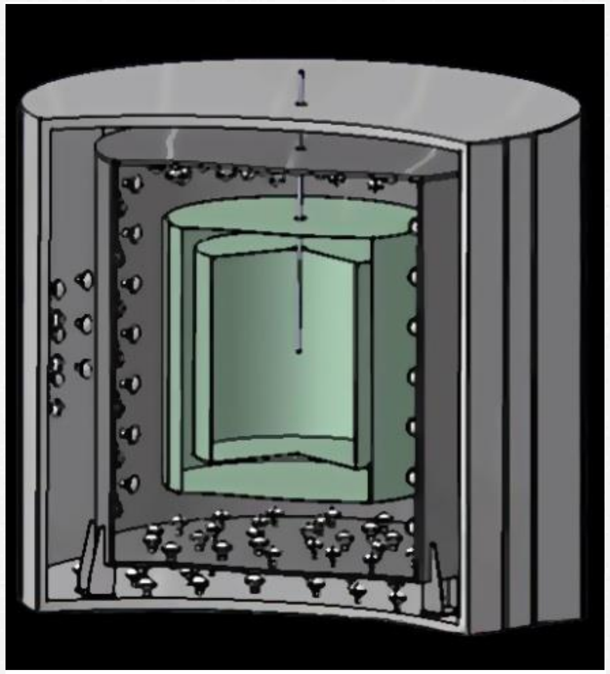


Double Chooz

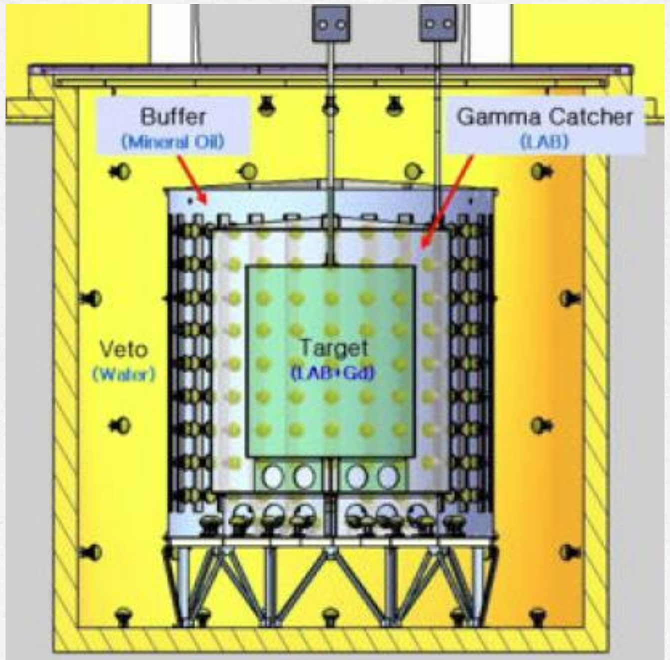
Reno



Daya Bay



Double Chooz



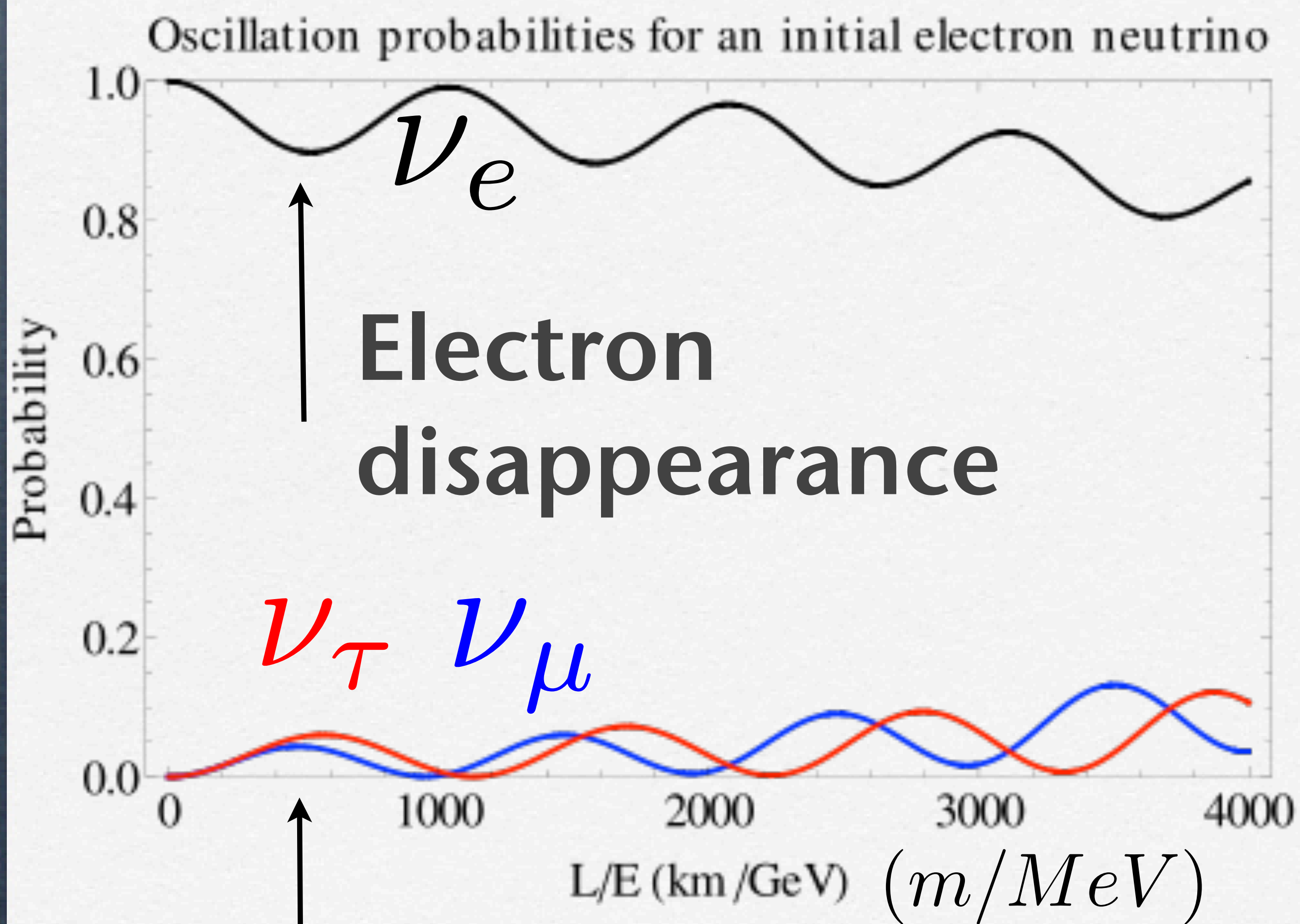
RENO

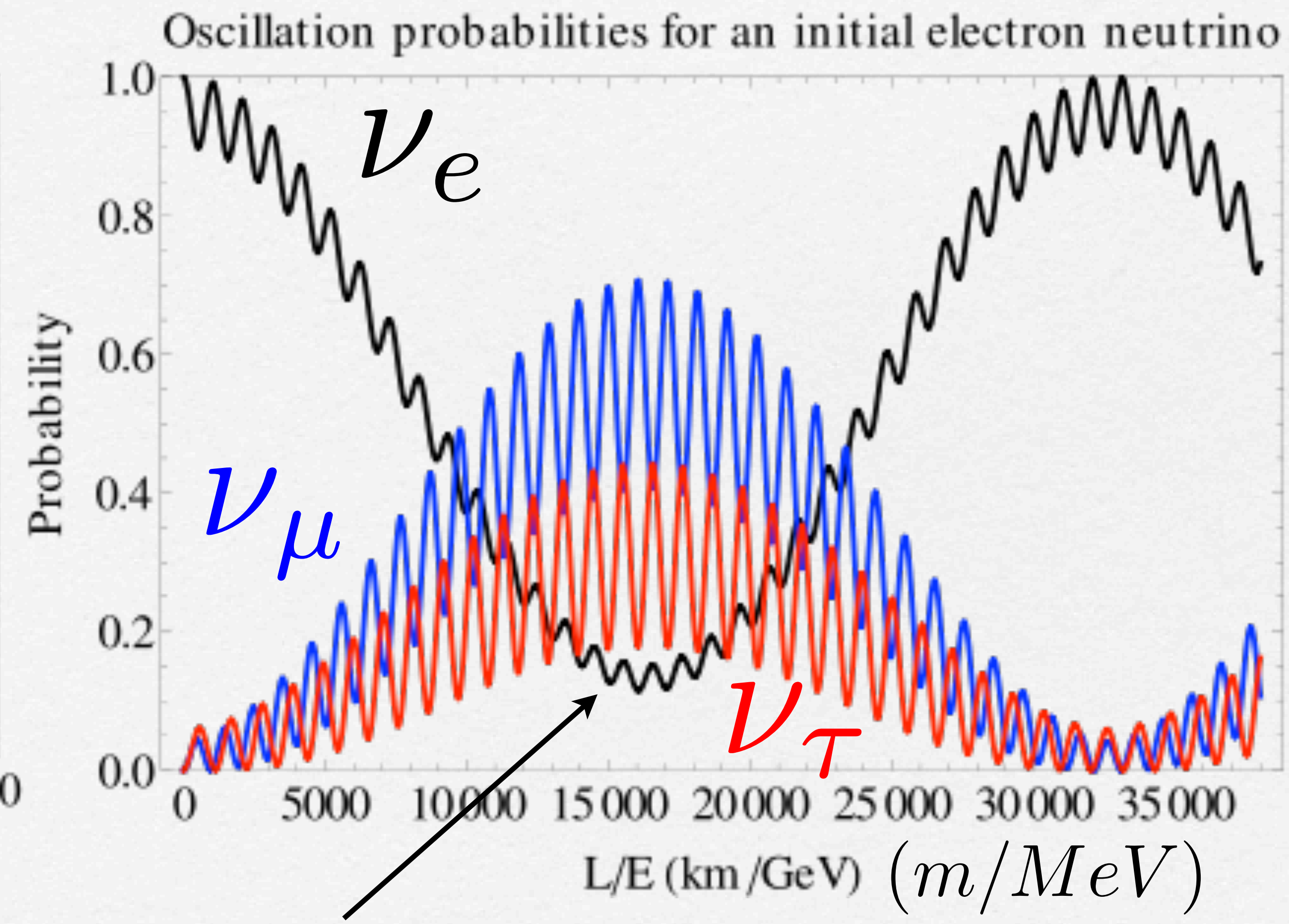
Farewell Daya Bay, hello JUNO (coming soon)

Electron Neutrino Oscillations

$$P(\bar{\nu}_e \to \bar{\nu}_e; E, L) = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21}$$
$$- \sin^2 2\theta_{13}(\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32})$$

$$\Delta_{ij} = \frac{\Delta m^2_{ij} L}{4E}$$





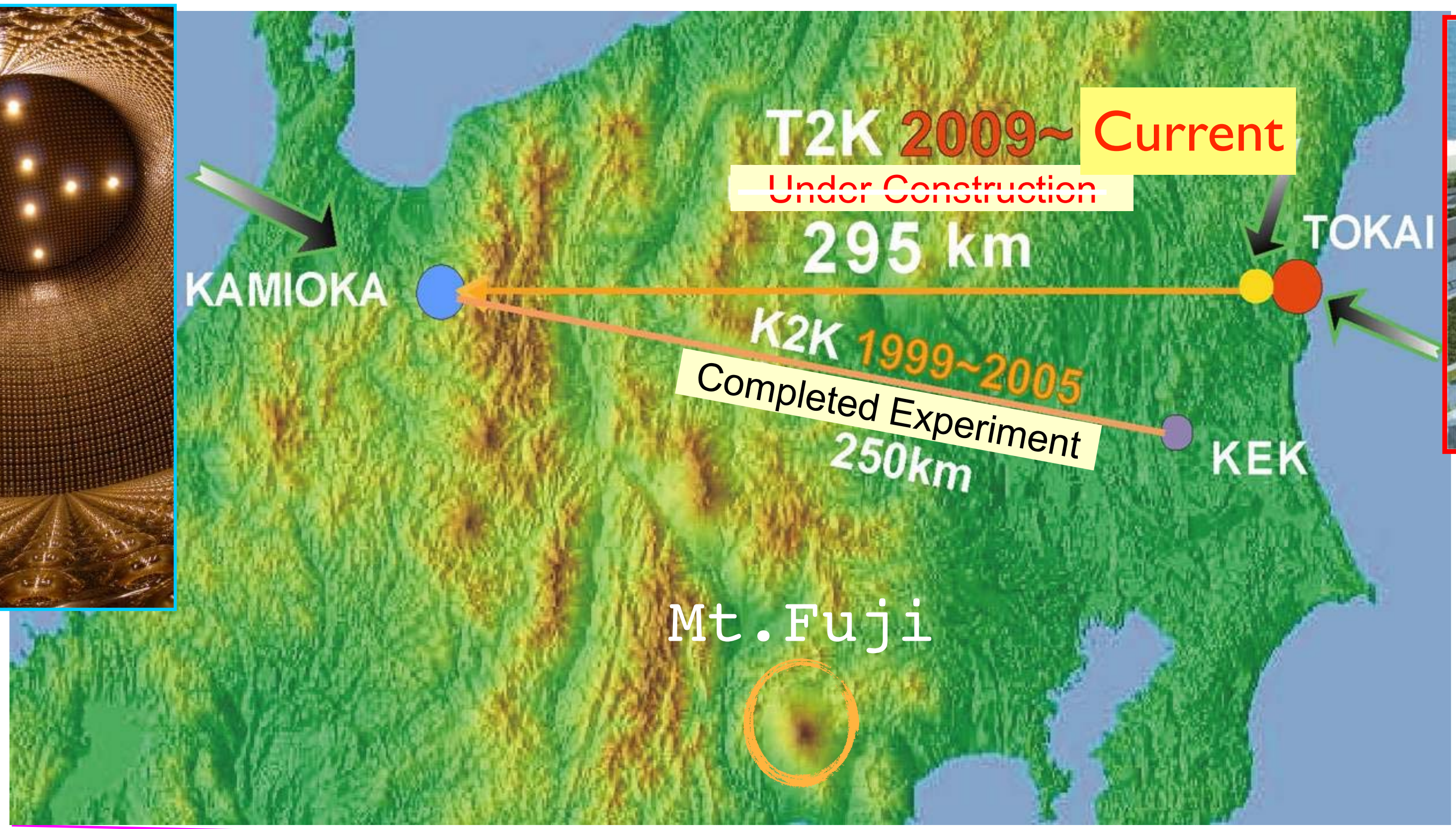
Daya Bay RENO 2km (1st atm max)

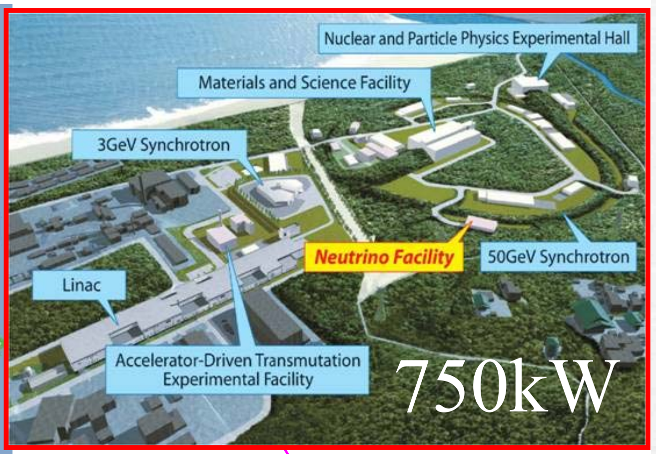
$$\frac{\Delta m^2_{31} L}{4E} = \frac{\pi}{2}$$

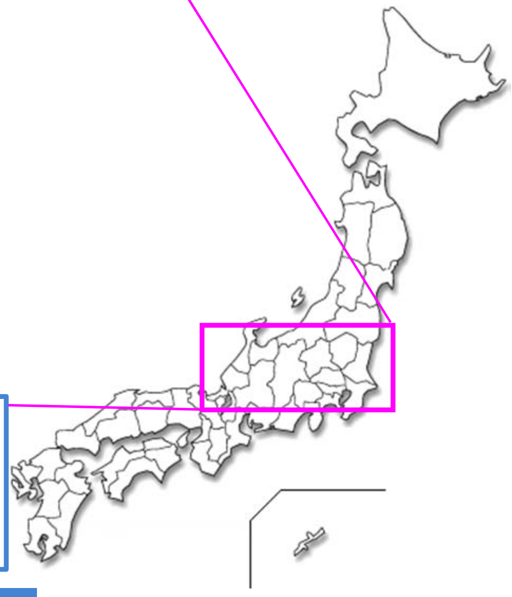
JUNO RENO50km (1st sol max)

$$\frac{\Delta m^2_{21} L}{4E} = \frac{\pi}{2}$$

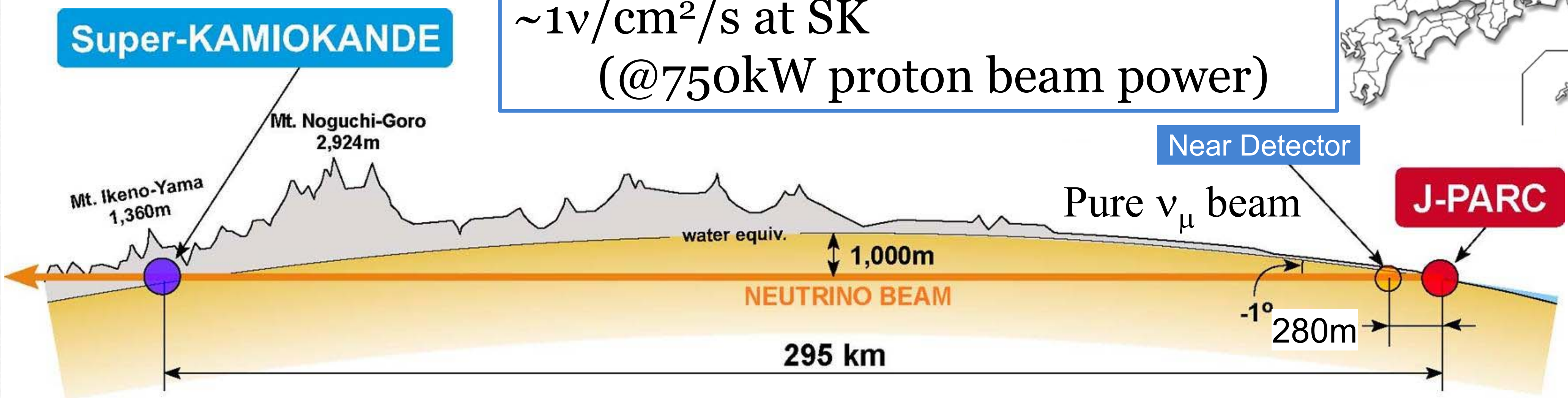
T2K (Tokai to Kamioka)
Long Baseline ν experiment







~1ν/cm²/s at SK
(@750kW proton beam power)

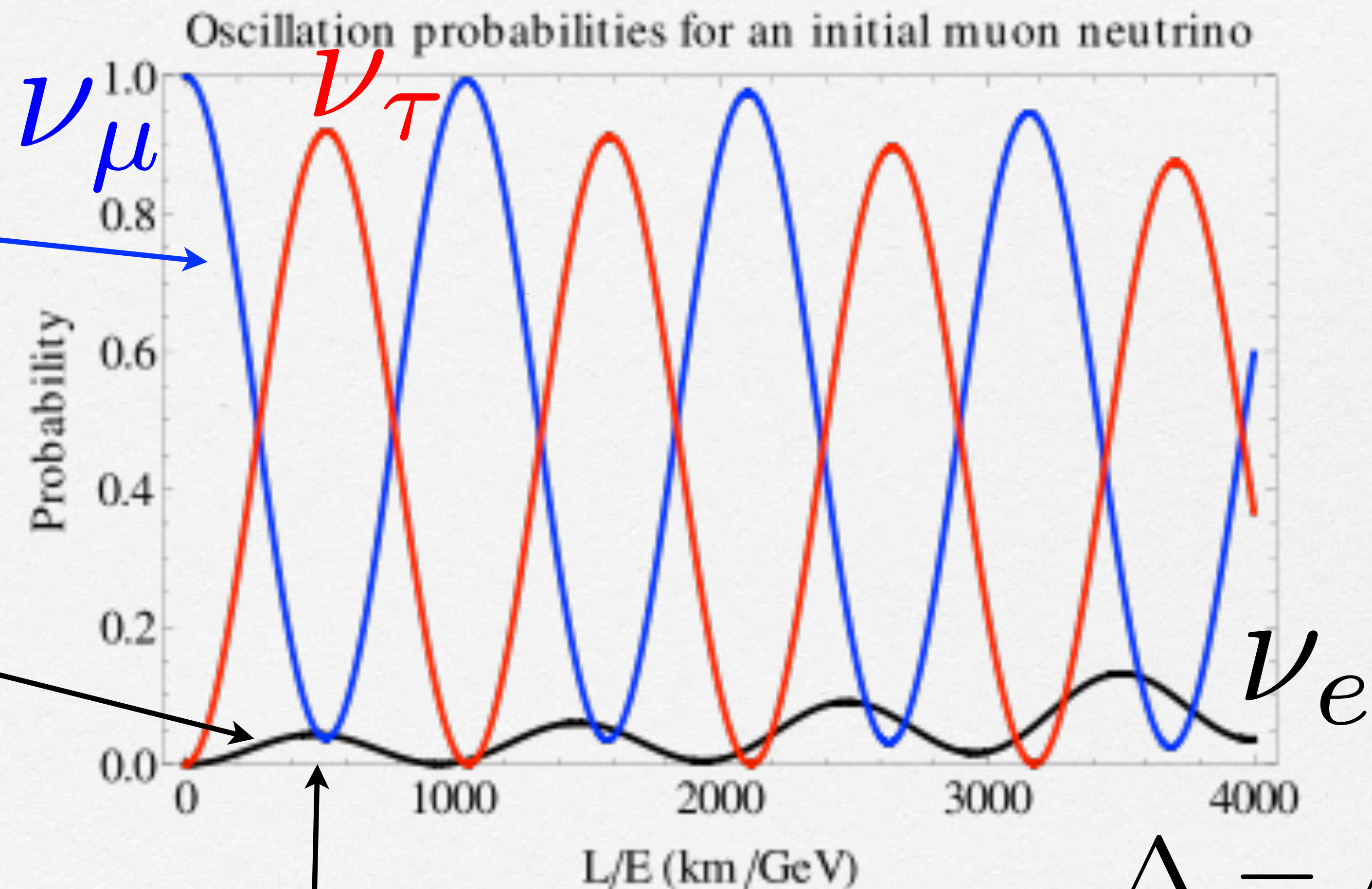


Muon Neutrino Oscillations

$$P(\nu_\mu \to \nu_\mu; E, L) = 1 - \sin^2(2\theta_{23}) \sin^2\left(\frac{\Delta L}{2}\right) + \mathcal{O}(\epsilon)$$

Muon disappearance

Electron appearance



Accelerator LBL (1st atm max)

$$\frac{\Delta m^2_{31} L}{4E} = \frac{\pi}{2}$$

$$\Delta = \Delta m^2_{31}/2E$$

$$\epsilon \equiv \Delta m^2_{21}/\Delta m^2_{31} \approx 0.03$$

Electron Neutrino Appearance

$$P(\nu_\mu \to \nu_e; E, L) \equiv P_1 + P_{\frac{3}{2}} + \mathcal{O}\left(\epsilon^2\right)$$

$$P_1 = \frac{4}{(1-r_A)^2} \sin^2\theta_{23} \sin^2\theta_{13} \sin^2\left(\frac{(1-r_A)\Delta L}{2}\right),$$

$$P_{\frac{3}{2}} = 8 J_r \frac{\epsilon}{r_A(1-r_A)} \cos\left(\delta + \frac{\Delta L}{2}\right) \sin\left(\frac{r_A \Delta L}{2}\right) \sin\left(\frac{(1-r_A)\Delta L}{2}\right)$$

CP phase **Matter effect**

Electron appearance depends on CP phase

$$\Delta = \Delta m^2_{31}/2E$$

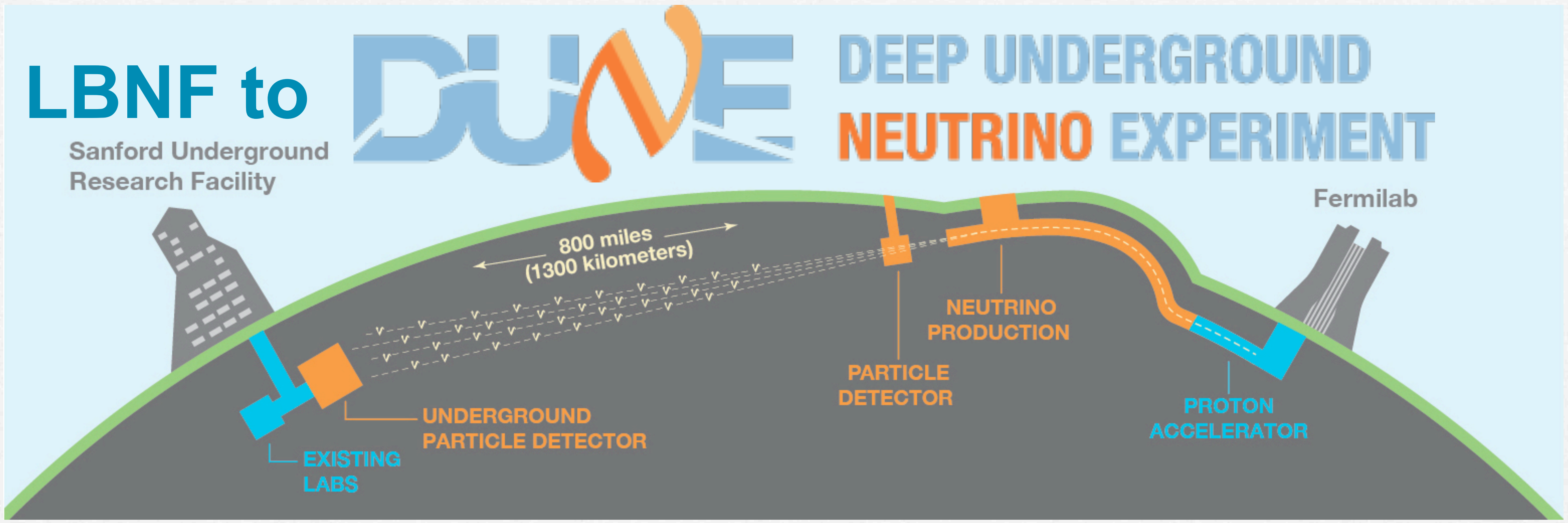
$$\epsilon \equiv \Delta m^2_{21}/\Delta m^2_{31} \approx 0.03$$

r_A, δ change sign for antineutrinos

$$r_A = 2\sqrt{2} G_F N_e E/\Delta m^2_{31}$$

$$J_r = \cos\theta_{12} \sin\theta_{12} \cos\theta_{23} \sin\theta_{23} \sin\theta_{13}$$

