Standard model irreps as an extension of 4-momentum: our first attempt

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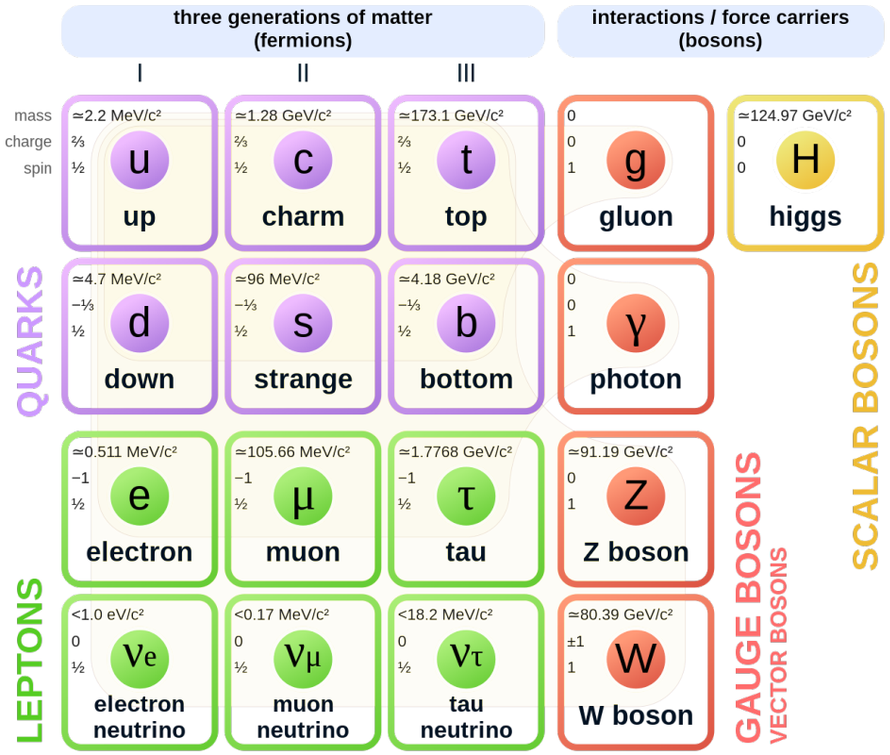
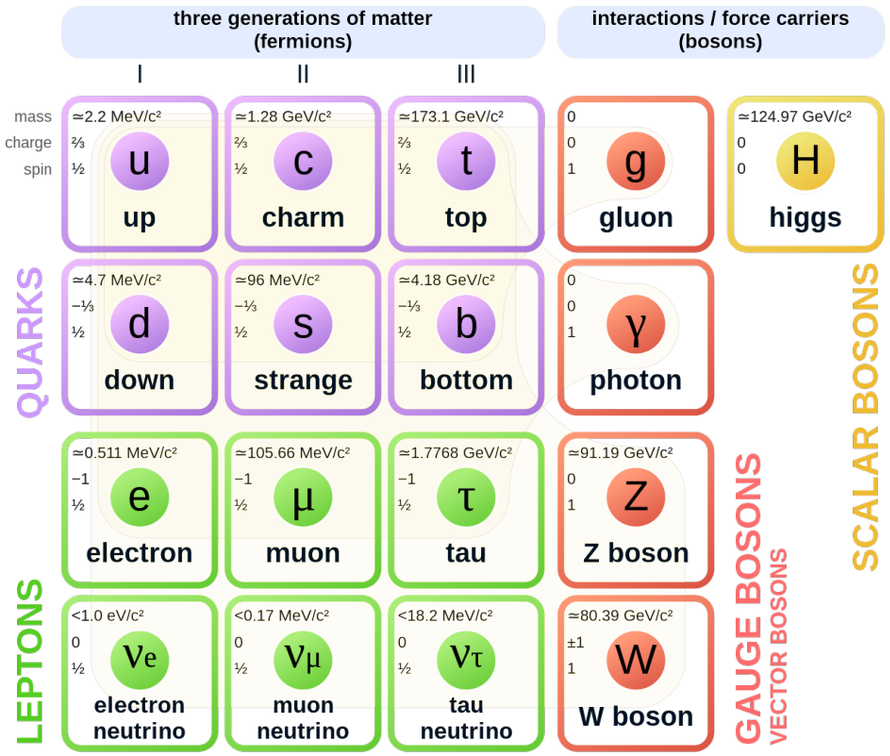
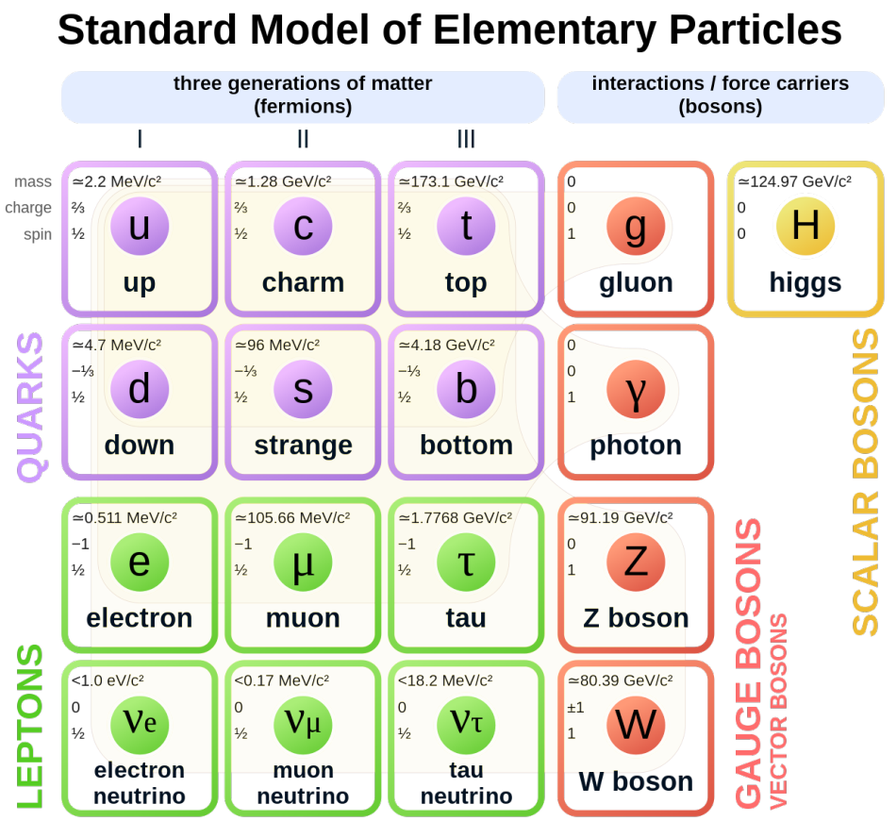
Standard Model of Elementary Particles



