Universal Deformations

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Deformations of QFTs

$$S_{\text{new}} = S + \Lambda^{d-\Delta} \int d^d x\, \mathcal{O}(x)$$

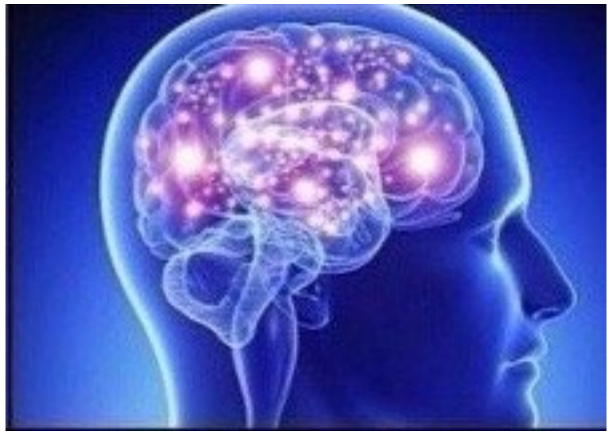
If the scaling dimension $\Delta > d$, negligible at long distance, but if $\Delta < d$, the deformation is relevant, interesting effects!

Usually we can't determine dependence on Λ exactly.

- Perturbation theory tends to be the best we can do.

Punchlines

- There are non-SUSY interacting QFTs with exactly solvable relevant deformations



- In 2d QFTs, some fun implications for confinement

- Examples of QFTs that violate the EFT naturalness principle.

Symmetry = topological operators

U(1) continuous symmetry example:

$$S = \int d^4x \left(|\partial\phi|^2 + m^2 |\phi|^2 \right)$$

No change if $\phi \to e^{i\alpha}\phi \;\Rightarrow\; \partial^\mu j_\mu = 0$, and $U_\alpha = e^{i\alpha \int_{\text{space}} j_0} = e^{i\alpha Q}$ generates the U(1) symmetry

$$U_\alpha = e^{i\alpha \int_{\text{space}} j_0} \quad \Rightarrow \quad U_\alpha(M_3) = \exp\left(i\alpha \int_{M_3} \star j \right)$$

$d \star j = 0 \Rightarrow U_\alpha$ only has topological dependence on M_3

U(1) symmetry ⇒ co-dim 1 topological operators

Gaiotto, Kapustin, Seiberg, Willett, 2014

Symmetry as topology

Gaiotto, Kapustin, Seiberg, Willett, 2014

Modern definition:

existence of (d-1)-dimensional **topological operators** $\Rightarrow$ Existence of a symmetry

Everything we do with normal symmetries can be rephrased in terms of manipulations of topological operators.

Symmetry as topology

Benefit of fancy topological language is generalizations!

existence of (d-1-p)-dimensional topological operators $\Rightarrow$ Existence of a 'p-form' symmetry

Topological operators need to satisfy some 'fusion rule' like

$$U_i(M)U_j(M) = \sum_k N_{ij}^k U_k(M)$$

and have some consistent topological action on charged **p-dimensional** operators.

Wild consequences, such as symmetry $\neq$ symmetry group

Chang, Lin, Shao, Yin 2018

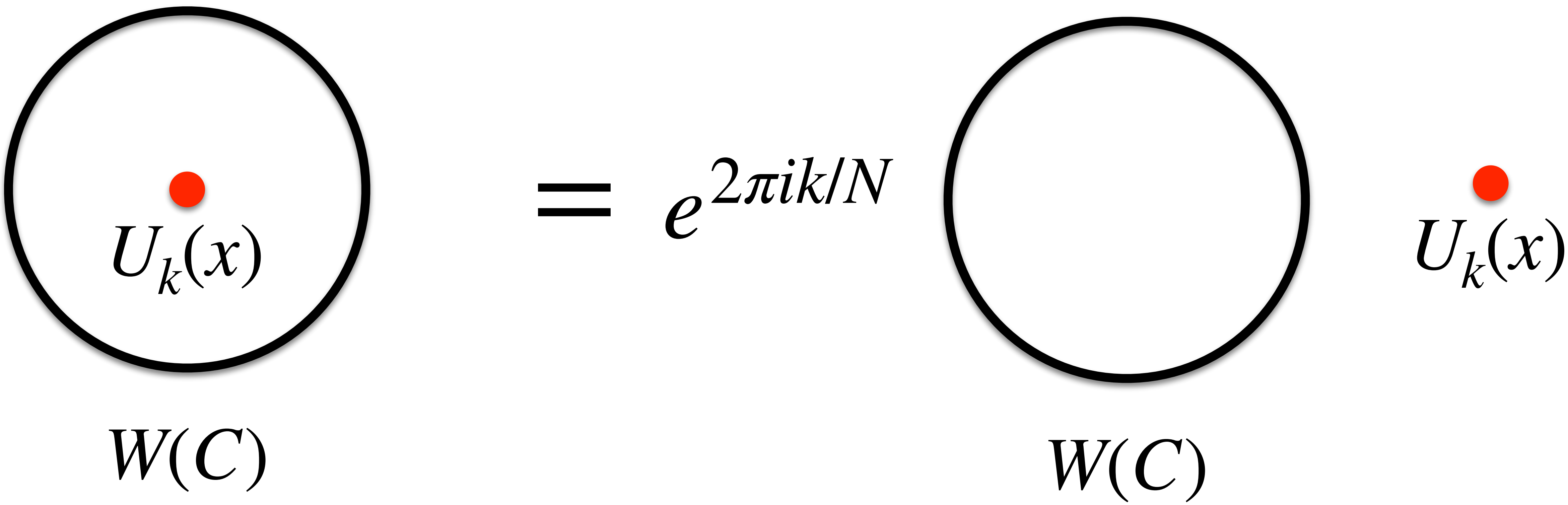
QFT in 1+1d

Focus on 1+1d because it makes for nice pictures and connects to recent literature.

``1-form symmetry" ⇒ local topological operators $U_k(x)$.

- They act on charged line operators (Wilson loops)
  - Schwinger model (2d QED) with fermions of charge N
  - 2d SU(N) YM theory
  - 2d adjoint QCD
  - …

$\mathbb{Z}_N$ 1-form symmetry in 2d



$$U_1(x)^N = 1$$

$$U_1(x) \cdot U_1(x) = U_2(x)$$

Universes

Expectation values of $\mathbb{Z}_n$ LTOs are tightly constrained:

$$\langle U_1(x) \rangle = e^{2\pi i k/N}$$

Hellerman et al, 2006;
Tanizaki, Unsal 2019;
Komargodski et al, 2020

k labels sectors of the QFT called 'universes'.

- Universe domain walls have *infinite* tension
  - $\langle U_1(x) \rangle$ is topological, so can't change smoothly
- Excitations can't take you from one universe to another.
- Physically, universe walls are just probe Wilson lines.
  - Wilson loops ⇔ infinitely-heavy probe particles, useful to explore phase structure.