Future LBL experiments

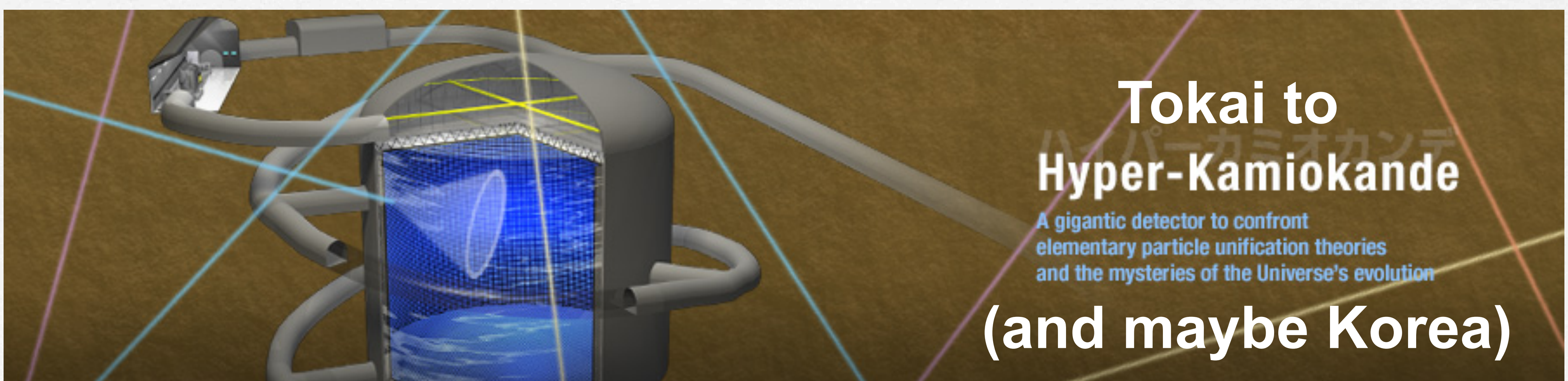


Beams of

$\nu_\mu \quad \bar{\nu}_\mu$

from

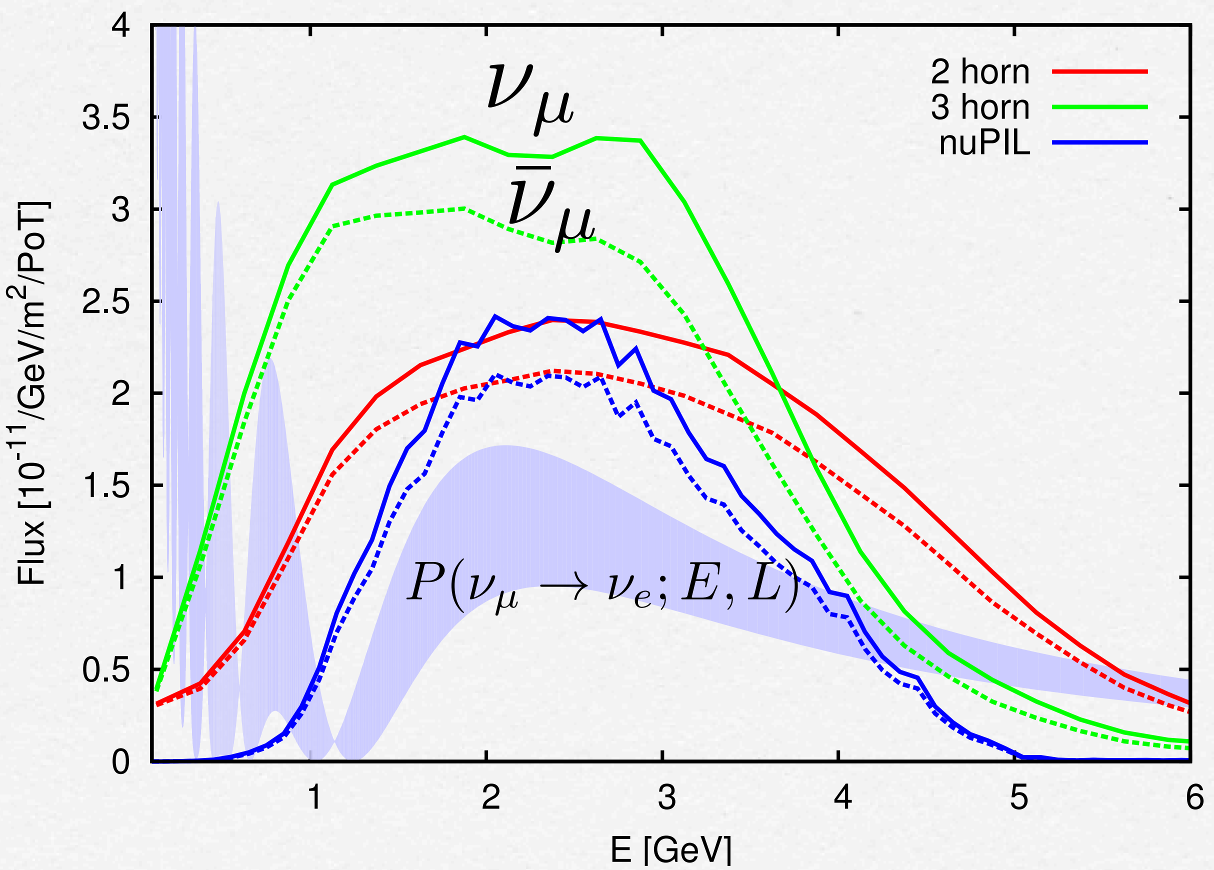
$$\pi^+ \to \mu^+ + \nu_\mu$$
$$\pi^- \to \mu^- + \bar{\nu}_\mu$$



Highly complementary experiments:

DUNE

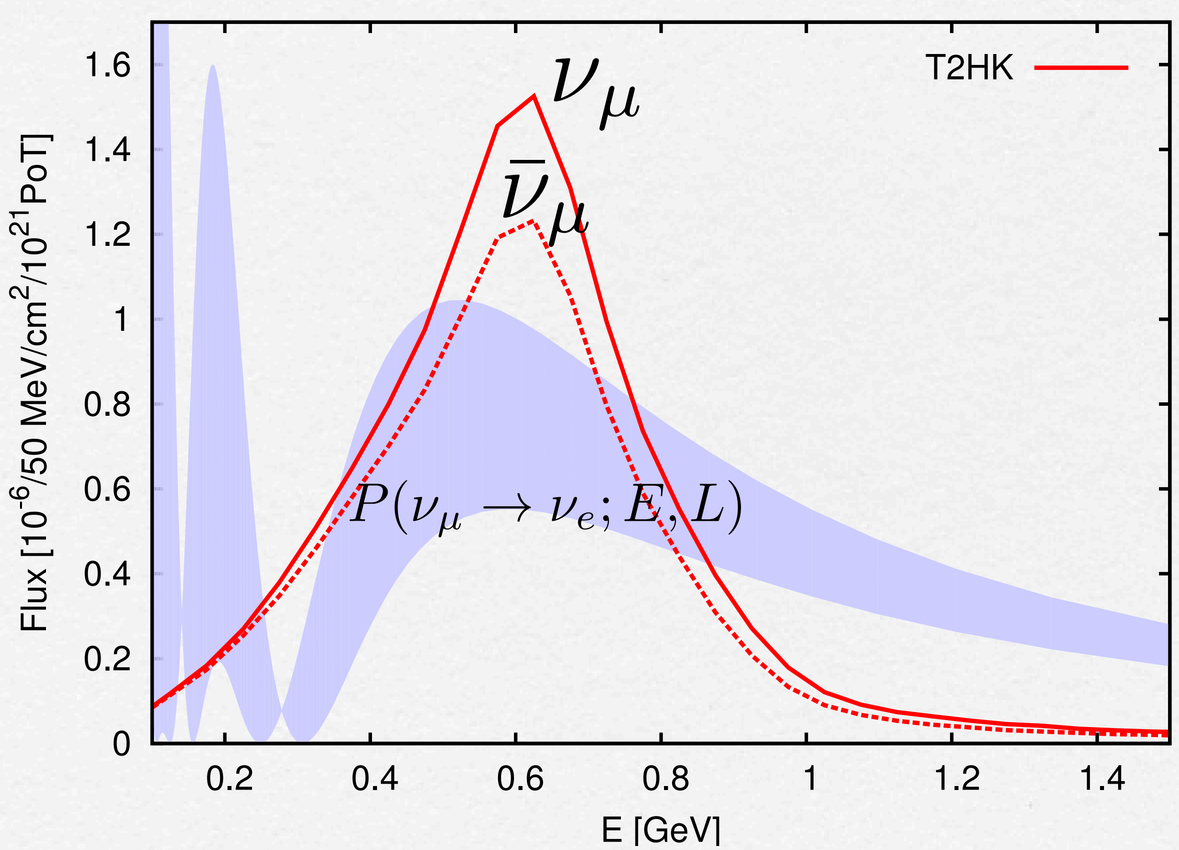
$L = 1300km$



Wide Band Beam
LAr detector

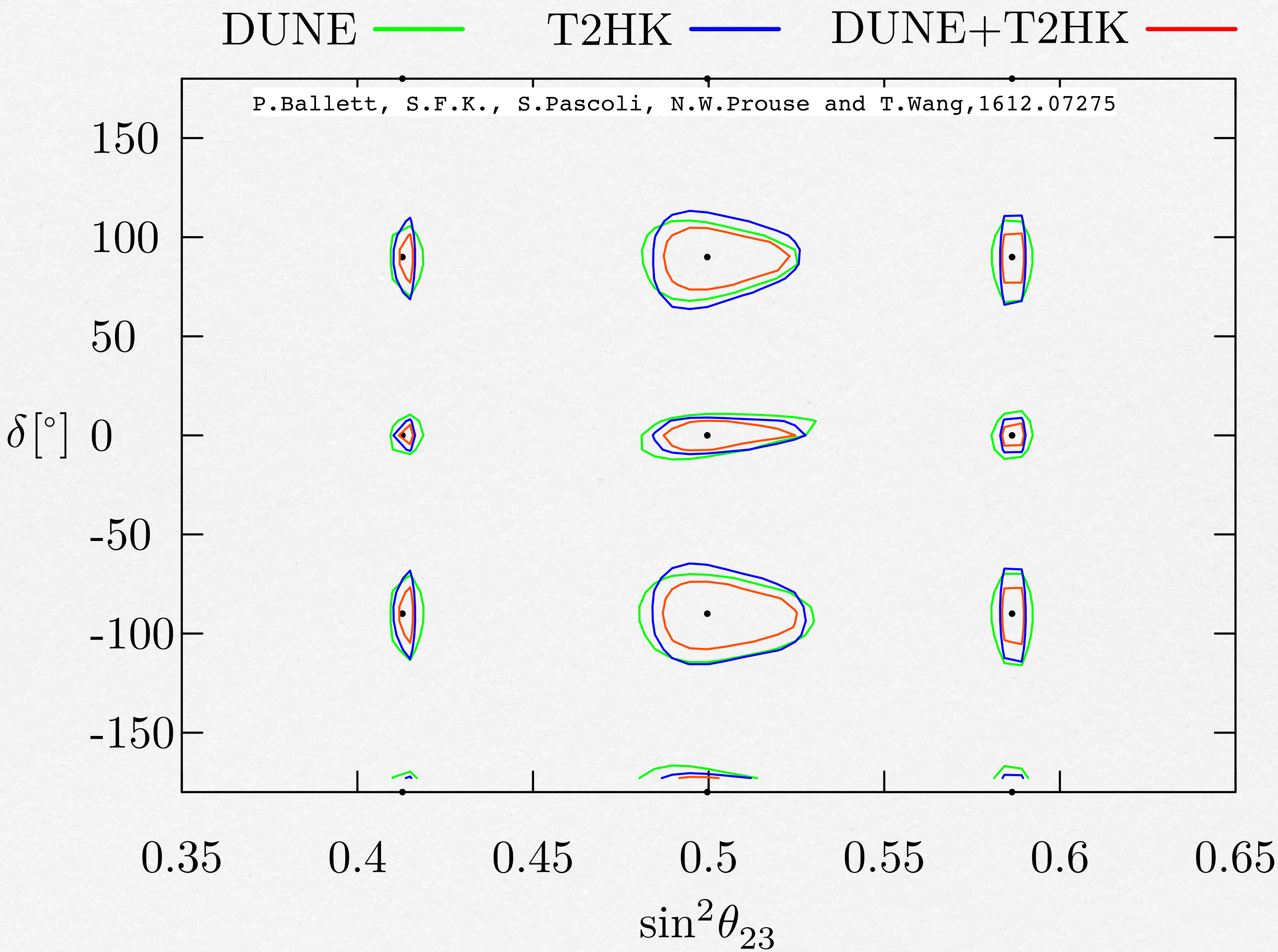
T2HK

$L = 295km$



Narrow Band Beam (off-axis)
Water detector

Precision measurements



1 sigma contours in future

Parameters	Neutrino Oscillation Experiments
Δm^2_{21}	KamLAND ($\overline{\nu}_e \to \overline{\nu}_e$)[21]
Δm^2_{31}	T2K ($\nu_\mu \to \nu_\mu$)[22] MINOS ($\overline{\nu}_\mu \to \overline{\nu}_\mu$, $\nu_\mu \to \nu_\mu$)[23]
θ_{12}	solar neutrinos ($\nu_e \to \nu_e$) Borexino[24], SNO[25,26], Super-Kamionkande I-IV[27]
θ_{13}	Daya Bay ($\overline{\nu}_e \to \overline{\nu}_e$)[28] RENO ($\overline{\nu}_e \to \overline{\nu}_e$)[29]
θ_{23}	atmospheric neutrinos ($\overline{\nu}_\mu \to \overline{\nu}_\mu$, $\nu_\mu \to \nu_\mu$) Super-Kamiokande I-IV[30]
δ	–

What we know as of now

NuFIT 5.1 (2021)

with SK atmospheric data

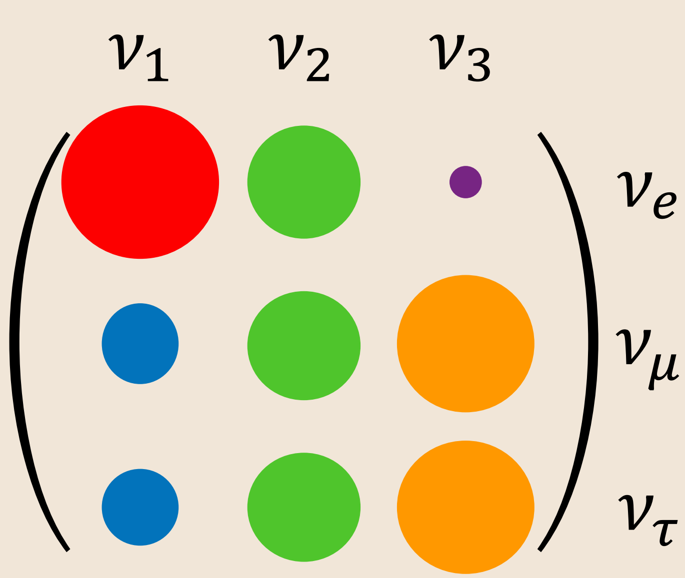
	Normal Ordering (best fit)	
	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.45^{+0.77}_{-0.75}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.450^{+0.019}_{-0.016}$	$0.408 \to 0.603$
$\theta_{23}/^\circ$	$42.1^{+1.1}_{-0.9}$	$39.7 \to 50.9$
$\sin^2\theta_{13}$	$0.02246^{+0.00062}_{-0.00062}$	$0.02060 \to 0.02435$
$\theta_{13}/^\circ$	$8.62^{+0.12}_{-0.12}$	$8.25 \to 8.98$
$\delta_{\rm CP}/^\circ$	230^{+36}_{-25}	$144 \to 350$
$\dfrac{\Delta m^2_{21}}{10^{-5}\ {\rm eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\dfrac{\Delta m^2_{3\ell}}{10^{-3}\ {\rm eV}^2}$	$+2.510^{+0.027}_{-0.027}$	$+2.430 \to +2.593$

Inverted Ordering ($\Delta\chi^2 = 7.0$)

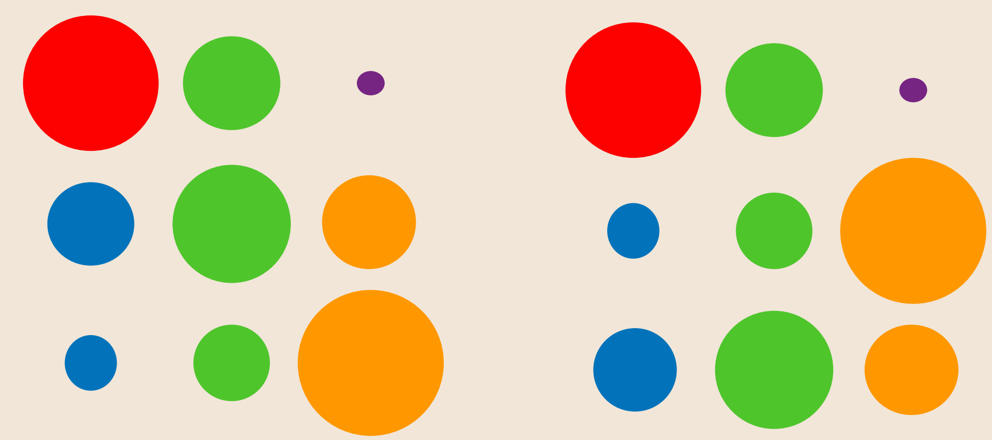
Phill Litchfield

Experimental open questions for neutrino mixing

$|U_{\mathrm{PMNS}}|^2 \simeq$

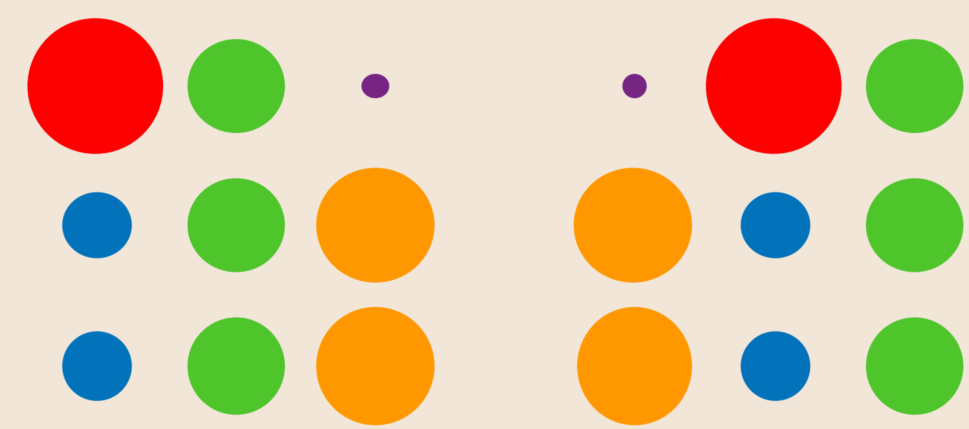


Octant degeneracy



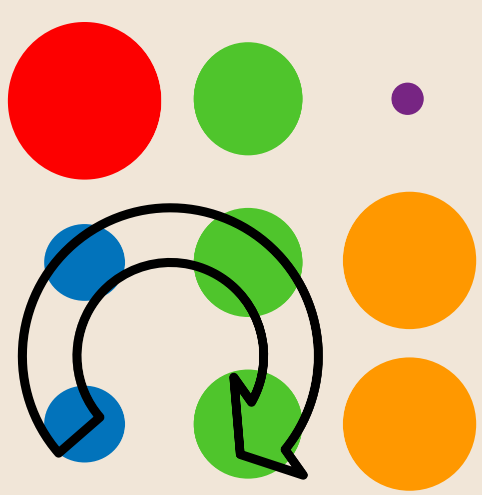
Lower ($\theta_{23} < 45°$) Upper ($\theta_{23} > 45°$)

Mass Ordering (Hierarchy)



Normal (NO) Inverted (IO)

CP Violation



Complex mixing of these 4 elements causes

$$P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

Key parameter: $\boldsymbol{\delta_{CP}}$

Theory of the mixing matrices

$$\mathcal{L} = -v^u Y^u_{ij} \overline{u}^i_{\rm L} u^j_{\rm R} - v^d Y^d_{ij} \overline{d}^i_{\rm L} d^j_{\rm R} + h.c.$$

Quark sector

$$U_{u_{\rm L}}\, Y^u\, U^\dagger_{u_{\rm R}} = \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}, \qquad U_{d_{\rm L}}\, Y^d\, U^\dagger_{d_{\rm R}} = \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}$$

$$\mathcal{L}^{CC} = -\frac{g}{\sqrt{2}} \begin{pmatrix} \bar{u}_L & \bar{c}_L & \bar{t}_L \end{pmatrix} U_{\rm CKM} \gamma^\mu W^+_\mu \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

$$U_{\rm CKM} = U_{u_{\rm L}} U^\dagger_{d_{\rm L}}$$

5 phases removed

$$L = -\frac{1}{2} m^\nu \overline{\nu}^i_{\rm L} \nu^{cj}_{\rm L} - v^d Y^e_{ij} \overline{e}^i_{\rm L} e^j_{\rm R} + h.c.$$

Lepton sector

$$U_{\nu_{\rm L}}\, m^\nu\, U^T_{\nu_{\rm L}} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \qquad U_{e_{\rm L}}\, Y^e\, U^\dagger_{e_{\rm R}} = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

$$\mathcal{L}^{CC} = -\frac{g}{\sqrt{2}} \begin{pmatrix} \bar{e}_L & \bar{\mu}_L & \bar{\tau}_L \end{pmatrix} U_{\rm PMNS} \gamma^\mu W^-_\mu \begin{pmatrix} \nu_{1\,L} \\ \nu_{2\,L} \\ \nu_{3\,L} \end{pmatrix}$$

$$U_{\rm PMNS} = U_{e_{\rm L}} U^\dagger_{\nu_{\rm L}}$$

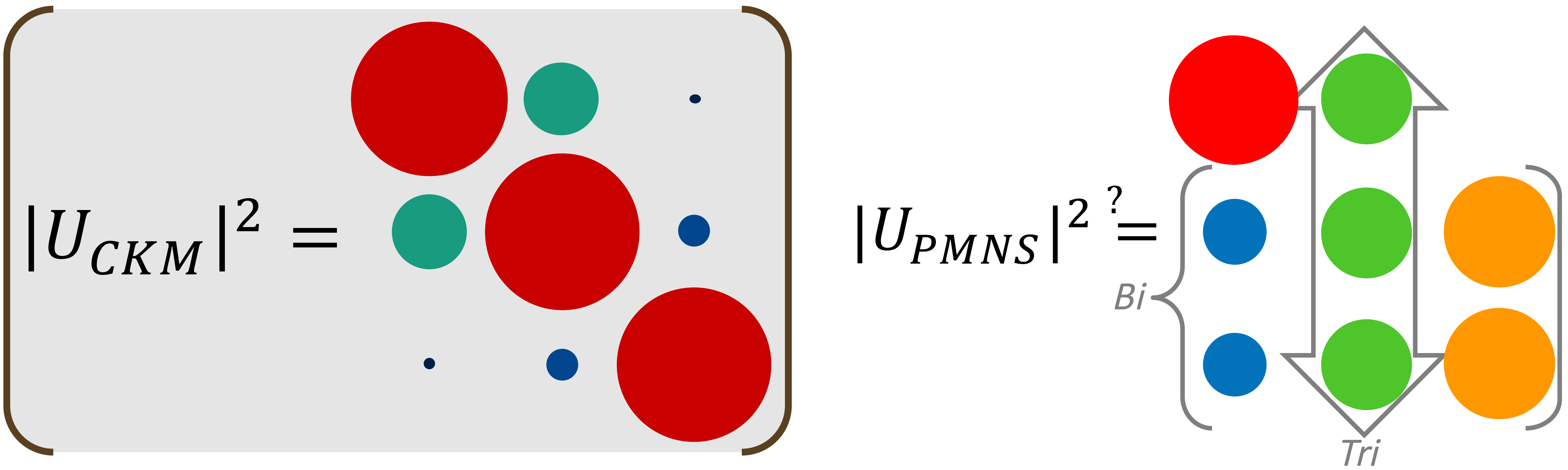
3 phases removed

Phill Litchfield

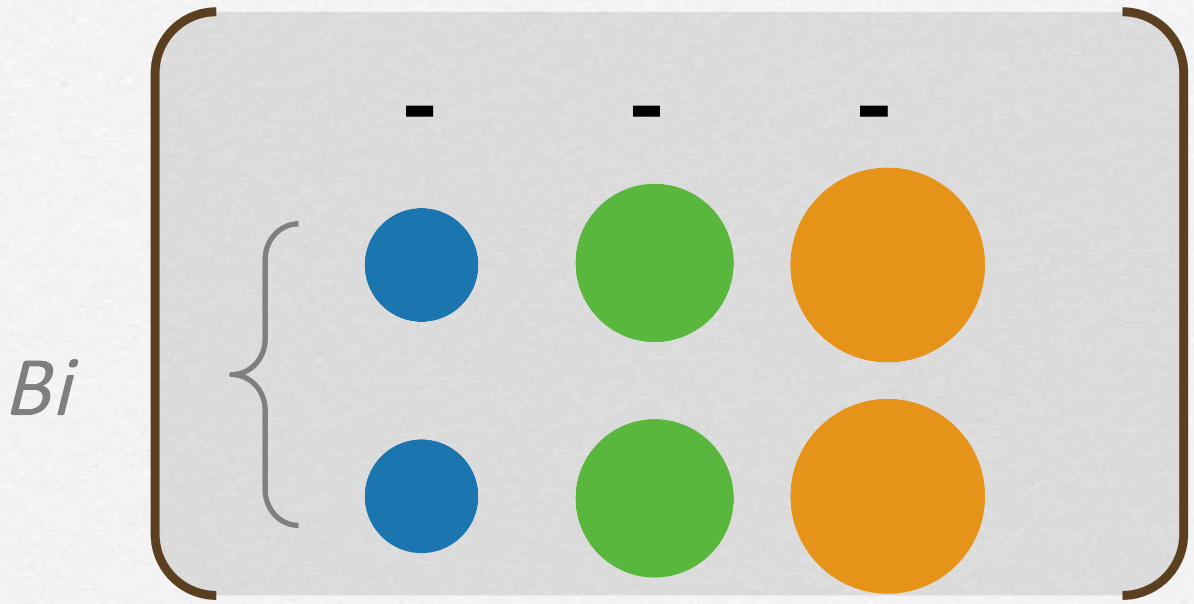
CKM vs PMNS

CKM Matrix

PMNS Matrix



Mu-Tau Symmetry $\nu_\mu \leftrightarrow \nu_\tau^*$



Basic Idea:

→ Two rows have
→ equal magnitudes

Z.z.Xing and S.Zhou, 0804.3512

→ $\theta_{13} \neq 0\,, \quad \theta_{23} = 45^\circ, \quad \delta_{\rm CP} = \pm 90^\circ$

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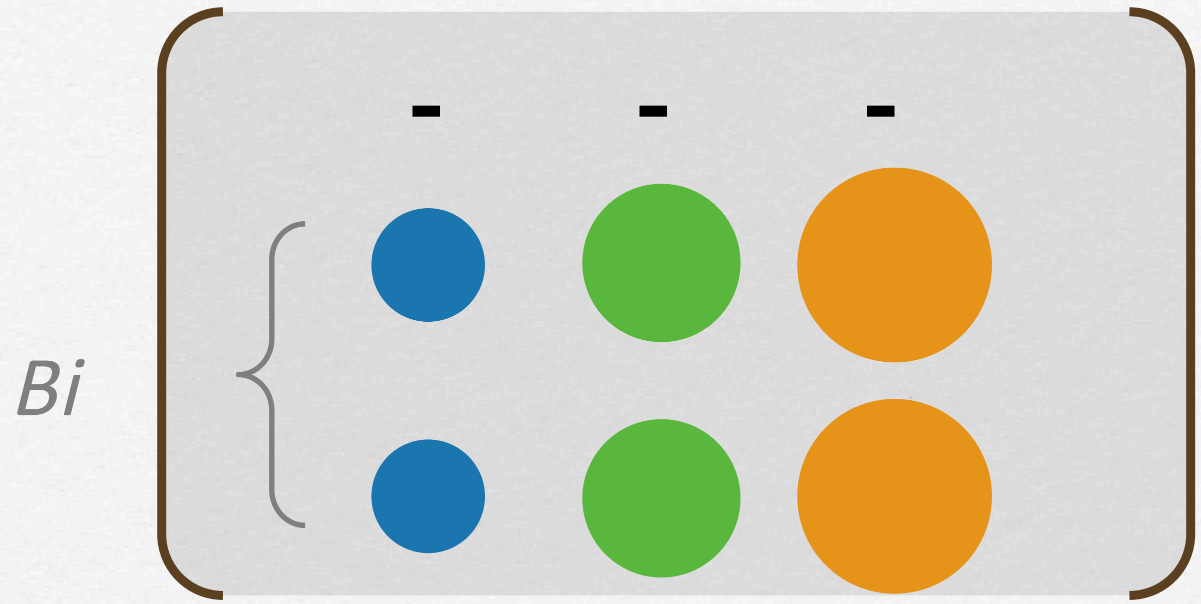


Basic Idea:

→ Two rows have
→ equal magnitudes

Z.z.Xing and S.Zhou, 0804.3512

$$\rightarrow \quad \theta_{13} \neq 0\,, \quad \theta_{23} = 45^\circ, \quad \delta_{\mathrm{CP}} = \pm 90^\circ$$

$$V_0 = \begin{pmatrix} |V_{e1}| & |V_{e2}| & |V_{e3}| \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\mu 1}^* & V_{\mu 2}^* & V_{\mu 3}^* \end{pmatrix}$$

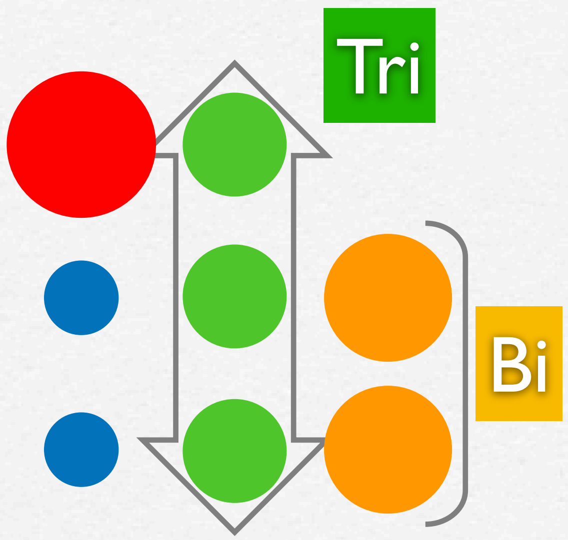
Generalisation of:

Mu-tau reflection symmetry

P.F.Harrison and W.G.Scott, hep-ph/0210197

P.F.Harrison, D.H.Perkins and W.G.Scott, hep-ph/0202074

Tri-Bimaximal Mixing



$$\begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sin\theta_{12} = \frac{1}{\sqrt{3}}$$

Allowed at 3 sigma

$$\sin\theta_{23} = \frac{1}{\sqrt{2}}$$

Allowed at 3 sigma

$$\sin\theta_{13} = 0$$

Excluded at many sigma

P.F.Harrison, D.H.Perkins and W.G.Scott, hep-ph/0202074

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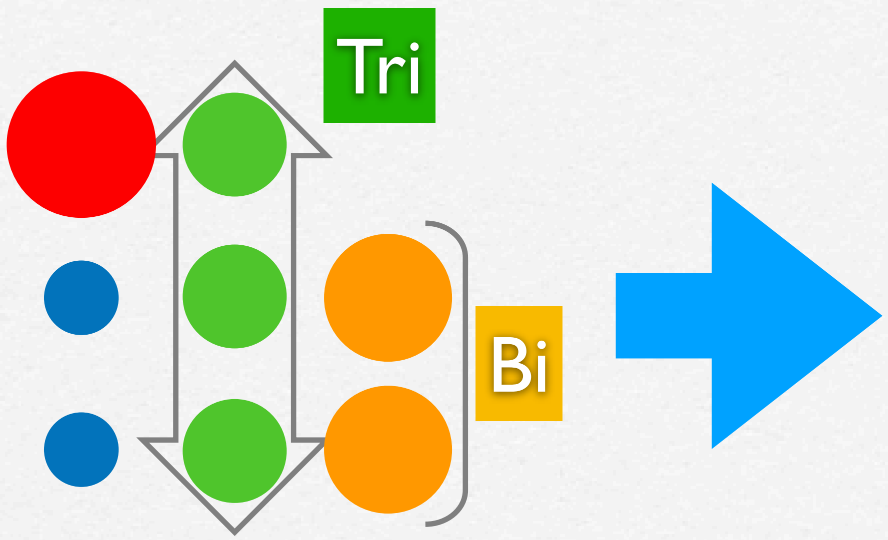

$$\begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

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Allowed at 3 sigma

$$\sin\theta_{23} = \frac{1}{\sqrt{2}}$$

Allowed at 3 sigma

$$\sin\theta_{13} = 0$$

Excluded at many sigma

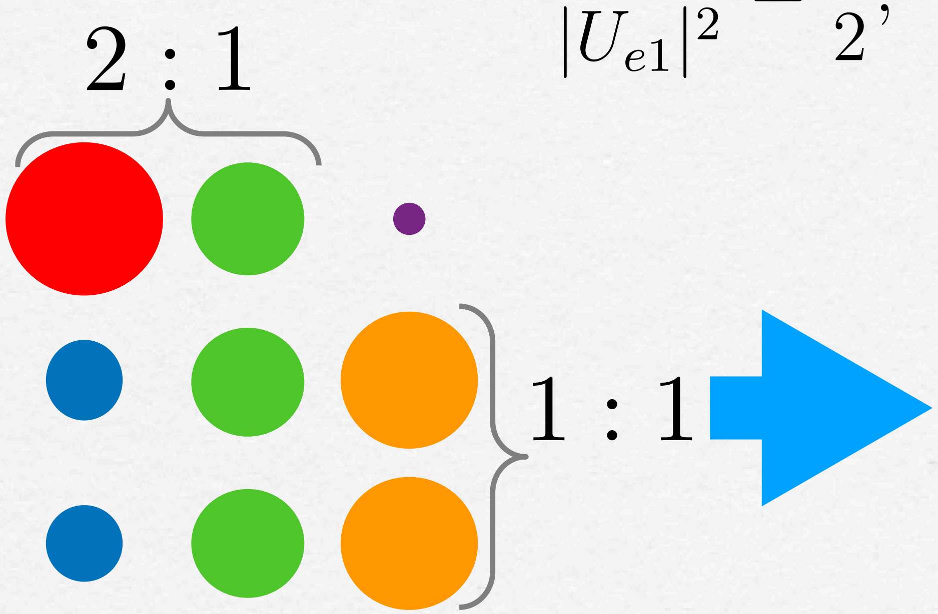
Best Fit Preference:

$$s_{12}^2 < \frac{1}{3}$$

SFK 0903.3199, 1205.0506

Tri-Bimaximal-Reactor

$$\frac{|U_{e2}|^2}{|U_{e1}|^2} = \frac{1}{2}, \quad \frac{|U_{\mu 3}|^2}{|U_{\tau 3}|^2} = 1.$$



$$\begin{pmatrix} \sqrt{\frac{2}{3}}(1-\frac{1}{4}\lambda^2) & \frac{1}{\sqrt{3}}(1-\frac{1}{4}\lambda^2) & \frac{1}{\sqrt{2}}\lambda e^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+\lambda e^{i\delta}) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}\lambda e^{i\delta}) & \frac{1}{\sqrt{2}}(1-\frac{1}{4}\lambda^2) \\ \frac{1}{\sqrt{6}}(1-\lambda e^{i\delta}) & -\frac{1}{\sqrt{3}}(1+\frac{1}{2}\lambda e^{i\delta}) & \frac{1}{\sqrt{2}}(1-\frac{1}{4}\lambda^2) \end{pmatrix}$$

$$\sin\theta_{12} = \frac{1}{\sqrt{3}}$$

Allowed at 3 sigma

$$\sin\theta_{23} = \frac{1}{\sqrt{2}}$$

Allowed at 3 sigma

$$\sin\theta_{13} = \frac{\lambda}{\sqrt{2}}$$

Allowed ✓

Charged lepton corrections

Charged lepton rotation — Tri-bimaximal neutrinos

$$U_{\rm PMNS} = \begin{pmatrix} c^e_{12} & s^e_{12}e^{-i\delta^e_{12}} & 0 \\ -s^e_{12}e^{i\delta^e_{12}} & c^e_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$= \begin{pmatrix} \cdots & \cdots & \frac{s^e_{12}}{\sqrt{2}}e^{-i\delta^e_{12}} \\ \cdots & \cdots & \frac{c^e_{12}}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\rightarrow s_{13} = \frac{s^e_{12}}{\sqrt{2}}$$

Suggests $\theta^e_{12} \approx \theta_C$

$$\rightarrow c_{23}c_{13} = \frac{1}{\sqrt{2}} \rightarrow s^2_{23} < \frac{1}{2}$$

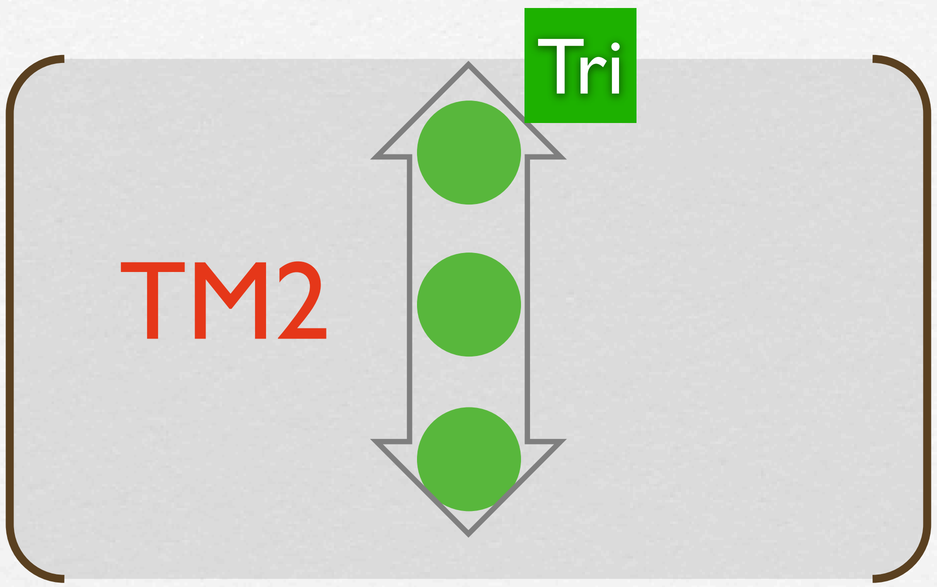
Prediction for CP phase

$$\frac{|U_{\tau 1}|}{|U_{\tau 2}|} = \frac{|s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta}|}{|-c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta}|} = \frac{1}{\sqrt{2}} \rightarrow \cos\delta = \frac{t_{23}s^2_{12} + s^2_{13}c^2_{12}/t_{23} - \frac{1}{3}(t_{23} + s^2_{13}/t_{23})}{\sin 2\theta_{12}s_{13}}$$

This derivation: P.Ballett, S.F.King, C.Luhn, S.Pascoli and M.A.Schmidt, 1410.7573

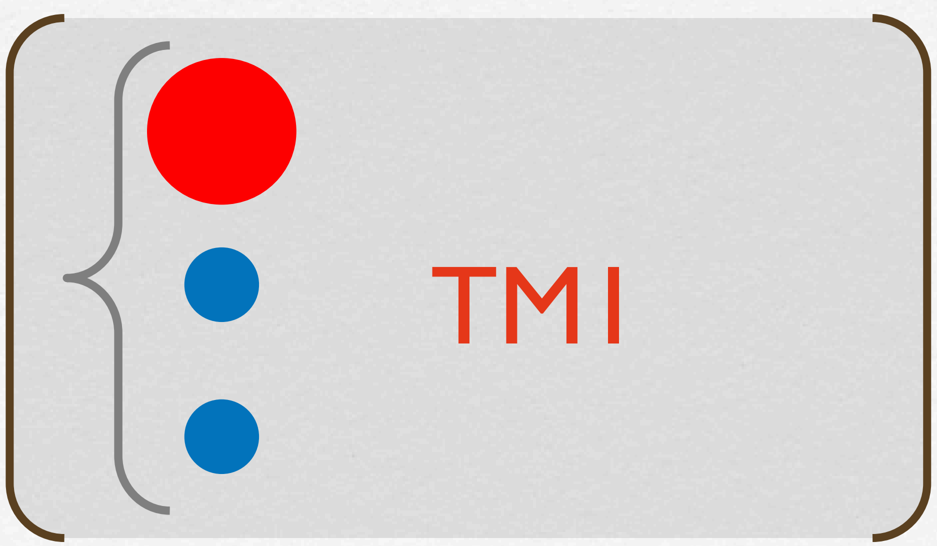
Tri-maximal Mixing

C.H.Albright and W.Rodejohann, 0812.0436; C.H.Albright, A.Dueck and W.Rodejohann, 1004.2798



Second column of TBM

$$U_{\mathrm{TM2}} \approx \begin{pmatrix} - & \frac{1}{\sqrt{3}} & - \\ - & \frac{1}{\sqrt{3}} & - \\ - & -\frac{1}{\sqrt{3}} & - \end{pmatrix}$$



First column of TBM

$$U_{\mathrm{TM1}} \approx \begin{pmatrix} \sqrt{\frac{2}{3}} & - & - \\ -\frac{1}{\sqrt{6}} & - & - \\ \frac{1}{\sqrt{6}} & - & - \end{pmatrix}$$

Tri-maximal Mixing

C.H.Albright and W.Rodejohann, 0812.0436; C.H.Albright, A.Dueck and W.Rodejohann, 1004.2798

Disfavoured

$$U_{\mathrm{TM2}} \approx \begin{pmatrix} - & \frac{1}{\sqrt{3}} & - \\ - & \frac{1}{\sqrt{3}} & - \\ - & -\frac{1}{\sqrt{3}} & \end{pmatrix}$$

$$|U_{e2}| = s_{12}c_{13} = \sqrt{\frac{1}{3}} \rightarrow s_{12}^2 > \frac{1}{3}$$

$$|U_{\mu 2}| = |c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta}| = \sqrt{\frac{1}{3}}$$

$$|U_{\tau 2}| = |-c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta}| = \sqrt{\frac{1}{3}}$$

$$\rightarrow \cos\delta = \frac{2c_{13}\cot 2\theta_{23}\cot 2\theta_{13}}{\sqrt{2-3s_{13}^2}}$$

Tri-maximal Mixing

C.H.Albright and W.Rodejohann, 0812.0436; C.H.Albright, A.Dueck and W.Rodejohann, 1004.2798

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Favoured

$$U_{\rm TM1} \approx \begin{pmatrix} \sqrt{\frac{2}{3}} & - & - \\ -\frac{1}{\sqrt{6}} & - & - \\ \frac{1}{\sqrt{6}} & - & - \end{pmatrix}$$

$$\rightarrow |U_{e1}| = c_{12}c_{13} = \sqrt{\frac{2}{3}} \rightarrow s_{12}^2 < \frac{1}{3}$$

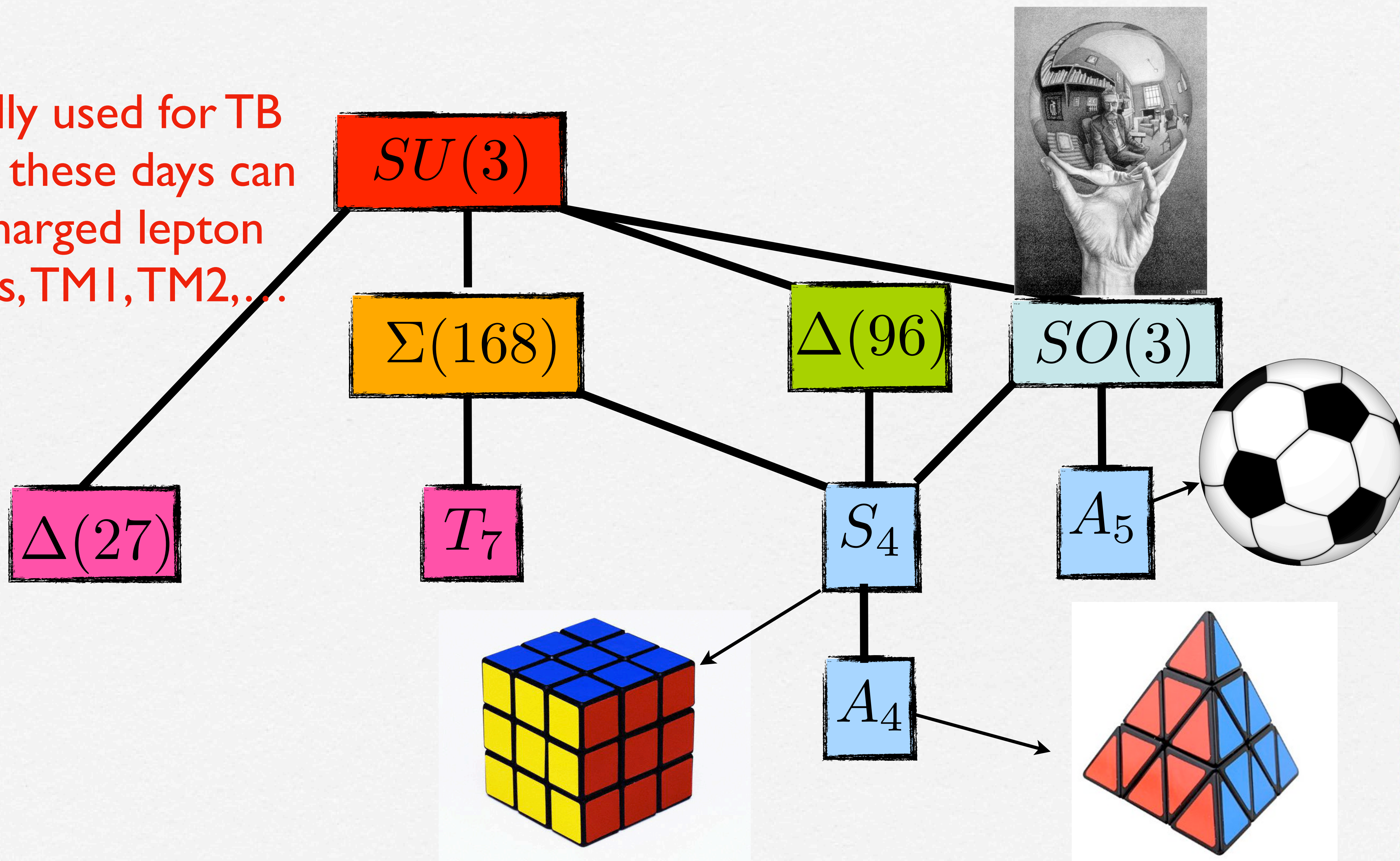
$$\rightarrow |U_{\mu 1}| = |-s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta}| = \sqrt{\frac{1}{6}}$$

$$\rightarrow |U_{\tau 1}| = |s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta}| = \sqrt{\frac{1}{6}}$$

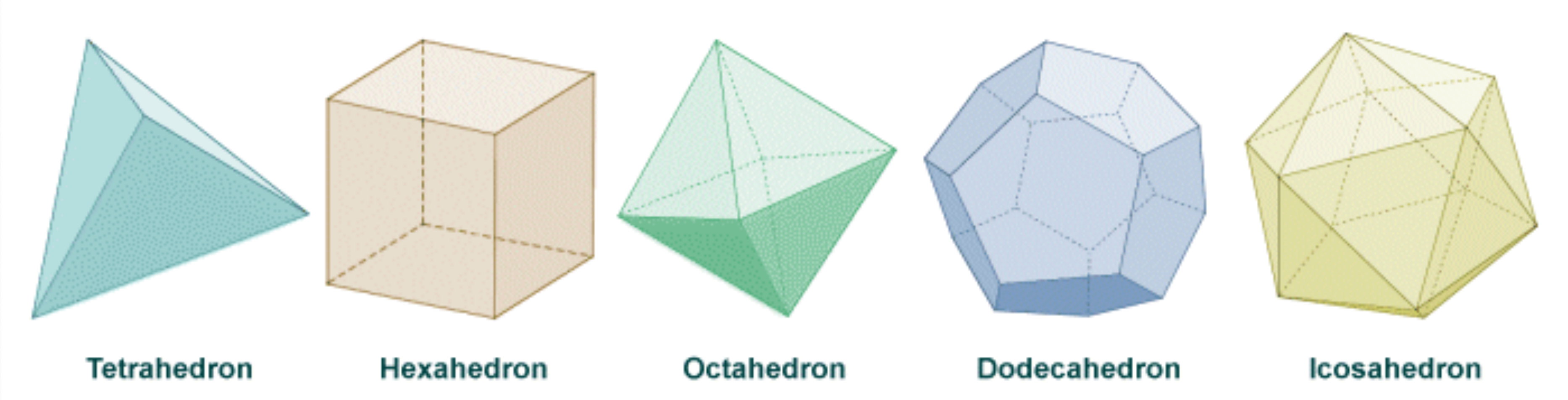
$$\rightarrow \cos\delta = -\frac{\cot 2\theta_{23}(1-5s_{13}^2)}{2\sqrt{2}s_{13}\sqrt{1-3s_{13}^2}}$$

Family Symmetry

Traditionally used for TB mixing, but these days can explain charged lepton corrections, TM1, TM2,….



Platonic Solids



solid	faces	vertices	Plato	Group
tetrahedron	4	4	fire	A_4
octahedron	8	6	air	S_4
icosahedron	20	12	water	A_5
hexahedron	6	8	earth	S_4
dodecahedron	12	20	?	A_5

Plato's fire
A4 can explain
Tri-bimaximal
Mixing

E.Ma and G.Rajasekaran, hep-ph/0106291;
K.S.Babu, E.Ma, J.W.F.Valle, hep-ph/0206292;
G.Altarelli and F.Feruglio, hep-ph/0504165,hep-ph/0512103

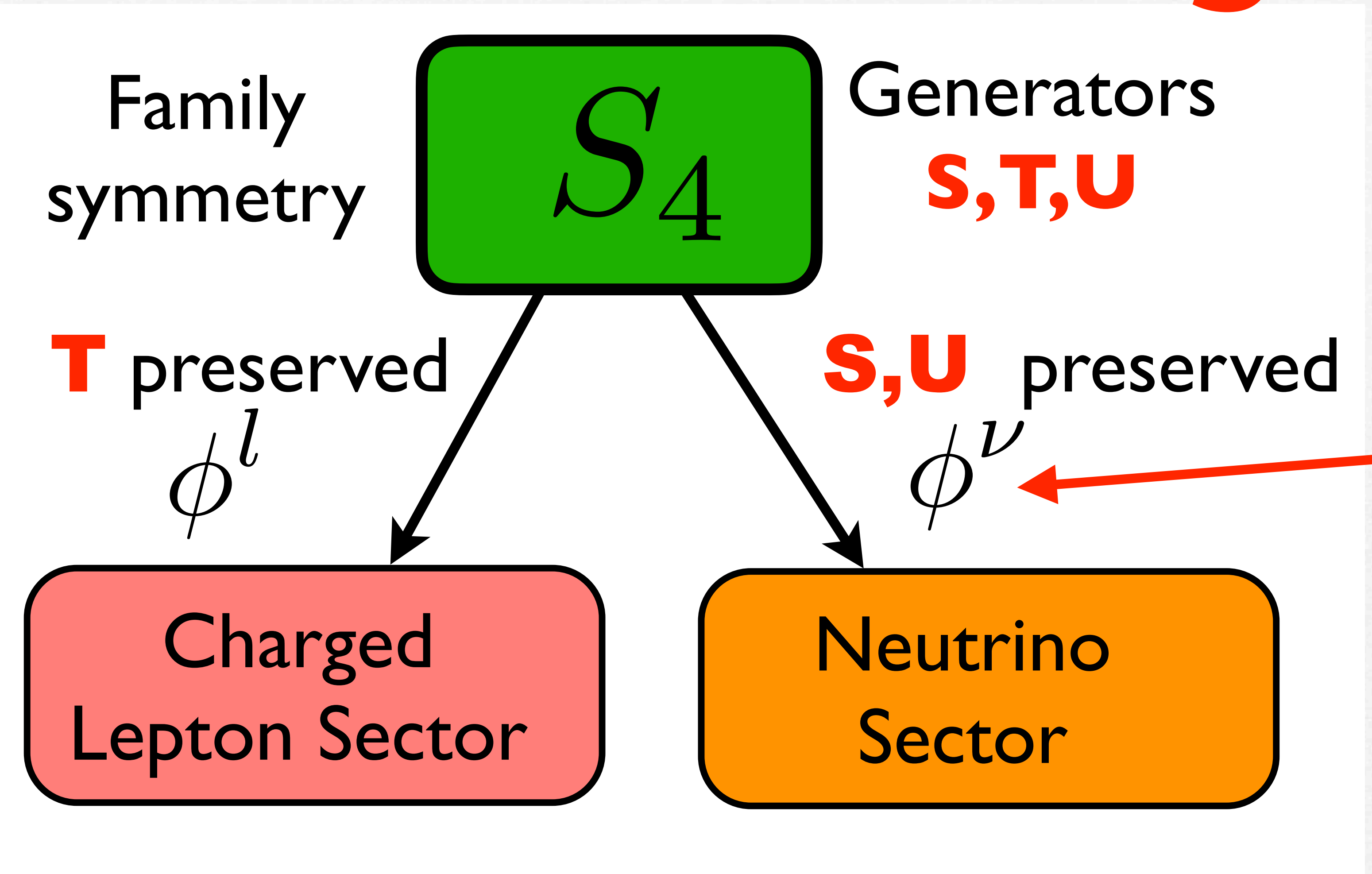
S.F.K., C.Luhn, 1301.1340

A_4 and S_4 Group Theory

S_4	A_4	S	T	U
$\mathbf{1}, \mathbf{1}'$	$\mathbf{1}$	1	1	± 1
$\mathbf{2}$	$\begin{pmatrix} \mathbf{1}'' \\ \mathbf{1}' \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\mathbf{3}, \mathbf{3}'$	$\mathbf{3}$	$\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$	$\mp \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

Diagonalised by TB matrix

Tri-bimaximal mixing from S_4

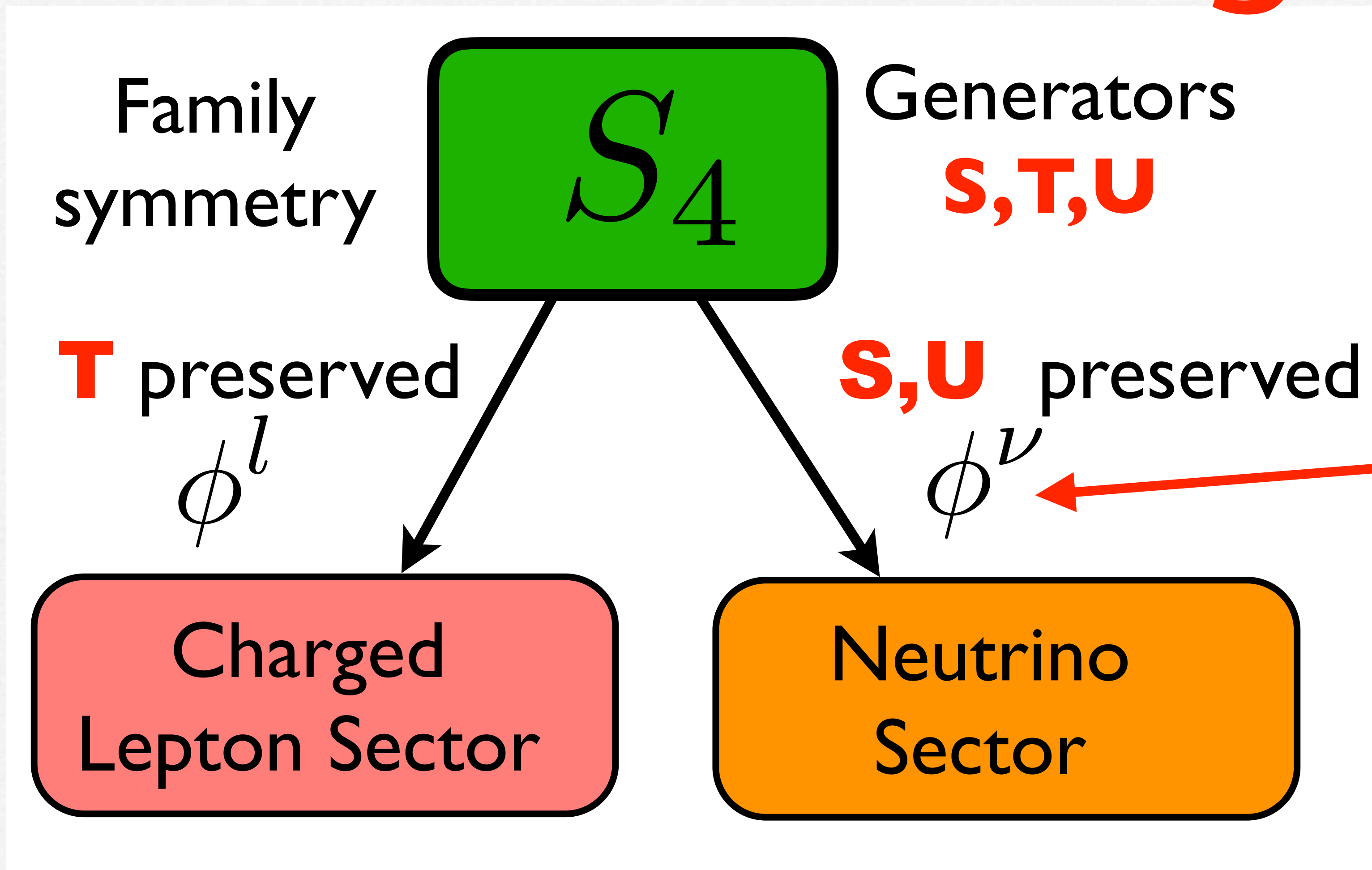


S.F.K., C.Luhn, 1301.1340

Flavons are new Higgs fields which break the flavour symmetry

Tri-bimaximal mixing from S_4

S.F.K., C.Luhn, 1301.1340



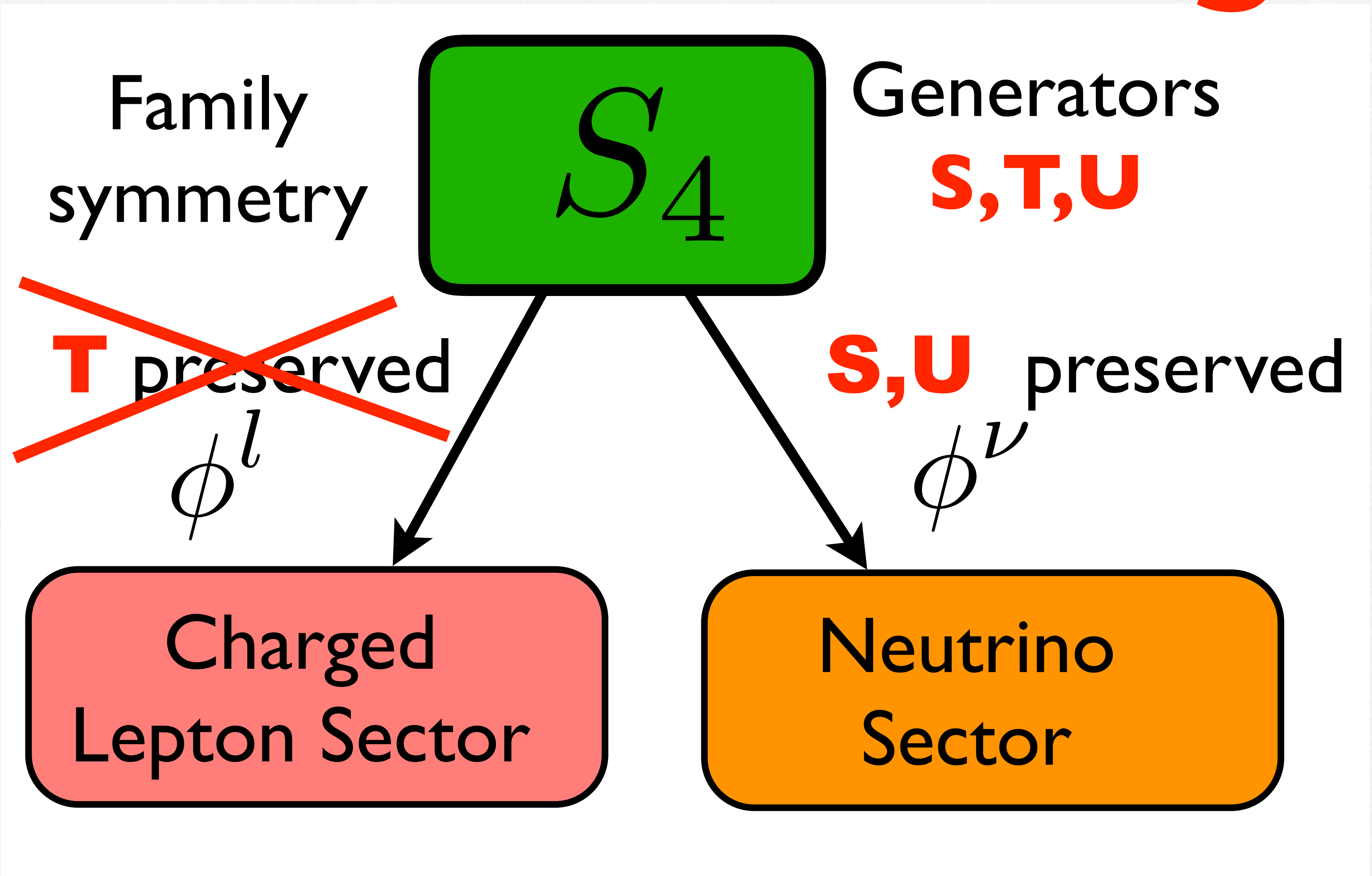
Flavons are new Higgs fields which break the flavour symmetry

