Field-space Surprises in Multi-field Preheating

Evangelos Sfakianakis

DIAS, March 31, 2022

Supported by the “la Caixa” Foundation and EU’s Horizon 2020 programme under the Marie Skłodowska-Curie grant agreement

I. The Big Bang and Cosmic Inflation

$\Rightarrow$ From the Big Bang to inflation: successes and prospects

II. Reheating: A Critical Epoch

$\Rightarrow$ The necessity, importance of reheating. A big unknown of cosmic evolution

III. Realistic Reheating Scenarios

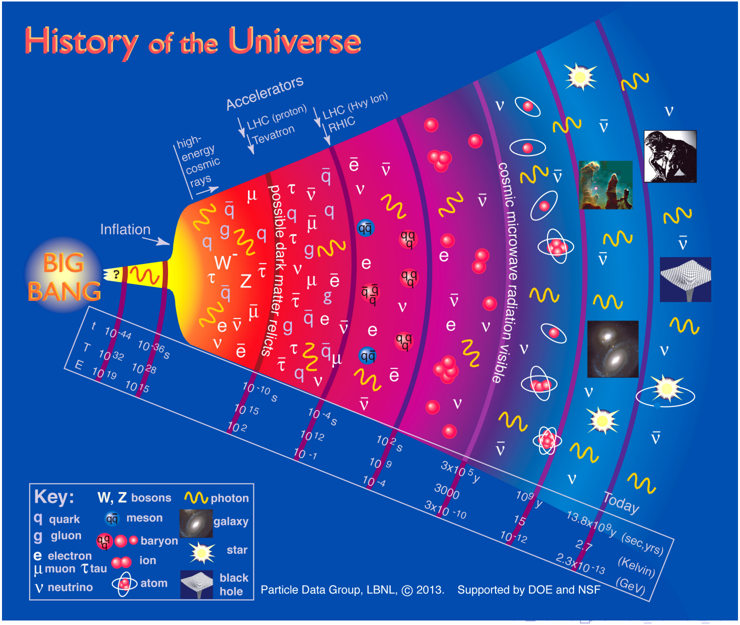
$\Rightarrow$ Multiple fields with non-canonical kinetic terms lead to drastically different behavior

I. The Big Bang and Cosmic Inflation
$\Rightarrow$ From the Big Bang to inflation: successes and prospects

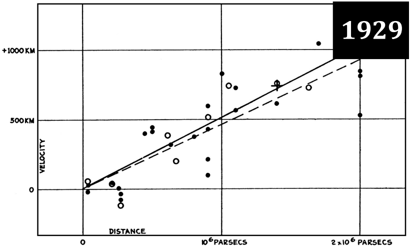
II. Reheating: A Critical Epoch
$\Rightarrow$ The necessity, importance of reheating. A big unknown of cosmic evolution

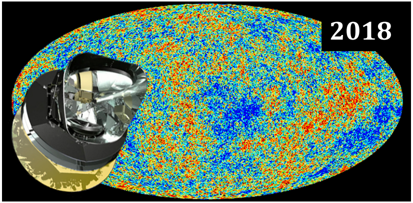
III. Realistic Reheating Scenarios
$\Rightarrow$ Multiple fields with non-canonical kinetic terms lead to drastically different behavior

The Big picture



Cosmology: modern tools for age-old questions



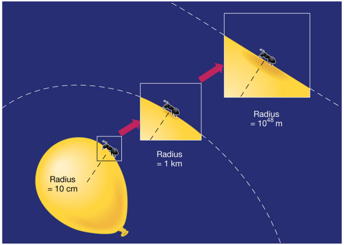


Inflation: two birds with one ... scalar field

PHYSICAL REVIEW D VOLUME 23, NUMBER 2 15 JANUARY 1981

Inflationary universe: A possible solution to the horizon and flatness problems

Alan H. Guth*
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 11 August 1980)

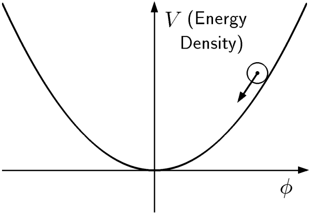


A scalar field called the "inflaton", slowly rolling down its potential.

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2(t)}\nabla^2\phi + \frac{\partial V}{\partial \phi} = 0$$

The motion is **dominated by a drag coefficient** caused by the expansion of the universe

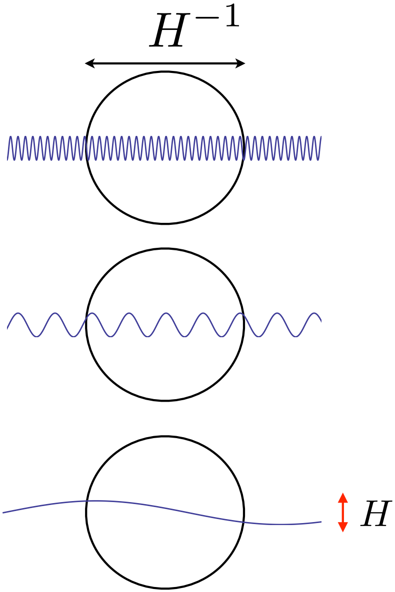
$$H(t) = \frac{\dot{a}(t)}{a(t)}.$$



fluctuations are always present in **quantum fields** and in the **metric** itself

they are stretched **beyond the horizon** by the expansion

they freeze out and become **classical fluctuations**: density perturbations & gravitational waves

