Vacuum Phases of Gauge Theories

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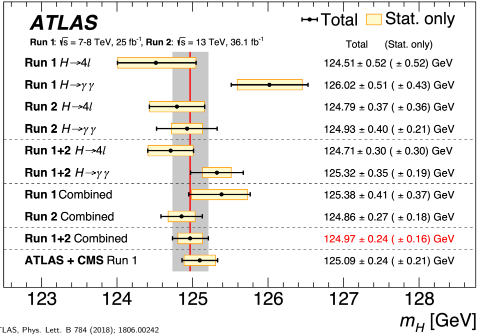
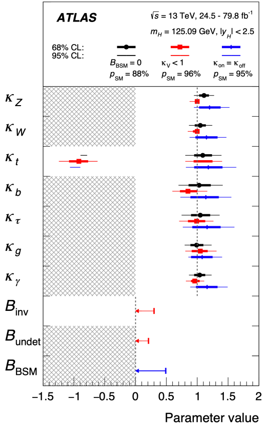
SM: Fantastically successful, glaringly incomplete

$m_h = 125$ GeV



ATLAS, Phys. Lett. B 784 (2018); 1806.00242

SM-like couplings



ATLAS, Phys. Rev. D 101 (2020); 1909.02845

SM: Fantastically successful, glaringly incomplete

- Origin of EW scale?
- Dark matter?
- Strong CP violation?
- Baryogenesis?
- Patterns of fermion masses?
- Why three generations?
- Why one Higgs doublet?

Beyond the Standard model?

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- Patterns of fermion masses?
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- Why one Higgs doublet?

Beyond the Standard model?

- How well do we understand gauge theory dynamics?
- i.e. given a theory, how does it look like at large length scales?
- i.e. what is its vacuum phase?

The bedrock: QCD vacuum

'QCD lite':

$$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \bar{q}_L(i\not{D})q_L + \bar{q}_R(i\not{D})q_R.$$

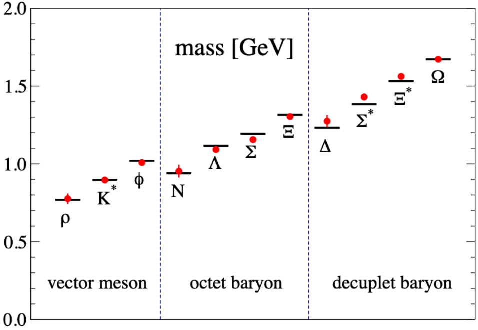
- $G_c = SU(3)$ gauge, $N_f = 3$ massless Dirac fermions.
- Asymptotic freedom. Weak coupling at short distances.
- $G = SU(3)_L \times SU(3)_R \times U(1)_V$ global symmetry.
- Vacuum condensate, strong coupling at large distances:

$$\langle \bar{q}^F_L q^{F'}_R \rangle \sim \Delta\delta^{FF'}$$

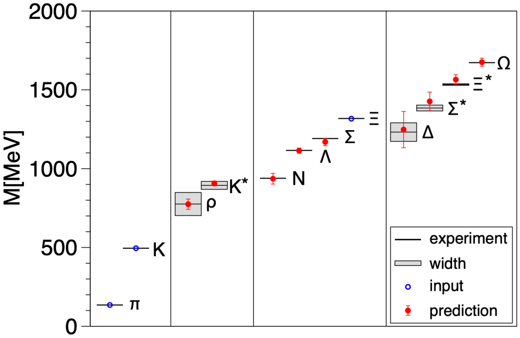
breaks G to $SU(3)_V \times U(1)_V$. Approximation for QCD with 3 light flavors.

The bedrock: QCD vacuum

Vacuum spectrum with 2+1 flavors:



Aoki *et al.* (2009) Phys. Rev. **D73**,



Durr *et al.* (2008) Science **322**.

The bedrock: QCD vacuum

In QCD all bits fall nicely together, but what happens for other G_c and G?