$$\Rightarrow \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

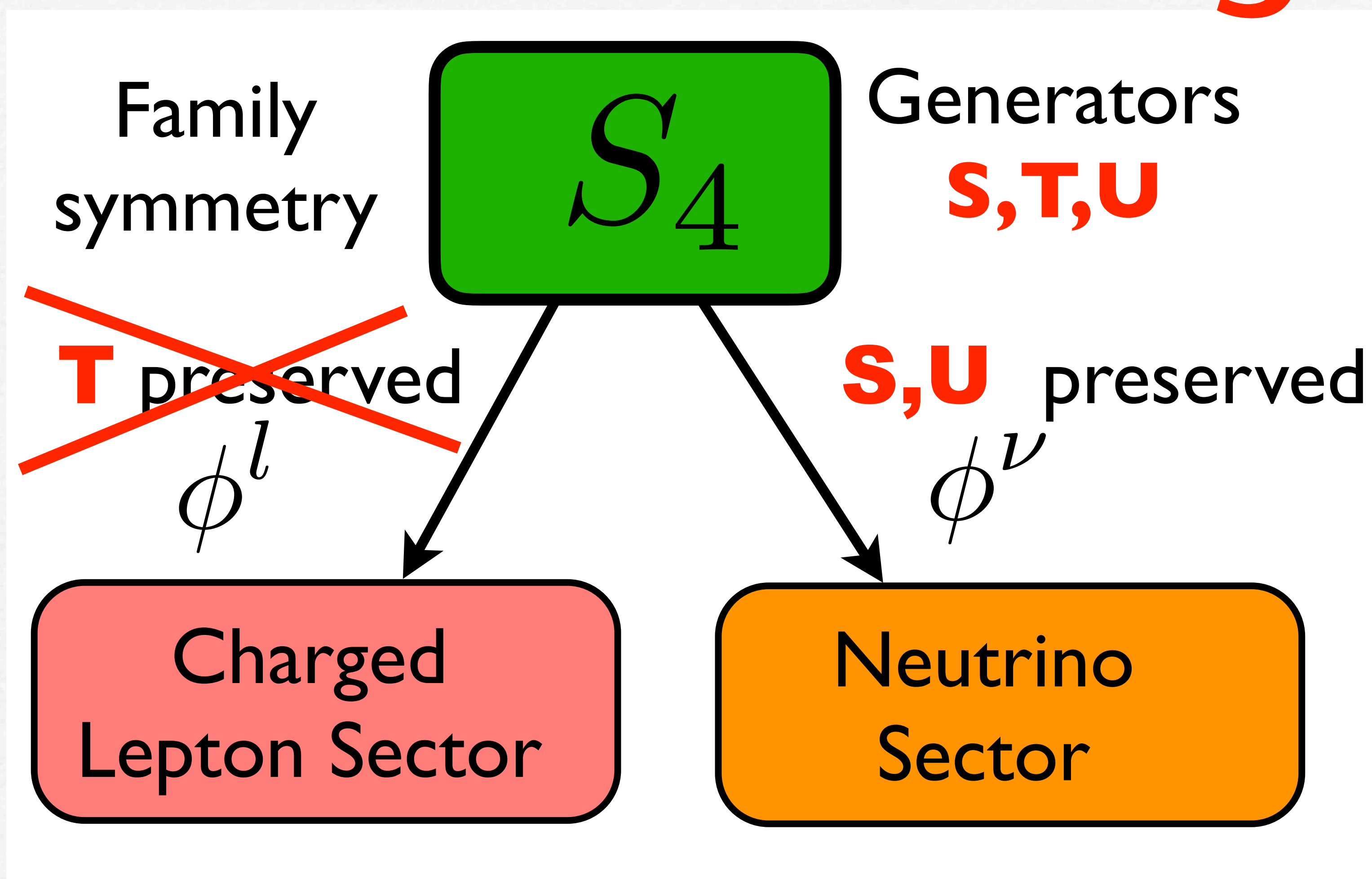
TB mixing excluded so need to break S,T,U

Tri-bimaximal mixing from S_4

break T



Tri-bimaximal mixing from S_4

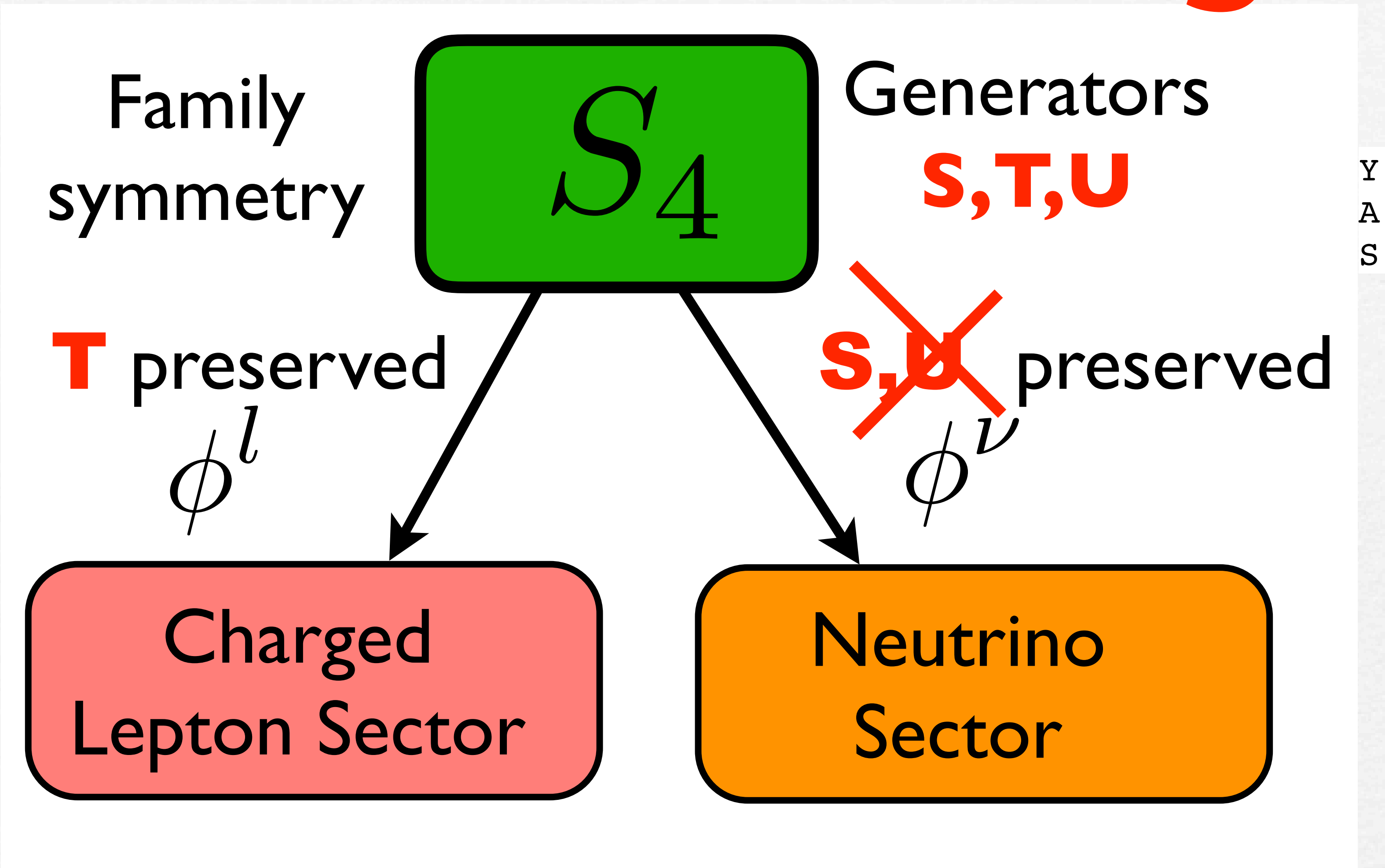


break T

$$U_{\rm PMNS} = \begin{pmatrix} c^e_{12} & s^e_{12}e^{-i\delta^e_{12}} & 0 \\ -s^e_{12}e^{i\delta^e_{12}} & c^e_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Charged lepton rotation

Tri-bimaximal mixing from S_4

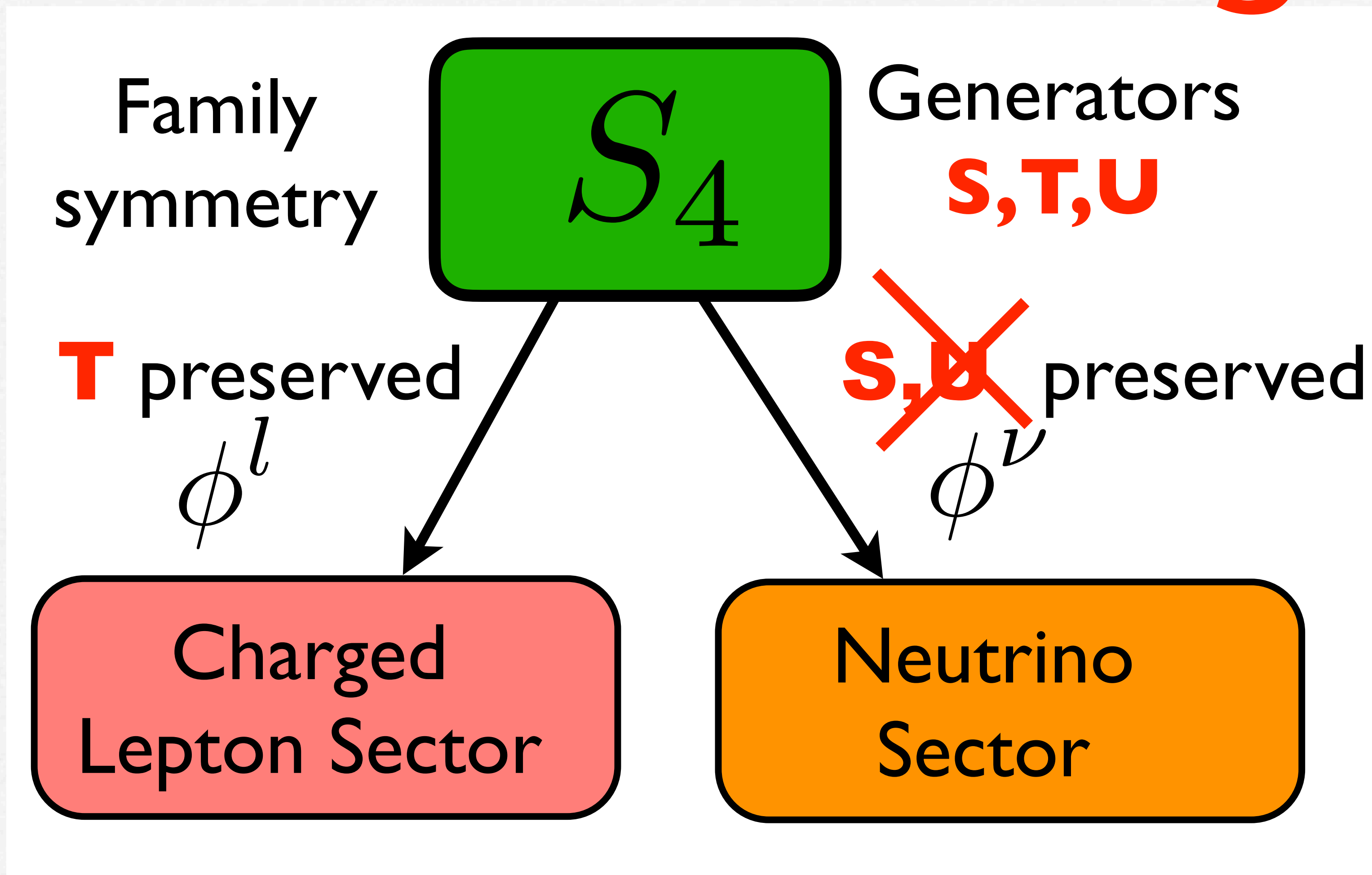


S.F.K., C.Luhn, 1301.1340

Y.Shimizu, M.Tanimoto, A.Watanabe, 1105.2929; S.F.K.,C.Luhn,1107.5332

break U

Tri-bimaximal mixing from S_4



S.F.K., C.Luhn, 1301.1340

Y.Shimizu, M.Tanimoto, A.Watanabe, 1105.2929; S.F.K.,C.Luhn,1107.5332

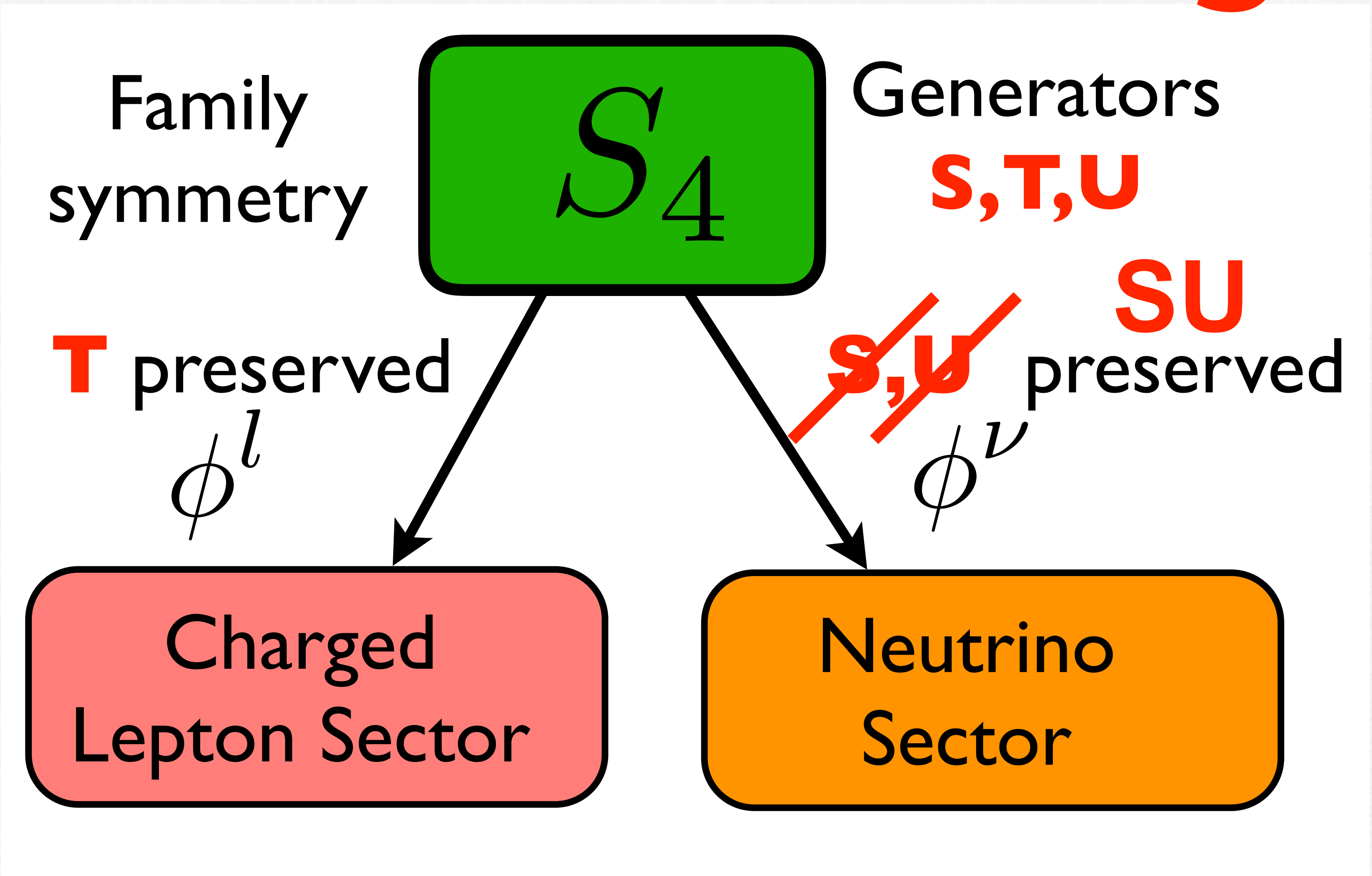
break U

$$U_{\rm TM2} \approx \begin{pmatrix} - & \frac{1}{\sqrt{3}} & - \\ - & \frac{1}{\sqrt{3}} & - \\ - & -\frac{1}{\sqrt{3}} & - \end{pmatrix}$$

TM2 as A4 with just S and T

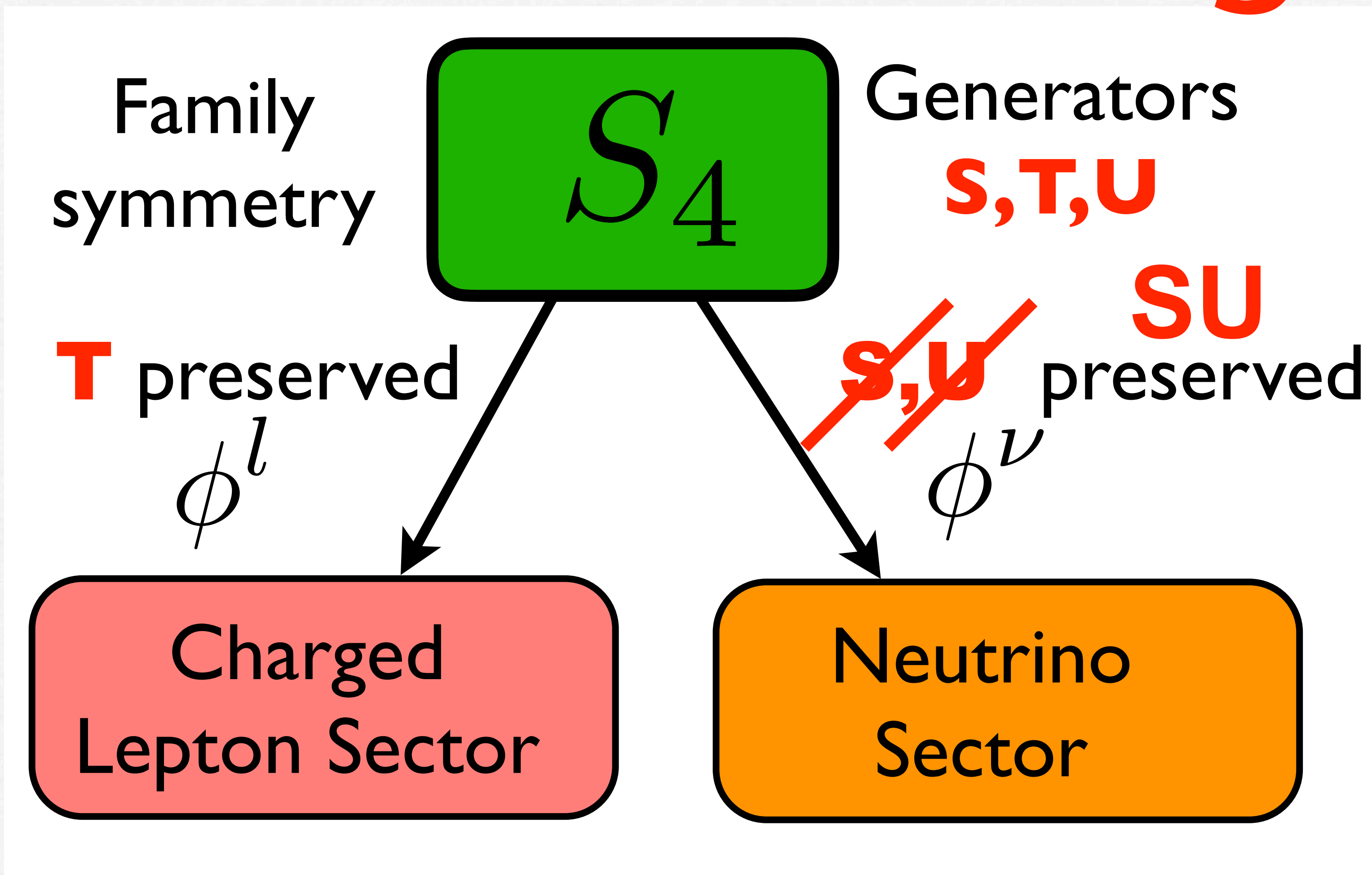
Tri-bimaximal mixing from S_4

S.F.K., C.Luhn, 1301.1340



break S,U separately preserve SU

Tri-bimaximal mixing from S_4



S.F.K., C.Luhn,
1301.1340

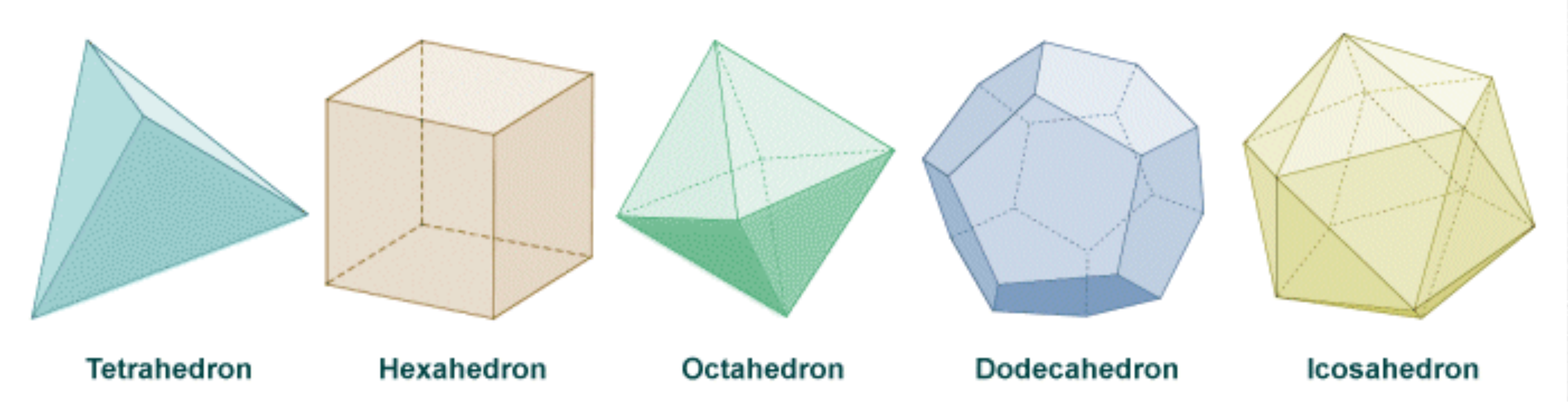
break S,U
separately
preserve SU

$$U_{\text{TM1}} \approx \begin{pmatrix} \sqrt{\frac{2}{3}} & - & - \\ -\frac{1}{\sqrt{6}} & - & - \\ \frac{1}{\sqrt{6}} & - & - \end{pmatrix}$$

TM1 with
SU and T

D.Hernandez and A.Y.Smirnov
1204.0445,1212.2149,1304.7738;
C.Luhn, 1306.2358
S.F.K.,C.Luhn,1607.05276

Origin of Plato's symmetry?