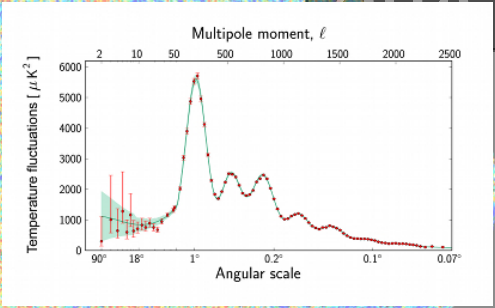


$$\mathcal{P} = A_s (k/k_*)^{n_s - 1}$$

2018

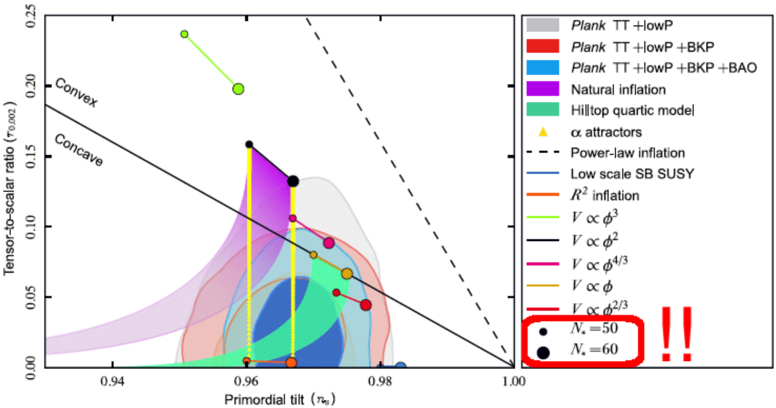
(Simple) Single field inflation:

- **Solves** horizon, flatness, monopole **problems**
- **Explains fluctuations** as stretched quantum mechanical perturbations
- Predicts a **nearly scale invariant** spectrum (of tunable amplitude)
- Predicts **Gaussian** perturbations



- Spectral index not flat by 5σ
- Spectral index running is small
- $|f_{NL}| \lesssim \mathcal{O}(1)$

Hints from the sky



Many models with **different motivation**.

⇓

They all share the **same uncertainty**.

I. The Big Bang and Cosmic Inflation

$\Rightarrow$ From the Big Bang to inflation: successes and prospects

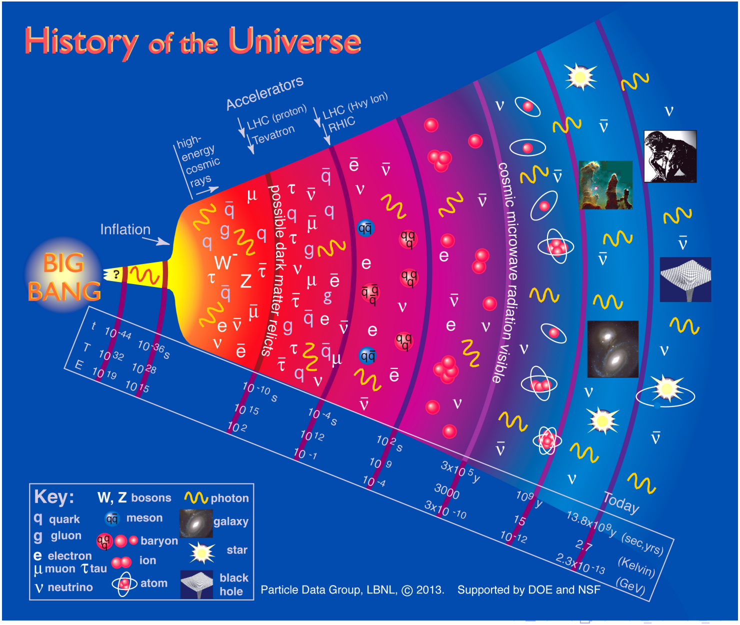
II. Reheating: A Critical Epoch

$\Rightarrow$ The necessity, importance of reheating. A big unknown of cosmic evolution

III. Realistic Reheating Scenarios

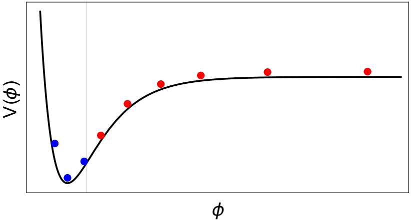
$\Rightarrow$ Multiple fields with non-canonical kinetic terms lead to drastically different behavior

Big picture reminder

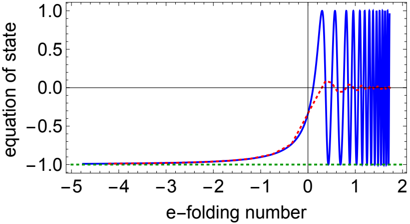


Inflation must end

- The inflaton rolls on a flat potential.
- The inflaton oscillates.