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We will focus on:

- G_c =SU(N_c).
- N_f massless Dirac fermions
- in single representation R of G_c.

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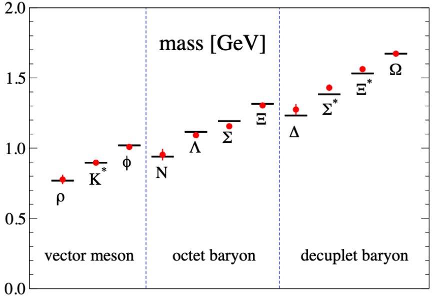
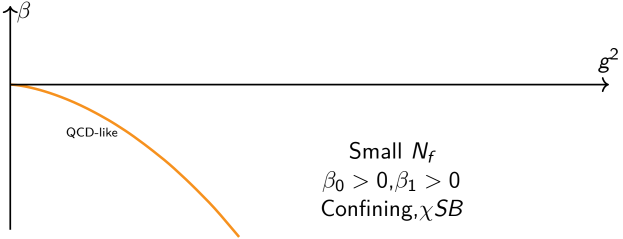
First, seek guidance from perturbation theory.

N_f -dependence: Small N_f

Strongly coupled in IR, asymptotically free in UV:





$$\beta(g) = -\frac{\beta_0}{(4\pi)^2} g^3 - \frac{\beta_1}{(4\pi)^4} g^5 + \ldots$$

Aoki *et al.* (2009) Phys. Rev. **D73**

N_f -dependence: Large (enough) N_f

The beta function:

$$\beta(g) = \mu \frac{dg}{d\mu} = -\frac{\beta_0}{(4\pi)^2} g^3 - \frac{\beta_1}{(4\pi)^4} g^5 + \ldots, \qquad \beta_0 = \frac{11}{3} C_2(G) - \frac{4}{3} T(R) N_f,$$

Interplay between screening (matter) and antiscreening (gauge).

e.g. $R =$Fund.: $\beta_0 = \frac{11}{3} N_c - \frac{2}{3} N_f$, so asymptotic freedom is lost above $N_f^{\mathrm{as}} = \frac{11}{2} N_c$.

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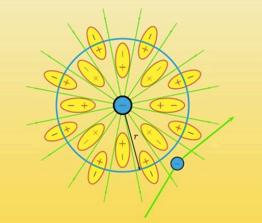
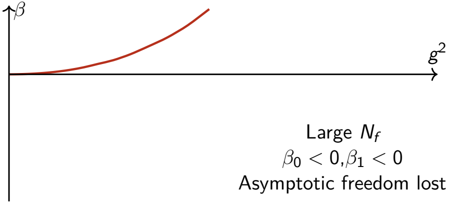
QED-like for $N_f > N_f^{\mathrm{as}}$

$$V(r) \sim \frac{1}{r \ln(r \Lambda_{\mathrm{UV}})}.$$





Large N_f
$\beta_0 < 0, \beta_1 < 0$
Asymptotic freedom lost

N_f -dependence: Intermediate N_f

The beta function: $\beta(g) = \mu\frac{dg}{d\mu} = -\frac{\beta_0}{(4\pi)^2}g^3 - \frac{\beta_1}{(4\pi)^4}g^5 + \ldots,$

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$\beta_0 > 0$ and $\beta_1 < 0$ between

$$N_f^{\mathrm{as}} = \frac{11 C_2(G)}{4 T(R)} > N_f > \frac{34 C_2(G)^2}{(20 C_2(G) + 12 C_2(R)) T(R)} = N_f^*.$$

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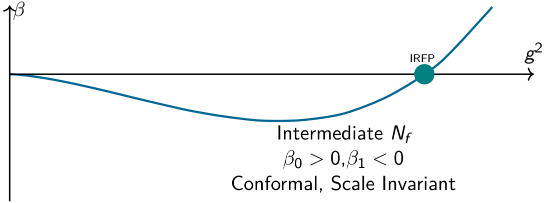
$\Rightarrow$ fixed point $g_*^2 = -\frac{\beta_0}{\beta_1}(4\pi)^2,$



N_f -dependence

Estimate the onset of chiral symmetry breaking by the gap equation:

$$\alpha \geq \alpha_c = \frac{\pi}{3C_2(R)}.$$

Appelquist & Terning, (1996) PRL77.

