Stückelberged Unimodular vs Goldstino Brane Approach to Pure dS Supergravity

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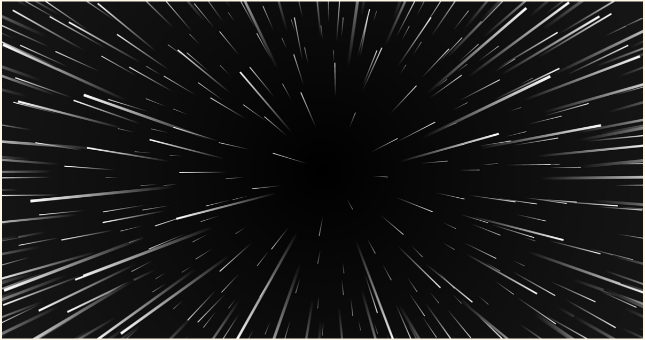
Introduction



- Universe is accelerating $\equiv$ de-Sitter spacetime
- Linearly realised SUSY w/o scalar fields does not allow positive cosmological constant

Linear SUSY Algebras for dS/AdS Spacetimes

In dS/AdS algebras translations have a non-zero commutator:

$$[P_\mu, P_\nu] = s\,\frac{1}{4L^2}\,M_{\mu\nu} \quad \text{where} \quad s = \begin{cases} -1 & \text{for dS} \\ +1 & \text{for AdS} \end{cases}$$

Linear SUSY algebra:

$$[P_\mu, Q_\alpha] = \frac{1}{4L}(\gamma_\mu Q)_\alpha \qquad [M_{\mu\nu}, Q_\alpha] = -\,(\gamma_{\mu\nu})_\alpha{}^\beta Q_\beta$$

$$\{Q_\alpha, Q_\beta\} = -\,\frac{1}{2}\,(\gamma^\mu)_{\alpha\beta}P_\mu - \frac{1}{8L}(\gamma^{\mu\nu})_{\alpha\beta}M_{\mu\nu} \qquad [P_\mu, P_\nu] = 0$$

On embedding the dS/AdS algebra in the above linear SUSY algebra, the Jacobi identity

$$[P_\mu, P_\nu, Q] = 0$$

fixes $s = 1$. Therefore, linear $\mathcal{N} = 1$ super-dS algebra does not exist.

Move to non-linear SUSY

- Pure dS SUGRA was constructed by realising $\mathcal{N}$=1 SUSY non-linearly

- Different methods to realise SUSY non-linearly:
  - Nilpotent superfields
  - Goldstino brane action
  - Stückelberging unimodular supergravity

Motivation

- Different methods for realising $\mathcal{N}$=1 SUSY non-linearly
- Do different constructions give the same action?
- How to compare the actions?

Brief Historical Review

- In 2015 Bergshoeff, Freedman, Kallosh & Proeyen presented dS SUGRA for the first time using superconformal methods.
- Later in 2015 Bandos, Martucci, Sorokin & Tonin presented dS SUGRA by coupling a goldstino 3-brane to minimal supergravity.
- A lot of work has been done on dS SUGRA in recent years [Antoniadis, Dudas, Farakos, Ferrara, Hasegawa, Kehagias, Kuzenko, Porrati, Sagnotti, Scalisi, Wrase, Yamada, ...'15-'21]
- Cosmological and inflationary models in dS SUGRA [Andriot, Antoniadis, Dudas, Ferrara, Sagnotti, Buchmuller, Heurtier, Wieck, Ferrara, Kallosh, Linde, Thaler, Zavala, Zwirner, ...'15-'21]
- Brane models [Angelantonj, Antoniadis, Dudas, Mourad, Parameswaran, Pradisi, Riccioni, Sagnotti, Uranga, Vercnocke, Zavala, ...'99-'21]

Outline

Nilpotent Superfield Construction

In superconformal model we use 3 multiplets:

1) chiral compensating multiplet $\{X^0, \chi^0, F^0\}$,
2) nilpotent chiral multiplet $S = \{X^1, \mathcal{G}, F^1\}$,
3) Lagrange multiplier multiplet $\{\Lambda, \chi^\Lambda, F^\Lambda\}$.

The Lagrangian is [E Bergshoeff, D Freedman, R Kallosh & A Proeyen '15]

$$\mathcal{L} = [\tfrac{1}{2}\eta_{IJ} X^I \bar{F}^J]_F + [\mathcal{W}(X^I)]_F + [\Lambda (X^1)^2]_F$$

where $I, J = 0, 1$; $\eta_{IJ} = \mathrm{diag}(-1, 1)$ and the superpotential $\mathcal{W}$ is

$$\mathcal{W} = a\,(X^0)^3 + b\,(X^0)^2 X^1$$

where a and b are arbitrary constants.

Nilpotent Superfield

$$S = X^1 + \sqrt{2}\,\theta\,\mathcal{G} + \theta^2 F^1$$

Nilpotency constraint: $S^2 = 0 \quad \Rightarrow \quad X^1 = \dfrac{\mathcal{G}^2}{2F^1}$

This eliminates the fundamental scalar partner of the goldstino $\mathcal{G}$ and hence SUSY is realised non-linearly.

It gives solutions with the cosmological constant

$$\Lambda = |b|^2 - |a|^2$$

Goldstino Brane Action in Supergravity

$$S_B = S_{SG} + S_{VA}$$ [I Bandos, L Martucci, D Sorokin, M Tonin '15]

$$= \underbrace{-\frac{3}{2\kappa^2}\int d^8z\,\mathrm{Ber}\,E}_{\text{Standard pure SUGRA}} \; - \; \underbrace{\frac{2\,m^{(B)}}{\kappa^2}\left(\int d^6\zeta_L\mathcal{E} + \mathrm{h.c.}\right)}_{\text{AdS cosmological constant term}}$$

$$\underbrace{-f^2\int d^4\xi\,\det\boldsymbol{E}(z(\xi))}_{\text{Goldstino brane coupled to SUGRA}}$$

ξ^i are the 3-brane worldvolume coordinates with $i = 0, 1, 2, 3$. Coupling to supergravity is given via the embedding in the bulk superspace as

$$\xi^i \to z^M(\xi) = (x^\mu(\xi), \theta^\alpha(\xi), \bar{\theta}^{\dot{\beta}}(\xi))$$

Goldstino Brane Action in Supergravity

The solutions to the equations of motion of the auxilliary fields show that they belong to nilpotent superfields.