Standard Model of Elementary Particles





$$\mathbb{R},\ \mathbb{C},\ \mathbb{H},\ \mathbb{O}$$

Symmetry:

Symmetry:

$$\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)\ /\ \mathbb{Z}_6$$

$$G_{sm} :=$$

$$\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1) \;/\; \mathbb{Z}_6$$

$G_{sm} :=$

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Particles:

Irreps

$G_{sm} :=$

$\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1) \;/\; \mathbb{Z}_6$

Particles:

Irreps

Which?

Standard Model Irreps
$(SU(3)_C, SU(2)_L, U(1)_Y)$

LH fermions

$$
\begin{array}{ll}
(u,\ d)_L & \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \tfrac{1}{6} \right)_2 \\
(c,\ s)_L & \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \tfrac{1}{6} \right)_2 \\
(t,\ b)_L & \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \tfrac{1}{6} \right)_2
\end{array}
$$

$$
\begin{array}{ll}
(\nu_e,\ e)_L & \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\tfrac{1}{2} \right)_2 \\
(\nu_\mu,\ \mu)_L & \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\tfrac{1}{2} \right)_2 \\
(\nu_\tau,\ \tau)_L & \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\tfrac{1}{2} \right)_2
\end{array}
$$

Gauge bosons

$$
\begin{array}{ll}
G_\mu & \left(\underline{\mathbf{8}},\ \underline{\mathbf{1}},\ 0 \right)_4 \\
W_\mu & \left(\underline{\mathbf{1}},\ \underline{\mathbf{3}},\ 0 \right)_4 \\
B_\mu & \left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0 \right)_4
\end{array}
$$