Compare with $\alpha_* = g_*^2/(4\pi^2)$ to determine the lower boundary of the conformal window:

$$\alpha_c \geq \alpha_* \Rightarrow N_f^{\mathrm{crit}} = \frac{(66C_2(R) + 17C_2(G))C_2(G)}{10T(R)(3C_2(R) + C_2(G))}$$

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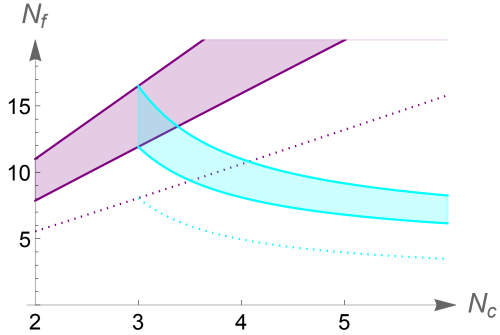
Only few relevant representations: $R = F$, G, $2S$ and $2AS$.

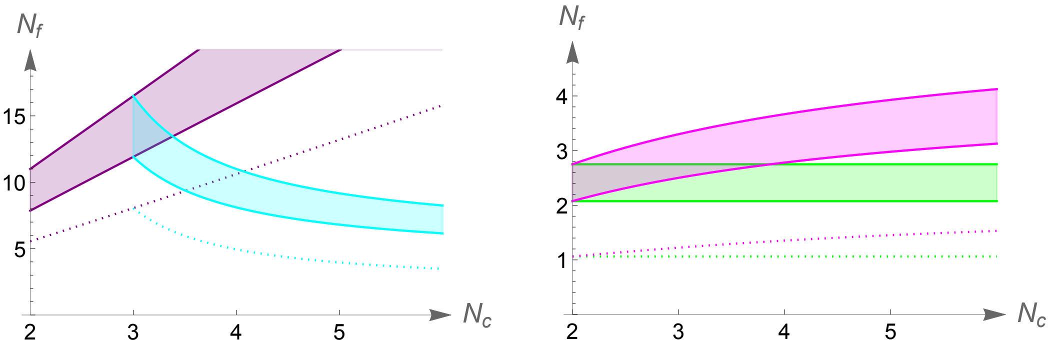
F. Sannino & K. Tuominen, (2005) Phys. Rev. **D71**

N_f and R dependence



- Left: $R = F$ (purple) and $R = 2AS$ (cyan)
- Upper boundary, $\beta_0 = 0$, Dashed: $\beta_1 = 0$

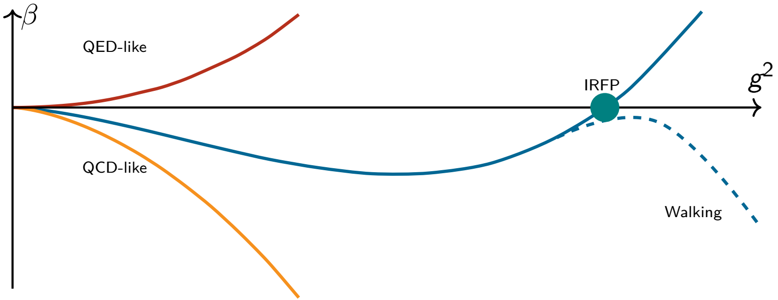
N_f and R dependence



- Left: $R = F$ (purple) and $R = 2AS$ (cyan)
- Right: $R = G$ (green) and $R = 2S$ (magenta)
- Upper boundary, $\beta_0 = 0$, Dashed: $\beta_1 = 0$

Motivations: summary

- SU(N_c) dynamics at strong coupling?
- Draw phase diagrams with ink.
- Phenomenology motivation: light composite Higgs from near conformal dynamics.
- Phenomenology motivation: composite dark sectors.