S_B perturbed up to the 3rd order in fluctuations, is:

$$S = \frac{1}{2\kappa^2}\int d^4x \left[\left\{ \sqrt{-g}\,R - \varepsilon^{\mu\nu\rho\lambda}\left(\psi_\mu\sigma_\nu\mathcal{D}_\rho\bar{\psi}_\lambda + h.c.\right)\right\}^{(3)} - \left\{ 2\sqrt{-g}\left(\mathbf{\Lambda}_2 - \frac{m^2}{3}\right)\right\}^{(3)} \right.$$
$$+ \left\{ \tfrac{2}{3}m\psi_\mu\sigma^{\mu\nu}\psi_\nu + 2i\mathbf{\Lambda}_2\breve{\mathcal{G}}\sigma^\mu\bar{\psi}_\mu - 2i\mathbf{\Lambda}_2\breve{\mathcal{G}}\sigma^\mu\mathbf{D}_\mu\bar{\breve{\mathcal{G}}} + \tfrac{4}{3}m\mathbf{\Lambda}_2\breve{\mathcal{G}}^2 \right.$$
$$+ \tfrac{1}{3}hm\psi_\mu\sigma^{\mu\nu}\psi_\nu + \tfrac{2}{3}m\psi_\mu h_\rho{}^{[\mu}\sigma^{\nu]\rho}\psi_\nu + \mathbf{\Lambda}_2(ih\breve{\mathcal{G}}\sigma^\mu\bar{\psi}_\mu - i\breve{\mathcal{G}}h^\mu{}_\nu\sigma^\nu\bar{\psi}_\mu$$
$$\left.\left. - ih\breve{\mathcal{G}}\sigma^\mu\mathbf{D}_\mu\bar{\breve{\mathcal{G}}} + i\breve{\mathcal{G}}h^\mu{}_\nu\sigma^\nu\mathbf{D}_\mu\bar{\breve{\mathcal{G}}} + 2i\breve{\mathcal{G}}\sigma^\mu\bar{\omega}^{(1)}_\mu\bar{\breve{\mathcal{G}}} + \tfrac{2}{3}hm\breve{\mathcal{G}}^2) + h.c.\right\}\right]$$

$\breve{\mathcal{G}}$ is the goldstino.

Constructing Volkov-Akulov Action

$\mathcal{N} = 1$ in $D = 4$

SUSY transformations:

$$\delta x^a = i\left(\epsilon\,\sigma^a\,\bar\theta - \bar\epsilon\,\sigma^a\,\theta\right), \quad \delta\theta^\alpha = \epsilon^\alpha$$

Replace the Grassman coordinate θ with the field $\chi(x)$.

$$\theta^\alpha \to \kappa\,\chi^\alpha(x)$$

$$\delta x^a = i\,\kappa^2\left(\epsilon\,\sigma^a\,\bar\chi - \chi\,\sigma^a\,\bar\epsilon\right), \quad \delta\chi^\alpha = \epsilon^\alpha + i\,\kappa^2\left(\epsilon\,\sigma^a\,\bar\chi - \chi\,\sigma^a\,\bar\epsilon\right)\partial_a\chi^\alpha$$

It is easy to check that the above transformation realises SUSY algebra.

$$\left(\delta_\epsilon\delta_\eta - \delta_\eta\delta_\epsilon\right)\chi = 2i\kappa^2\left(\epsilon\,\sigma^a\,\bar\eta - \eta\,\sigma^a\,\bar\epsilon\right)\partial_a\chi^\alpha$$

$$\{Q_\alpha\,,\,\bar Q_{\dot\beta}\} = 2\,\sigma_{\alpha\dot\beta}{}^a\,P_a$$

Q_α $(\alpha = 1, 2)$ are Weyl spinor generators
P_a are translation generators.

Constructing Volkov-Akulov Action

Find a SUSY-invariant Cartan one-form:

$$g = e^{i\,x^a P_a}\, e^{i\,\theta Q}\, e^{-i\,\bar{\theta}\bar{Q}}$$

$$\Omega = -ig^{-1}dg = E^a P_a + E^\alpha Q_\alpha + E_{\dot{\alpha}} Q^{\dot{\alpha}}$$

$$E^a = dx^a + i\kappa^2 \left(\chi\,\sigma^a\, d\bar{\chi} - d\chi\,\sigma^a\,\bar{\chi}\right)$$

Construct Volkov-Akulov Action

In D dimensions:

$$S = \frac{1}{\kappa^2 D!} \int \varepsilon_{a_1,\ldots a_D}\, E^{a_1} \wedge E^{a_2} \ldots \wedge E^{a_D} = -\frac{1}{\kappa^2} \int d^D x \,\det E_m^a$$

Unimodular Gravity

Unimodular Gravity

Unimodular Gravity Action:

$$S = \frac{1}{16\pi G_N}\int d^4x \left[\sqrt{-g}R - 2\Lambda(x)\left(\sqrt{-g} - \epsilon_0\right)\right]$$

where $\Lambda(x)$ is a Lagrange multiplier field imposing the unimodularity condition:

$$\sqrt{-g} = \epsilon_0$$

ϵ_0 is a constant, traditionally set to unity. $\Lambda'(x') = \Lambda(x)$. But diffeomorphism symmetry is broken because of the term

$$\int d^4x \, \Lambda \, \epsilon_0 \, .$$

Unimodular Gravity and the Stückelberg Procedure

Under active diffeomorphism tranformation, Λ transforms as

$$\Lambda \to \Lambda' = \Lambda - \xi^\mu \partial_\mu \Lambda + \tfrac{1}{2}\xi^\nu \partial_\nu \left(\xi^\mu \partial_\mu \Lambda\right) + \ldots$$

We promote the diffeo parameter ξ^μ to the field ϕ^μ. Then the action becomes

$$S_\phi = \tfrac{1}{16\pi G_N} \int d^4x \Big[\sqrt{-g}R - 2\Lambda\sqrt{-g} + 2\,\Lambda\,\epsilon_0 \Big[1 + \partial_\mu \phi^\mu + \tfrac{1}{2}\phi^\nu \partial_\nu \partial_\mu \phi^\mu + \tfrac{1}{2}\left(\partial_\mu \phi^\mu\right)\left(\partial_\nu \phi^\nu\right) + \ldots\Big]\Big]$$

It is invariant up to the relevant order, when ϕ^μ transforms as

$$\delta\phi^\mu = -\,\xi^\mu - \tfrac{1}{2}\xi^\nu \partial_\nu \phi^\mu + \tfrac{1}{2}\phi^\nu \partial_\nu \xi^\mu + \ldots$$

We can also do passive diffeomorphism transformation.

$$x^\mu \to \hat{x}^\mu(x) \xrightarrow{\text{Stückelberg}} s^\mu(x)$$

Then the Stückelberged action is

$$S_s = \frac{1}{16\pi G_N} \int d^4x \left[\sqrt{-g}\mathcal{R} - 2\Lambda \left(\sqrt{-g} - \mathrm{Det}\left(\frac{\partial s}{\partial x} \right) \epsilon_0 \right) \right] ,$$

Using $s^\mu = x^\mu + \phi^\mu + \dots$ the two Stückelberg actions are identical order by order in ϕ^μ.