RH fermions

$$
\begin{array}{ll}
u_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \tfrac{2}{3} \right)_2 \\
c_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \tfrac{2}{3} \right)_2 \\
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\end{array}
$$

$$
\begin{array}{ll}
d_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\tfrac{1}{3} \right)_2 \\
s_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\tfrac{1}{3} \right)_2 \\
b_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\tfrac{1}{3} \right)_2
\end{array}
$$

$$
\begin{array}{ll}
e_R & \left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1 \right)_2 \\
\mu_R & \left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1 \right)_2 \\
\tau_R & \left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1 \right)_2
\end{array}
$$

Higgs

$$
H \quad \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\tfrac{1}{2} \right)_1
$$

Standard Model Irreps

(SU(3)$_C$, SU(2)$_L$, U(1)$_Y$)

LH fermions

$$\begin{array}{ll} (\,u,\,d\,)_L & (\,\underline{\mathbf{3}},\,\underline{\mathbf{2}},\,\tfrac{1}{6}\,)_2 \\ (\,c,\,s\,)_L & (\,\underline{\mathbf{3}},\,\underline{\mathbf{2}},\,\tfrac{1}{6}\,)_2 \\ (\,t,\,b\,)_L & (\,\underline{\mathbf{3}},\,\underline{\mathbf{2}},\,\tfrac{1}{6}\,)_2 \end{array}$$

$$\begin{array}{ll} (\,\nu_e,\,e\,)_L & (\,\underline{\mathbf{1}},\,\underline{\mathbf{2}},\,-\tfrac{1}{2}\,)_2 \\ (\,\nu_\mu,\,\mu\,)_L & (\,\underline{\mathbf{1}},\,\underline{\mathbf{2}},\,-\tfrac{1}{2}\,)_2 \\ (\,\nu_\tau,\,\tau\,)_L & (\,\underline{\mathbf{1}},\,\underline{\mathbf{2}},\,-\tfrac{1}{2}\,)_2 \end{array}$$

Gauge bosons

$$\begin{array}{ll} G_\mu & (\,\underline{\mathbf{8}},\,\underline{\mathbf{1}},\,0\,)_4 \\ W_\mu & (\,\underline{\mathbf{1}},\,\underline{\mathbf{3}},\,0\,)_4 \\ B_\mu & (\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_4 \end{array}$$

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$$\begin{array}{ll} e_R & (\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,-1\,)_2 \\ \mu_R & (\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,-1\,)_2 \\ \tau_R & (\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,-1\,)_2 \end{array}$$

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Standard Model Irreps

$(SU(3)_C, SU(2)_L, U(1)_Y)$

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Standard Model Irreps
($SU(3)_C$, $SU(2)_L$, $U(1)_Y$)

LH fermions

$(u,\ d)_L \qquad \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6} \right)_2$

Standard Model Irreps
($SU(3)_C$, $SU(2)_L$, $U(1)_Y$)

LH fermions

$(u,\ d)_L \qquad \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6} \right)_2$

RH fermions

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Standard Model Irreps
($SU(3)_C$, $SU(2)_L$, $U(1)_Y$)

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$(u,\ d)_L \qquad \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6} \right)_2$

$(\nu_e,\ e)_L \qquad \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2} \right)_2$

RH fermions

$u_R \qquad \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3} \right)_2$

$d_R \qquad \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3} \right)_2$

$e_R \qquad \left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1 \right)_2$

Standard Model Irreps
($SU(3)_C$, $SU(2)_L$, $U(1)_Y$)

LH fermions

$(u,\ d)_L \quad \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6} \right)_2$

$(\nu_e,\ e)_L \quad \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2} \right)_2$

RH fermions

$u_R \quad \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3} \right)_2$

$d_R \quad \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3} \right)_2$

$e_R \quad \left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1 \right)_2$

1 generation

Standard Model Irreps
$(SU(3)_C, SU(2)_L, U(1)_Y)$

LH fermions

$$(u,\ d)_L \qquad \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \tfrac{1}{6} \right)_2$$
$$(c,\ s)_L \qquad \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \tfrac{1}{6} \right)_2$$

$$(\nu_e,\ e)_L \qquad \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\tfrac{1}{2} \right)_2$$
$$(\nu_\mu,\ \mu)_L \qquad \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\tfrac{1}{2} \right)_2$$