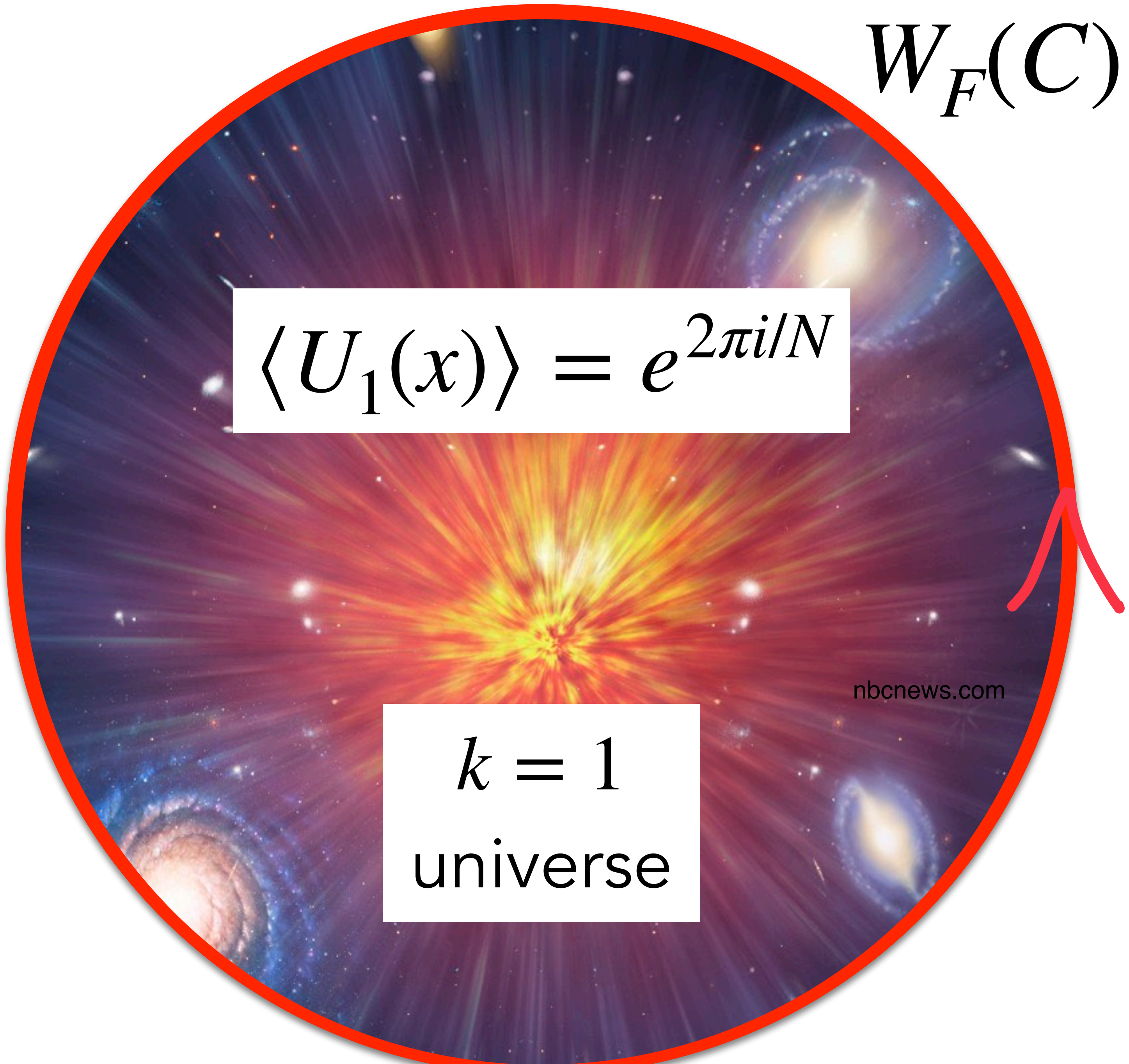
Universes

$$W_F(C) \sim e^{i\int_C a}$$

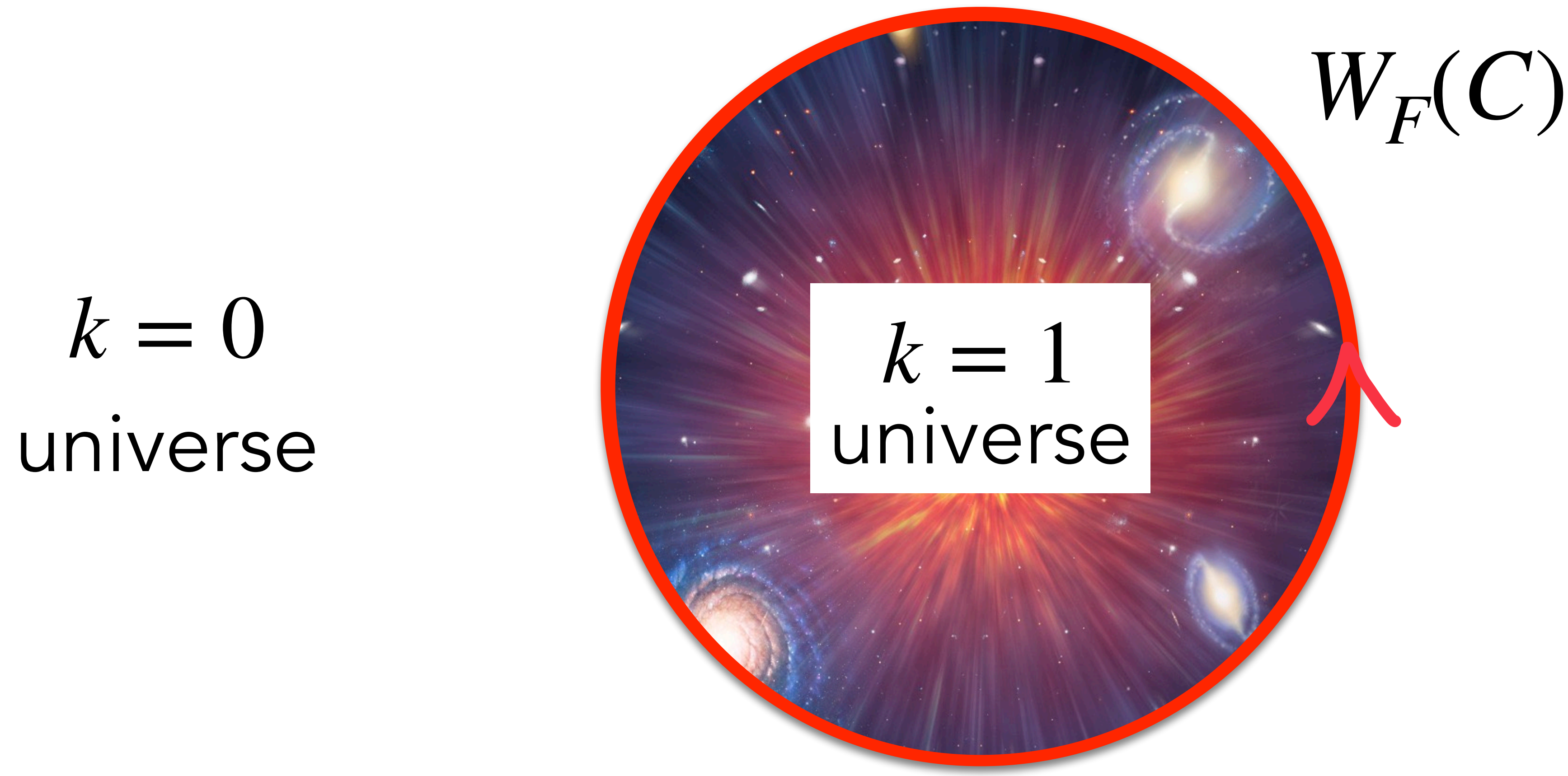
$$\langle U_1(x) \rangle = 1$$

$$k = 0$$

universe



Universes and confinement



If vacuum energy density inside is bigger than outside, gain energy by shrinking C

- Then $\langle W_F(C) \rangle \sim e^{-TA}$ - this is quark/charge confinement.