Unimodular Supergravity

$\mathcal{N} = 1$ Supergravity Action

Now we supersymmetrise the theory and see what solutions we get.

$\mathcal{N} = 1$ supergravity action

$$S = -\frac{6}{8\pi G_N} \int d^4x\, d^2\Theta\, \mathcal{E}\mathcal{R} + h.c.$$

Volume element $\mathcal{E}$ is

$$\begin{aligned} \mathcal{E} &= \mathcal{F}_0 + \sqrt{2}\,\Theta\mathcal{F}_1 + \Theta\Theta\mathcal{F}_2, \qquad \text{with} \\ \mathcal{F}_0 &= \tfrac{1}{2}e\,, \\ \mathcal{F}_1 &= \tfrac{i\sqrt{2}}{4}e\sigma^\mu\bar{\psi}_\mu\,, \\ \mathcal{F}_2 &= -\tfrac{1}{2}eM^* - \tfrac{1}{8}e\bar{\psi}_\mu\left(\bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\mu\right)\bar{\psi}_\nu\,. \end{aligned}$$

$\mathcal{N} = 1$ Supergravity Action

$\mathcal{N} = 1$ supergravity action

$$S = -\tfrac{6}{8\pi G_N} \int d^4x\, d^2\Theta\, \mathcal{E}\mathcal{R} + h.c.$$

Superfield $\mathcal{R}$ is

$$\mathcal{R} = -\frac{1}{6}(\mathcal{R}_0 \;+\; \Theta\mathcal{R}_1 \;+\; \Theta\Theta\mathcal{R}_2), \qquad \text{with}$$

$$\mathcal{R}_0 = M\,,$$

$$\mathcal{R}_1 = \sigma^\mu \bar{\sigma}^\nu \psi_{\mu\nu} - i\sigma^\mu \bar{\psi}_\mu M + i\psi_\mu b^\mu\,,$$

$$\mathcal{R}_2 = \tfrac{1}{2}R + i\bar{\psi}^\mu \bar{\sigma}^\nu \psi_{\mu\nu}\,.$$

$\mathcal{N} = 1$ Unimodular Supergravity Action

$\mathcal{N} = 1$ unimodular supergravity action [S. Nagy, A. Padilla, I. Zavala '19]

$$S = -\frac{6}{8\pi G_N} \int d^4x\, d^2\Theta \left[\mathcal{E}\,\mathcal{R} + \frac{1}{6}\Lambda\left(\mathcal{E} - \mathcal{E}_0\right)\right] + h.c.$$

where

$$\mathcal{E}_0 = \epsilon_0 - \frac{1}{2} m\Theta^2 \,.$$

The Lagrange multiplier field Λ is

$$\Lambda = \Lambda_0 + \sqrt{2}\,\Theta\Lambda_1 + \Lambda_2\Theta^2$$

Varying over Λ, we get

$$\mathcal{E} = \mathcal{E}_0 \,.$$

The symmetry breaking part of the action is

$$\frac{1}{16\pi G_N} \int d^4x \left[2\Lambda_2\epsilon_0 - m\Lambda_0 + h.c.\right] \subset S \,.$$

Super-Stückelberg Procedure

Stückelberg trick is performed up to the 2nd order in Stückelberg fields, so the diffeo and SUSY transformations of the superfield components are derived up to the 2nd order in ξ^μ and ϵ.

Then we promote the diffeo and SUSY transformation parameters to fields:

$$\xi^\mu \;\rightarrow\; \phi^\mu \quad \text{and} \quad \epsilon \;\rightarrow\; \zeta$$

The action can then be constructed perturbatively as:

$$\begin{aligned}\mathcal{L} =& \sqrt{-g}\Big[R - \tfrac{2}{3}M^* M + \tfrac{2}{3}b^\mu b_\mu + \varepsilon^{\mu\nu\rho\sigma}\left(\bar{\psi}_\mu\bar{\sigma}_\nu\tilde{\mathcal{D}}_\rho\psi_\sigma - \psi_\mu\sigma_\nu\tilde{\mathcal{D}}_\rho\bar{\psi}_\sigma\right)\Big] \\ &+ \tfrac{1}{2}\sqrt{-g}\Big[-2\Lambda_2 + \sqrt{2}i\Lambda_1\sigma^\mu\bar{\psi}_\mu + 2\Lambda_0\left(\bar{\psi}_\mu\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu + M^*\right) + h.c.\Big] \\ &+ 2\left[\Lambda_2 + \delta^{(\epsilon\rightarrow\zeta,\,\xi^\mu\rightarrow\phi^\mu)}\Lambda_2\right]\epsilon_0 - m\left[\Lambda_0 + \delta^{(\epsilon\rightarrow\zeta,\,\xi^\mu\rightarrow\phi^\mu)}\Lambda_0\right] + h.c.\end{aligned}$$

Super-Stückelberg Procedure

Initial boundary conditions on the components of Λ:

$$\Lambda_0\Big|_\infty = K_0\,, \qquad \Lambda_1\Big|_\infty = 0\,, \qquad \Lambda_2\Big|_\infty = K_2\,,$$

K_0 and K_2 are some constants.

Boundary conditions on Λ get modified as:

$$\begin{aligned}
&\left[\Lambda_0 - \phi^\mu\partial_\mu\Lambda_0 - \sqrt{2}\zeta\Lambda_1\right]\Big|_\infty = K_0\,,\\
&\left[\Lambda_1 - \phi^\mu\partial_\mu\Lambda_1 - \sqrt{2}\zeta\Lambda_2 + \tfrac{\sqrt{2}}{2}y_1^\mu(\zeta)\partial_\mu\Lambda_0\right]\Big|_\infty = 0\,,\\
&\left[\Lambda_2 - \phi^\mu\partial_\mu\Lambda_2 - \tfrac{\sqrt{2}}{2}y_1^\mu(\zeta)\partial_\mu\Lambda_1 - \sqrt{2}\Gamma_2(\zeta)\Lambda_1 - Tr(\Gamma_1(\zeta))\Lambda_2 - y_2^\mu(\zeta)\partial_\mu\Lambda_0\right]\Big|_\infty = K_2\,,
\end{aligned}$$