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icosahedron	20	12	water	A_5
hexahedron	6	8	earth	S_4
dodecahedron	12	20	?	A_5

Two possibilities:

1. Subgroup of gauge group SU(3), SO(3)
2. Extra-dimensional superstring theory (modular symmetry)

F.Feruglio and A.Romanino, 1912.06028

Modular invariant theory

Gui-Jun Ding

For $\mathcal{N}$=1 global SUSY, the modular invariant action

$$S=\int d^4x d^2\theta d^2\bar{\theta}\; K(\Phi_I,\bar{\Phi}_I,\tau,\bar{\tau})+\int d^4x d^2\theta\, W(\Phi_I,\tau)+\text{h.c.}$$

[Ferrara et al, 1989; Feruglio, 1706.08749]

- Kahler potential (**not fixed by symmetry**) [Chen,Sanchez,Ratz,1909.06910]

Minimal: $K=-h\Lambda^2\ln(-i\tau+i\bar{\tau})+\sum_I(-i\tau+i\bar{\tau})^{-k_I}|\Phi_I|^2$ → kinetic terms

- Modular invariant superpotential

$$W=\sum_n Y_{I_1 I_2\ldots I_n}(\tau)\Phi_{I_1}\Phi_{I_2}\ldots\Phi_{I_n}$$

$Y_{I_1 I_2\cdots I_n}(\tau)$ are modular forms

$$\tau\to\gamma\tau=\frac{a\tau+b}{c\tau+d},$$

$$\Phi_I\to(c\tau+d)^{-k_I}\rho_I(\gamma)\Phi_I$$

$$Y_{I_1 I_2\ldots I_n}(\tau)\to Y_{I_1 I_2\ldots I_n}(\gamma\tau)=(c\tau+d)^{k_Y}\rho_Y(\gamma)Y_{I_1 I_2\ldots I_n}(\tau)$$

Modular invariance requires

$$\begin{cases} k_Y=k_{I_1}+k_{I_2}+\ldots+k_{I_n} \\ \rho_Y\otimes\rho_{I_1}\otimes\cdots\otimes\rho_{I_n}\supset 1\end{cases}$$

Modular weights

Reps of finite modular group

Modular forms

Modular forms are **holomorphic** functions transforming under

$$Y(\gamma\tau)=(c\tau+d)^k Y(\tau), \quad \forall\, \gamma\in\overline{\Gamma}(N)$$

N: level, positive integer

k: modular weight, **even integer**

$$\overline{\Gamma}(N)=\left\{\gamma=\begin{pmatrix} a & b \\ c & d\end{pmatrix}\middle|\, \gamma\in\overline{\Gamma},\ \gamma=I(\text{mod N})\right\}$$

Modular forms of weight k and level N form a linear space, they can be decomposed into irreducible representations of finite modular group,

$$Y_i(\gamma\tau)=(c\tau+d)^k \rho_{ij}(\gamma)Y_j(\tau), \quad \gamma\in\overline{\Gamma}$$

[Feruglio, 1706.08749]

ρ is unitary representation of $\Gamma_N\equiv\overline{\Gamma}/\overline{\Gamma}(N)=\left\{S,T\,|\,S^2=(ST)^3=T^N=1\right\}$

- **Inhomogeneous** finite modular group Γ_N

$$S:\tau\to-\frac{1}{\tau}, \qquad T:\tau\to\tau+1$$

N	$d_{2k}(\Gamma(N))$	$\|\Gamma_N\|$	Γ_N
2	$k+1$	6	S_3
3	$2k+1$	12	A_4
4	$4k+1$	24	S_4
5	$10k+1$	60	A_5
6	$12k$	72	$S_3\times A_4$
7	$28k-2$	168	$\Sigma(168)$

[Kobayashi et al, arXiv:1803.10391]

[Feruglio, 1706.08749]

[Penedo, Petcov, arXiv:1806.11040]

[Novichkov,Penedo,Petcov,Titov,arXiv:1812.02158; Ding,King,Liu,arXiv:1903.12588]

[Ding,King, Li,Zhou,arXiv:2004.12662]

F.Feruglio, 1706.08749

A4 Example of Modular Forms

Yukawa couplings involving twisted states whose modular weights do not add up to zero are modular forms

Level 3 Weight 2 acts as A4 triplet:

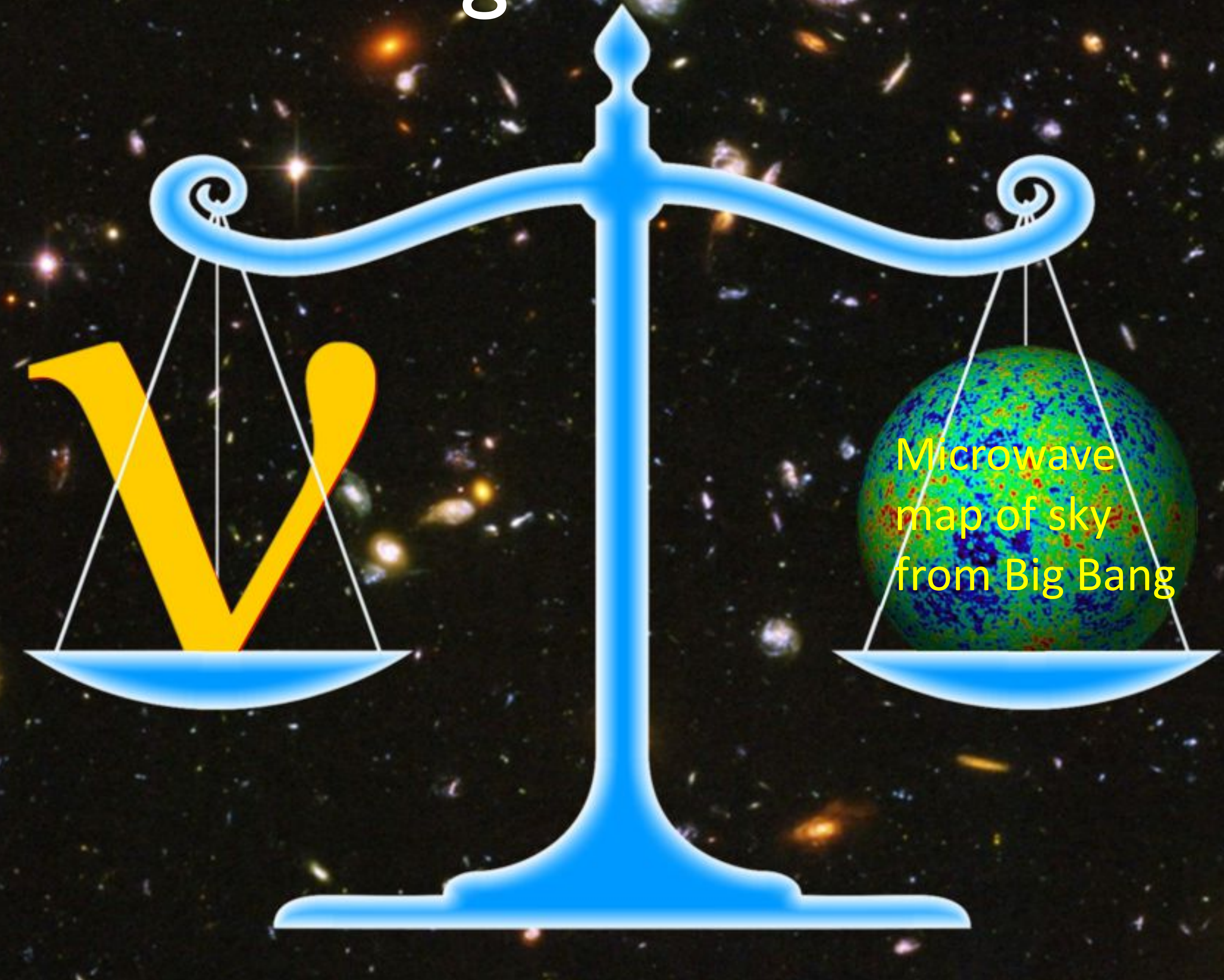
$$Y = \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix} = \begin{pmatrix} 1 + 12q + 36q^2 + 12q^3 + 84q^4 + 72q^5 + \dots \\ -6q^{1/3}(1 + 7q + 8q^2 + 18q^3 + 14q^4 + \dots) \\ -18q^{2/3}(1 + 2q + 5q^2 + 4q^3 + 8q^4 + \dots) \end{pmatrix}$$

$q \equiv e^{i2\pi\tau}$ ← **free modulus** $\tau = \dfrac{\omega_2}{\omega_1}$

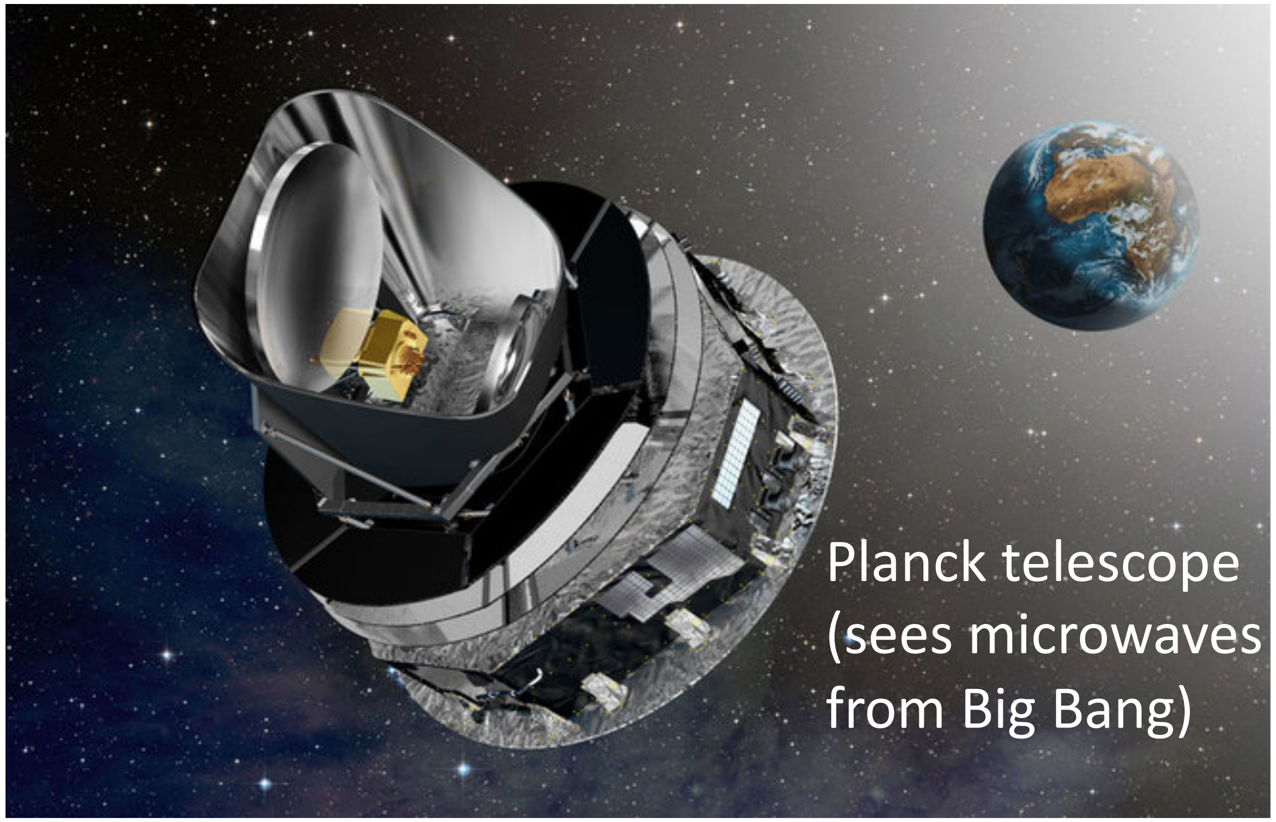
Weinberg operator $\dfrac{1}{\Lambda}(H_u H_u \; LL \; Y) \longrightarrow m_\nu = \begin{pmatrix} 2Y_1 & -Y_3 & -Y_2 \\ -Y_3 & 2Y_2 & -Y_1 \\ -Y_2 & -Y_1 & 2Y_3 \end{pmatrix} \dfrac{v_u^2}{\Lambda}$

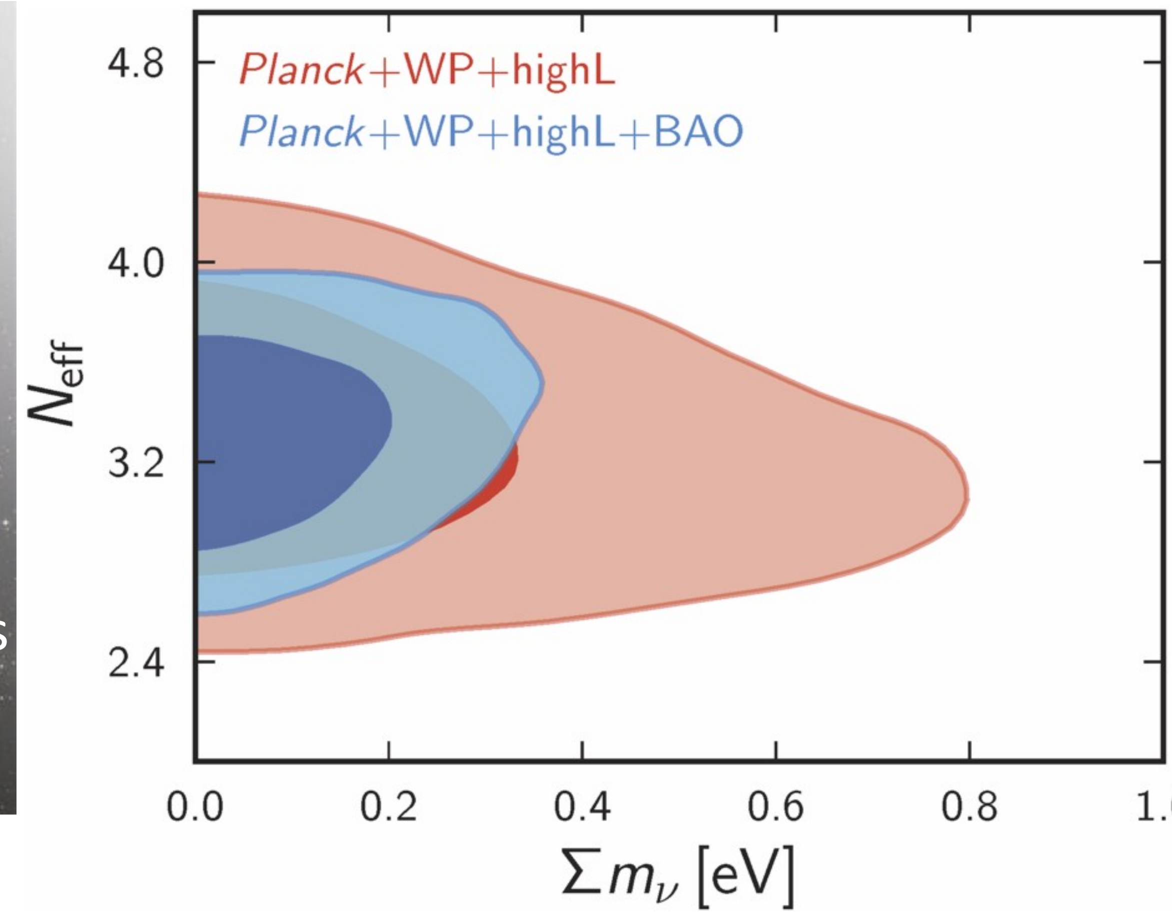
A_4: 3 3 3

Measuring Neutrino Mass



Neutrino Mass Limits from cosmology (2013)





CMB + BAO limit: $\Sigma m_\nu < 0.23$ eV (95% CL)
c.f. electron mass me=511,000 eV

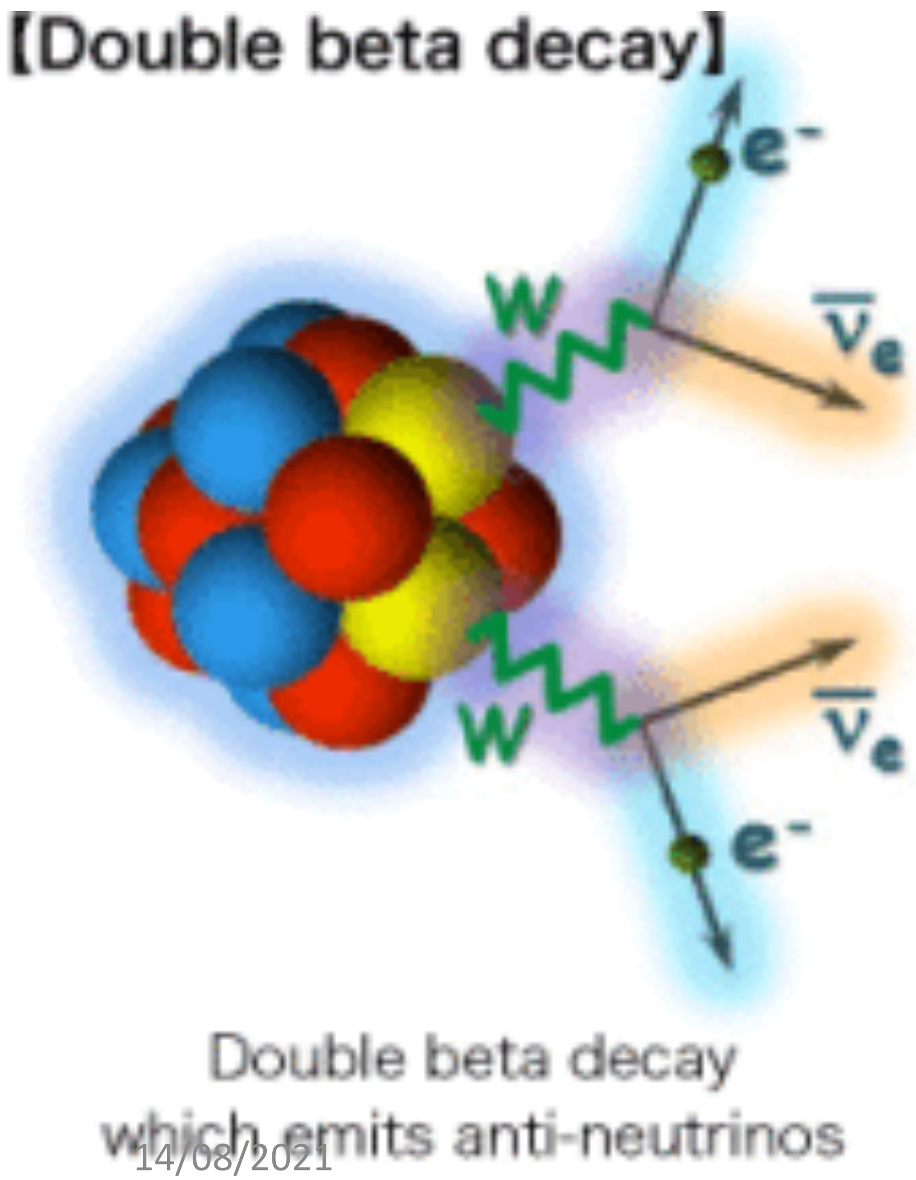
Ade et al. (Planck Collaboration), arXiv:1303.5076

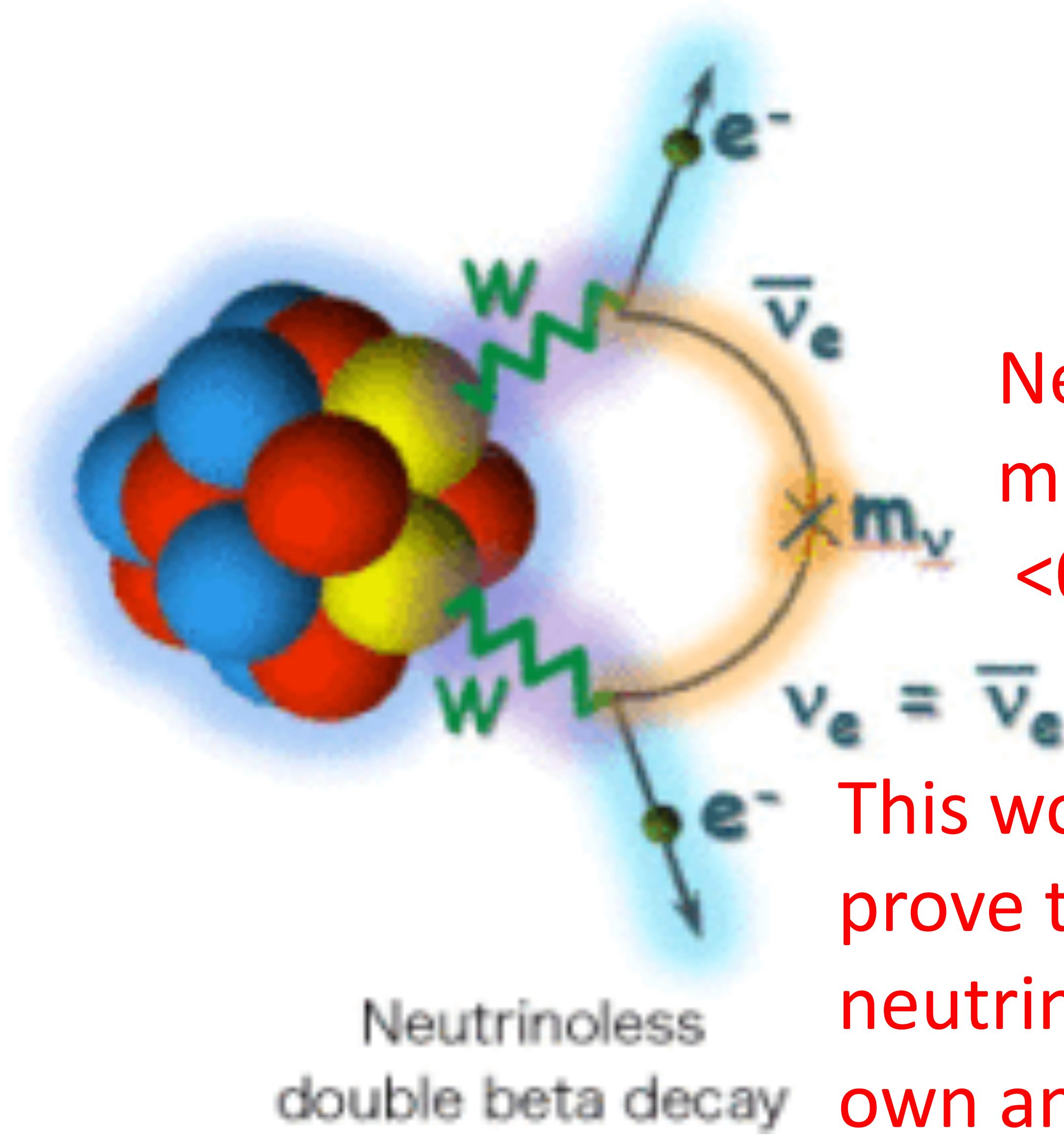
Neutrino Mass Limits from the Laboratory

Many currently running experiments: GERDA, Majorana, EXO, CUORE, Kamland-Zen

This decay (on the left) is commonly observed

The rarest form of beta decay, if observed, would give a precise mass measurement





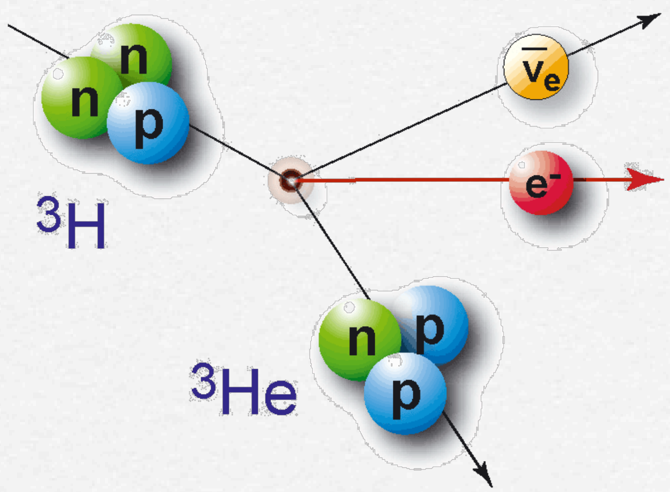
Neutrino mass <0.2 eV

This would also prove that the neutrino is its own antiparticle

Experimental determination of neutrino mass

Majorana only
(no signal if Dirac)

Tritium beta decay



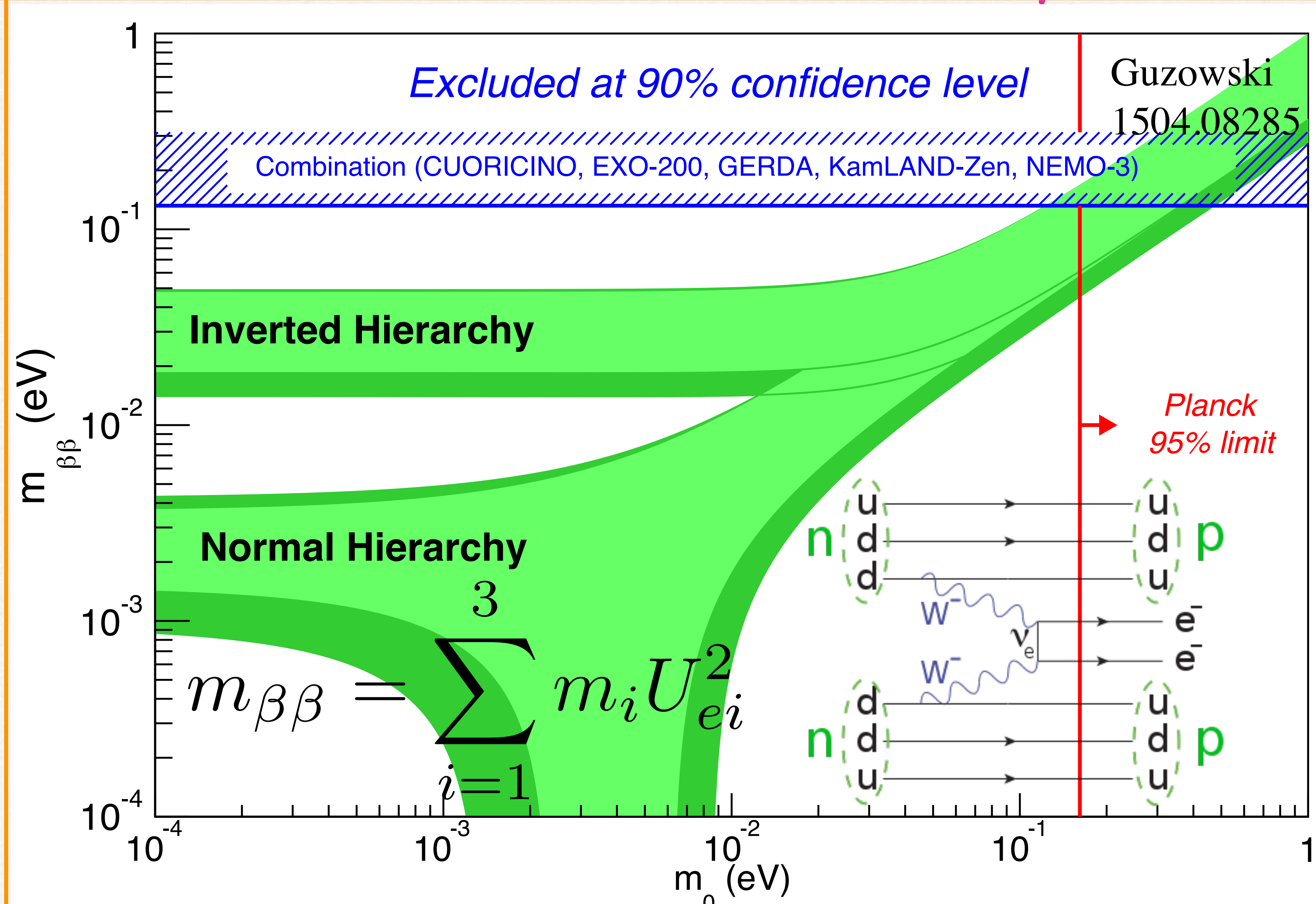
$$m_{\nu_e}^2 = \sum_{i=1}^{3} |U_{ei}|^2 m_i^2$$

Present Mainz < 2.2 eV

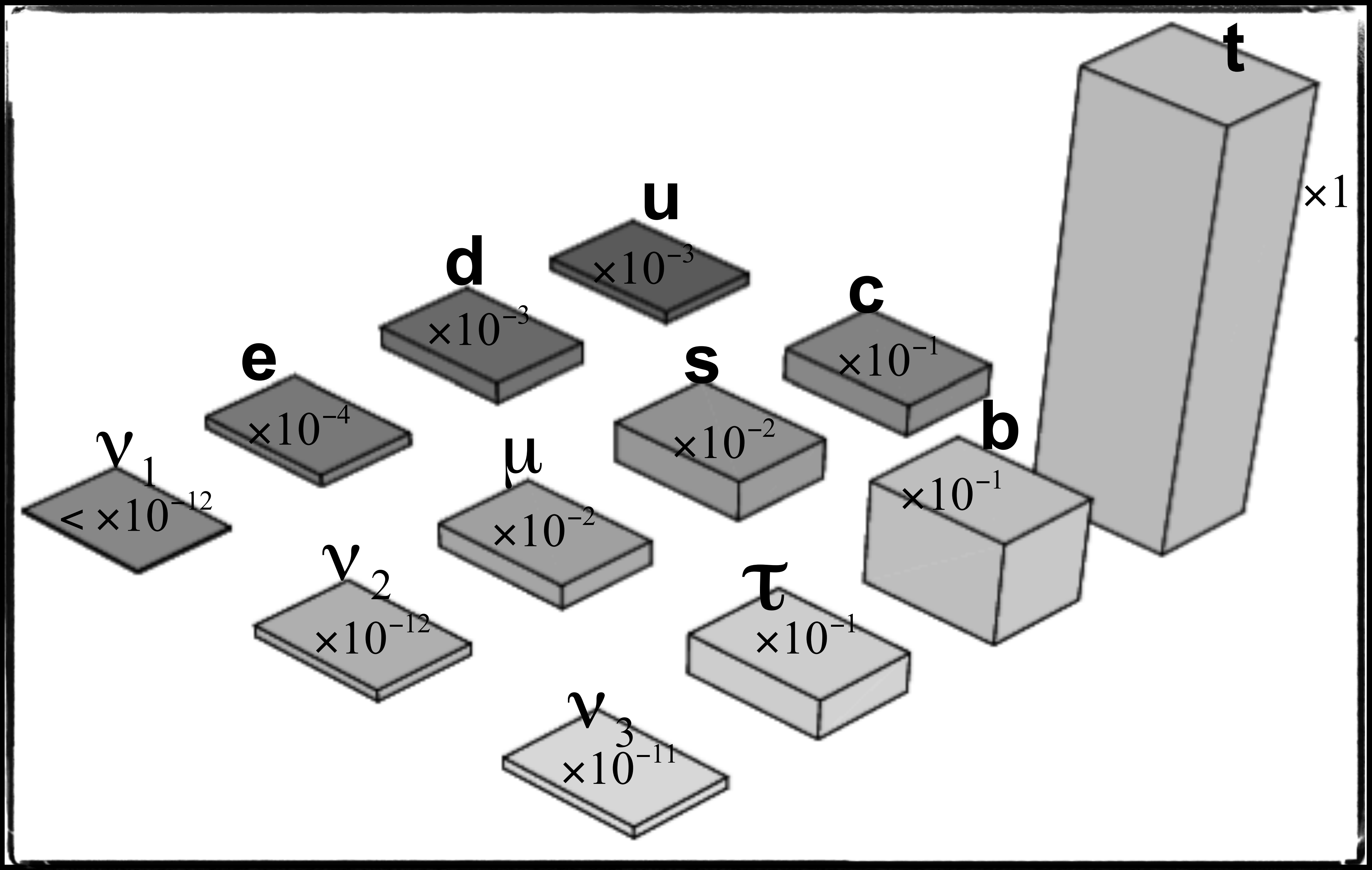
Present KATRIN < 1.1eV

future KATRIN~0.35eV

Neutrinoless double beta decay

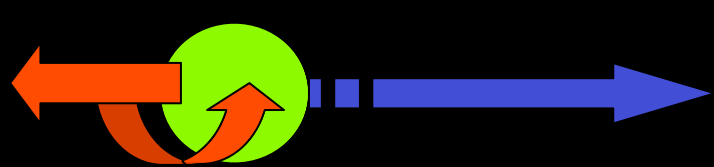


Why nu mass small?