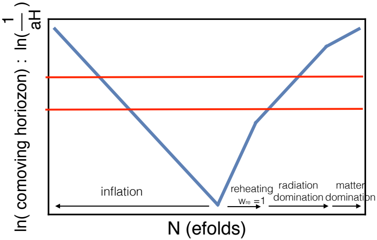
- During inflation: $p \simeq -\rho$
- After inflation: $V(\phi) \approx \frac{1}{2} m^2 \phi^2$ and $p \to 0$

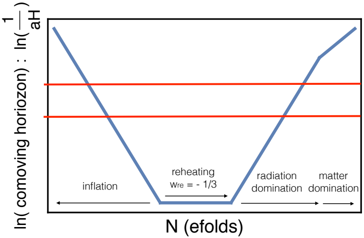


The inflaton **must** transfer its energy to radiative degrees of freedom, setting the stage for BBN.

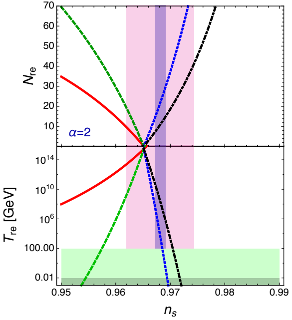
This process is called **reheating**.

Reheating effects





Cook et al. 2015



The **reheating history** connects the times of horizon exit & re-entry of perturbations $\Rightarrow$ **shifts CMB observables**

"The value of $\mathcal{N}_*$ is not well constrained and depends on unknown details of reheating"

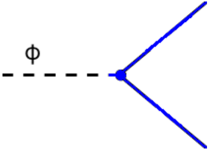
CMB-S4 Science Book, 2016

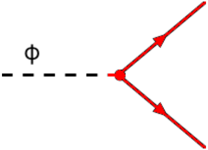
Perturbative reheating

Introduce couplings $\mu\phi\chi^2$ or $h\phi\bar{\psi}\psi$ and assume $m_\phi \gg m_\chi, m_\psi$

$$\Gamma_{\phi\to\chi\chi} = \frac{\mu^2}{8\pi m_\phi}$$

$$\Gamma_{\phi\to\bar{\psi}\psi} = \frac{h^2 m_\phi}{8\pi}$$





We can describe the decays as an extra friction term

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma\dot{\phi} + m^2\phi = 0$$

Reheating occurs for $\Gamma > H$.

Parametric resonance: preheating

Bose enhancement changes the game. Take $\mathcal{L} \subset -\frac{1}{2} g \phi^2 \chi^2$

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{a^2} + 2g\phi^2 \right) \chi_k = 0$$

Neglect the expansion $(H = 0)$ and take $\phi(t) = \Phi_0 sin(mt)$

$$\boxed{\ddot{\chi}_k + \left[k^2 + g\Phi_0^2 \sin^2(mt) \right] \chi_k = 0}$$

An equation of the form $\dot{x} = A(t)x$, where $A(t)$ is **periodic**, $A(t+T) = A(t)$, has solutions of the form

$$x(t) = c_1 P(t) e^{\mu t} + c_2 P(t) e^{-\mu t}$$

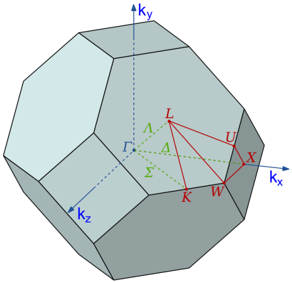
where μ is called the **Floquet exponent**.

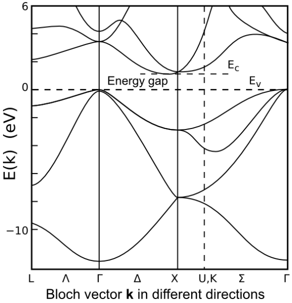
Solid-state analogue

In crystals the potential is **periodic in space** $V(\vec{x}) = V(\vec{x} + \vec{x}_0)$

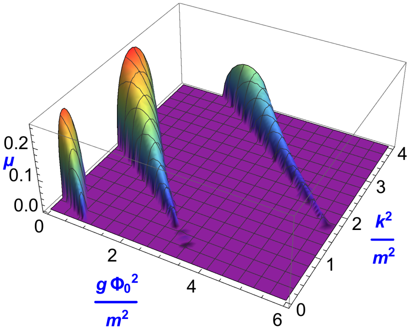
$$\Downarrow$$

The Schroedinger equation has solutions $\psi(x) \propto e^{\mu x}$
leading to **bands** and **band-gaps**





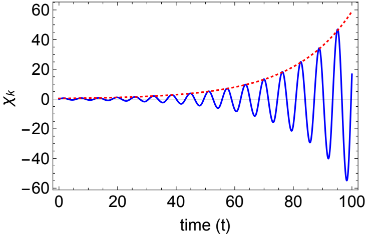
Floquet charts



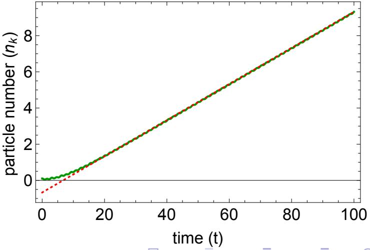
We can read off the regions where the Floquet chart leads to amplification $\chi_k(t) \sim e^{\mu_k t}$.