where the following notation is used for conciseness:

$$\begin{aligned}
y^\mu_{1\alpha}(\zeta) =& 2\,i\left(\sigma^\mu\bar{\zeta}\right)_\alpha\,,\\
y_2^\mu(\zeta) =& \bar{\psi}_\nu\bar{\sigma}^\mu\sigma^\nu\bar{\zeta}\,,\\
\Gamma^\alpha_{1\beta}(\zeta) =& -i\left(\sigma^\mu\bar{\zeta}\right)_\beta\psi^\alpha_\mu\,,\\
\Gamma_2^\alpha(\zeta) =& -i\,\omega_\mu^{\alpha\beta}\left(\sigma^\mu\bar{\zeta}\right)_\beta + \tfrac{1}{3}M^*\zeta^\alpha - \tfrac{1}{2}\psi^\alpha_\nu\left(\bar{\psi}_\mu\bar{\sigma}^\nu\sigma^\mu\bar{\zeta}\right) + \tfrac{1}{6}b_\mu\left(\varepsilon\sigma^\mu\bar{\zeta}\right)^\alpha\,.
\end{aligned}$$

Cosmological Constant and Boundary Conditions

We are now interested in the backgrounds allowed by our model, in particular the range of values taken by the cosmological constant.

Upon computing the equations of motion, we find that they admit the solution

$$
\begin{aligned}
\langle g_{\mu\nu}\rangle &= \bar{g}_{\mu\nu}, \quad \text{with} \quad \sqrt{-\bar{g}} = 1\,,\\
\langle M\rangle &= m\,,\\
\langle \Lambda_0\rangle &= \tfrac{2}{3}m = K_0\,,\\
\langle \Lambda_2\rangle &= \mathbf{\Lambda}_2 = K_2, \quad \text{with} \quad \mathrm{Im}(\Lambda_2) = 0\,,
\end{aligned}
$$

with all other fields vanishing. The cosmological constant is

$$
c.c. = \mathbf{\Lambda}_2 - \tfrac{1}{3}m^2 \;=\; K_2 - \tfrac{3}{4}K_0^2\,.
$$

Cosmological constant can have either sign.

Comparing the two dS Supergravity Actions

dS SUGRA action obtained via passive transformation in super Stückelberg trick [S Bansal, S Nagy, A Padilla, I Zavala '20]:

$$S = -\frac{1}{\kappa^2}\int d^4x\, d^2\Theta \left[\mathcal{E}\left(6\mathcal{R} + \tfrac{2}{3}m\right) + \hat{\Lambda}\left(\mathcal{E} - \mathrm{Ber}\left(\frac{\partial \Phi}{\partial X}\right)\mathcal{E}_0\left(\Phi\right)\right)\right] + h.c.$$

where $\hat{\Lambda} = \Lambda - \frac{2m}{3}$.

dS SUGRA action obtained using nilpotent superfield and goldstino brane [I Bandos, L Martucci, D Sorokin, M Tonin '15]:

$$S_B = -\frac{1}{\kappa^2}\int d^6\zeta_L\, \mathcal{E}\left(6\mathcal{R} + \tfrac{2}{3}m + \mathrm{h.c.}\right) - f^2\int d^4\xi\, \det\mathbb{E}(z(\xi))$$

The green terms are pure AdS SUGRA action and the black terms give positive contribution to the cosmological constant.

Unimodular Supergravity at Cubic Order

The fluctuations of the fields around the background solution are:

$$\begin{aligned}
g_{\mu\nu} &= \overline{g}_{\mu\nu} + h_{\mu\nu}\,, \\
\omega_\mu{}^{\alpha\beta} &= \boldsymbol{\omega}_\mu{}^{\alpha\beta} + \omega_\mu^{(1)\alpha\beta}\,, \\
\psi_\mu^\alpha &= 0 + \psi_\mu^\alpha\,, \\
b_\mu &= 0 + b_\mu\,, \\
M &= m + M\,, \\
\Lambda_0 &= \tfrac{2}{3} m + \lambda_0\,, \\
\Lambda_1 &= 0 + \lambda_1\,, \\
\Lambda_2 &= \boldsymbol{\Lambda}_2 + \lambda_2, \qquad \mathrm{Im}(\boldsymbol{\Lambda}_2) = 0\,, \\
\phi^\mu &= 0 + t^\mu\,, \\
\zeta &= 0 + \chi\,.
\end{aligned}$$

We perturb the unimodular supergravity action around the background solution.

Action Perturbed up to 2^{nd} Order

We look at the fermionic part of the action.

$$S_f = \frac{1}{2\kappa^2}\int d^4x\Big[\varepsilon^{\mu\nu\rho\sigma}\left(\bar{\psi}_\mu\bar{\sigma}_\nu \boldsymbol{D}_\rho\psi_\sigma - \psi_\mu\sigma_\nu\boldsymbol{D}_\rho\bar{\psi}_\sigma\right) + \frac{2}{3}m\bar{\psi}_\mu\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu + \frac{2}{3}m\psi_\mu\sigma^{\mu\nu}\psi_\nu$$
$$+ i\boldsymbol{\Lambda}_2\left(\chi + \frac{\sqrt{2}}{2\boldsymbol{\Lambda_2}}\lambda_1\right)\sigma^\mu\bar{\psi}_\mu - i\boldsymbol{\Lambda}_2\,\psi_\mu\sigma^\mu\left(\bar{\chi} + \frac{\sqrt{2}}{2\boldsymbol{\Lambda_2}}\bar{\lambda}_1\right)$$
$$+ i\sqrt{2}\mathcal{D}_\mu\lambda_1\sigma^\mu\bar{\chi} - i\sqrt{2}\chi\sigma^\mu\mathcal{D}_\mu\bar{\lambda}_1 + \frac{2\sqrt{2}}{3}m\lambda_1\chi + \frac{2\sqrt{2}}{3}m\bar{\lambda}_1\bar{\chi}$$

So we identify the goldstino, denoted by $\mathcal{G}$, with the combination

$$\mathcal{G} = \frac{1}{2}\left(\chi + \frac{\sqrt{2}}{2\boldsymbol{\Lambda_2}}\lambda_1\right),$$

and the orthogonal mode to $\mathcal{G}$, denoted by τ, as

$$\tau = \frac{1}{2}\left(\chi - \frac{\sqrt{2}}{2\boldsymbol{\Lambda_2}}\lambda_1\right).$$