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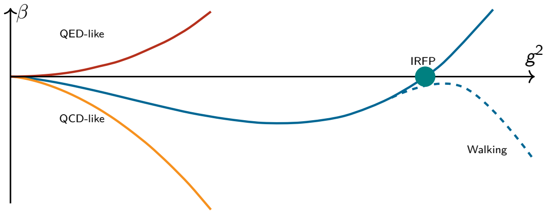
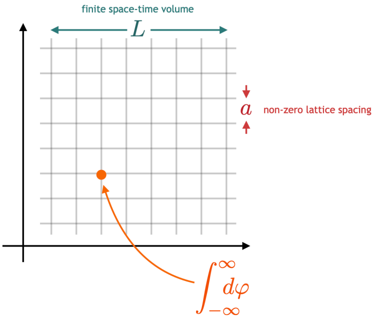


- Lower boundary of conformal window at strong coupling: first principle methods.
- Established a new field: Lattice-BSM.
- Currently an international effort involving $\mathcal{O}(50-100)$ scientists.

Lattice: general remarks

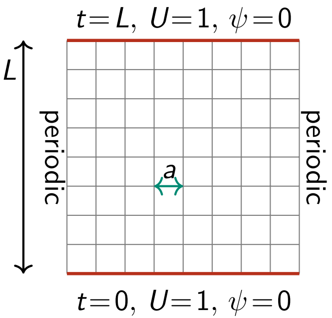
Compute observables: $\langle \mathcal{O} \rangle = \int [d\phi] \mathcal{O} e^{-S}$.
Discretize spacetime, generate field configurations to evaluate the path integral.



- Hypercubic grid, lattice spacing a.
- Extrapolate $a \to 0$.
- Lattice size $\left(\frac{L}{a}\right)^4$, $L = 10, 20, \ldots$
- Computational grand challenge.

- Gauge fields $\to$ "link variables" $U_\mu(x) = \exp iaA_\mu(x)$.
- Fermions: many realizations. We will consider Wilson fermions.
- Lots of methodology/ intuition developed for QCD over past 30+ years.

Lattice: action



- HEX-smearing and mixing: c_g
- Chiral limit: tune m_0.
- Clover improvement: $c_{SW} = 1$.

The "plaquette" action: $(\beta = 2N_c/g_0^2)$

$$S_G = \beta \sum_{\square} (1 - \frac{1}{2}\mathrm{Tr}\square(x)) \sim \frac{1}{4g_0^2} \sum_x \sum_{\mu>\nu} a^4 F_{\mu\nu}^2 + \mathcal{O}(a^2)$$

Wilson fermion action: $S_F = \underbrace{a^4 \sum_{N_f} \sum_x (\bar{\psi}(iD + m_0)\psi}_{\mathcal{O}(a)\text{ lattice artifacts}}$

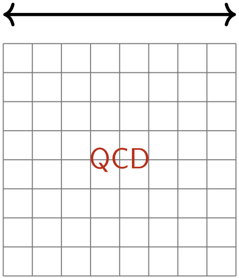
Clover: $S_{SW} = \sum_{N_f} \sum_x \frac{ai}{4} c_{sw} \bar{\psi} \sigma_{\mu\nu} F_{\mu\nu} \psi$

$$S = (1 - c_g) S_G(U) + c_g S_G(V) + S_F(V) + S_{SW}(V)$$

Lattice: challenges ahead

We aim to measure the coupling. This can be a very different task than in QCD.