If vacuum energy densities inside = outside, only cost is from the edge, so $\langle W_F(C) \rangle \sim e^{-\mu L_C}$ - deconfinement!

Universal deformations

Consider any 2d theory with a $\mathbb{Z}_N$ 1-form symmetry.

- Means it has local topological operators (LTOs) $U_n(x)$
- Local operators can be added to the Lagrangian - defines a deformation of the theory!

$$S_{\text{new}} = S + \sum_n \Lambda_n^{2-\Delta} \int d^d x\, U_n + \text{h.c.}$$

What is Δ? Work near UV fixed point, then:

$$\langle U_n^\dagger(x) U_n(0) \rangle \longrightarrow \lambda^{-2\Delta} \langle U_n^\dagger(\lambda^{-1} x) U_n(0) \rangle = \lambda^{-2\Delta} \langle U_n^\dagger(x) U_n(0) \rangle$$

$$\Rightarrow \Delta = 0$$

This deformation is maximally relevant!

$\Delta = 0 \Rightarrow$ boring?

We've all heard of one famous dimension-0 deformation before:

$$S = \int d^4x \sqrt{-|g|} \left(\frac{1}{\kappa} R - \Lambda^4 + \mathcal{L}_{\text{matter}} \right)$$

Cosmological constant term is a deformation by $\mathbf{1}$, and has $\Delta = 0$, but boring within QFT without gravity!

hcamag.com

LTO deformations have physical effects within QFT, like driving phase transitions.

ere.net

Concrete example

2d QED: U(1) gauge theory with charge N Dirac fermion.

$$S = \int d^2x \left(\frac{1}{4e^2} f_{\mu\nu}^2 + \frac{i\theta}{2\pi} \epsilon^{\mu\nu} \partial_\mu a_\nu + \bar{\psi}(\partial\!\!\!/ + iN a\!\!\!/ + m)\psi \right)$$

$$\frac{1}{2\pi} \int_{M_2} da \in \mathbb{Z}$$

Charge N Schwinger model!

Famous playground for exploring confinement

- Do charge 1 probes feel linear potential?
- Analytic control in $m \ll e$ regime.

Concrete example

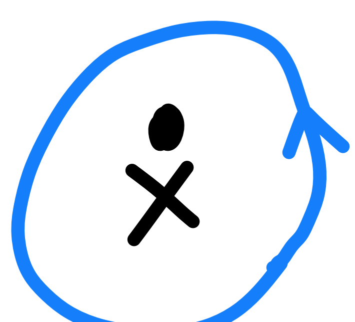
2d QED: U(1) gauge theory with charge N Dirac fermion.

$$S = \int d^2x \left(\frac{1}{4e^2} f_{\mu\nu}^2 + \frac{i\theta}{2\pi} \epsilon^{\mu\nu} \partial_\mu a_\nu + \bar{\psi}(\partial\!\!\!/ + iN a\!\!\!/ + m)\psi \right)$$

Known to have a $\mathbb{Z}_N$ 1-form symmetry, so it has local topological operators!

They are just flux insertion operators: $da \to da + \frac{2\pi}{N}\delta^{(2)}(x)$

$U_1(x) =$



$e^{i\int a} = e^{2\pi i/N}$

Bosonized Schwinger model

Equivalent bosonic theory:

$$S_\varphi = \int_M \left(\frac{1}{2e^2} ||da||^2 + \frac{1}{8\pi} ||d\varphi||^2 + \frac{i}{2\pi} (N\varphi + \theta) \wedge da \right.$$
$$\left. - m\mu \cos\varphi \right)$$

φ is compact, $\varphi \simeq \varphi + 2\pi$, and $\mu e^{i\varphi} \simeq \bar{\psi}_L \psi_R$

Nice feature of bosonization is it allows us to write the local topological operators as standard-looking local operators.

Local topological operators

Equivalent bosonic theory:

$$S_\varphi = \int_M \left(\frac{1}{2e^2} ||f||^2 + \frac{1}{8\pi} ||d\varphi||^2 + \frac{iN}{2\pi} \varphi \wedge da - m\mu \cos\varphi \right)$$

$$da \to da + \frac{2\pi k}{N} \delta^{(2)}(x)$$

$$\Updownarrow$$

$$U(x) = \exp\left(\frac{2\pi\, k}{N} \left[\frac{1}{e^2} \star da + \frac{iN}{2\pi} \varphi \right] \right) \quad ?$$

Local topological operators, first try

$$U_k(x) = \exp\left(\frac{2\pi k}{N}\left[\frac{1}{e^2}\star da + \frac{iN}{2\pi}\varphi\right]\right)$$

Good news: EoMs imply that $d\left(\frac{1}{e^2}\star da + \frac{iN}{2\pi}\varphi\right) = 0$

- So $U(x)$ is topological!

Bad news: expect $\langle U_k(x)\rangle = 1$ in the trivial universe, but instead

$$\langle U_k(x)\rangle = \exp\left(\left(\frac{2\pi k}{N}\right)^2 \frac{1}{2e^2}\int_M ||\delta^{(2)}(x)||^2\right)$$

Local topological operators, second try

Good news: if we redefine $U_k(x) \to U'_k(x)$ to absorb the divergence, then in the trivial universe

$$\langle U'_k(x) \rangle = 1$$

Bad news: this messes up the $\mathbb{Z}_N$ fusion rule

$$\langle U'_k(x) U'_m(x) \rangle \neq \langle U'_{k+m \bmod N}(x) \rangle$$

Also, if we add $U'_k(x)$ to the action we change gauge field EoMs $\Rightarrow U'_k(x)$ doesn't remain topological.

There's a much better fix…

Local topological operators done right

Integrating in a new field removes the problems:

$$S_\varphi = \int_M \left(\frac{e^2}{2} \|b\|^2 + \frac{1}{8\pi} \|d\varphi\|^2 + \frac{i}{2\pi} (N\varphi + 2\pi b + \theta) \wedge da - m\mu \cos\varphi \right)$$

$$U_n(x) = \exp\left[i \frac{2\pi n}{N} \left(b + \frac{N}{2\pi}\varphi + \frac{\theta}{2\pi} \right) \right]$$

This $U_n(n)$ has all the expected properties. Add it to the action:

$$S_{\text{new}} = S_{\text{old}} + \sum_{n=1}^{N-1} \Lambda_n^2 \int d^2x \left(U_n(x) + \text{h.c.} \right)$$

$U_n(x)$ **remains** a topological operator when $\Lambda_n \neq 0$!