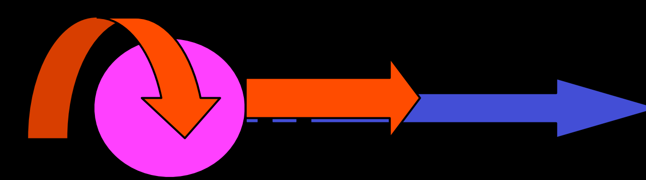


Dirac or Majorana?

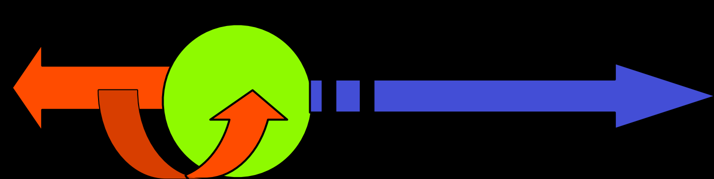
ν_L **Dirac** ν_R



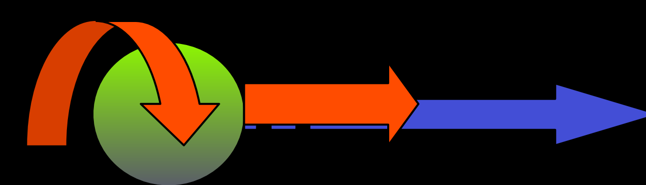
$m_{\mathrm{D}}\bar{\nu}_L\nu_R$



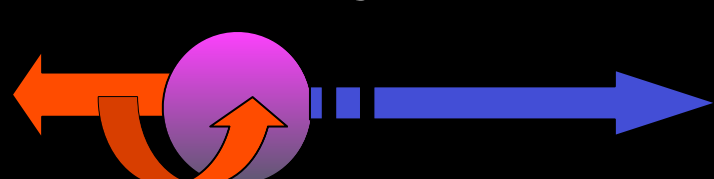
ν_L **Majorana** ν_L^c



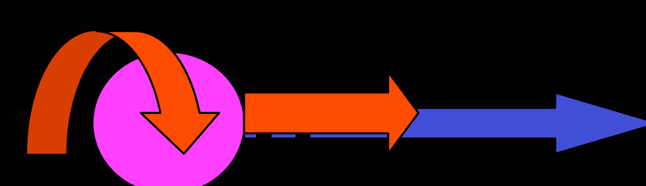
$m_{\mathrm{L}}\bar{\nu}_L\nu_L^c$



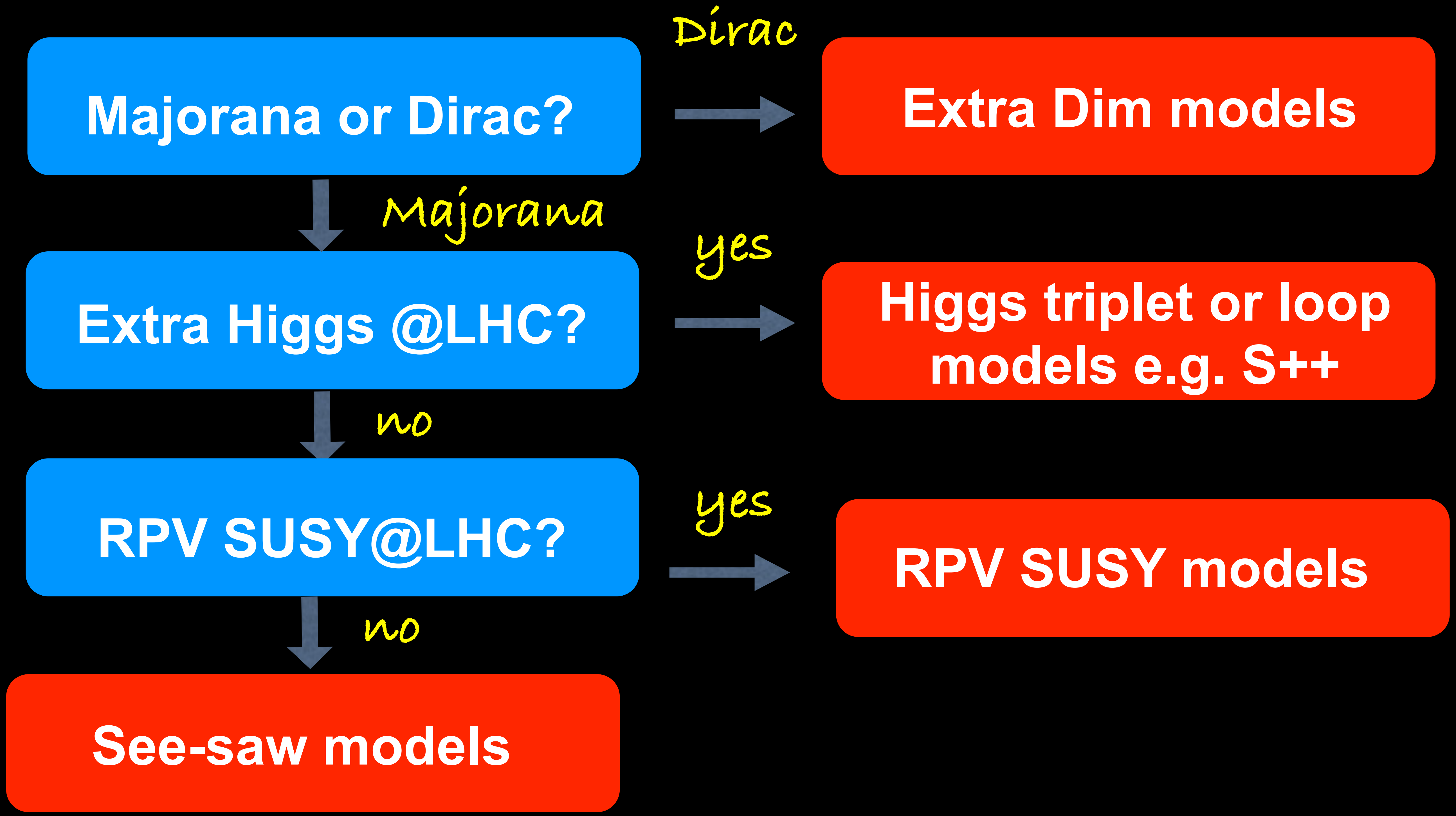
ν_R^c **Majorana** ν_R



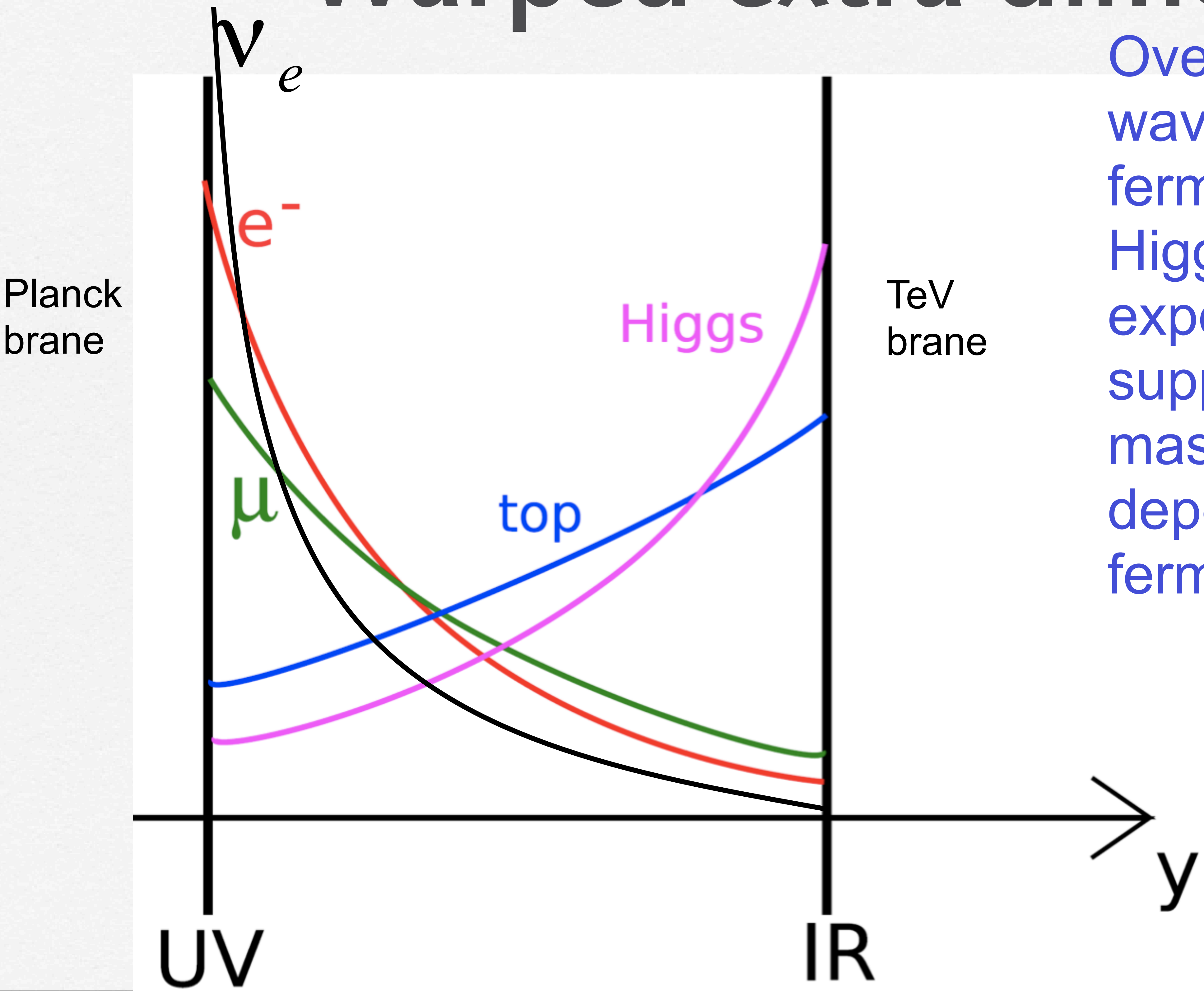
$M_{\mathrm{R}}\bar{\nu}_R^c\nu_R$



Roadmap of neutrino mass

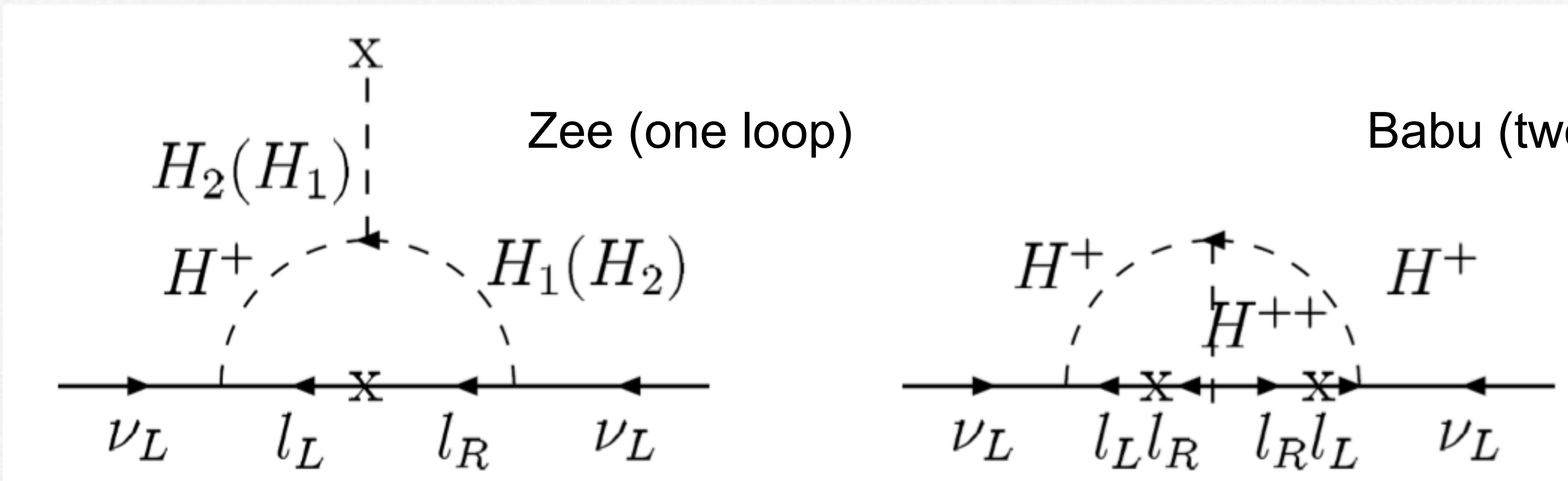


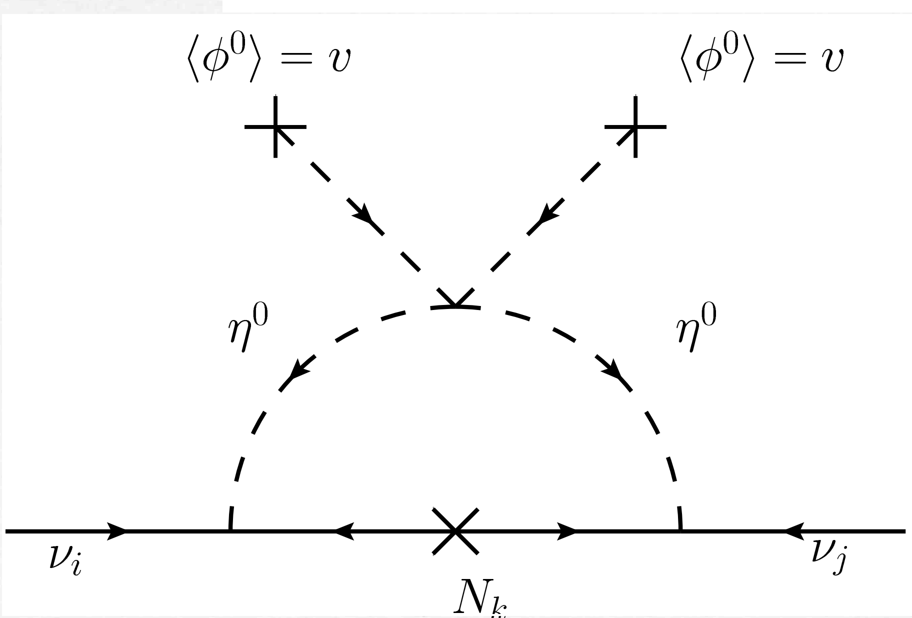
Warped extra dimensions

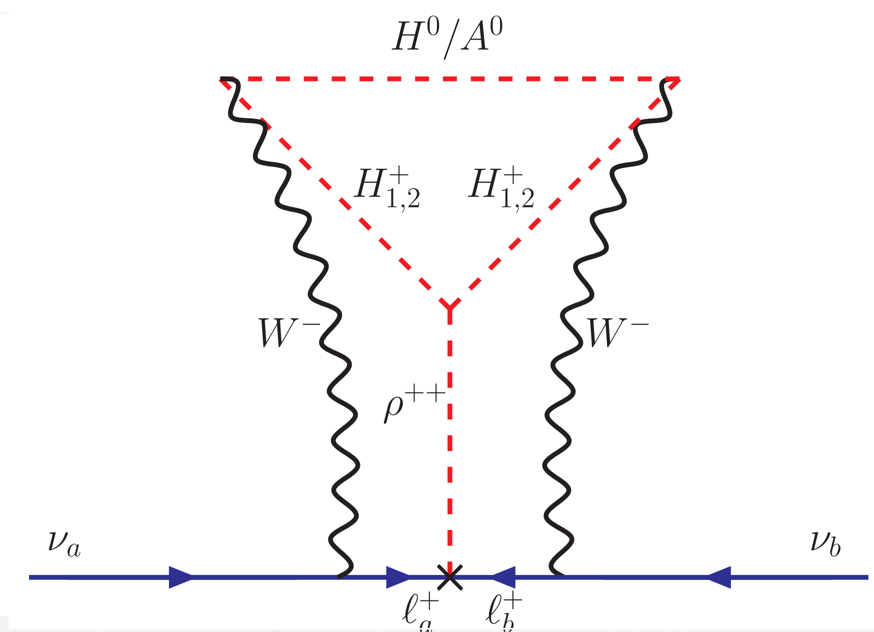


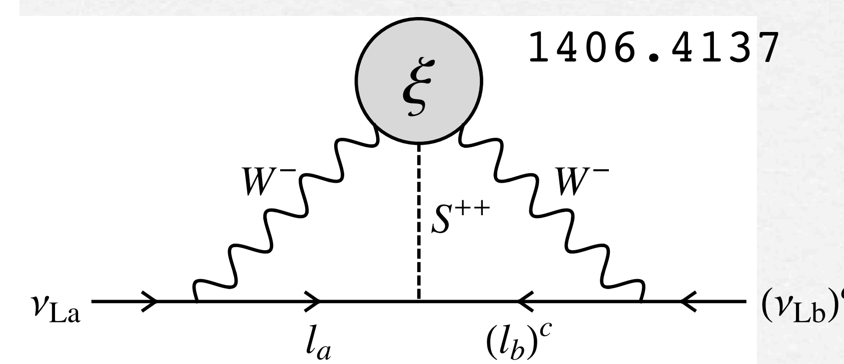
Overlap wavefunction of fermions with Higgs gives exponentially suppressed Dirac masses, depending on the fermion profile

Loop Models of Neutrino Mass









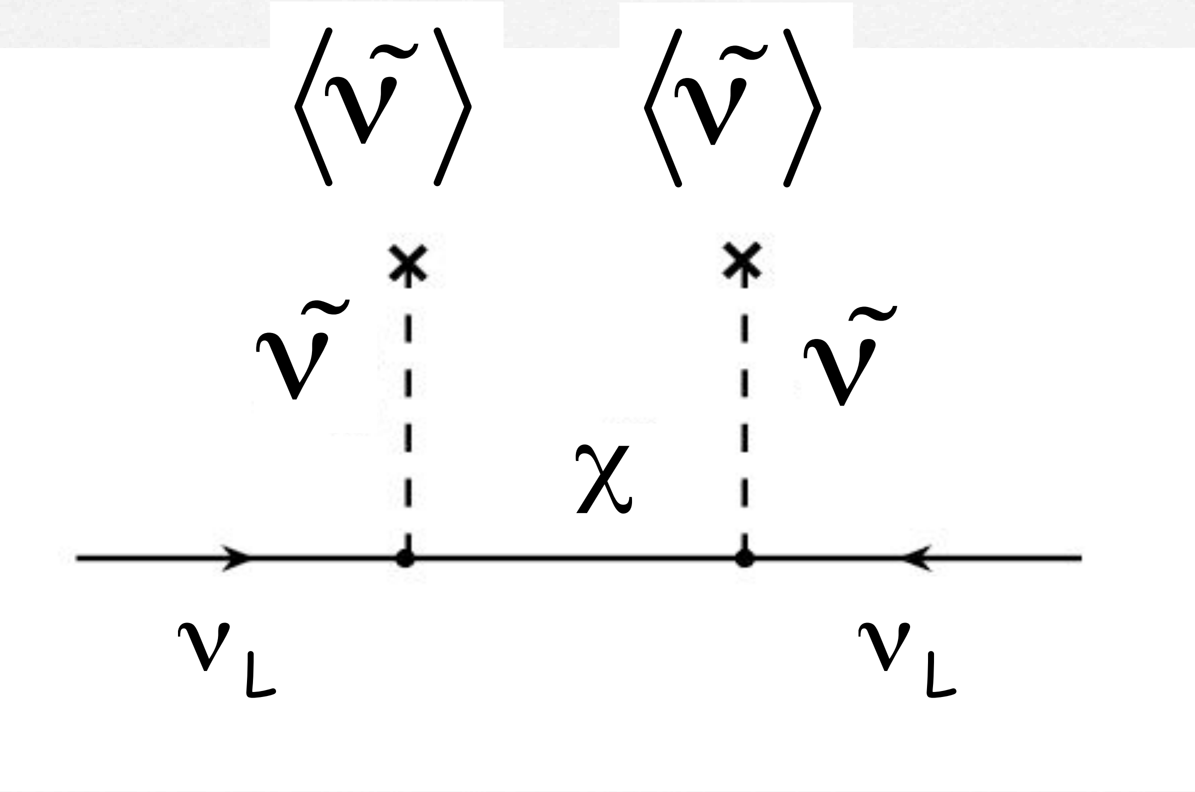
Scotogenic model

Cocktail model

Effective theory

R-Parity Violating SUSY

- Majorana masses can be generated via RPV SUSY
- Scalar partners of lepton doublets (slepton doublets) have same quantum numbers as Higgs doublets
- If R-parity is violated then sneutrinos may get (small) VEVs inducing a mixing between neutrinos and neutralinos χ



$$m^{\nu}_{LL} \approx \frac{\langle\tilde{\nu}\rangle^2}{M_{\chi}} \approx \frac{MeV^2}{TeV} \approx eV$$

Is Majorana mass renormalisable?

Renormalisable
$\Delta L = 2$ operator $\lambda_\nu LL\Delta$

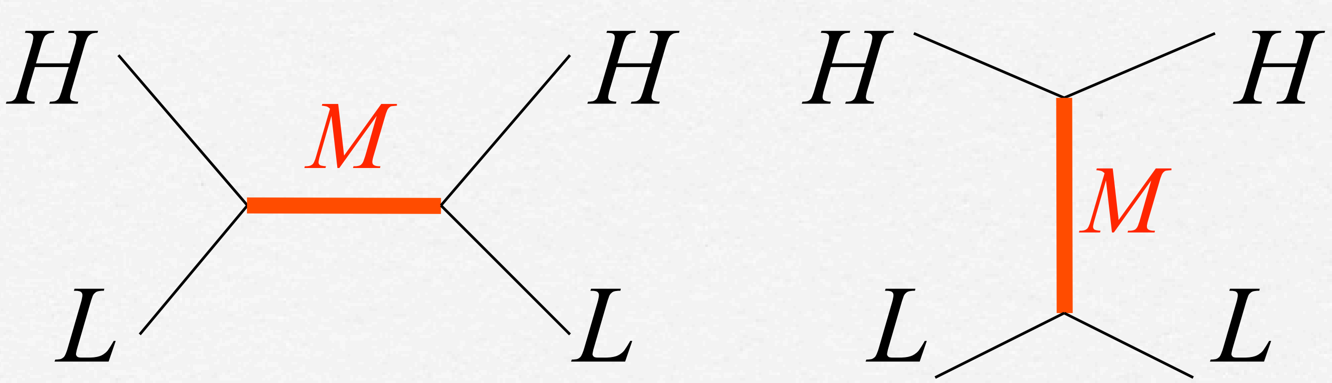
where Δ is light Higgs triplet with VEV < 8GeV from ρ parameter

Non-renormalisable
$\Delta L = 2$ operator

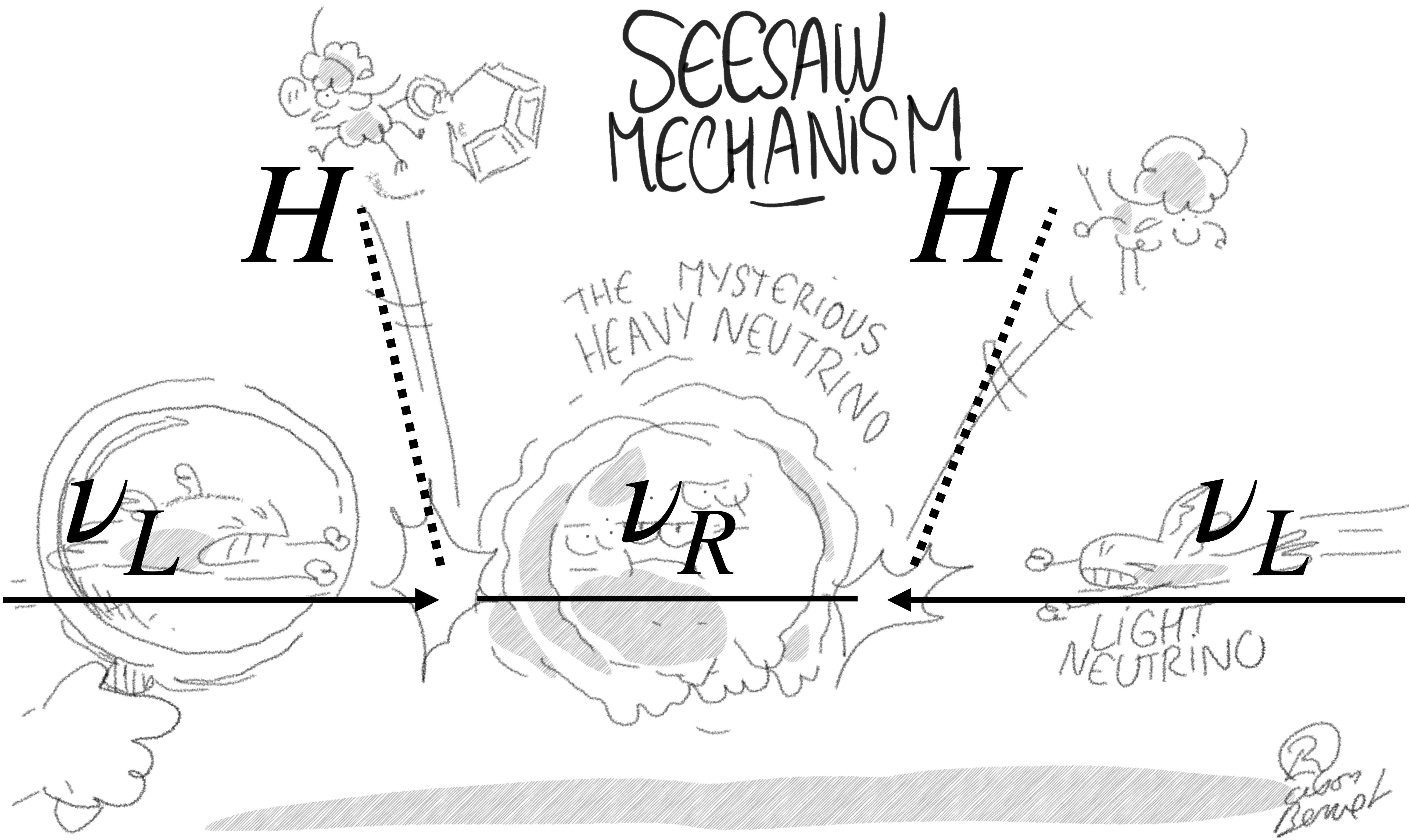
$$\frac{\lambda_\nu}{M} LLHH = \frac{\lambda_\nu}{M}\left\langle H^0 \right\rangle^2 \overline{\nu}_{eL} \nu^c_{eL}$$ Weinberg

This is nice because it gives naturally small Majorana neutrino masses $m_{LL} \sim \langle H^0\rangle^2/M$ where M is some high energy scale

The high mass scale can be associated with some heavy particle of mass M being exchanged (can be singlet or triplet)

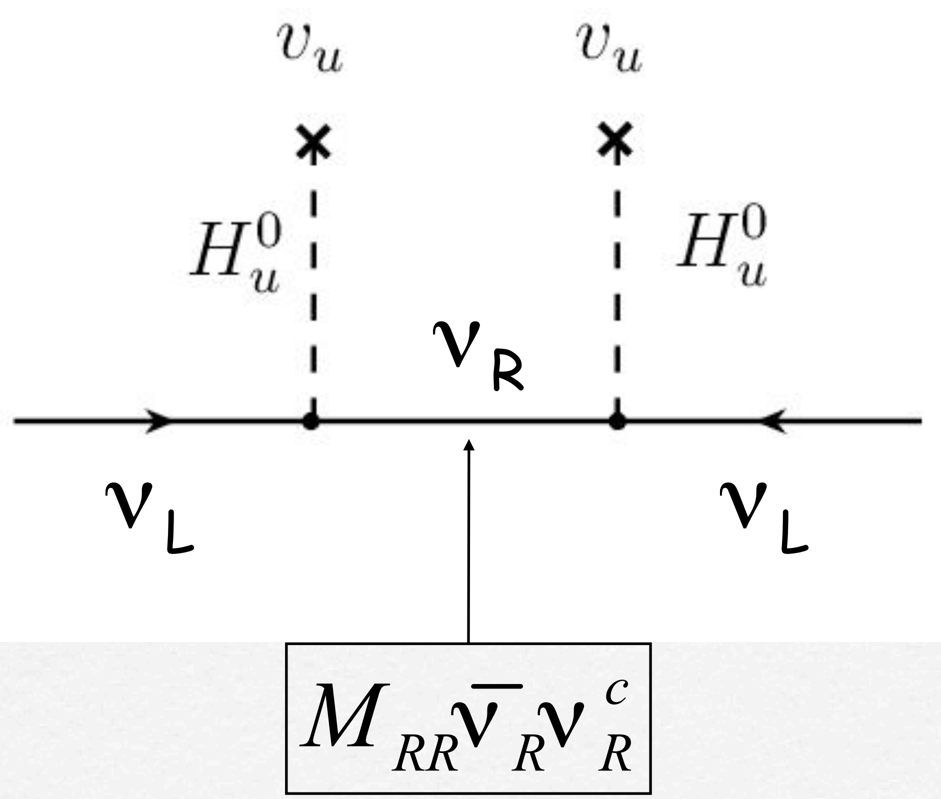


See-saw mechanisms


SEESAW MECHANISM
H
H
THE MYSTERIOUS HEAVY NEUTRINO
ν_L
ν_R
ν_L
LIGHT NEUTRINO

Type Ia see-saw mechanism

P. Minkowski (1977), Gell-Mann, Glashow, Mohapatra, Ramond, Senjanovic, Slanski, Yanagida (1979/1980), Schechter and Valle (1980)...