Coupling is large here

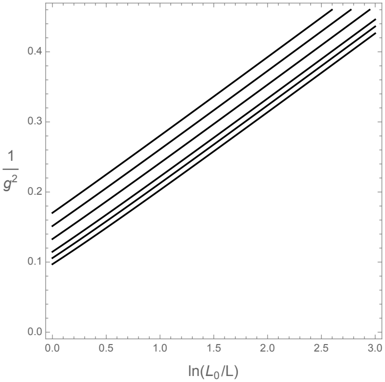


$\leftrightarrow$ But small here

- Precocious onset of as. freedom:

$$\frac{1}{g^2(s)} - \frac{1}{g^2(1)} = -\frac{\beta_1}{8\pi^2} \ln(s)$$

- $s = L/L_0 = \mu_0/\mu = 10$ to get from weak (0.1 fm) to strong (1 fm) coupling.
- Access by $L = 20 - 40$.



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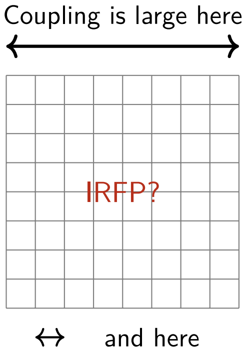
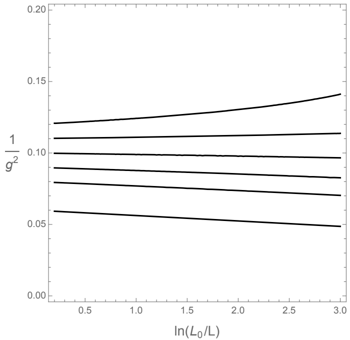


- A linear zero,

$$g^2(\mu) - g_*^2 = (g^2(\mu_0) - g_*^2)\left(\frac{\mu}{\mu_0}\right)^{y_g}$$

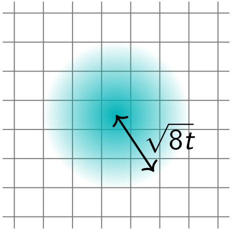
- Need to learn to live at strong coupling.
- Continuum limit?



Lattice: Definition of the coupling

Using the gradient flow method:

$$\partial_t B_{t,\mu} = -\frac{\delta S_{YM}}{\delta B} = D_{t,\mu} G_{t,\mu\nu} \,,$$
$$G_{t,\mu\nu} = \partial_\mu B_{t,\nu} - \partial_\nu B_{t,\mu} + [B_{t,\mu}, B_{t,\nu}] \,.$$
$$B_{0,\mu} = A_\mu \quad \leftarrow \text{ the original gauge field}$$

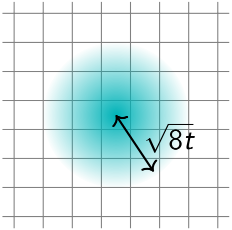


- Introduce fictitious time coordinate t and evolve the gauge field
- Smoothens the initial gauge field within radius $\sqrt{8t}$

Lattice: Definition of the coupling

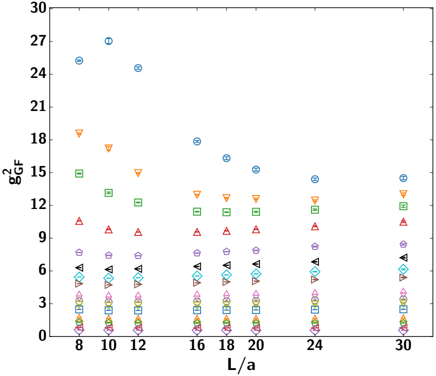
Using the gradient flow method:

$$\langle E(t)\rangle = \frac{1}{4}\langle G_{\mu\nu}(t)G_{\mu\nu}(t)\rangle$$
$$= \frac{3(N^2-1)g_0^2}{128\pi^2 t^2} + \mathcal{O}(g_0^4)\,,$$
$$g^2_{\mathrm{GF}}(\mu) = \mathcal{N}^{-1} t^2 \langle E(t)\rangle \,|_{x_0=L/2\,,\,t=1/8\mu^2}\,,$$