Chiral symmetry

$$U_n(x) = \exp\left[i\frac{2\pi n}{N}\left(b + \frac{N}{2\pi}\varphi + \frac{\theta}{2\pi}\right)\right]$$

If $m = 0$, then there is a $\mathbb{Z}_N$ chiral symmetry:

$$\bar{\psi}_L \psi_R \to e^{2\pi i/N}\bar{\psi}_L \psi_R$$

$$\Updownarrow \qquad\qquad \Updownarrow$$

$$e^{i\varphi} \to e^{2\pi i/N} e^{i\varphi}$$

$U_1(x) \to e^{2\pi i/N} U_1(x)$ under chiral symmetry.

- Mixed 't Hooft anomaly for 1-form and 0-form symmetries.
- Adding $U_n(x)$ to the action breaks chiral symmetry!

Effect of LTO deformation

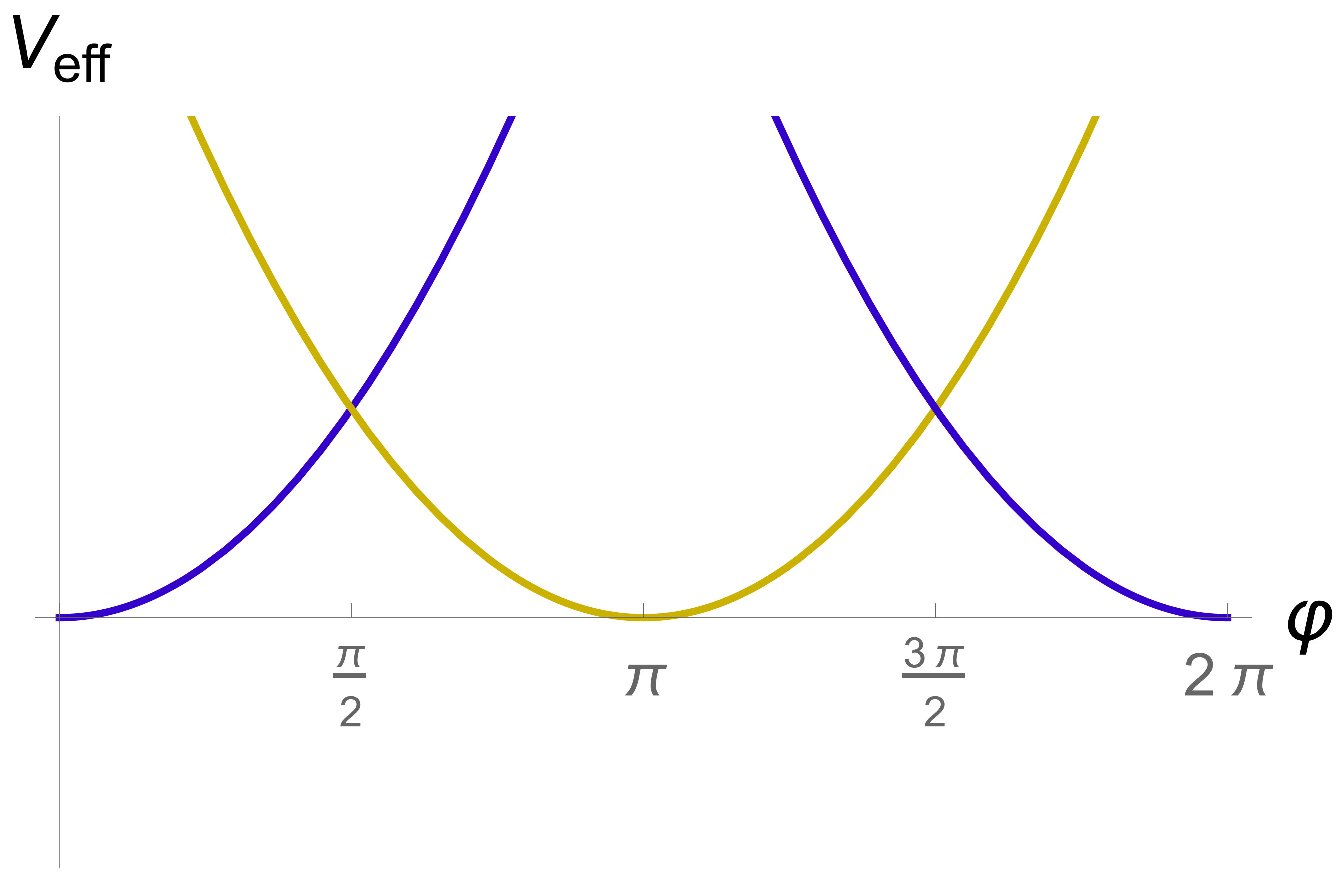
If e.g. N=2, and we integrate out a and b, the effective potential becomes