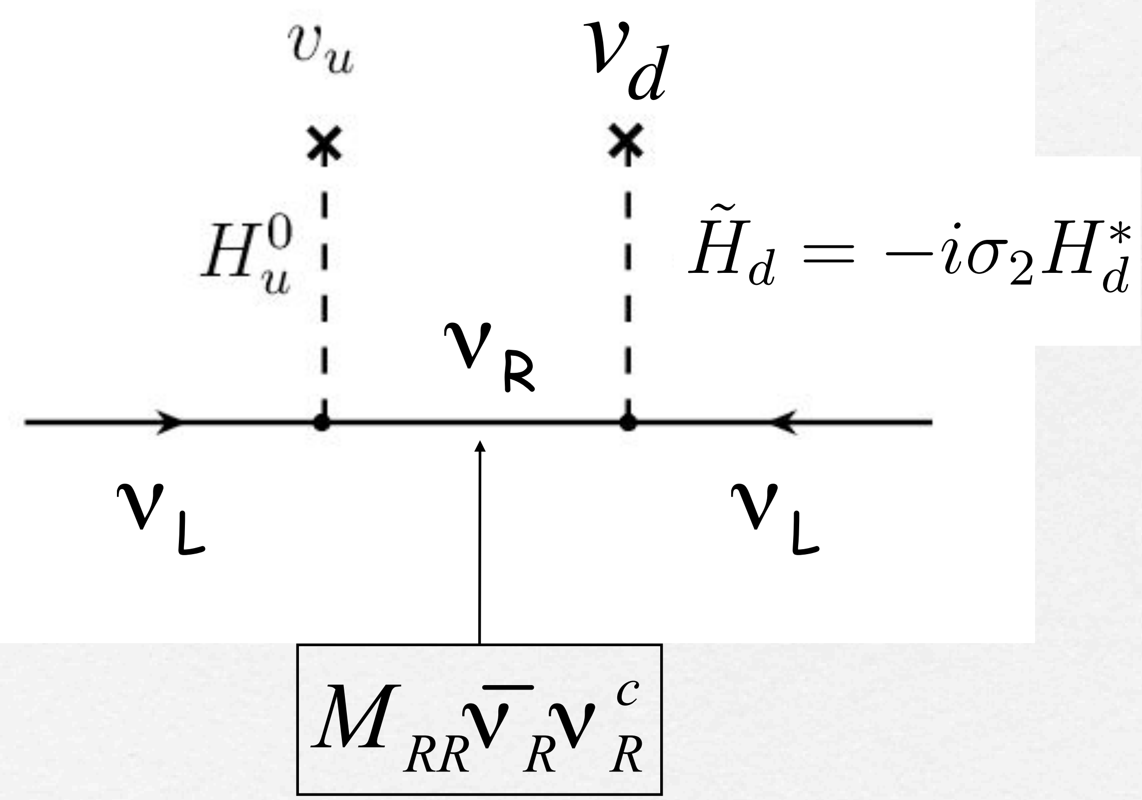


$$m_{LL}^{I} \approx -m_{LR} M_{RR}^{-1} m_{LR}^{T}$$

Type Ia

Type Ib see-saw mechanism

Hernandez-Garcia and SFK 1903.01474

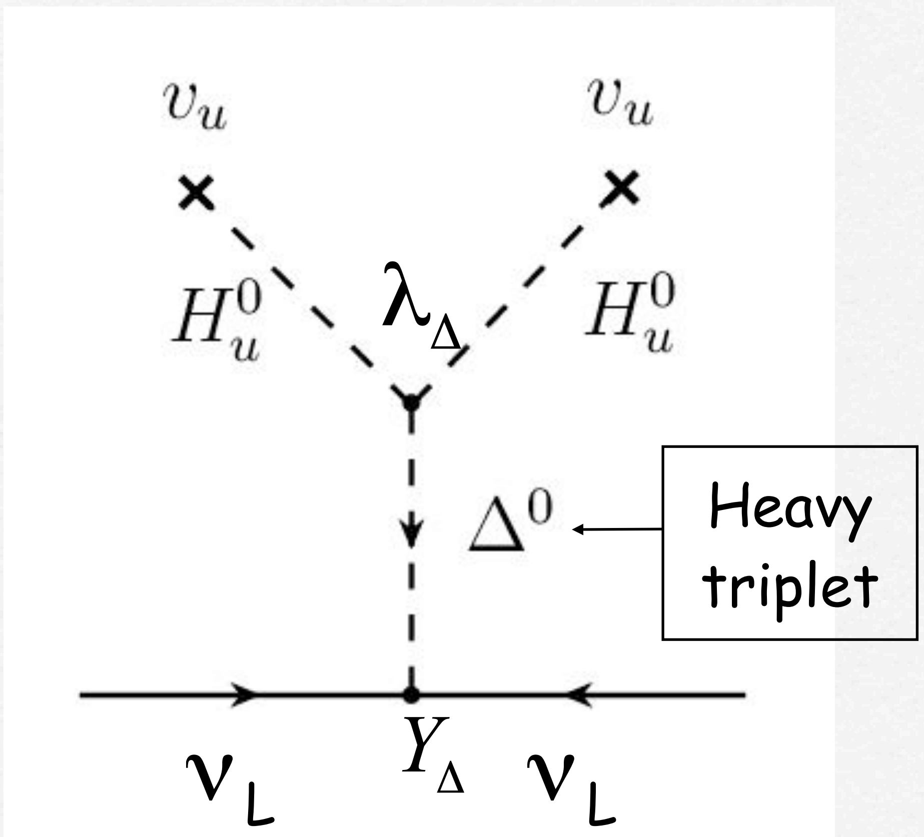


$$m_{LL}^{Ib} = -\, m_{LR1} M_{RR}^{-1} m_{LR2}^{T}$$

Type Ib

Type II see-saw mechanism (SUSY)

Lazarides, Magg, Mohapatra, Senjanovic, Shafi, Wetterich, Schechter and Valle…



$$m_{LL}^{II} \approx \lambda_\Delta Y_\Delta \frac{v_u^2}{M_\Delta}$$

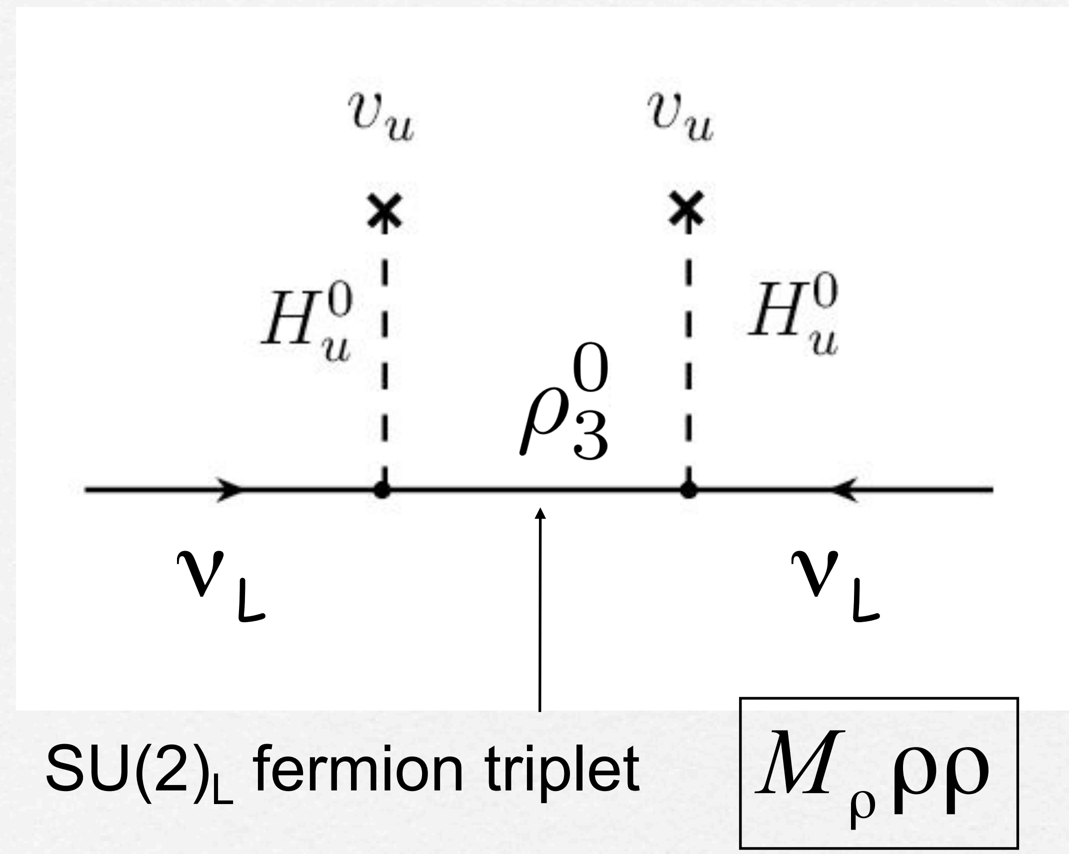
Type II

Type III see-saw mechanism

Foot, Lew, He, Joshi; Ma…

Supersymmetric adjoint SU(5)

Perez et al; Cooper, SFK, Luhn,…



$$m_{LL}^{III} \approx -m_{LR} M_\rho^{-1} m_{LR}^T$$

Type III

See-saw w/extra singlets S

Inverse see-saw

Wyler, Wolferstein; Mohapatra, Valle

$$\begin{pmatrix} 0 & M_D & 0 \\ M_D^T & 0 & M \\ 0 & M^T & \mu \end{pmatrix}$$

M ≈ TeV➔LHC

$$M_\nu = M_D M^{T^{-1}} \mu M^{-1} M_D^T$$

Linear see-saw

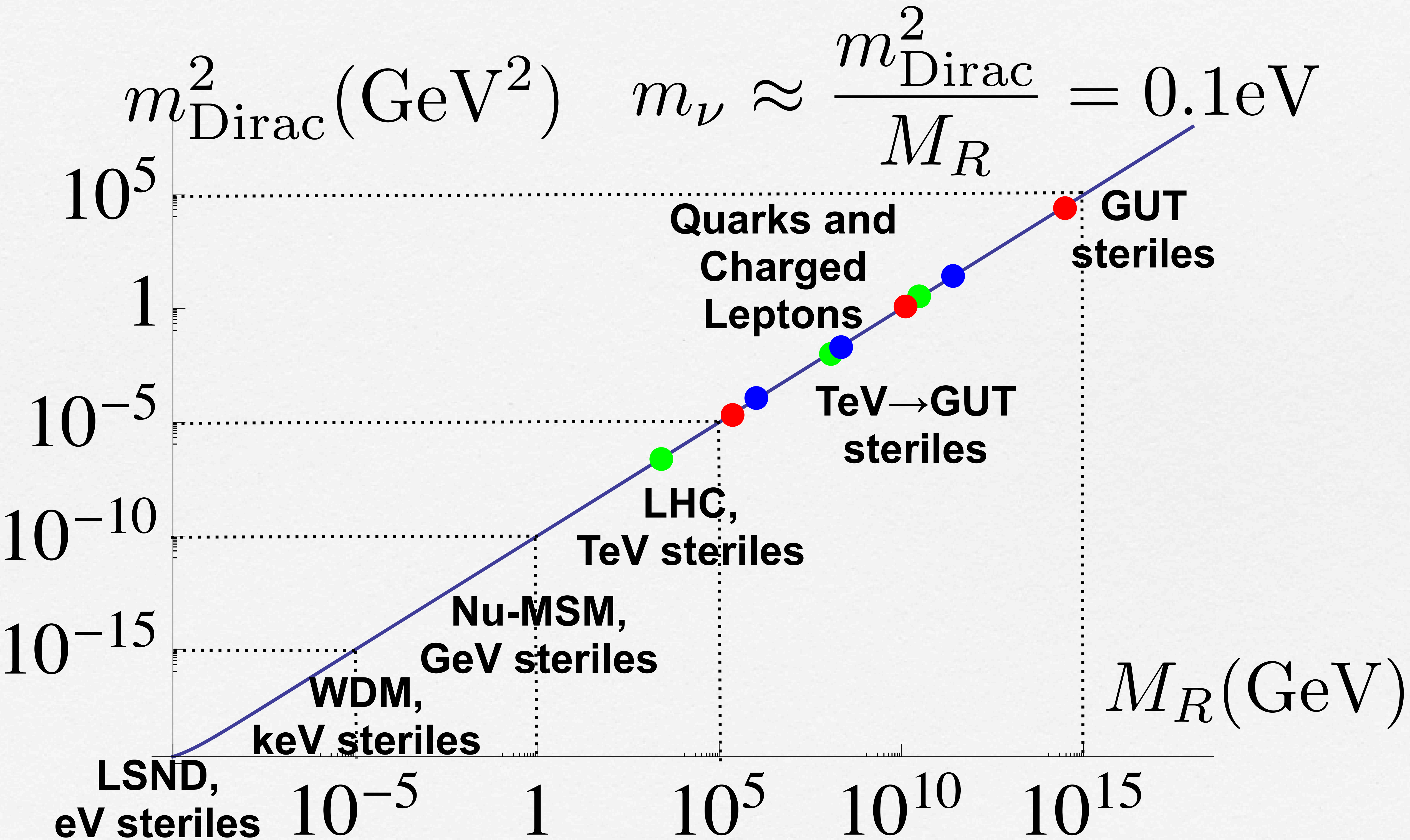
$$\begin{pmatrix} 0 & M_D & M_L \\ M_D^T & 0 & M \\ M_L^T & M^T & 0 \end{pmatrix}$$

Malinsky, Romao, Valle

$$M_\nu = M_D (M_L M^{-1})^T + (M_L M^{-1}) M_D{}^T$$

LFV predictions

RHN masses in Type Ia Seesaw



Type Ia see-saw in diagonal RHN basis

Heavy Majorana → $M_{RR} = \begin{pmatrix} Y & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X' \end{pmatrix}$

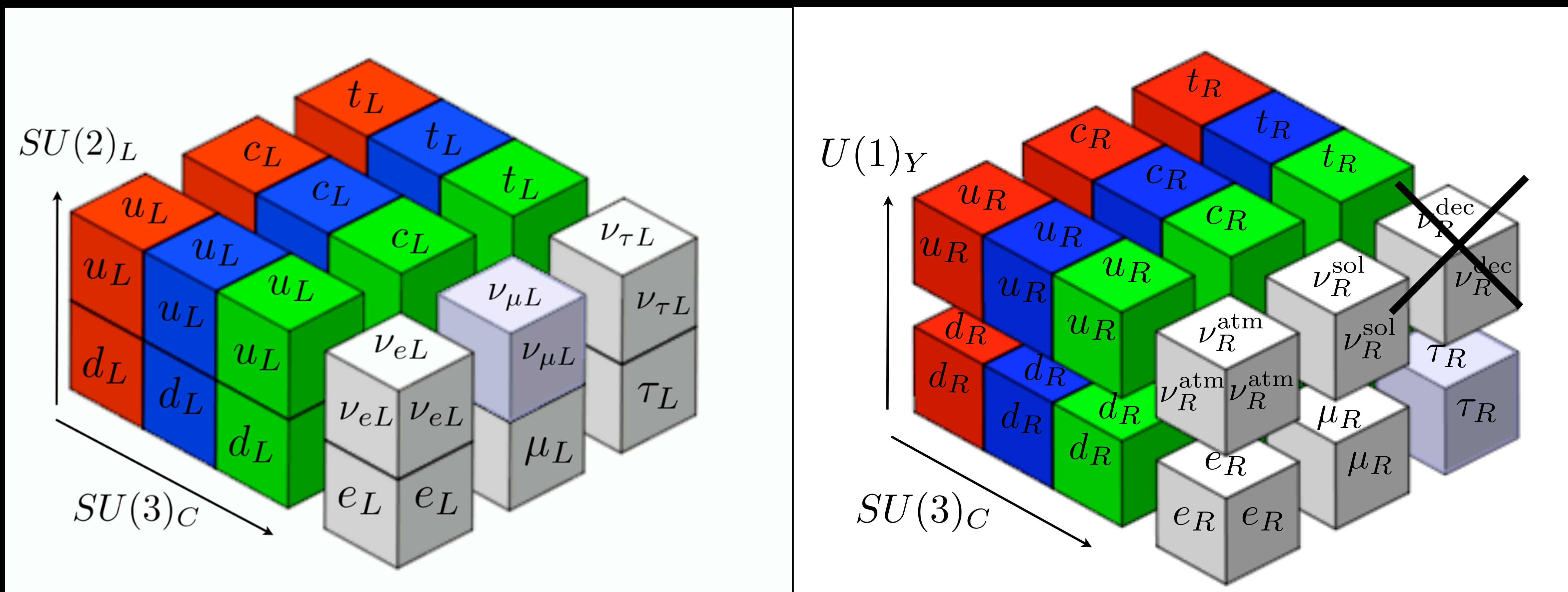
Dirac → $m_{LR} = \begin{pmatrix} d & a & a' \\ e & b & b' \\ f & c & c' \end{pmatrix}$

Light Majorana →

$$-m_{LL} = m_{LR} M_{RR}^{-1} m_{LR}^{T} = \begin{pmatrix} \left(\frac{a'^2}{X'} + \frac{a^2}{X} + \frac{d^2}{Y}\right) & \left(\frac{a'b'}{X'} + \frac{ab}{X} + \frac{de}{Y}\right) & \left(\frac{a'c'}{X'} + \frac{ac}{X} + \frac{df}{Y}\right) \\ . & \left(\frac{b'^2}{X'} + \frac{b^2}{X} + \frac{e^2}{Y}\right) & \left(\frac{b'c'}{X'} + \frac{bc}{X} + \frac{ef}{Y}\right) \\ . & . & \left(\frac{c'^2}{X'} + \frac{c^2}{X} + \frac{f^2}{Y}\right) \end{pmatrix}$$

Each element has three contributions, one from each right-handed neutrino - sequential dominance with d=0, red terms dominant, primed terms subdominant, gives simple analytic formulae (9806440, 0204360)

Two right-handed neutrinos is viable (drop the prime terms completely)



Consistent with data, predicts a massless physical neutrino

S.F.K,hep-ph/9912492
Frampton,Glashow,
Yanagida,hep-ph/0208157

Littlest Seesaw

SFK, Molina Sedgwick, Rowley, 1808.01005

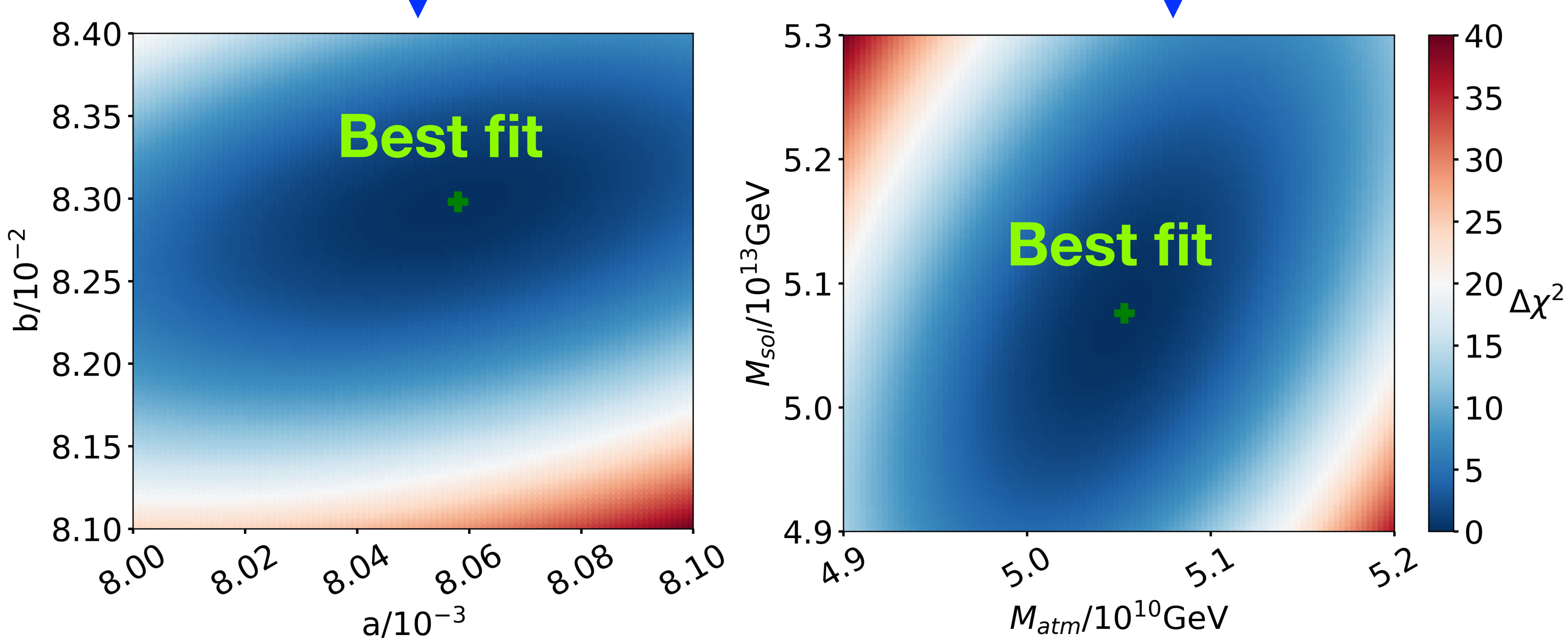
Dirac texture zero

2 RHNs

$$Y^\nu = \begin{pmatrix} 0 & be^{i\pi/3} \\ a & 3be^{i\pi/3} \\ a & be^{i\pi/3} \end{pmatrix} \qquad M_R = \begin{pmatrix} M_{\rm atm} & 0 \\ 0 & M_{\rm sol} \end{pmatrix}$$

Constrained couplings

4 real input parameters



- Fit includes effects of RG corrections
- Determines the RHN masses!

4 real input parameters

Describes:

3 neutrino masses (m_1=0),
3 mixing angles,
1 Dirac CP phase,
2 Majorana phases (1 zero)
1 BAU parameter Y_B
= 10 observables
of which 7 are constrained

Predictions	$1\,\sigma$ range
$\theta_{12}/^\circ$	$34.254 \to 34.350$
$\theta_{13}/^\circ$	$8.370 \to 8.803$
$\theta_{23}/^\circ$	$45.405 \to 45.834$
$\Delta m_{12}{}^2/10^{-5}\mathrm{eV}^2$	$7.030 \to 7.673$
$\Delta m_{31}{}^2/10^{-3}\mathrm{eV}^2$	$2.434 \to 2.561$
$\delta/^\circ$	$-88.284 \to -86.568$
$Y_B/10^{-10}$	$0.839 \to 0.881$

Also predicts NO and m_1=0

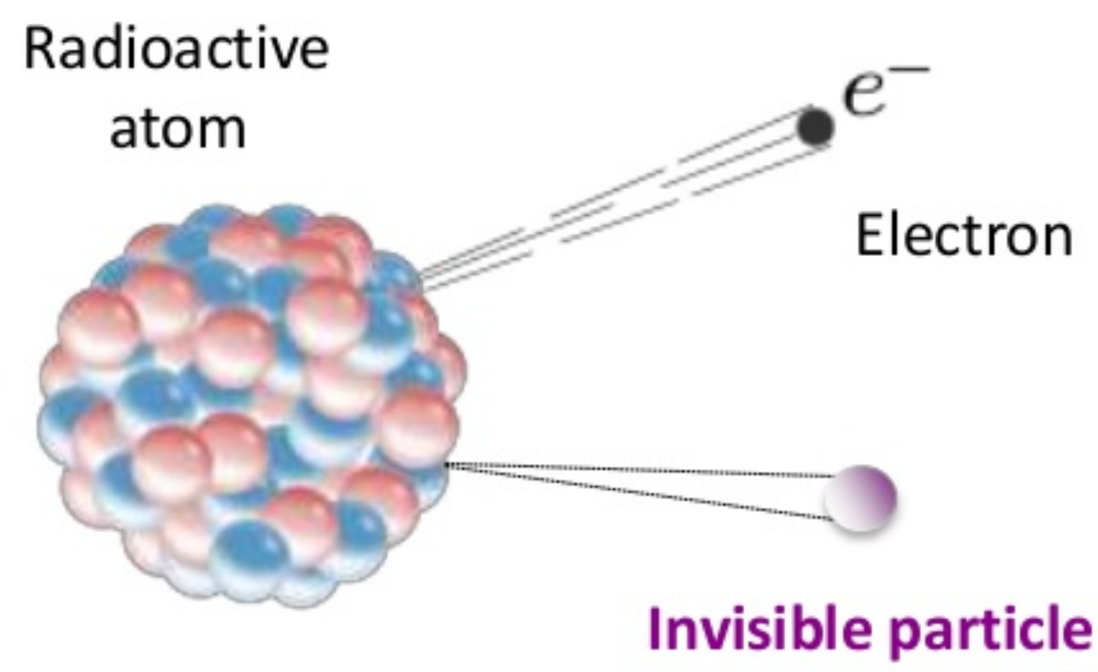
Conclusions

- Most parameters well measured in oscillation experiments…but…CP phase, octant, ordering? Also: Dirac or Majorana? Absolute masses?
- TB mixing explained by S_4…excluded by reactor angle…but…S_4 violations allow: charged lepton corrections, or TM1,TM2, with testable sum rules
- Origin of Plato's symmetry - modular symmetry?
- Origin of neutrino mass is unknown! Theoretical prejudice favours type Ia seesaw, experiment will decide (but high scale seesaw hard to test!)

Backup slides

So why are neutrinos required?