Kofman, Linde, Starobinsky [9704452]

Exponentially growing mode-function



and particle number



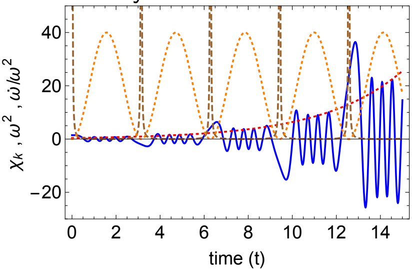
$$\ddot{\chi} + \omega^2(t)\chi = 0$$

For $\omega^2 \gg 1/T$ and $\frac{\dot{\omega}}{\omega^2} \ll 1$

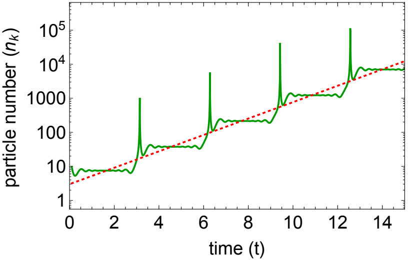
$$\chi \simeq \frac{1}{\sqrt{\omega}} \exp\left[\pm i \int \omega dt\right]$$

When the **adiabaticity condition** is violated, we get a sudden **burst of particle production**.

Adiabaticity violation:

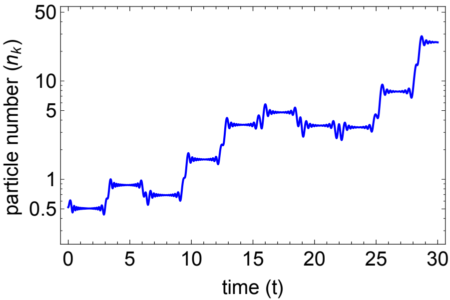


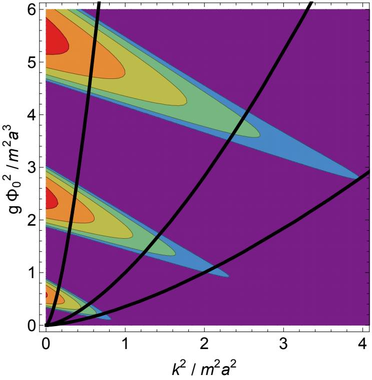
Particle production at $\dot{\omega}/\omega^2 \gg 1$



$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{a^2} + 2g\phi^2\right)\chi_k = 0$$

Stochastic resonance:

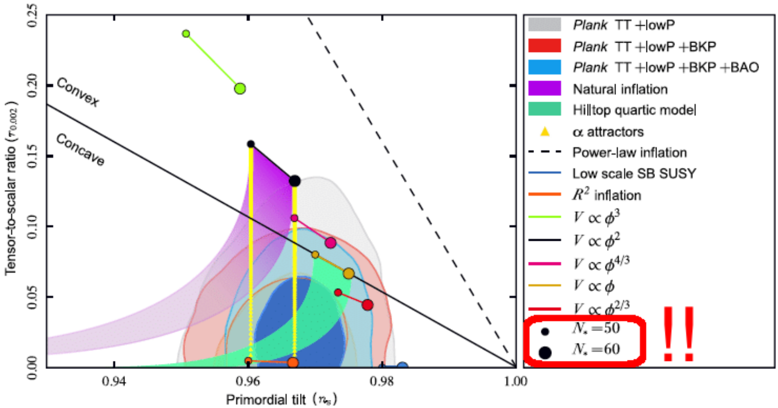




Quantitative differences and qualitative similarities

$\Rightarrow$ **Floquet theory is still useful**

Anticipating upcoming data



The time of horizon-exit is being constrained, begging for a **better understanding of reheating**.

Outline

I. The Big Bang and Cosmic Inflation

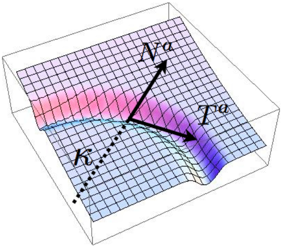
$\Rightarrow$ From the Big Bang to inflation: successes and prospects

II. Reheating: A Critical Epoch

$\Rightarrow$ The necessity, importance of reheating. A big unknown of cosmic evolution

III. Realistic Reheating Scenarios

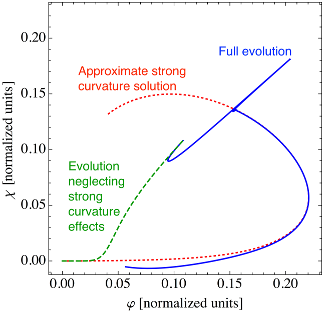
$\Rightarrow$ Multiple fields with non-canonical kinetic terms lead to drastically different behavior



At high energies, we expect

- multiple fields and
- more complicated couplings, e.g. $\mathcal{L} \subset f(\phi)(\partial\chi)^2 + \tilde{f}(\chi)(\partial\phi)^2$

leading to interesting inflationary dynamics.



During inflation, field-space features received significant attention (van Tent et al 2003, Achucarro et al 2010, ...).

Recent **novel trajectories** supported by field-space curvature reveal interesting connections to the Swampland program (a **whole other talk !**)

"Family tree" of this work

inflation (80's)

$\Downarrow$

preheating (late 90's)

$\Downarrow$

field-space effects (2000's - ...)