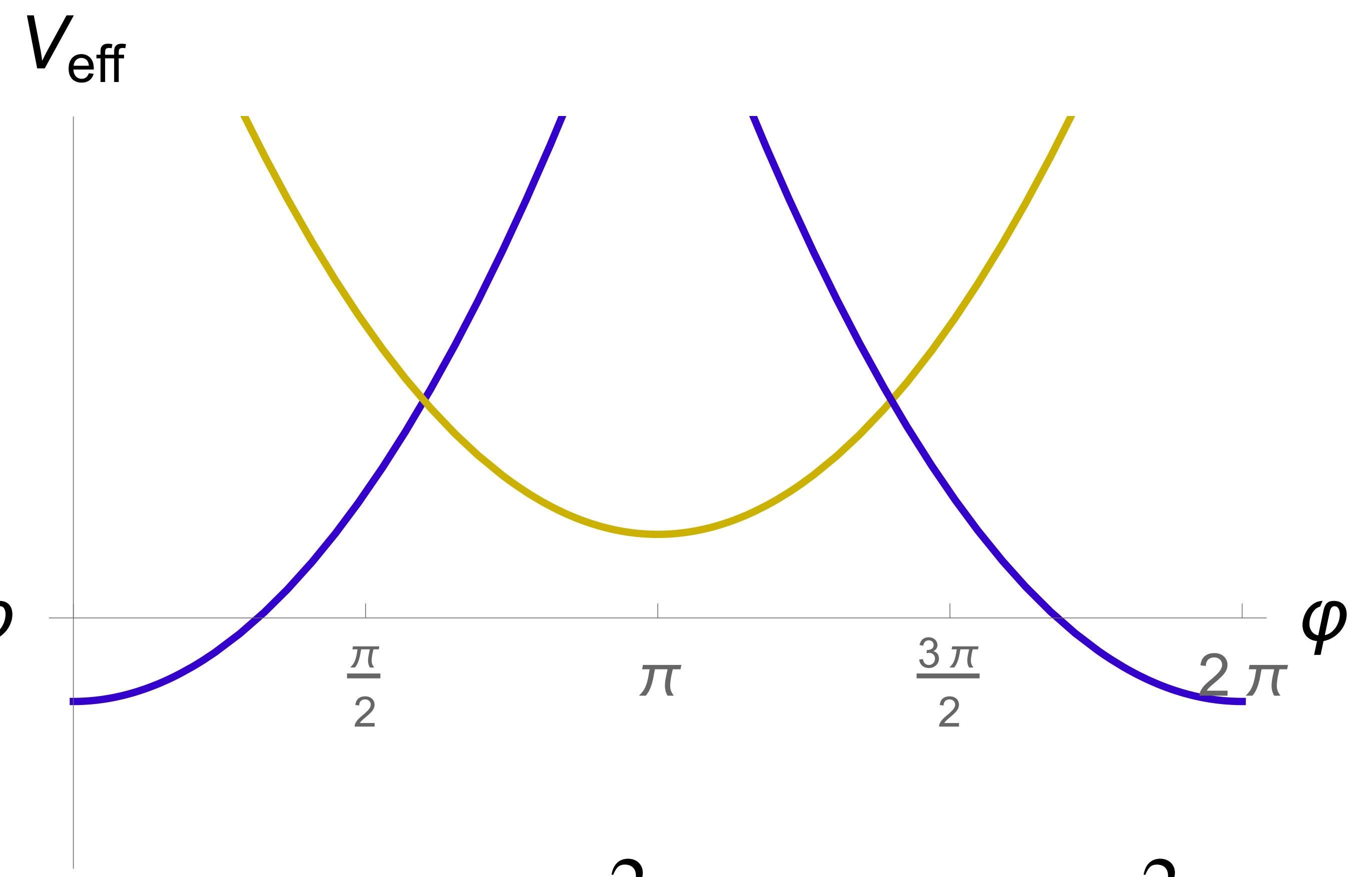
$$V_k(\varphi) = \frac{1}{2}\left(\frac{2e}{2\pi}\right)\left(\varphi - \frac{2\pi k}{2}\right)^2 - m\mu\cos\varphi + \Lambda^2\cos\left(\frac{2\pi k}{N}\right)$$

Dialing Λ dials relative vacuum energies of universes.

- Controls the fate of confinement
- $\Lambda \neq 0$ breaks chiral symmetry at $m = 0$

Charge-2 Schwinger model universes





$$m = \Lambda = 0$$

Spontaneously broken $\mathbb{Z}_2$ chiral symmetry

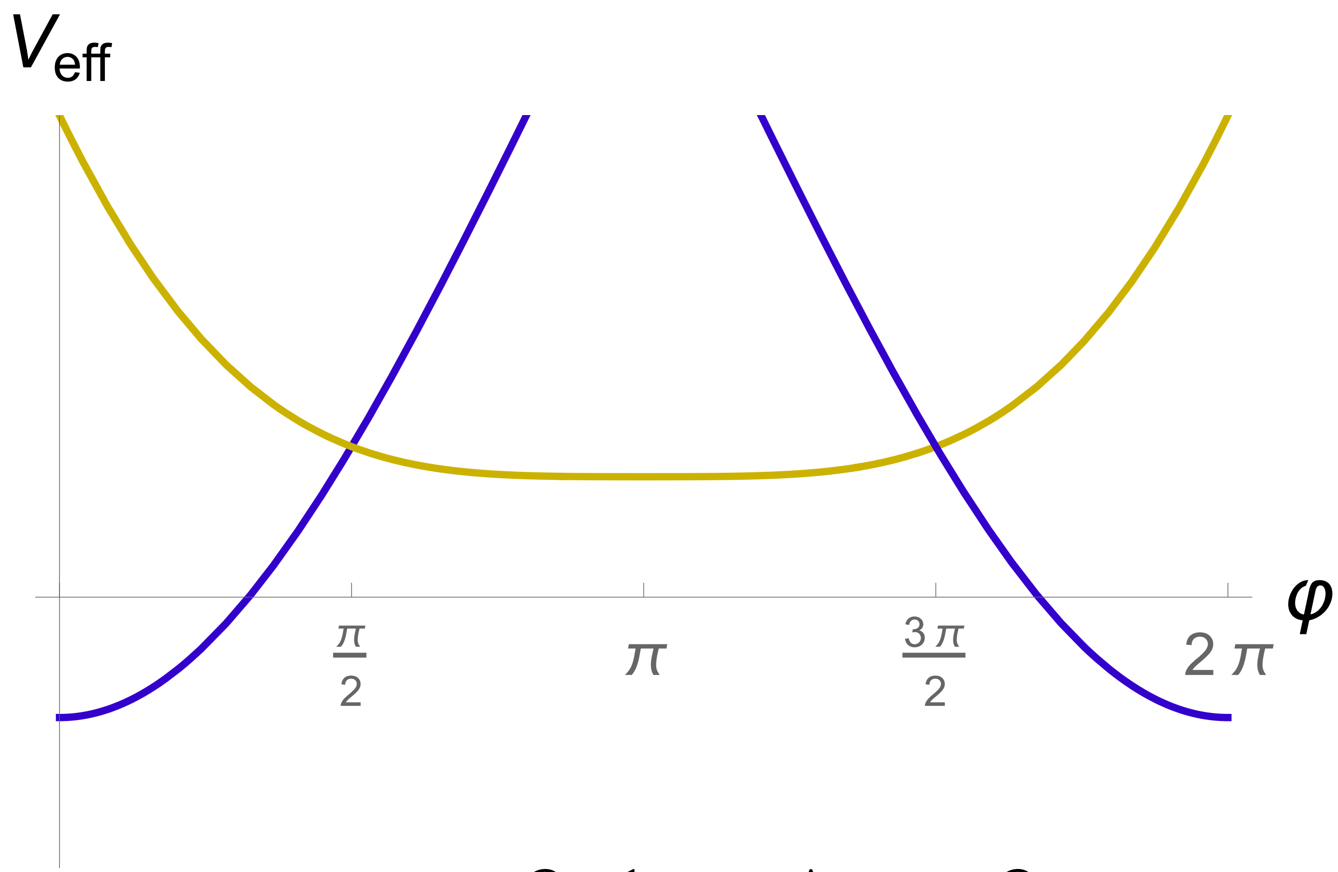
Spontaneously broken $\mathbb{Z}_2$ 1-form symmetry

$$m = 0,\ \Lambda^2 = -0.05e^2$$

Explicitly broken $\mathbb{Z}_2$ chiral symmetry

Unbroken $\mathbb{Z}_2$ 1-form symmetry: confinement.