RH fermions

$$u_R \qquad \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \tfrac{2}{3} \right)_2$$
$$c_R \qquad \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \tfrac{2}{3} \right)_2$$

$$d_R \qquad \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\tfrac{1}{3} \right)_2$$
$$s_R \qquad \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\tfrac{1}{3} \right)_2$$

$$e_R \qquad \left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1 \right)_2$$
$$\mu_R \qquad \left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1 \right)_2$$

2 generations

Standard Model Irreps
$(SU(3)_C, SU(2)_L, U(1)_Y)$

LH fermions

$$
\begin{array}{ll}
(u,\ d)_L & \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \tfrac{1}{6} \right)_2 \\
(c,\ s)_L & \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \tfrac{1}{6} \right)_2 \\
(t,\ b)_L & \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \tfrac{1}{6} \right)_2
\end{array}
$$

$$
\begin{array}{ll}
(\nu_e,\ e)_L & \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\tfrac{1}{2} \right)_2 \\
(\nu_\mu,\ \mu)_L & \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\tfrac{1}{2} \right)_2 \\
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\end{array}
$$

RH fermions

$$
\begin{array}{ll}
u_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \tfrac{2}{3} \right)_2 \\
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\end{array}
$$

3 generations

Standard Model Irreps
$(SU(3)_C, SU(2)_L, U(1)_Y)$

LH fermions

$$
\begin{array}{ll}
(u,\ d)_L & \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \tfrac{1}{6} \right)_2 \\
(c,\ s)_L & \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \tfrac{1}{6} \right)_2 \\
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\end{array}
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\begin{array}{ll}
(\nu_e,\ e)_L & \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\tfrac{1}{2} \right)_2 \\
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RH fermions

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\end{array}
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Fermion content

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$(SU(3)_C, SU(2)_L, U(1)_Y)$

LH fermions

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$$t_R \qquad \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \tfrac{2}{3} \right)_2$$

$$d_R \qquad \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\tfrac{1}{3} \right)_2$$
$$s_R \qquad \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\tfrac{1}{3} \right)_2$$
$$b_R \qquad \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\tfrac{1}{3} \right)_2$$

$$e_R \qquad \left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1 \right)_2$$
$$\mu_R \qquad \left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1 \right)_2$$
$$\tau_R \qquad \left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1 \right)_2$$

Standard Model Irreps
$(SU(3)_C, SU(2)_L, U(1)_Y)$

LH fermions

$$
\begin{array}{ll}
(u,\ d)_L & \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \tfrac{1}{6} \right)_2 \\
(c,\ s)_L & \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \tfrac{1}{6} \right)_2 \\
(t,\ b)_L & \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \tfrac{1}{6} \right)_2
\end{array}
$$

$$
\begin{array}{ll}
(\nu_e,\ e)_L & \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\tfrac{1}{2} \right)_2 \\
(\nu_\mu,\ \mu)_L & \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\tfrac{1}{2} \right)_2 \\
(\nu_\tau,\ \tau)_L & \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\tfrac{1}{2} \right)_2
\end{array}
$$

Gauge bosons

$$
\begin{array}{ll}
G_\mu & \left(\underline{\mathbf{8}},\ \underline{\mathbf{1}},\ 0 \right)_4 \\
W_\mu & \left(\underline{\mathbf{1}},\ \underline{\mathbf{3}},\ 0 \right)_4 \\
B_\mu & \left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0 \right)_4
\end{array}
$$

RH fermions

$$
\begin{array}{ll}
u_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \tfrac{2}{3} \right)_2 \\
c_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \tfrac{2}{3} \right)_2 \\
t_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \tfrac{2}{3} \right)_2
\end{array}
$$

$$
\begin{array}{ll}
d_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\tfrac{1}{3} \right)_2 \\
s_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\tfrac{1}{3} \right)_2 \\
b_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\tfrac{1}{3} \right)_2
\end{array}
$$

$$
\begin{array}{ll}
e_R & \left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1 \right)_2 \\
\mu_R & \left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1 \right)_2 \\
\tau_R & \left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1 \right)_2
\end{array}
$$

Higgs

$$
H \quad \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\tfrac{1}{2} \right)_1
$$