90 years ago:
A common type of radioactive decay seemed to indicate that energy was disappearing

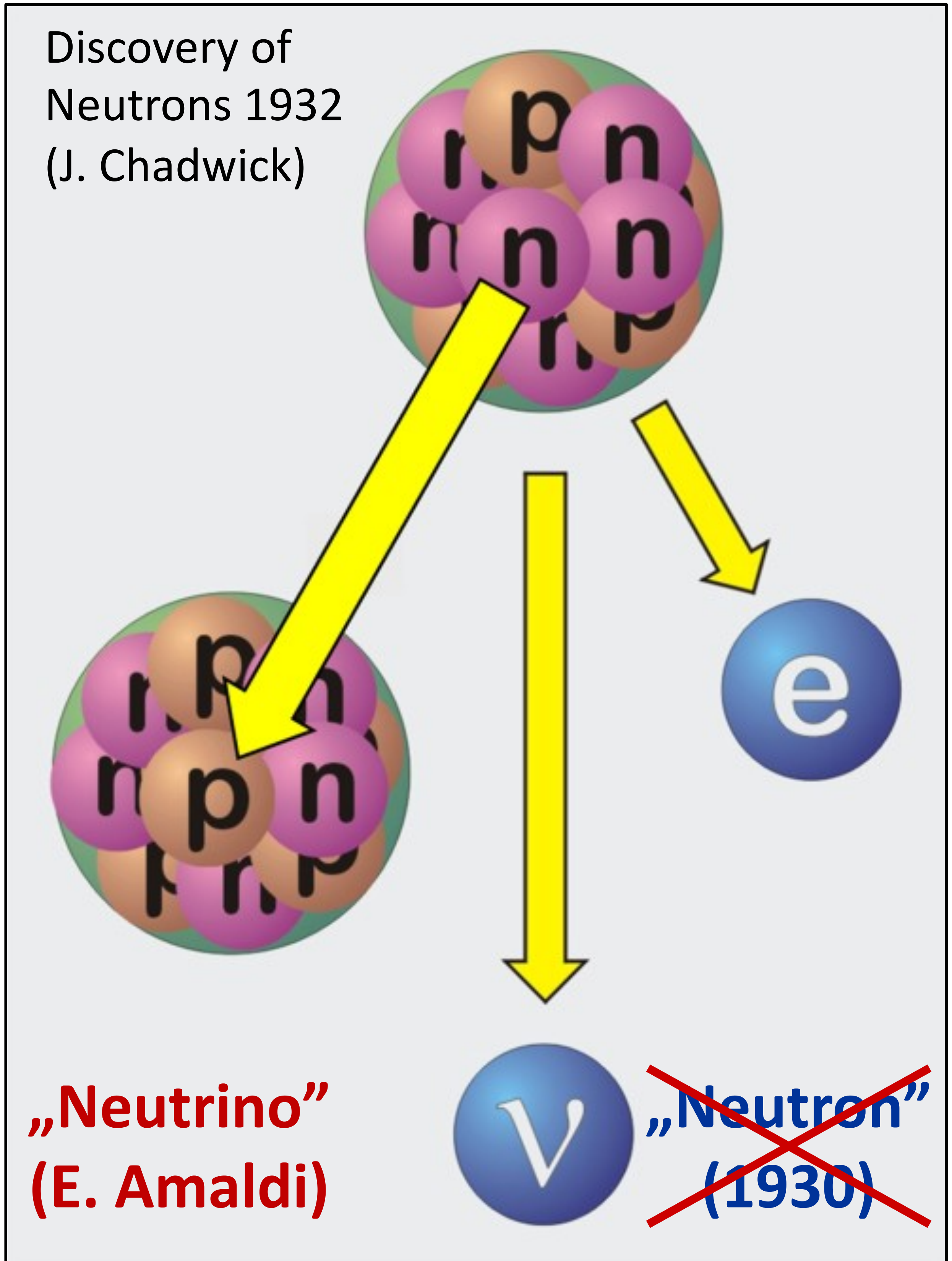


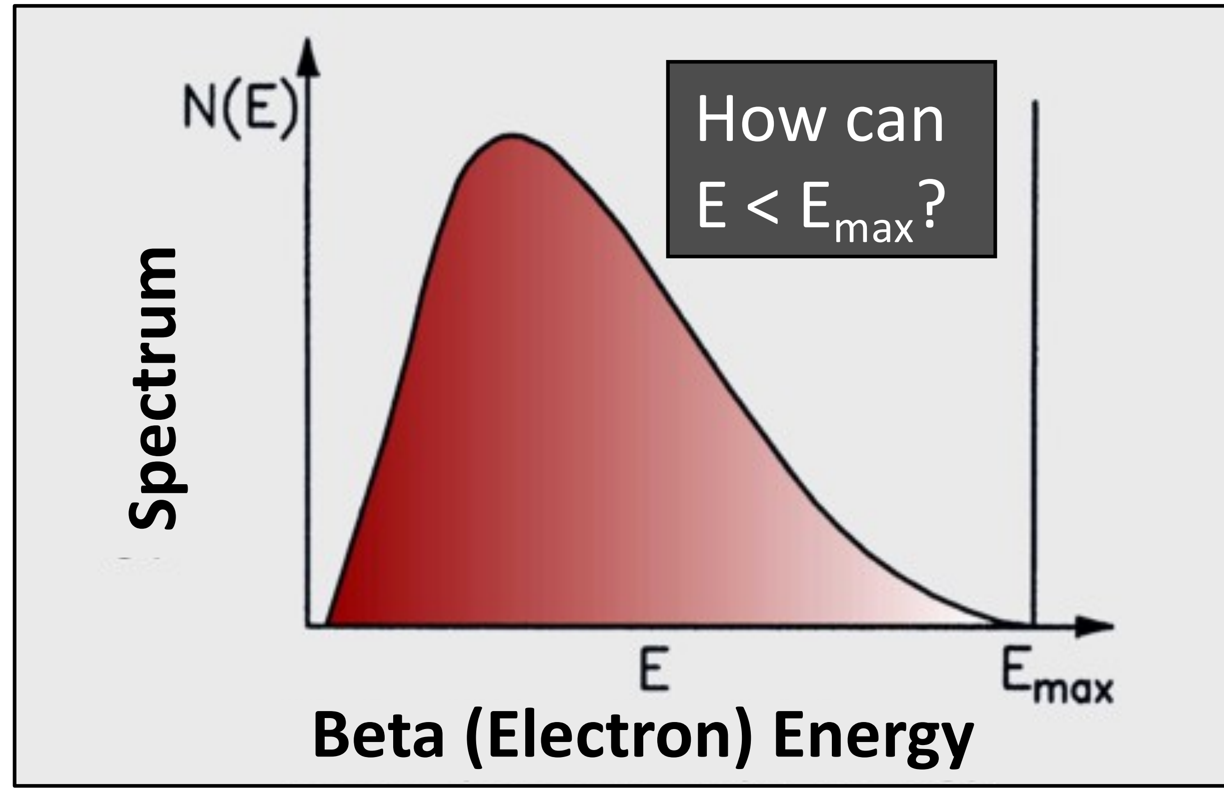
From Wolfgang Pauli's letter on 4 Dec 1930:

"Dear Radioactive Ladies and Gentlemen,
...
I have hit upon a desperate remedy to save the [...] law of conservation of energy."

In Pauli's journal:
"I have done something very bad today by proposing a particle that cannot be detected. It is something no theorist should ever do."

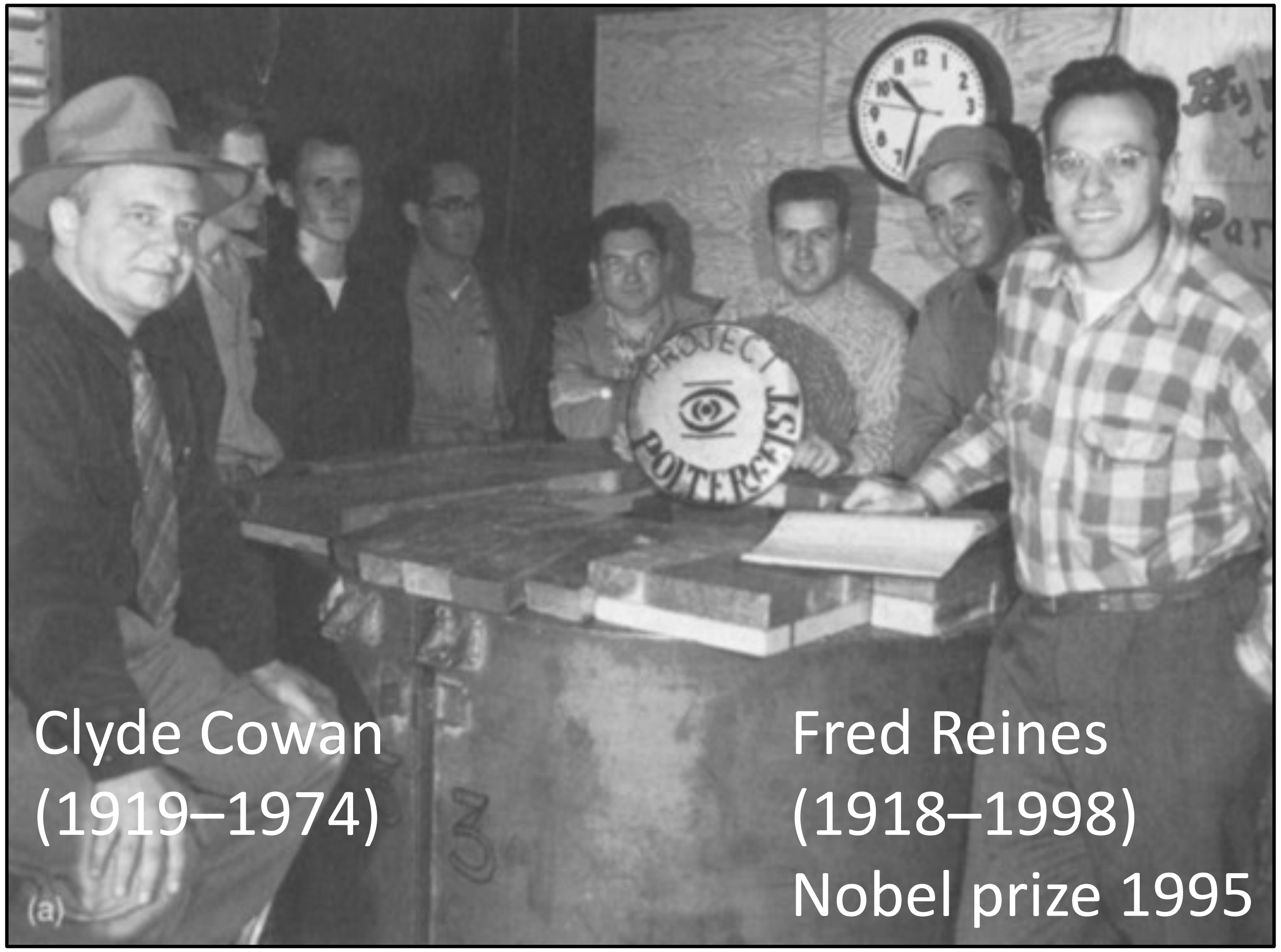
Pauli's explanation of the beta spectrum (1930)

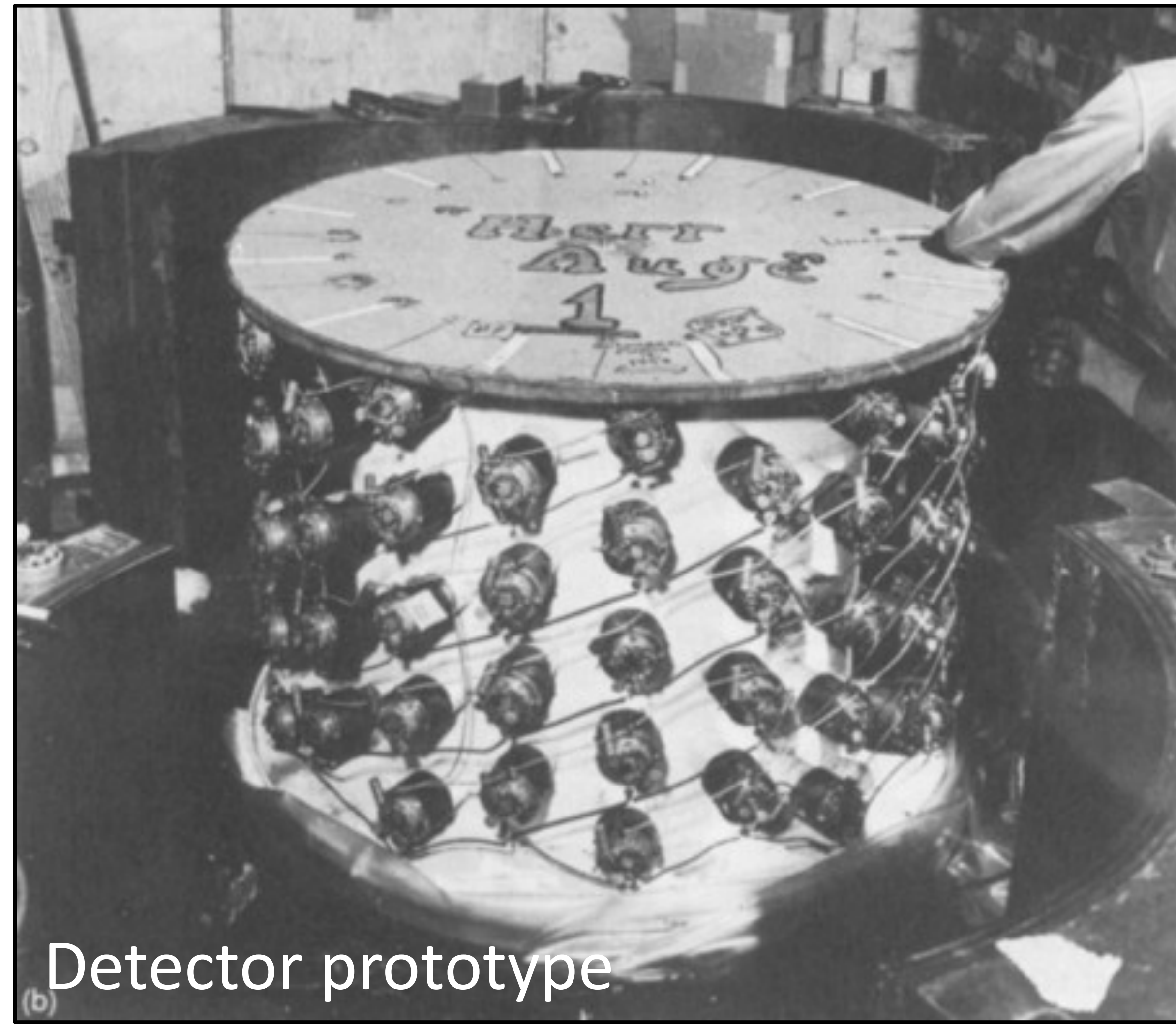




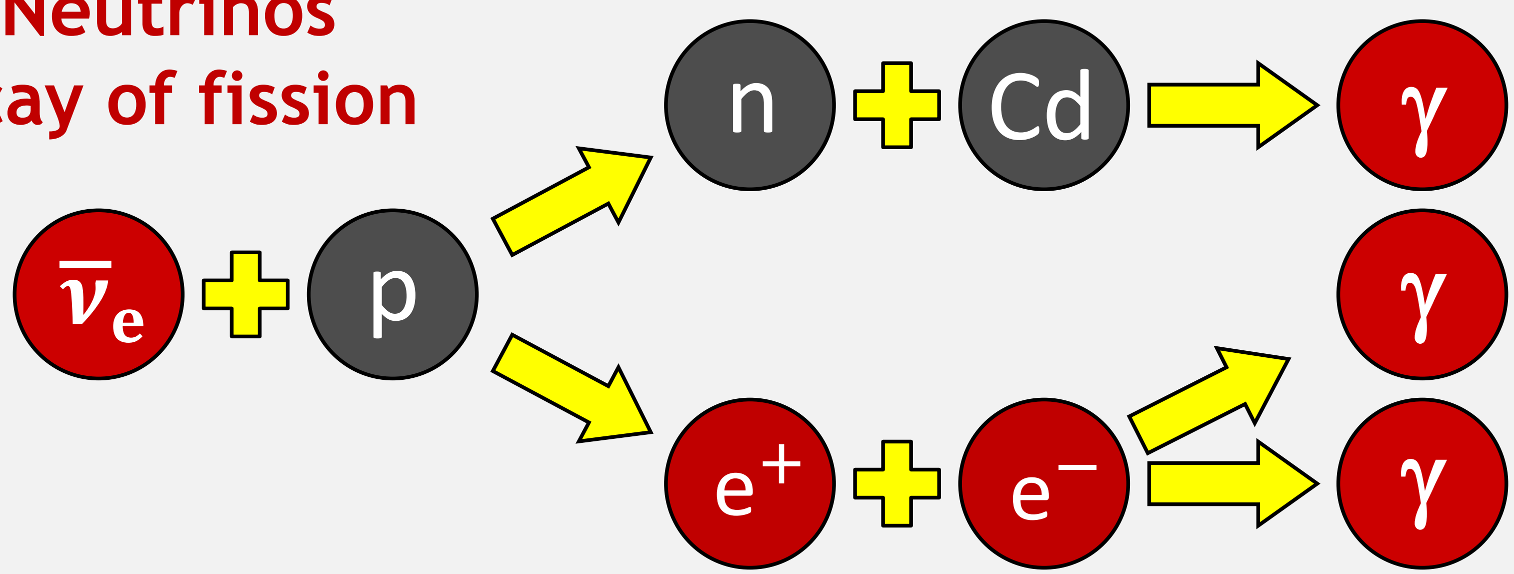
Wolfgang Pauli
(1900–1958)
Nobel Prize 1945

First neutrinos from nuclear reactors (20th July 1956)





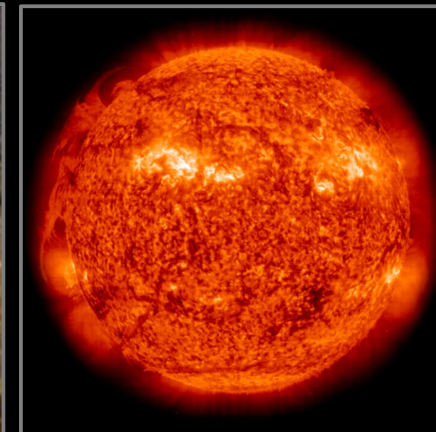
Anti-Electron Neutrinos from beta decay of fission products in Hanford Nuclear reactor



3 Gammas in coincidence

Where do neutrinos appear in nature?

✓ Nuclear Reactors

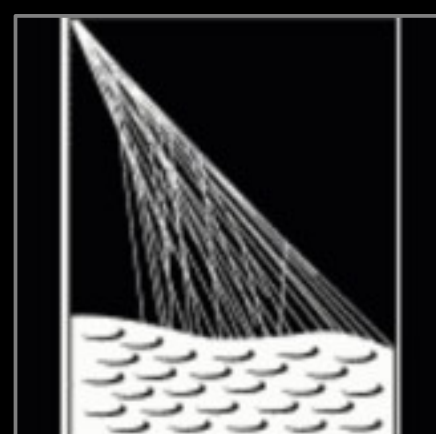


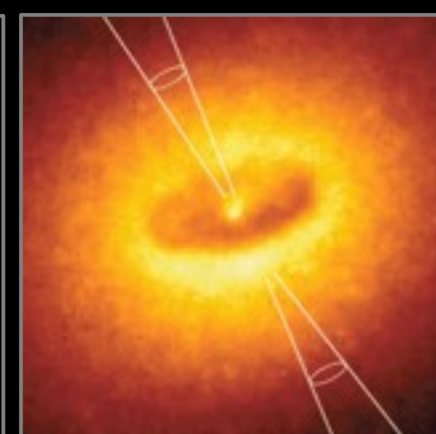
Sun ✓

✓ Particle accelerator

Supernova
(Star collapse)
SN 1987A ✓

✓ The atmosphere
(Cosmic Rays)





Astrophysical
accelerator

✓ Earth's crust
(Natural radioactivity)

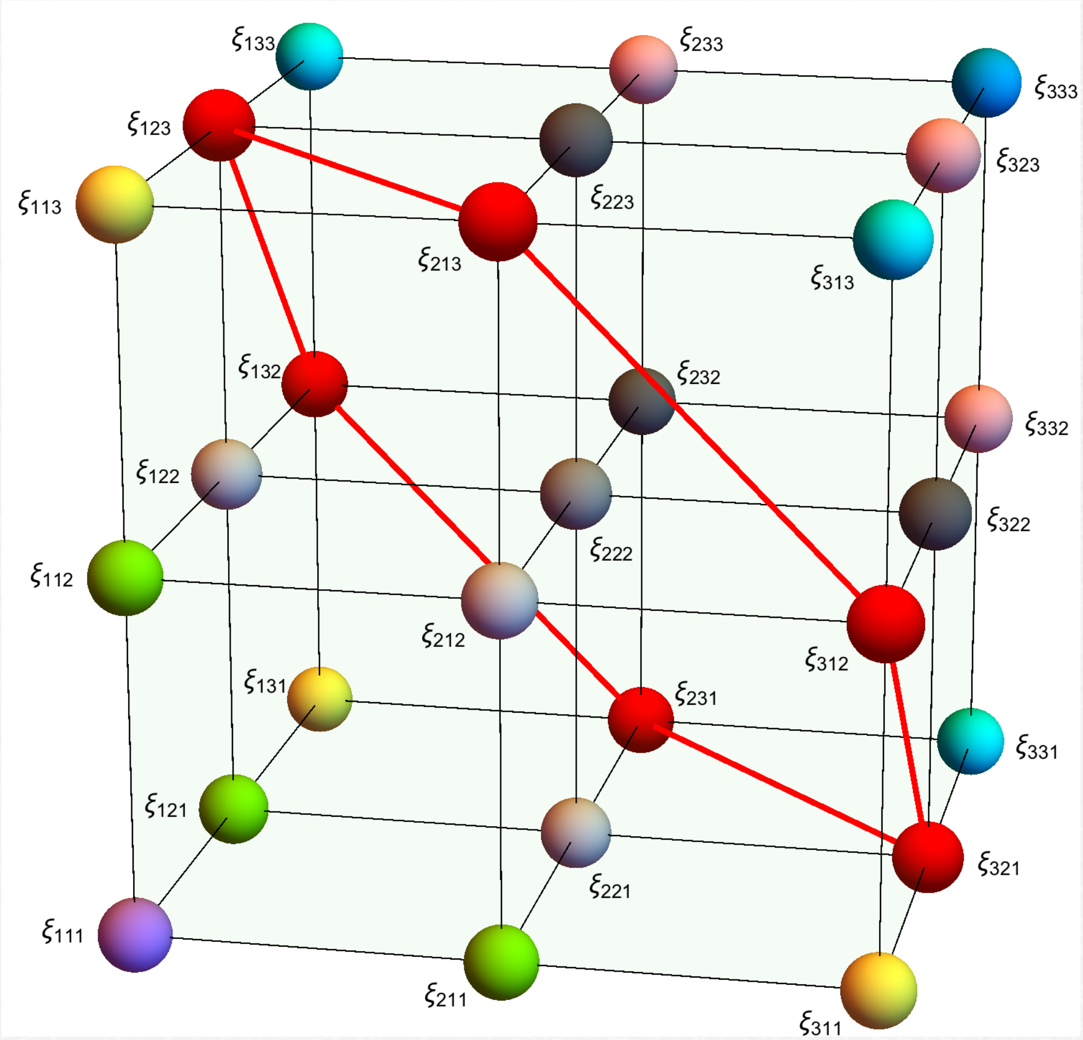
Origin of Plato's symmetry?

Y.Koide,0705.2275; T.Banks and N.Seiberg,1011.5120;
Y.L.Wu,1203.2382; A.Merle and R.Zwicky,1110.4891;
B.L.Rachlin and T.W.Kephart,1702.08073; C. Luhn, 1101.2417;
S.F.K. and Ye-Ling Zhou, 1809.10292

Possibility 1:

Break SO(3) using large Higgs reps E.g. 7-plet

irrep	$\underline{1}$	$\underline{3}$	$\underline{5}$	$\underline{7}$
subgroups	$SO(3)$	$SO(2)$	$Z_2 \times Z_2$	$\mathbf{1}$
		$SO(3)$	$SO(2)$	A_4
			$SO(3)$	Z_3
				D_4
				$SO(2)$
				$SO(3)$



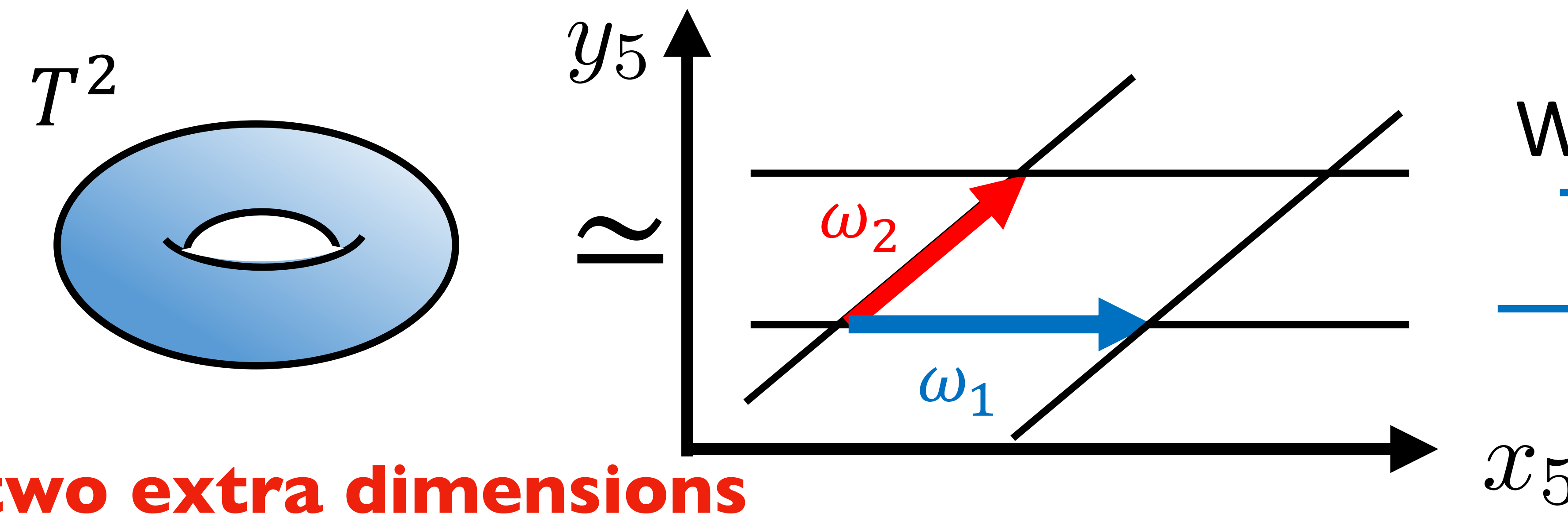
A4 preserving direction of 7-plet VEV

$$\langle \xi_{123} \rangle \equiv \frac{v_\xi}{\sqrt{6}}, \quad \langle \xi_{111} \rangle = \langle \xi_{112} \rangle = \langle \xi_{113} \rangle = \langle \xi_{133} \rangle = \langle \xi_{233} \rangle = \langle \xi_{333} \rangle = 0$$

Possibility 2: Extra dimensions (string theory)

G.Altarelli and F.Feruglio, hep-ph/0512103
R.de Adelhart Toorop, F.Feruglio and C.Hagedorn, 1112.1340
F.Feruglio, 1706.08749; J.C.Criado and F.Feruglio, 1807.01125; J.T.Penedo and S.T.Petcov 1806.11040;
P.P.Novichkov, J.T.Penedo, S.T.Petcov and A.V.Titov, 1811.04933, 1812.02158;
T.Kobayashi, K.Tanaka and T.H.Tatsuishi,1803.10391; F.de Anda,S.F.K.,E.Perdomo,1812.05620
T.Kobayashi, N.Omoto, Y.Shimizu, K.Takagi, M.Tanimoto and T.H.Tatsuishi,1808.03012;
G.J.Ding, S.F.King and X.G.Liu, 1903.12588

The structure of a torus $T^2 \simeq$ The structure of a lattice on $\mathbb{C}$-plane



With identification

T.H.Tatsuishi

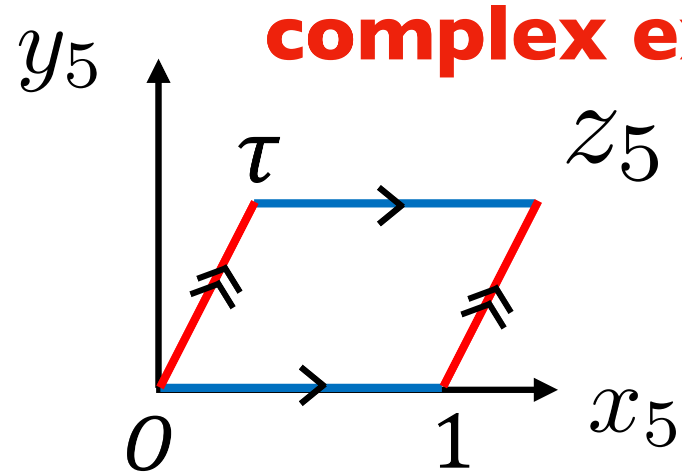
two extra dimensions compactified on torus

Without loss of generality,

$$(\omega_1, \omega_2) \rightarrow \left(1, \frac{\omega_2}{\omega_1}\right) \equiv (1, \tau)$$

modulus

complex extra dimension



$$z_5 = x_5 + i y_5$$

$$(\mathrm{Im}(\tau) > 0)$$