$\Downarrow$

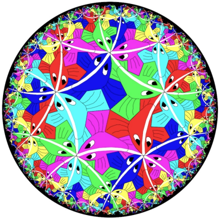
Higgs inflation (2008) + α-attractors (2010's)

$\Downarrow$

Field-space effects in multi-field preheating, focusing on Higgs(-like) inflation & α-attractors

Hyperbolic manifolds & α-attractors

Hyperbolic space on an "Escher disk"



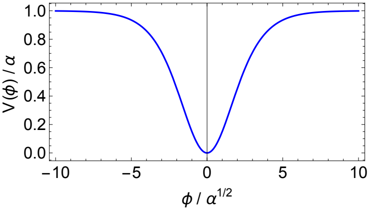
$$\mathcal{L} = \frac{\alpha}{2}\frac{(\partial r)^2 + r^2(\partial\theta)^2}{(1-r^2)^2} + V(r,\theta)$$
$$= \frac{1}{2}(\partial\Phi)^2 + \tilde{V}(\Phi) + [\ldots\theta\ldots]$$

where

$$V(r) = \frac{1}{2}m^2r^2 + \ldots \Rightarrow \tilde{V}(\Phi) \sim \tanh^2(\Phi/\sqrt{\alpha}) + \ldots$$

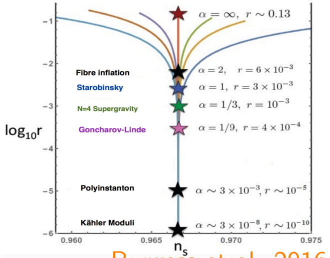
α-attractors lead to "universal" predictions

$$n_s \simeq 1 - \frac{2}{N}, \quad r \simeq \frac{12\alpha}{N^2}$$



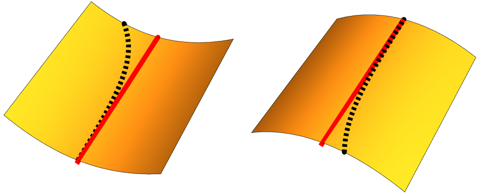
α-attractors and geodesics

- **String theory** compactifications: Fibre inflation
- **Supergravity**, e.g. E- and T-model

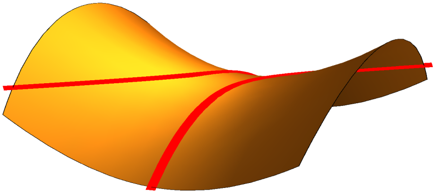


Burgess et al. 2016

"Usual" behavior for $m^2 > 0$ & $m^2 < 0$



Geodesics diverge, similarly to a $m^2 < 0$



Equations of motion

Background fields:

$$\mathcal{D}_t \dot{\phi}^I + 3H\dot{\phi}^I + \mathcal{G}^{IK} V_{,K} = 0$$

where $\mathcal{D}_t A^I \equiv \dot{A}^I$ for our choice of variables

Fluctuations:

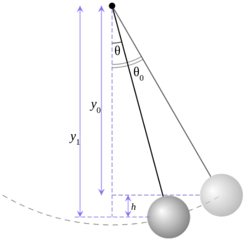
$$\ddot{Q}^I_k + 3H\dot{Q}^I_k + \left[\frac{k^2}{a^2}\delta^I_J + \mathcal{M}^I_J\right] Q^I_k = 0$$

where

$$\mathcal{M}^I_J = \mathcal{G}^{IK}\mathcal{D}_J\mathcal{D}_K V - \mathcal{R}^I_{LMJ}\dot{\phi}^L\dot{\phi}^M - \frac{1}{M_{Pl}^2 a^3}\mathcal{D}_t\left(\frac{a^3}{H}\dot{\phi}^I\dot{\phi}_J\right)$$

Effective Mass-squared: Ingredients

For motion **along a single-field attractor** ϕ, quantization is simple for the second field χ



$$\ddot{\chi}_k + 3H\dot{\chi}_k \left(\frac{k^2}{a^2} + m^2_{\mathrm{eff},\chi} \right) \chi_k = 0$$

$$\boxed{m^2_{\mathrm{eff},\chi} \simeq m^2_{1,\chi} + m^2_{2,\chi}}$$

$m^2_{1,\chi} \equiv \mathcal{G}^{\chi K} \left(\mathcal{D}_\chi \mathcal{D}_K V \right) \longleftrightarrow$ potential gradient – "traditional" mass

$$\boxed{m^2_{2,\chi} \equiv \frac{1}{2} \mathcal{R} \dot{\phi}^2 \longleftrightarrow \text{non-trivial field-space manifold}}$$

Complex fields in supergravity lead to the **2-field Lagrangian**

$$\mathcal{L} = -\frac{1}{2}\left(\partial_\mu \chi \partial^\mu \chi + e^{2b(\chi)} \partial_\mu \phi \partial^\mu \phi\right) - V(\phi, \chi)$$