Action Perturbed up to 2^{nd} Order

The action becomes:

$$S_f = \frac{1}{2\kappa^2} \int d^4x \Big[\varepsilon^{\mu\nu\rho\sigma} \left(\bar{\psi}_\mu \bar{\sigma}_\nu \boldsymbol{D}_\rho \psi_\sigma - \psi_\mu \sigma_\nu \boldsymbol{D}_\rho \bar{\psi}_\sigma \right) + \tfrac{2}{3} m \bar{\psi}_\mu \bar{\sigma}^{\mu\nu} \bar{\psi}_\nu + \tfrac{2}{3} m \psi_\mu \sigma^{\mu\nu} \psi_\nu$$
$$+ 2i\boldsymbol{\Lambda}_2 \mathcal{G} \sigma^\mu \bar{\psi}_\mu - 2i\boldsymbol{\Lambda}_2 \psi_\mu \sigma^\mu \bar{\mathcal{G}} - 4i\boldsymbol{\Lambda}_2 \mathcal{G} \sigma^\mu \mathcal{D}_\mu \bar{\mathcal{G}} + \tfrac{4}{3} m \boldsymbol{\Lambda}_2 \mathcal{G}^2$$
$$+ \tfrac{4}{3} m \boldsymbol{\Lambda}_2 \bar{\mathcal{G}}^2 + 4i\boldsymbol{\Lambda}_2 \tau \sigma^\mu \mathcal{D}_\mu \bar{\tau} - \tfrac{4}{3} m \boldsymbol{\Lambda}_2 \tau^2 - \tfrac{4}{3} m \boldsymbol{\Lambda}_2 \bar{\tau}^2 \Big]$$

The boundary conditions on the Lagrange multiplier superfield are now:

$$\lambda_0 \Big|_\infty = 0, \quad \left(\lambda_1 - \sqrt{2} \boldsymbol{\Lambda}_2 \chi \right) \Big|_\infty = 0, \quad \lambda_2 = 0 \Big|_\infty$$

So τ gets eliminated leaving us with

$$S_f = \frac{1}{2\kappa^2} \int d^4x \Big[\varepsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu \bar{\sigma}_\nu \boldsymbol{D}_\rho \psi_\sigma + \tfrac{2}{3} m \bar{\psi}_\mu \bar{\sigma}^{\mu\nu} \bar{\psi}_\nu + 2i\boldsymbol{\Lambda}_2 \mathcal{G} \sigma^\mu \bar{\psi}_\mu - 2i\boldsymbol{\Lambda}_2 \mathcal{G} \sigma^\mu \mathcal{D}_\mu \bar{\mathcal{G}}$$
$$+ \tfrac{4}{3} m \boldsymbol{\Lambda}_2 \mathcal{G}^2 + h.c. \Big]$$

which is the same as the fermionic part of the action with goldstino brane.

Boundary Conditions and Field Redefinitions

Diffeo and SUSY transformations are perturbed up to the 2nd order in fluctuations.

Boundary conditions perturbed up to the 2nd order:

$$\left[\lambda_0 - t^\mu \partial_\mu \lambda_0 + \mathbf{\Lambda}_2(-\mathcal{G}^2 + 2\,\mathcal{G}\tau + 3\,\tau^2)\right]|_\infty = 0\,,$$

$$\left[\tau + \frac{1}{4} t^\mu \partial_\mu(\mathcal{G} - 3\tau) + \tfrac{1}{2\mathbf{\Lambda_2}}\lambda_2(\mathcal{G} + \tau) + \tfrac{i}{2\mathbf{\Lambda_2}}\sigma^\mu(\bar{\mathcal{G}} + \bar{\tau})\partial_\mu\lambda_0\right]|_\infty = 0\,,$$

$$\Big[\lambda_2 - t^\mu \partial_\mu \lambda_2 + 2i\mathbf{\Lambda}_2\sigma^\mu(\bar{\mathcal{G}} + \bar{\tau})\partial_\mu(\mathcal{G} - \tau) - i\mathbf{\Lambda}_2\psi_\mu{}^\alpha(\sigma^\mu(\bar{\mathcal{G}} + \bar{\tau}))_\alpha$$
$$+2\,i\mathbf{\Lambda}_2\,\boldsymbol{\omega}_\mu{}^\alpha(\sigma^\mu(\bar{\mathcal{G}} + \bar{\tau}))_\alpha(\mathcal{G} - \tau) + \frac{m\mathbf{\Lambda}_2}{3}(-\mathcal{G}^2 + 2\,\mathcal{G}\tau + 3\,\tau^2 + 2\,(\bar{\mathcal{G}} + \bar{\tau})^2)\Big]|_\infty = 0\,.$$

By redefining the fields the above conditions can be expressed simply as:

$$r_0|_\infty = 0, \quad \rho_1|_\infty = 0, \quad r_2|_\infty = 0\,.$$

Boundary Conditions and Field Redefinitions

$$
\begin{aligned}
S = \frac{1}{2\kappa^2} \int d^4x \{ [(& \sqrt{-g}\, [R - \tfrac{2}{3} M^* M + \tfrac{2}{3} b^\mu b_\mu + \varepsilon^{\mu\nu\eta\lambda} (\bar\psi_\mu \bar\sigma_\nu \mathcal{D}_\eta \psi_\lambda - \psi_\mu \sigma_\nu \mathcal{D}_\eta \bar\psi_\lambda)] \}^{(3)} \\
& + \{ - (2\sqrt{-g}\, (\Lambda_2 - \tfrac{2}{3}\, m^2))^{(3)} + \tfrac{2}{3} m \psi_\mu \sigma^{\mu\nu} \psi_\nu + 2i\Lambda_2 \mathcal{G} \sigma^\mu (\bar\psi_\mu - \partial_\mu \bar{\mathcal{G}}) + \tfrac{4}{3} m \Lambda_2 (\mathcal{G}^2 - \rho_1^2) \\
& + 2i\Lambda_2 \rho_1 \sigma^\mu \partial_\mu \bar\rho_1 + (-m r_0 + r_2) \left(\partial_\nu t^\nu - \tfrac{1}{2} h \right) + r_0 M^* + i\Lambda_2 [(\mathcal{G} - 3\rho_1) \partial_\nu (t^\nu \sigma^\mu \partial_\mu \bar\rho_1) \\
& - \partial_\nu (t^\nu \partial_\mu (\mathcal{G} - \rho_1)) \sigma^\mu (\bar{\mathcal{G}} + \bar\rho_1) + t^\nu \partial_\nu (\mathcal{G} - 3\rho_1) \sigma^\mu \bar{\boldsymbol\omega}_\mu \bar\rho_1 + \tfrac{2}{\Lambda_2} r_2 (\mathcal{G} + \rho_1) \sigma^\mu \bar{\boldsymbol\omega}_\mu \bar\rho_1 \\
& - \tfrac{2i}{\Lambda_2} (\mathcal{G} + \rho_1) \bar\sigma^\nu \partial_\nu r_0 \sigma^\mu \bar{\boldsymbol\omega}_\mu \bar\rho_1] + i r_2 (\mathcal{G} + \rho_1) \sigma^\mu \partial_\mu (\bar{\mathcal{G}} - \bar\rho_1) + i r_2 \partial_\mu (\mathcal{G} + \rho_1) \sigma^\mu (\bar{\mathcal{G}} + \bar\rho_1) \\
& + (\mathcal{G} + \rho_1) \bar\sigma^\mu \partial_\mu r_0 \sigma^\nu \partial_\nu (\bar{\mathcal{G}} - \bar\rho_1) + \tfrac{2}{3} m \Lambda_2 [\partial_\mu t^\mu (\tfrac{1}{2} \mathcal{G}^2 + 3\rho_1^2 + \mathcal{G}\rho_1) - \partial_\mu \mathcal{G} t^\mu \rho_1] \\
& - \tfrac{2}{3} m r_2 [(\mathcal{G}^2 - \rho_1^2) - (\bar{\mathcal{G}} + \bar\rho_1)^2] - \tfrac{1}{3} m i (\mathcal{G} - 3\rho_1) \sigma^\mu (\bar{\mathcal{G}} + \bar\rho_1) \partial_\mu r_0 - M^* r_0 \left(\partial_\nu t^\nu - \tfrac{1}{2} h \right) \\
& - \tfrac{2}{3} m \Lambda_2 (\mathcal{G} + \rho_1)^2\, \partial_\nu t^\nu + \tfrac{2}{3} m h \Lambda_2 (\mathcal{G}^2 - \rho_1^2) + i\Lambda_2 \partial_\mu (\mathcal{G} - \rho_1) (h \sigma^\mu - h^\mu{}_\nu \sigma^\nu) (\sigma^\mu (\bar{\mathcal{G}} + \bar\rho_1)) \\
& - \tfrac{1}{2} h t^\mu \partial_\mu r_2 + i h \Lambda_2 \mathcal{G} \sigma^\mu \bar\psi - \tfrac{i}{2} \Lambda_2 t^\mu \psi_\rho \sigma^\rho \partial_\mu (\bar{\mathcal{G}} + \bar\rho_1) + \tfrac{2}{3} \Lambda_2 (\mathcal{G}^2 - 2\,\mathcal{G}\rho_1 - 3\,\rho_1^2) M^* \\
& - t^\mu r_0 \partial_\mu M^* - \tfrac{1}{2} m t^\mu \partial_\mu r_0 (\partial_\nu t^\nu - h) + \tfrac{1}{2} (\tfrac{1}{2} h^2 - h_{\mu\nu} h^{\mu\nu}) (\tfrac{1}{3} m M^* + \tfrac{1}{2} r_0 m - \tfrac{1}{2} r_2) \\
& + i\Lambda_2 \psi_\mu h^\mu{}_\nu \sigma^\nu \bar{\mathcal{G}} + (r_0 + \tfrac{1}{3} h m) \bar\psi_\mu \bar\sigma^{\mu\nu} \bar\psi_\nu + \tfrac{2}{3}\, m \bar\psi_\mu h^{[\mu}{}_\rho \bar\sigma^{\nu]\rho} \bar\psi_\nu + r_0\, \partial_\mu [\bar\psi_\nu \bar\sigma^\mu \sigma^\nu (\bar{\mathcal{G}} + \bar\rho_1)] \\
& - i r_2 \psi_\mu \sigma^\mu (\bar{\mathcal{G}} + \bar\rho_1) - 2\,\Lambda_2 (\mathcal{G} - \rho_1) [\tfrac{i}{2}\, \boldsymbol\omega_\mu h^\mu{}_\rho \sigma^\rho (\bar{\mathcal{G}} + \bar\rho_1) - i \omega_\mu^{(1)} \sigma^\mu (\bar{\mathcal{G}} + \bar\rho_1)] \\
& - \tfrac{1}{2} r_2\, \partial_\mu (t^\mu \partial_\nu t^\nu) - \tfrac{1}{6} \Lambda_2 b_\mu \sigma^\mu (\mathcal{G}\bar{\mathcal{G}} + \mathcal{G}\bar\rho_1 - 3\rho_1 \bar{\mathcal{G}} - 3\rho_1 \bar\rho_1) + \tfrac{2}{3} \Lambda_2 M (\bar{\mathcal{G}} + \bar\rho_1)^2 \\
& + i\Lambda_2 (\mathcal{G} - \rho_1) [\partial_\mu (t^\mu \boldsymbol\omega_\rho \sigma^\rho (\bar{\mathcal{G}} + \bar\rho_1)) + \partial_\mu t^\mu \boldsymbol\omega_\rho \sigma^\rho (\bar{\mathcal{G}} + \bar\rho_1)] \\
& - i\Lambda_2 (\mathcal{G} - \rho_1) t^\nu \partial_\nu (\boldsymbol\omega_\mu \sigma^\mu) (\bar{\mathcal{G}} + \bar\rho_1) - r_0 \partial_\mu [(\mathcal{D}_\nu (\bar{\mathcal{G}} + \bar\rho_1)) \bar\sigma^\mu \sigma^\nu (\bar{\mathcal{G}} + \bar\rho_1) \\
& - (\bar{\mathcal{G}} + \bar\rho_1) \bar\sigma^\mu \boldsymbol\omega_\nu \sigma^\nu (\bar{\mathcal{G}} + \bar\rho_1)] + h.c. \} \,.
\end{aligned}
$$

[S Bansal, S Nagy, A Padilla, I Zavala '20]

Goldstino Dynamics

To zoom in on the goldstino dynamics in the action, we integrate out the auxiliary fields b_μ, M, M^* and the Lagrange multipliers r_0, r_0^*, ρ_1, $\bar{\rho}_1$, r_2 and r_2^*.