Standard Model Irreps
$(\mathrm{SU(3)_C}, \mathrm{SU(2)_L}, \mathrm{U(1)_Y})$

LH fermions

$$
\begin{array}{ll}
(u,\ d)_L & \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \tfrac{1}{6} \right)_2 \\
(c,\ s)_L & \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \tfrac{1}{6} \right)_2 \\
(t,\ b)_L & \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \tfrac{1}{6} \right)_2
\end{array}
$$

$$
\begin{array}{ll}
(\nu_e,\ e)_L & \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\tfrac{1}{2} \right)_2 \\
(\nu_\mu,\ \mu)_L & \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\tfrac{1}{2} \right)_2 \\
(\nu_\tau,\ \tau)_L & \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\tfrac{1}{2} \right)_2
\end{array}
$$

RH fermions

$$
\begin{array}{ll}
u_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \tfrac{2}{3} \right)_2 \\
c_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \tfrac{2}{3} \right)_2 \\
t_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \tfrac{2}{3} \right)_2
\end{array}
$$

$$
\begin{array}{ll}
d_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\tfrac{1}{3} \right)_2 \\
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G_\mu & \left(\underline{\mathbf{8}},\ \underline{\mathbf{1}},\ 0 \right)_4 \\
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B_\mu & \left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0 \right)_4
\end{array}
$$

Higgs

$$
H \qquad \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\tfrac{1}{2} \right)_1
$$

$$
+ 3\,\nu_R
$$

$$
\Rightarrow 244\,\mathbb{R}
$$

R, C, H, O

special

special $\longrightarrow$

special → ubiquitous

R, C, H, O

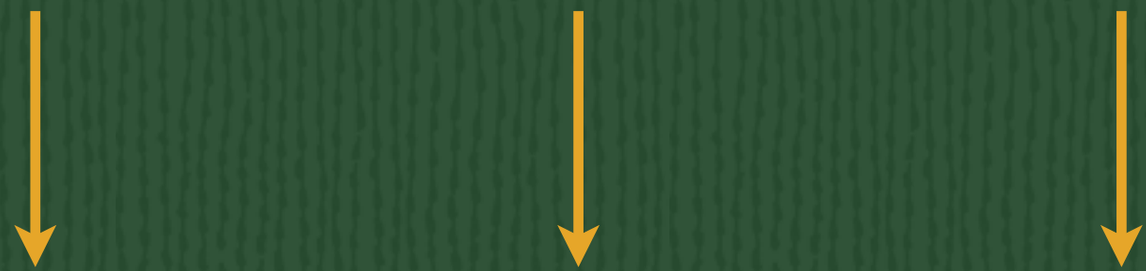
R, C, H, O

everywhere

R, C, H, O

everywhere quantum

R, **C,** **H,** **O**



everywhere quantum special relativity

R, **C,** **H,** **O**

everywhere quantum special relativity

electromagnetism

R, **C,** **H,** **O**

everywhere quantum special relativity strong nuclear

electromagnetism

N.F., M.J. Hughes,

One generation of standard model Weyl representations as a single copy of $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$,

Phys.Lett.B, 827 (2022) https://pirsa.org/21030013

$$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$$

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$$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O} \;=\; \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$$

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$$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O} \;=\; \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$$

G. Dixon

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Phys.Lett.B, 827 (2022) https://pirsa.org/21030013

$$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$$

$$64\ \mathbb{R}$$

N.F., M.J. Hughes,

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Phys.Lett.B, 827 (2022) https://pirsa.org/21030013

$$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$$

$$64\ \mathbb{R}$$

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Phys.Lett.B, 827 (2022) https://pirsa.org/21030013

$$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$$

64 $\mathbb{R}$

1 generation

N.F., M.J. Hughes,

One generation of standard model Weyl representations as a single copy of $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$,

Phys.Lett.B, 827 (2022) https://pirsa.org/21030013

$$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O} \quad \longleftrightarrow \quad 1 \text{ generation}$$

$64\ \mathbb{R}$

N.F., M.J. Hughes,

One generation of standard model Weyl representations as a single copy of $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$,

Phys.Lett.B, 827 (2022) https://pirsa.org/21030013

$$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O} \quad \longleftrightarrow \quad 1 \text{ generation}$$

$$64\ \mathbb{R}$$

N.F., M.J. Hughes,

One generation of standard model Weyl representations as a single copy of $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$,

Phys.Lett.B, 827 (2022) https://pirsa.org/21030013

$\subset \mathbb{O}$ $\longleftrightarrow$ up-quarks

$6\,\mathbb{R}$

M. Günaydin, F. Gürsey,

Quark structure and the octonions,

J. Math. Phys., 14, No.11 (1973)

Standard Model Irreps
($SU(3)_C$, $SU(2)_L$, $U(1)_Y$)