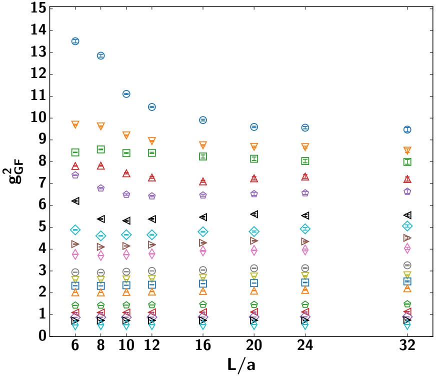


- To make scale free of lattice artifacts and finite volume effects: $\mu^{-1} = cL$ ($c \sim 0.3$).
- Evolve the flow equation to time t
- Coupling is at scale $\mu^{-1} = \sqrt{8t}$

Lattice: Raw couplings, SU(2) with $N_f = 6$ & 8



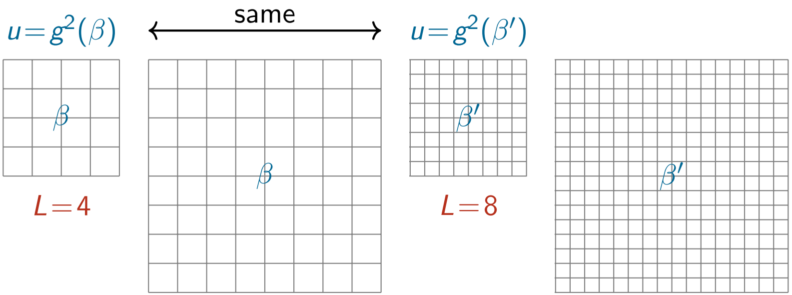
$N_f = 6$



$N_f = 8$

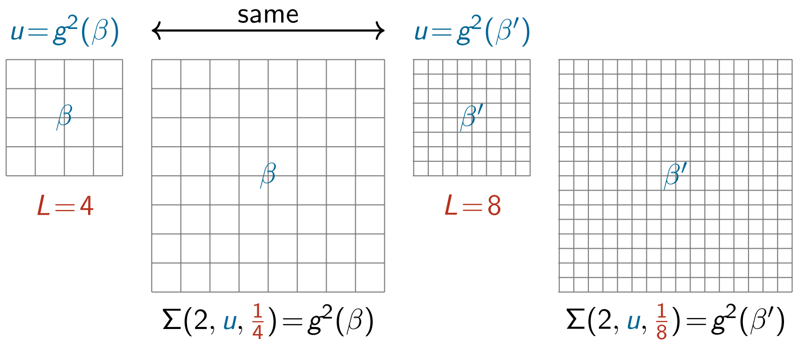
- Different symbols/ colors: different values of $\beta = 0.5 \ldots 8$
- Strong finite size effects on small lattices $\rightarrow$ Only use lattices of size 10 or bigger.

Quantifying running: Step scaling idea



$$\Sigma(2, u, \tfrac{1}{4}) = g^2(\beta) \qquad \Sigma(2, u, \tfrac{1}{8}) = g^2(\beta')$$

Quantifying running: Step scaling idea



- Choose $\beta = 4/g_0^2$ and L, measure $u = g^2_{GF}(L)$. Then choose stepsize $s = 2$.
- Double L, and measure $\Sigma(u, 1/L) = g^2_{GF}(sL)$. Take a bigger lattice, L'.
- Tune β' such that $g^2_{GF}(L') = u$. Double the lattice and measure $\Sigma(u, 1/L') = g^2_{GF}(sL')$
- Do for all lattice sizes, change u and repeat. (Difficult if running is slow!)