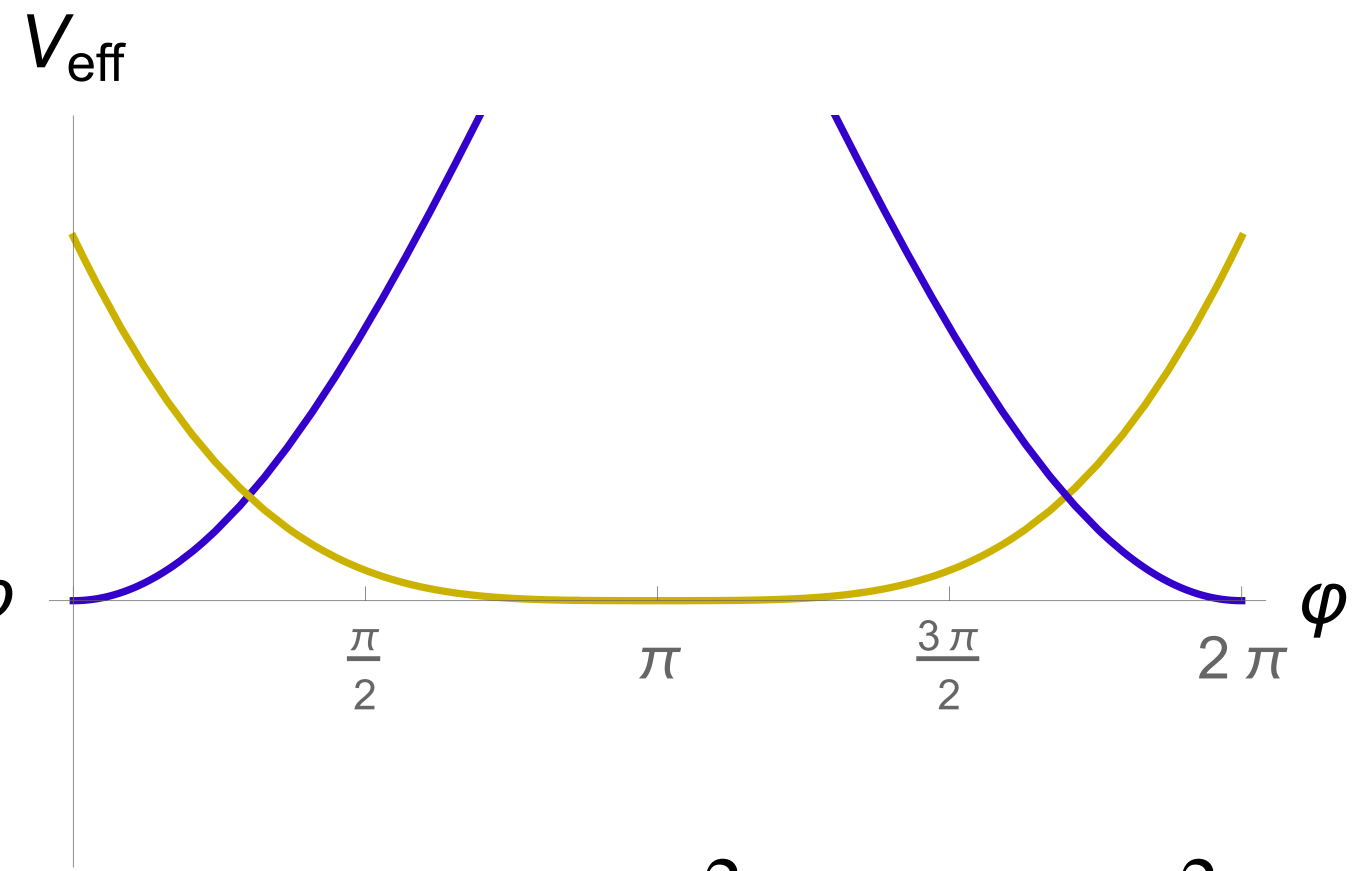
Massive Charge-2 Schwinger model



$$m = 0.1e,\ \Lambda = 0$$

Explicitly broken $\mathbb{Z}_2$ chiral symmetry

Unbroken $\mathbb{Z}_2$ 1-form symmetry

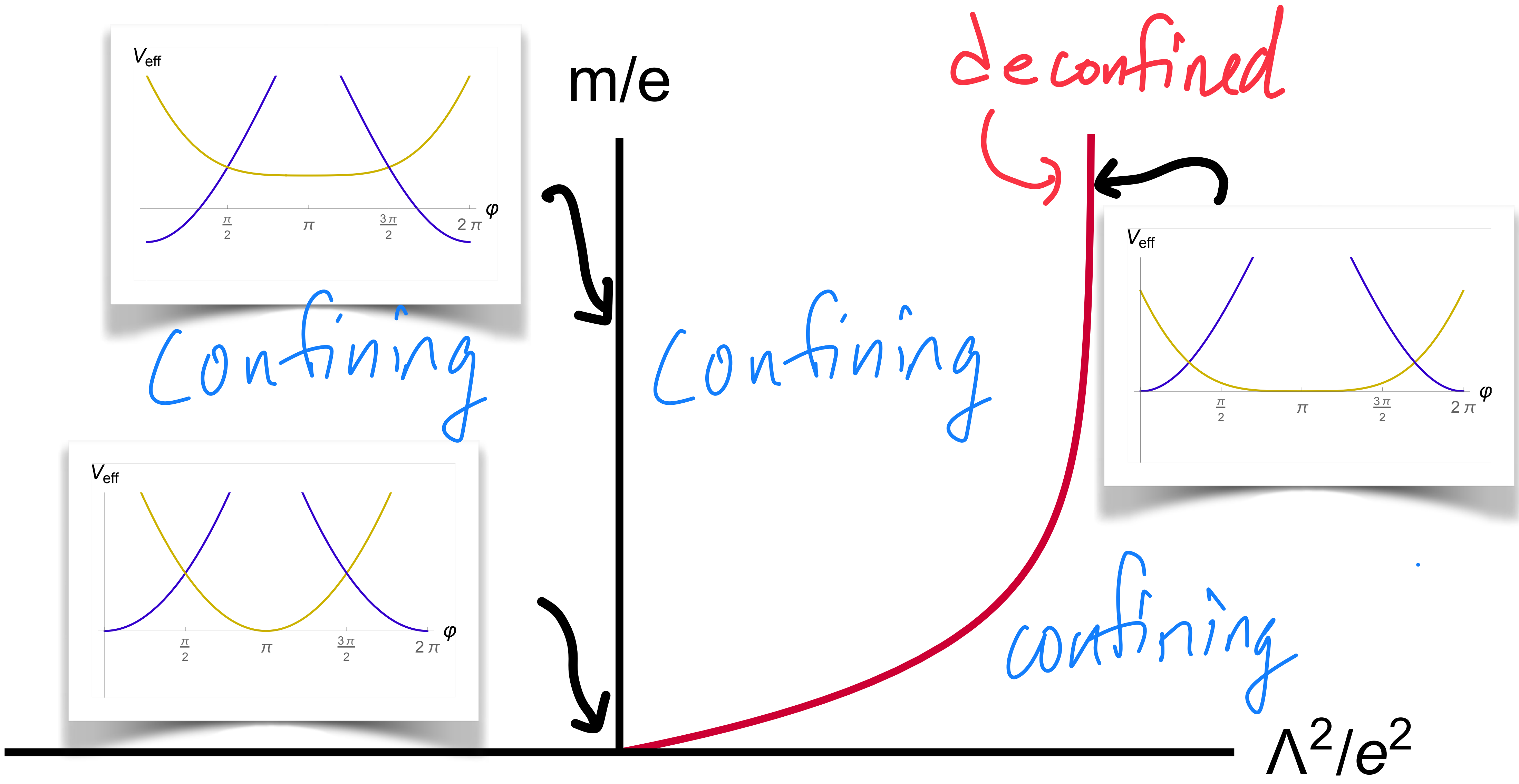


$$m = 0.1e,\ \Lambda^2 \simeq -0.1e^2$$

Explicitly broken $\mathbb{Z}_2$ chiral symmetry

Spontaneously broken $\mathbb{Z}_2$ 1-form symmetry

Phase diagram for $N = 2$



Assuming C symmetry and $\theta = 0$

LTO deformations are universal

These deformations are universal in two ways:

1. Exactly calculable in model-independent way.
2. Only effect is on the vacuum energy of `universes'.

Universal deformations

Can always do a formal expansion in powers of a deformation in the path integral:

$$Z_{\text{new}} = \int d[\text{fields}] e^{-S_{\text{old}}} e^{-\Lambda^2 \int d^2x U(x) + \text{h.c.}}$$

$$= Z_{\text{old}} \sum_{I,J=0}^{\infty} \int dx_i dy_j c_{I,J} \Lambda^{2I} \Lambda^{2J} \left\langle \prod_{i=1}^{I} \prod_{j=1}^{J} U_n(x_i) U_n^\dagger(y_j) \right\rangle_{\text{old}}$$

Normally useless, but here we know all the correlation functions!

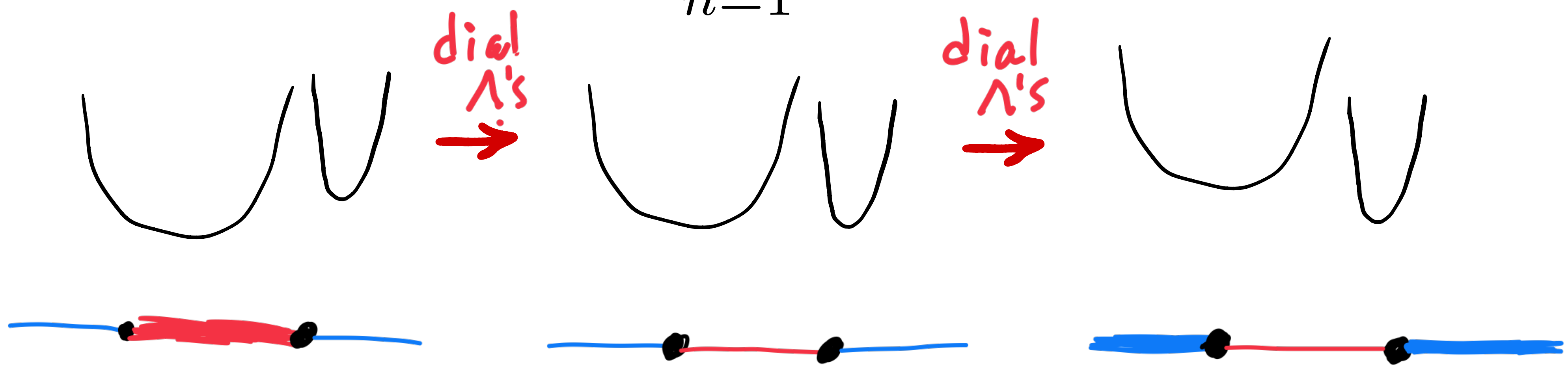
- Correlation functions can be read from fusion rules

Summing up, we get difference between Z_{old} and Z_{new} exactly.

Universal deformations

Within k-th universe, $\langle U_n \rangle = e^{2\pi i n k/N}$, so the effect of deformation is simply to shift all states by the same amount.

$$\mathcal{E}_{\text{new}} = \mathcal{E}_{k,\text{old}} + \sum_{n=1}^{N-1} \Lambda_n^2 \cos\left(2\pi k n/N\right)$$



Λ_n affects relative vacuum energy densities of universes.

- Makes their effects observable.
- Takes 2d QFTs through deconfinement phase transitions!

Consequences

Explicit example was in the Schwinger model, but results are complely general. Works in any 2d gauge theory.

- Also generalizes to higher-dim QFTs with LTOs.

I'll focus on highlighting two main points:

- General lesson on confinement in 1+1d
- Counterexample to EFT naturalness principle

Discuss one by one.

Confinement in 1+1d

Confinement is a question about the behavior of large Wilson loops on $\mathbb{R}^2$: do they have area law, or not?

- This is read off from comparison of vacuum energies
- Vacuum energies are sensitive to universal deformations, so confinement is as well.

In e.g. 2d SU(N) adjoint QCD with odd N, the LTOs are neutral under chiral symmetry.

- To claim that the theory deconfines, one must prove LTO deformation terms aren't generated radiatively.
- Only convincing way I know to do this is using non-invertible symmetries.

Komargodski, Ohmori, Roumpedakis, Seifnashri 2020

General picture of confinement in 1+1d

Here's a perspective suggested by M. Unsal and N. Nguyen.

Suppose one thinks a $\mathbb{Z}_N$ 1-form symmetry breaks spontaneously. Low-energy EFT would be

$$S = \frac{N}{2\pi} \int_{M_2} \phi \wedge da$$

Describes N degenerate states (more precisely, universes).

General picture of confinement in 1+1d

In contrast to BF theory in higher D, this TQFT is unstable!

- Has relevant deformations by LTOs, $e^{ia\phi}$ with integer a, that drive confinement

$$S = \frac{N}{2\pi}\int_{M_2} \phi \wedge da + \int d^2x \sum_{a=1}^{N-1} \left(\Lambda_a^2 e^{ia\phi} + \text{h.c.}\right)$$

Unless $e^{ia\phi}$ is charged under a UV symmetry, expect the deformation to be generated, and drive confinement in 2d.

This is the more precise version of a "Coleman-Mermin-Wagner" theorem for discrete 1-form symmetries

Gaiotto, Kapustin, Seiberg, Wilett, 2014

Confinement in 1+1d

Lesson: unless 2d QFT is fine-tuned, or has an 't Hooft anomaly involving the 1-form symmetry, confinement is generic in 2d!

Corollary: if theory is fine-tuned to deconfine, and hence the 1-form symmetry is spontaneously broken, then there's an emergent 0-form symmetry in IR which has a mixed 't Hooft anomaly with the 1-form symmetry.

This is the corrected version of a "Coleman-Mermin-Wagner" theorem for discrete 1-form symmetries

Gaiotto, Kapustin, Seiberg, Wilett, 2014

Naturalness Principle

Effective field theory naturalness principle:

- All operators not forbidden by a symmetry of IR theory will be generated by fluctuations, with scale set by scale of operators that break the symmetry in the UV.
- To avoid this, need to fine-tune UV parameters.