We perform the following field redefinitions:

$$
\begin{aligned}
b_\mu &= \mathcal{B}_\mu - \tfrac{1}{4}\mathcal{B}_\mu h + \tfrac{1}{4}\mathbf{\Lambda}_2\mathcal{G}\sigma_\mu\bar{\mathcal{G}} - \tfrac{1}{4}\mathbf{\Lambda}_2\rho_1\sigma_\mu\bar{\mathcal{G}} - \tfrac{1}{4}\mathbf{\Lambda}_2\mathcal{G}\sigma_\mu\bar{\rho}_1 - \tfrac{3}{4}\mathbf{\Lambda}_2\rho_1\sigma_\mu\bar{\rho}_1\,,\\
M &= \mathcal{M} - \tfrac{1}{4}h\mathcal{M} + 2\mathbf{\Lambda}_2\mathcal{G}^2 - 2\mathbf{\Lambda}_2\rho_1^2 + \tfrac{3}{2}t^\mu\partial_\mu + \tfrac{3}{2}r_0\,,\\
r_2 &= \mathcal{R}_2 + mr_0\,.\\
r_0 &= \mathcal{R}_0 - \tfrac{1}{2}\left(\tfrac{1}{2}h - \partial_\mu t^\mu\right)\mathcal{R}_0 - \tfrac{2}{3}\hat{F}_2[r_0^*]\,,\\
\mathcal{G} &= \breve{\mathcal{G}} - \tfrac{1}{4}t^\mu\partial_\mu(\breve{\mathcal{G}} + \rho_1) - \tfrac{1}{2\mathbf{\Lambda}_2}\mathcal{R}_2(\breve{\mathcal{G}} + \rho_1)\,,\\
\rho_1 &= \mathcal{P}_1 - t^\mu\partial_\mu\mathcal{P}_1\,.\\
\tilde{t}^\mu &= t^\mu + \frac{1}{2}ht^\mu - t^\mu\partial_\nu t^\nu\,.
\end{aligned}
$$

Goldstino Dynamics

Then solving the equations of motion of $\mathcal{B}_\mu, \mathcal{M}, \mathcal{R}_0, \tilde{t}^\mu$ and $\mathcal{P}_1$ and using the boundary conditions, we get the following solutions:

$$\begin{aligned} \mathcal{B}_\mu &= 0 + \mathcal{O}(3) \\ \mathcal{M} &= 0 + \mathcal{O}(3) \\ \mathcal{R}_0 &= 0 + \mathcal{O}(3) \\ Re[\mathcal{R}_2] &= 0 \\ \mathcal{P}_1 &= 0 + \mathcal{O}(3) \end{aligned}$$

We finally arrive at the following action:

$$\begin{aligned} S = \frac{1}{2\kappa^2} \int d^4x \, \Big[\big\{ \sqrt{-g}\, R - \varepsilon^{\mu\nu\rho\lambda}(\psi_\mu \sigma_\nu \mathcal{D}_\rho \bar{\psi}_\lambda + h.c.) \big\}^{(3)} - \Big\{ 2\sqrt{-g} \left(\mathbf{\Lambda}_2 - \frac{m^2}{3} \right) \Big\}^{(3)} \\ + \big\{ \tfrac{2}{3} m \psi_\mu \sigma^{\mu\nu} \psi_\nu + 2i \mathbf{\Lambda}_2 \breve{\mathcal{G}} \sigma^\mu \bar{\psi}_\mu - 2i \mathbf{\Lambda}_2 \breve{\mathcal{G}} \sigma^\mu \boldsymbol{D}_\mu \bar{\breve{\mathcal{G}}} + \tfrac{4}{3} m \mathbf{\Lambda}_2 \breve{\mathcal{G}}^2 \\ + \tfrac{1}{3} hm \psi_\mu \sigma^{\mu\nu} \psi_\nu + \tfrac{2}{3} m \psi_\mu h_\rho{}^{[\mu} \sigma^{\nu]\rho} \psi_\nu + \mathbf{\Lambda}_2 (ih \breve{\mathcal{G}} \sigma^\mu \bar{\psi}_\mu - i \breve{\mathcal{G}} h^\mu{}_\nu \sigma^\nu \bar{\psi}_\mu \\ - ih \breve{\mathcal{G}} \sigma^\mu \boldsymbol{D}_\mu \bar{\breve{\mathcal{G}}} + i \breve{\mathcal{G}} h^\mu{}_\nu \sigma^\nu \boldsymbol{D}_\mu \bar{\breve{\mathcal{G}}} + 2i \breve{\mathcal{G}} \sigma^\mu \bar{\omega}_\mu^{(1)} \bar{\breve{\mathcal{G}}} + \tfrac{2}{3} hm \breve{\mathcal{G}}^2) + h.c. \big\} \Big] \, . \end{aligned}$$

Same as Goldstino-brane action!

Absence of Pathological Terms

Possible extra term [Volkov, Soroka '73, '74] that didn't show up:

$$\sqrt{\tilde{e}}\,\tilde{g}^{\mu\nu}\tilde{\psi}_\mu\tilde{\psi}_\nu\,,$$

where the invariant vielbein and gravitino combinations are

$$\begin{aligned}\tilde{e}^a_\mu &= e^a_\mu + \tilde{\mathcal{D}}_\mu X^a + 2i\theta\sigma^a\bar{\psi}_\mu - 2i\psi_\mu\sigma^a\bar{\theta} + i\theta\sigma^a\tilde{\mathcal{D}}_\mu\bar{\theta} - i\tilde{\mathcal{D}}_\mu\theta\sigma^a\bar{\theta}\,,\\ \tilde{\psi}_\mu &= \psi_\mu + \tilde{\mathcal{D}}_\mu\theta\,.\end{aligned}$$

Ghost field?

Constructing Full Action: Active Diffeo-SUGRA Transformations

Infinitesimal Trasnformation:

$$\Lambda \to \Lambda' = \Lambda - K\Lambda + \frac{1}{2} K(K\Lambda) + \frac{1}{2} \delta^{(s)}(K\Lambda) + \ldots$$

where $K = K^M \partial_M$.
$\Lambda(X)$ transforms as a scalar, i.e. $\Lambda'(X') = \Lambda(X)$ with $(X^M)' = X^M + KX^M$.

It turns out we can define an operator for finite diffeo-SUGRA transformations.

We define a new operator $\hat{K} = K^M \partial_M + \delta^{(s)}$ which performs diffeo-SUGRA transformations on all the fields – SUGRA as well as Stückelberg fields.

Finite Trasnformation:

$$\Lambda \to \Lambda' = e^{-\hat{K}} \Lambda$$

Here $\Lambda(X)$ transforms as a scalar, i.e. $\Lambda'(X') = \Lambda(X)$ with $(X^M)' = e^{\hat{K}} X^M$.