Quantifying running: Step scaling theory

- Step scaling function in the lattice and continuum:

$$\Sigma(s, u, a/L) = g^2_{GF}(g_0, s\frac{L}{a})\Big|_{g^2_{GF}(g_0,\frac{L}{a})=u}\,, \quad \sigma(u, s) = \lim_{a/L\to 0} \Sigma(u, s, a/L)$$

- Interpolate $g^2_{GF}(g_0, L/a)$ for consistent u and do the limit as:

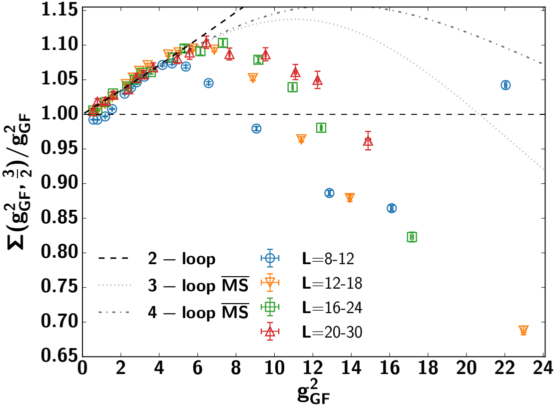
$$\Sigma(u, s, a/L) = \sigma(u, s) + c(u)(\frac{L}{a})^{-2}$$

- At fixed point $\sigma(u)/u = 1$
- Related to beta function:

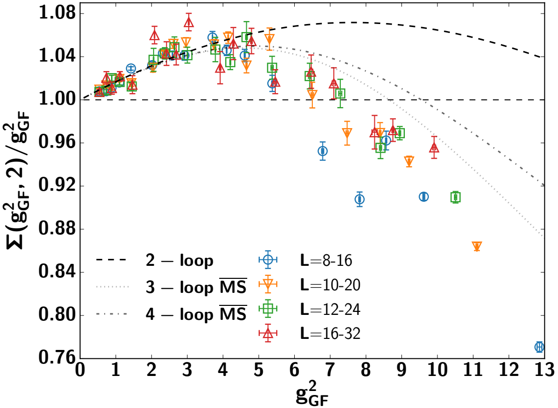
$$-2\ln(s) = \int_{\sqrt{u}}^{\sqrt{\sigma(u,s)}} \frac{\mathrm{d}x}{\beta(x)}\,, \quad \beta(g) \simeq \frac{g}{2\ln(s)}\left(1 - \frac{\sigma(g^2, s)}{g^2}\right)$$

- For $N_f = 8$ we use $s = 2$, Pairs: $8-16\,, 10-20\,, 12-24\,, 16-32$
- For $N_f = 6$ we use $s = 1.5$, Pairs: $8-12\,, 12-18\,, 16-24\,, 20-30$

Lattice: Raw step scaling function

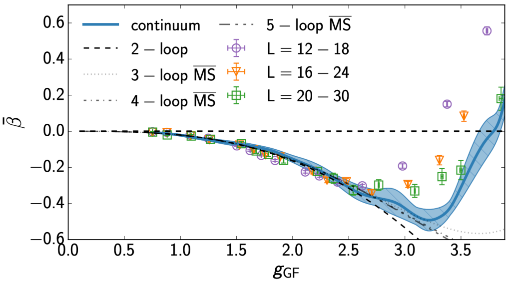


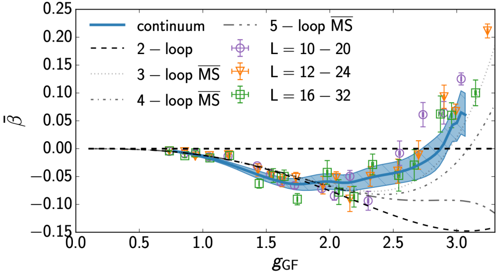
$N_f = 6$
$s = 3/2, \ c = 0.3$



$N_f = 8$
$s = 2, \ c = 0.4$

Results: SU(2) gauge + N_f fundamental fermions





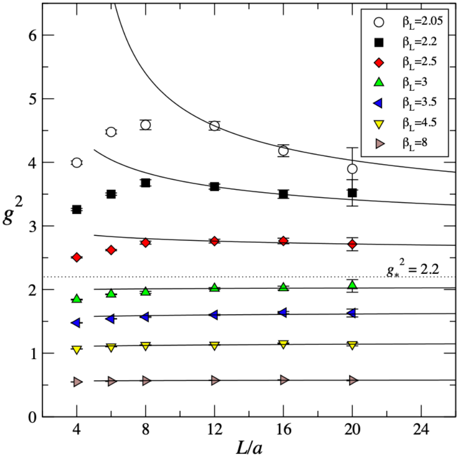
$N_f = 6$, V. Leino *et al.* (2018) Phys. Rev. **D97**
Slope of $\beta(g)$: $y_g^* = 0.65$