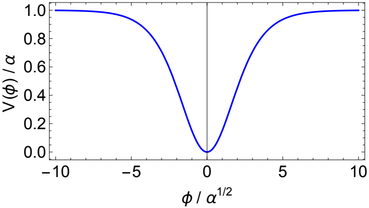
For single-field motion $\chi = 0$

$$V(\phi, \chi = 0) = \mu^2 \alpha \left| \tanh(\phi/\sqrt{6\alpha}) \right|^2$$

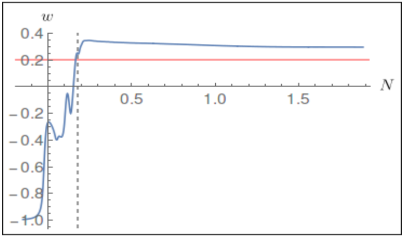
The **field-space Ricci scalar** is

$$\mathcal{R} = -\frac{4}{3\alpha}$$

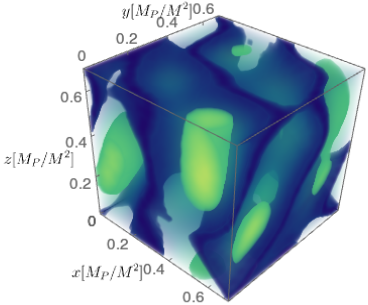
Smaller $\alpha \Rightarrow$ highly curved manifold

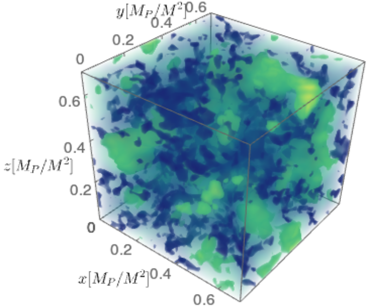


Lattice simulations

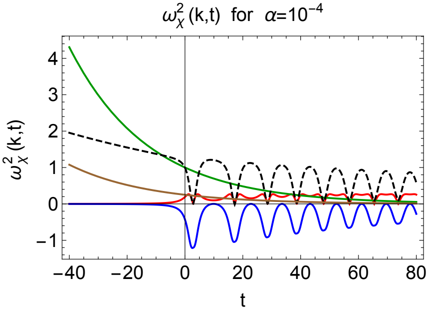


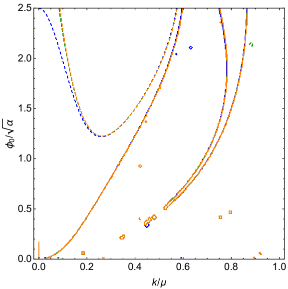
Lattice simulations
(Krajewski et al, 2018)
showed
very efficient preheating
for $\alpha \ll 1$





Effective frequency





$$m_{2,\chi}^2 = \frac{1}{2} R \left(\frac{d\phi}{dt}\right)^2 \propto -\frac{1}{\alpha} \times \left(\frac{\sqrt{\alpha}}{\mathcal{O}(1)}\right)^2 = -\mathcal{O}(1)$$

During each background oscillation
the χ field undergoes **tachyonic amplification**.

$\Rightarrow$ **Preheating is faster for larger curvature**.

The Standard Model Higgs boson as the inflaton

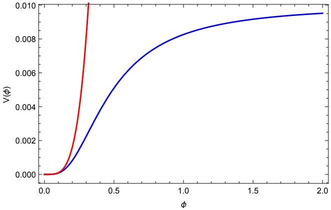
Fedor Bezrukov [a,b], Mikhail Shaposhnikov [a]

[a] *Institut de Théorie des Phénomènes Physiques, École Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland*

[b] *Institute for Nuclear Research of Russian Academy of Sciences, Prospect 60-letiya Oktyabrya 7a, Moscow 117312, Russia*

$$\mathcal{L} \subset \frac{1}{2} M_{\rm Pl}^2 R \to \frac{1}{2} M_{\rm Pl}^2 R + \xi H^\dagger H R \sim \frac{1}{2} M_{\rm Pl}^2 R + \xi h^2 R$$

The conformal transformation from the Jordan to the Einstein frame leads to a flat potential.



Multiple fields **necessarily** lead to

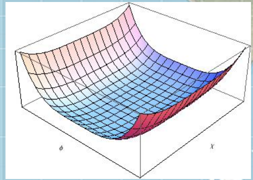
$$\mathcal{L} \subset \mathcal{G}_{IJ} \partial_\mu \phi^I \partial^\mu \phi^J \to \mathcal{R}_{\rm field-space}$$

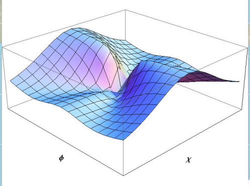
$$n_s \simeq 1 - \frac{2}{N}, \qquad r \simeq \frac{12}{N^2}$$

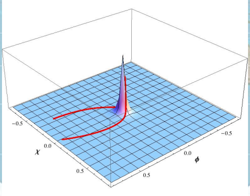
$$S_{\rm Jordan} = \int d^4x \sqrt{-\tilde{g}} \left[f(\phi^I)\tilde{R} - \frac{1}{2}\delta_{IJ}\tilde{g}^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J - \tilde{V}(\phi^I) \right]$$

$$\Downarrow \boxed{g_{\mu\nu}(x) \propto f(\phi^I(x))\, \tilde{g}_{\mu\nu}(x)} \Downarrow f(\phi^I) \subset \xi\phi^2$$

$$S_{\rm Einstein} = \int d^4x \sqrt{-g} \left[\frac{M_{\rm Pl}^2}{2} R - \frac{1}{2}\mathcal{G}_{IJ} g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J - V(\phi^I) \right]$$