Constructing Full Action: Active Diffeo-SUGRA Transformations

We promote the transformation parameters to Stückelberg fields:

$$K^M = (\xi^\mu + \eta^\mu(\epsilon), \eta^\alpha(\epsilon)) \quad \to \quad \varphi^M = (\phi^\mu + \eta^\mu(\zeta), \eta^\alpha(\zeta))$$
$$X^M(X, K) \quad \to \quad \Phi^M(X, \varphi)$$

The Stückelberg fields in turn transform as below, keeping the Stückelberged Lagrangian invariant:

Infinitesimal Trasnformation:

$$\delta\phi^\mu = -\xi^\mu + \frac{1}{2}\left[\zeta y_1^\mu(\epsilon) - \epsilon y_1^\mu(\zeta)\right] ,$$
$$\delta\zeta = -\epsilon + \frac{1}{2}\phi^\mu \partial_\mu \epsilon .$$

Finite Trasnformation:

$$\delta\Phi^M = \delta_K \Phi^M + \delta_{St}\Phi^M \quad \text{(work in progress)}$$

(Φ^M) is supposed to transform as a set of super-scalars, i.e. chiral multiplet:

$$\delta(\Phi^M) = -\hat{K}^N \partial_N (\Phi^M)$$

Constructing Full Action: Passive Diffeo-SUGRA Transformations

Infinitesimal Trasnformation:

$$X^M \to (X^M)' = X^M + K\,X^M + \tfrac{1}{2}\,K\,(K\,X^M) + \tfrac{1}{2}\,\delta^{(s)}(K\,X^M) + \ldots$$

Finite Trasnformation:

$$X^M \to e^{\hat{K}} X^M$$

where $\hat{K} = K^\mu \partial_\mu + \delta^{(s)}$.

We promote the transformation parameters to Stückelberg fields to get:

$$\begin{aligned} K^M = (\xi^\mu + \eta^\mu(\epsilon), \eta^\alpha(\epsilon)) \quad &\to \quad \varphi^M = (\phi^\mu + \eta^\mu(\zeta), \eta^\alpha(\zeta)) \\ (X^M)'(X, K) \quad &\to \quad \Phi^M(X, \varphi) \end{aligned}$$

At this point, we can construct the Lagrangian:

$$S = \frac{6}{8\pi G_N} \int d^4x d^2\Theta \left[\mathcal{E} R - 2\Lambda \left(\mathcal{E} - \mathrm{Ber}\left(\frac{\partial \Phi}{\partial X} \right) \mathcal{E}_0\left(\Phi\right) \right) \right] + h.c.$$

Constructing Full Action: Passive Diffeo-SUGRA Transformations

Superjacobian matrix:

$$sJ = \frac{\partial \Phi^M}{\partial X^N} = \begin{pmatrix} \frac{\partial \Phi^\mu}{\partial X^\nu} & \frac{\partial \Phi^\alpha}{\partial X^\nu} \\ \frac{\partial \Phi^\mu}{\partial X^\beta} & \frac{\partial \Phi^\alpha}{\partial X^\beta} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

We now need to compute the superdeterminant, i.e. the Berezinian, defined by

$$\mathrm{Ber}\,(sJ) = \mathrm{Det}\left(A - BD^{-1}C\right)\mathrm{Det}^{-1}(D)\,.$$

In components, the Berezinian superfield is given by

$$\mathrm{Ber}\,(sJ) = S_0 + \sqrt{2}\Theta S_1 + S_2\Theta^2$$

Henneaux-Teitelboim Formulation

Stückelberged unimodular supergravity Lagrangian:

$$S = \tfrac{6}{8\pi G_N} \int d^4x d^2\Theta \left[\mathcal{E} R - 2\Lambda\left(\mathcal{E} - \mathrm{Ber}\left(\frac{\partial \Phi}{\partial X}\right) \mathcal{E}_0\left(\Phi\right)\right)\right] + h.c.$$

Comparable with Henneaux-Teitelboim action in GR [M Henneaux and C Teitelboim '89], i.e.

$$S_{HT} = \int d^4x [\sqrt{-g}\,\frac{R}{16\pi G} - \lambda(\sqrt{-g} - \partial_\mu \tau^\mu)]$$

Henneaux-Teitelboim Formulation

Henneaux-Teitelboim action in GR:

$$S_{HT} = \int d^4x[\sqrt{-g}\,\frac{R}{16\pi G} - \lambda(\sqrt{-g} - \partial_\mu \tau^\mu)]$$

can also be written in terms of the 3-form $A_{\mu\nu\rho} = \epsilon_{\mu\nu\rho\sigma}\tau^\sigma$ dual to τ^σ, as following:

$$S_{HT} = \int d^4x[\sqrt{-g}\,\frac{R}{16\pi G} - \lambda\sqrt{-g} - A\,d\lambda]$$

The $|_0$ component of SUGRA chiral superfield $\mathcal{R}$ is M which we now express as

$$M = *F^{(4)} = *dC^{(3)}$$

Henneaux-Teitelboim Formulation

We propose:

$$\mathrm{Ber}\left(\frac{\partial\Phi^M}{\partial X^N}\right) = (-1)^M \partial_M T^M$$

for a set of superfields $T^M = (T^\mu, \tilde{T}^\alpha)$. In components:

$$\begin{aligned} T^\mu &= T_0^\mu + \sqrt{2}\,\Theta^\alpha T_{1\alpha}{}^\mu + T_2^\mu \Theta^2 \\ \tilde{T}^\alpha &= \tilde{T}_0^\alpha + \sqrt{2}\,\Theta^\beta \tilde{T}_{1\beta}{}^\alpha + \tilde{T}_2^\alpha \Theta^2 \end{aligned}$$

We have found

$$T^M = \tfrac{1}{2} x^\mu + \Phi^M + \tfrac{(-1)^N}{2}\,\Phi^M \frac{\partial\Phi^N}{\partial X^N} + \tfrac{1}{2}\,\Delta^{(s)}\eta^M + \eta^M + \ldots$$

T^M transforms as following:

$$\delta T^M = -2\,[\eta^N \partial_N T^M + (-1)^N \partial_N \eta^N T^M] - (-1)^N \eta^M \partial_N T^N + 2\,(-1)^{(N+MN)} \partial_N \eta^M T^N + G^M$$

where $\partial_M G^M = 0$.

Discussion

- If the full pure de Sitter actions obtained via different methods match up to all orders, it indicates the existence of an underlying comprehensive theory at higher energies, such as a string-theory model.
- The existence of such a string-theory model was corroborated in [I Bandos, M Heller, S Kuzenko, L Martucci, D Sorokin '16] where special versions of the pure dS SUGRA coupled to matter, were shown to match certain parts of the 4D effective action for an anti-D3-brane coming from the flux compactifications of 10D type IIB SUGRA.
- Find the $10D$ supergravity and string theory counterparts of this $4D$ pure dS supergravity, in search of dS vacua.
- It would also be worthwhile to use the pure dS SUGRA action for constructing phenomenological actions in inflationary cosmology.

Thank you!