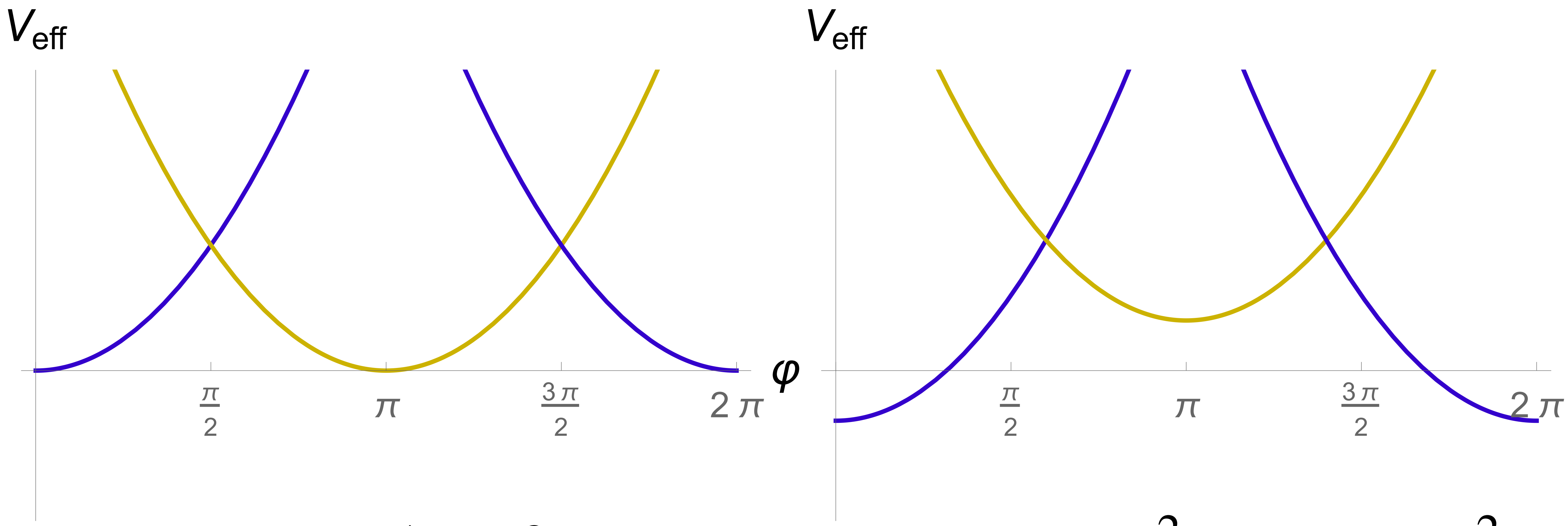
Huge fraction of modern particle physics research is dedicated to exploiting or fighting this principle.

- Higgs boson mass
- Cosmological constant
- Strong CP

Naturalness failure

- If 1-form and 0-form symmetries have ‘t Hooft anomaly, local topological operators are charged.
- When added to action, they are a relevant 0-form symmetry-breaking deformation.
- Effect of universal deformations is exactly calculable. Other 0-form breaking terms are not generated.
  - If they were, particle spectrum would be affected by deformations - and it isn’t!
  - The **only** effect is to shift relative vacuum energies.

Massive Charge-2 Schwinger model



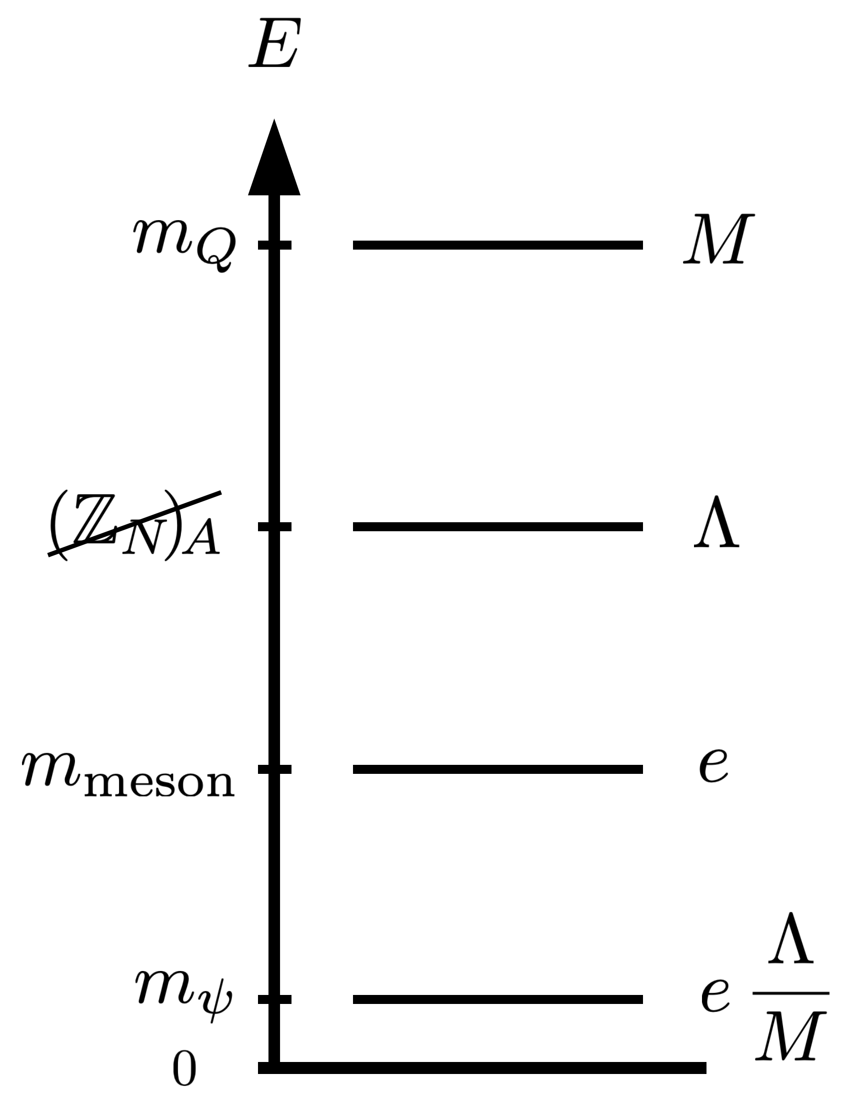
$m = \Lambda = 0$ $\qquad$ $m = 0,\ \Lambda^2 = -0.05e^2$

Particle spectrum comes from shape of potential curves — and shape doesn't depend on the universal deformation

Turning on $\Lambda^2 \int d^2x\, U(x)$ does **not** induce mass term!

Breaking symmetries

If 1-form symmetry is explicitly broken at a high scale M, expect QFTs with unnaturally-small mass scales



Understanding this in detail requires understanding emergent 1-form symmetries better

Conclusions

- There are interacting non-SUSY QFTs with **exactly solvable** relevant deformations!
    - All one needs is a (d-1)-form symmetry.

- In e.g. 2d QFTs, interesting implications for confinement.

- Curious violation of EFT naturalness principle.

Next steps:

- Explore the new naturalness violation further.
- Understand *emergent* 1-form symmetries better.

Thanks for listening!