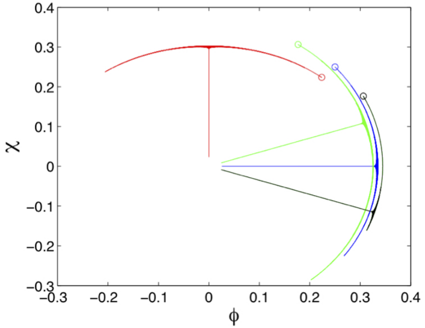




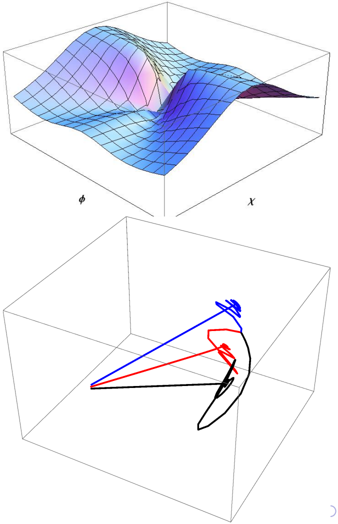
- $\frac{1}{2}\mathcal{G}_{IJ}\partial_\mu\phi^I\partial^\mu\phi^J$ leads to a **locally curved manifold**.
- **The potential has flat directions**, where inflation proceeds.

Non-minimal couplings and multiple fields

Non-minimal couplings $\mathcal{L} \subset \xi\phi^2 R$ are expected at high energies.
How does their existence affect multi-field models?



Kaiser & EIS 2014



- Non-minimal couplings lead to
  strong single-field attractors
- **Robust** Starobinsky-like **predictions**

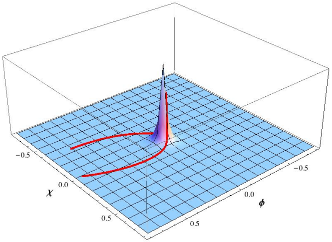
Effective Mass-squared reminder

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{a^2} + m^2_{\mathrm{eff},\chi}\right)\chi_k = 0$$

$$\boxed{m^2_{\mathrm{eff},\chi} \simeq m^2_{1,\chi} + m^2_{2,\chi}}$$

$$m^2_{1,\chi} \equiv \mathcal{G}^{\chi K}\left(\mathcal{D}_\chi \mathcal{D}_K V\right) \longleftrightarrow \text{potential gradient – “traditional” mass}$$

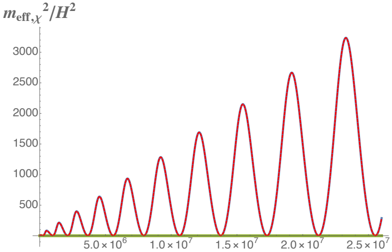
$$\boxed{m^2_{2,\chi} \equiv \frac{1}{2}\mathcal{R}\dot{\phi}^2 \longleftrightarrow \text{non-trivial field-space manifold}}$$



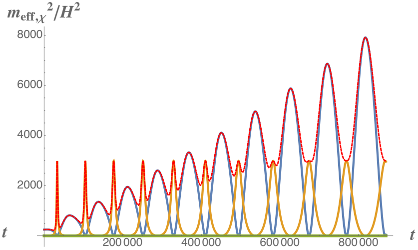
The field-space Ricci $\mathcal{R}$ “spikes” at the origin.

Effective Mass-squared for χ fluctuations

$\xi = 0.1 \ll 1$



$\xi = 10$

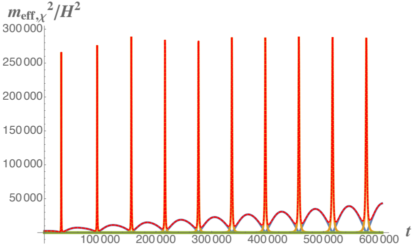


$$m_{\mathrm{eff}}^2 \approx m_1^2 + m_2^2 + m_3^3$$

$$m_{\mathrm{eff}}^2 \approx \textit{potential} + \textit{fieldspace} + \textit{metric}$$

Effective Mass-squared: $\xi \gg 1$

An "unusual" way for **adiabaticity violation** for $m_{2,\chi}^2 \propto \mathcal{R}\dot{\phi}^2$



We define

$$\mathcal{A} \equiv \frac{\Omega'}{\Omega^2}$$

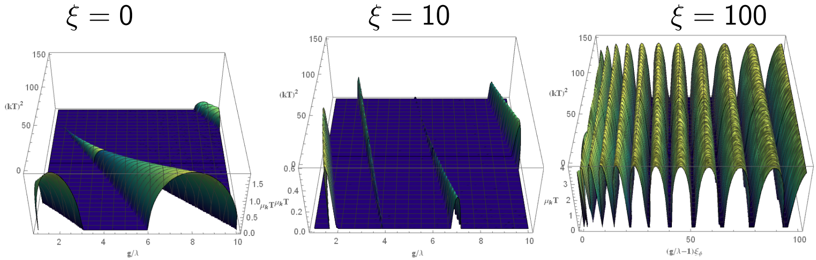
where

$$\Omega^2 = k^2 + a^2 m_{\mathrm{eff},\chi}^2$$

Adiabaticity is violated for $\Omega' \gg \Omega^2$, rather than $\Omega \approx 0$.