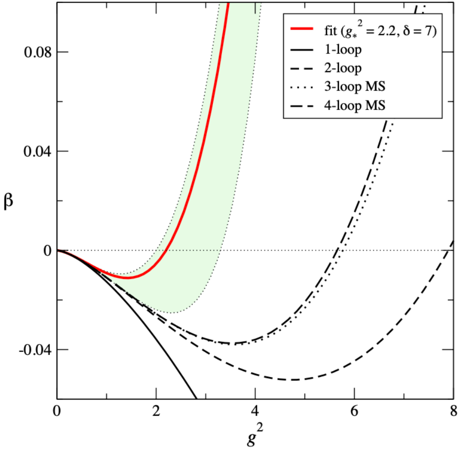
$N_f = 8$, V. Leino *et al.* (2017) Phys. Rev. **D95**
Slope of $\beta(g)$: $y_g^* = 0.20$
V. Leino *et al.* (2018) PRD 98

$N_f = 2, 4$ χsymmetry breaking, $N_f = 10$ has a fixed pointT. Karavirta *et al.* (2012) JHEP 05

Results: SU(2) gauge + 2 adjoint fermions

A. Hietanen, K. Rummukainen and K. Tuominen, (2009) Phys. Rev. **D80**

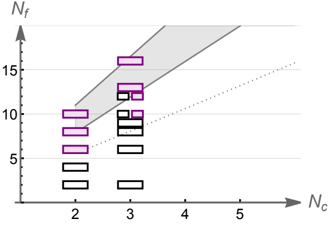




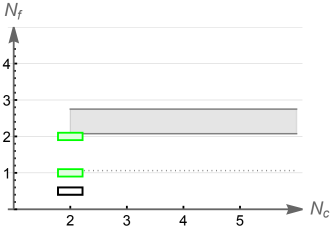
First observation of an IRFP in (non-susy) gauge theory.

Summary: drawing phase diagrams with ink

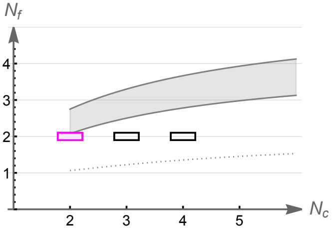
Fundamental:



Adjoint:



2-index symm.



(Compilation of results from several collaborations)

In addition: determination of anomalous dimensions γ_q^* & y_g and measurement of spectra.

Outlook

N_f	γ_q^*	SU(2) + adj.
1	0.72	Athenodorou et al. 2103.10485
2	0.2	Rantaharju et al. 1510.03335

N_f	γ_q^*	SU(2)+fund.
6	0.283	Leino et al. Phys. Rev. D95 (2017)
8	0.15	Leino et al. Phys. Rev. D97 (2018)
10	0.08	Banks-Zaks

N_f	γ_q^*	SU(3)+fund
10	1.10	Appelquist et al. 1204.6000
12	0.13	Appelquist et al. 0901.3766
13	0.197	Fodor et al. 1811.05024
16	0.03	Banks-Zaks

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Some further TODOs:

- CW boundary:
  - SU(3) $N_f = 10$ and $N_f = 12$ fundamentals.
  - $N_f = 2$ adjoints for $N_c > 2$.
- Multiple fermion representations
- Non-asymptotically free theories
  - SU(2)+24 (or 48) fundamentals
    V. Leino *et al.* (2020) PRD101
    J. Rantaharju *et al* (2021) PRD104
  - Adding scalars, asymptotic safety?
    Litim and Sannino, JHEP 12 (2014)