A broad range of wavenumbers is excited $k \lesssim \xi H_{\mathrm{end}}$

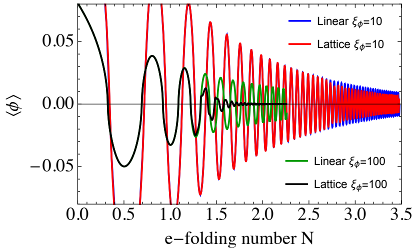
Linear analysis (VERY briefly)

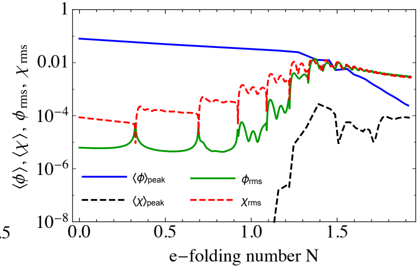


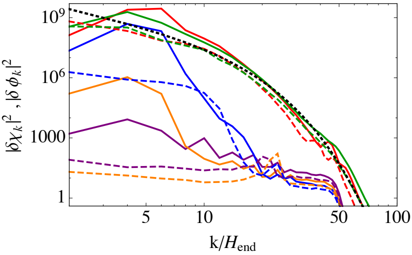
Dense instability bands hint at **efficient particle production**

Need for **lattice simulations**

Lattice results

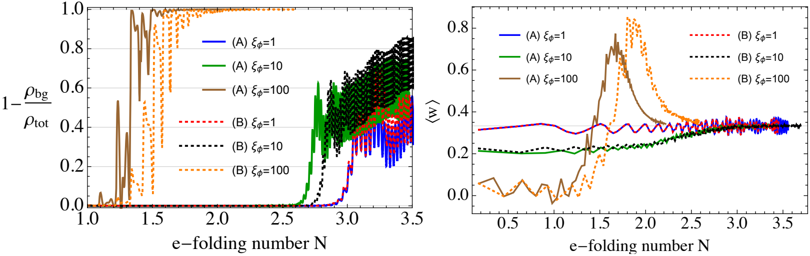






Non-minimal couplings quickly lead to a thermal radiation bath while preserving CMB predictions

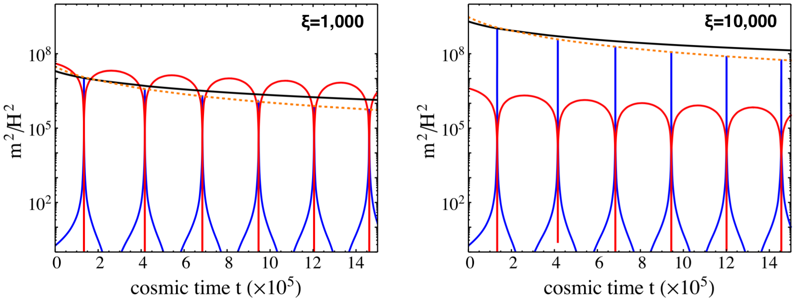
Energy transfer and e.o.s.



We see complete preheating
and a quick approach to radiation dominated expansion

Finally: Higgs inflation

Higgs inflation is a multi-field non-minimally coupled model with known SM couplings ⇒ the inflaton decays into W, Z bosons.

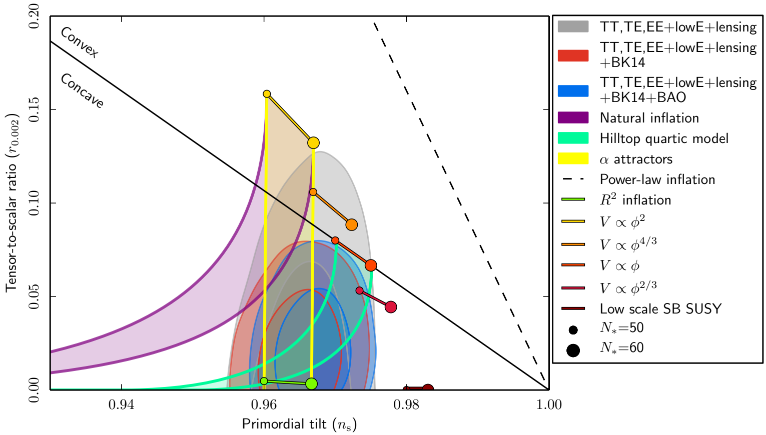


$$m_{\rm spike} \sim \xi H_{\rm end}$$

$$m_B \sim \frac{10^5}{\sqrt{\xi}} H_{\rm end}$$

For $\xi \gtrsim 10^3$ preheating completes within **ONE** oscillation

Thank you . . .



Understanding **preheating** in major plateau models
reduces theoretical **error-bars** of the $n_s - r$ plot
& allows for **comparison of Higgs inflation models**