LH fermions

$(u,\ d)_L \quad \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6} \right)_2$

$(\nu_e,\ e)_L \quad \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2} \right)_2$

RH fermions

$u_R \quad \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3} \right)_2$

$d_R \quad \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3} \right)_2$

$e_R \quad \left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1 \right)_2$

1 generation

Standard Model Irreps
$(SU(3)_C, SU(2)_L, U(1)_Y)$

LH fermions

$$
\begin{array}{ll}
(u,\ d)_L & \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \tfrac{1}{6} \right)_2 \\
(c,\ s)_L & \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \tfrac{1}{6} \right)_2 \\
(t,\ b)_L & \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \tfrac{1}{6} \right)_2
\end{array}
$$

$$
\begin{array}{ll}
(\nu_e,\ e)_L & \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\tfrac{1}{2} \right)_2 \\
(\nu_\mu,\ \mu)_L & \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\tfrac{1}{2} \right)_2 \\
(\nu_\tau,\ \tau)_L & \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\tfrac{1}{2} \right)_2
\end{array}
$$

RH fermions

$$
\begin{array}{ll}
u_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \tfrac{2}{3} \right)_2 \\
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t_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \tfrac{2}{3} \right)_2
\end{array}
$$

$$
\begin{array}{ll}
d_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\tfrac{1}{3} \right)_2 \\
s_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\tfrac{1}{3} \right)_2 \\
b_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\tfrac{1}{3} \right)_2
\end{array}
$$

$$
\begin{array}{ll}
e_R & \left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1 \right)_2 \\
\mu_R & \left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1 \right)_2 \\
\tau_R & \left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1 \right)_2
\end{array}
$$

3 generations

Standard Model Irreps
$(SU(3)_C, SU(2)_L, U(1)_Y)$

LH fermions

$$
\begin{array}{ll}
(u,\ d)_L & \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \tfrac{1}{6} \right)_2 \\
(c,\ s)_L & \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \tfrac{1}{6} \right)_2 \\
(t,\ b)_L & \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \tfrac{1}{6} \right)_2
\end{array}
$$

$$
\begin{array}{ll}
(\nu_e,\ e)_L & \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\tfrac{1}{2} \right)_2 \\
(\nu_\mu,\ \mu)_L & \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\tfrac{1}{2} \right)_2 \\
(\nu_\tau,\ \tau)_L & \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\tfrac{1}{2} \right)_2
\end{array}
$$

Gauge bosons

$$
\begin{array}{ll}
G_\mu & \left(\underline{\mathbf{8}},\ \underline{\mathbf{1}},\ 0 \right)_4 \\
W_\mu & \left(\underline{\mathbf{1}},\ \underline{\mathbf{3}},\ 0 \right)_4 \\
B_\mu & \left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0 \right)_4
\end{array}
$$

RH fermions

$$
\begin{array}{ll}
u_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \tfrac{2}{3} \right)_2 \\
c_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \tfrac{2}{3} \right)_2 \\
t_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \tfrac{2}{3} \right)_2
\end{array}
$$

$$
\begin{array}{ll}
d_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\tfrac{1}{3} \right)_2 \\
s_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\tfrac{1}{3} \right)_2 \\
b_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\tfrac{1}{3} \right)_2
\end{array}
$$

$$
\begin{array}{ll}
e_R & \left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1 \right)_2 \\
\mu_R & \left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1 \right)_2 \\
\tau_R & \left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1 \right)_2
\end{array}
$$

Higgs

$$
H \quad \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\tfrac{1}{2} \right)_1
$$


sugar phosphate
backbone
cytosine
C
T
thymine
ribose
deoxyribose
G
guanine
adenine
A
weak hydrogen bonds

sequence

$$\overbrace{w(x(yz))}$$

$$\overbrace{w(x(yz))}^{\text{sequence}}$$

$$w, x, y, z \ \in \ \mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$$

encode particles?

$$\overbrace{w(x(yz))}$$

$$w, x, y, z \ \in \ \mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$$

$$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}\text{'s}$$

left-multiplication algebra

encode particles?

$$\overbrace{w(x(yz))}$$

$$w, x, y, z \ \in \ \mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$$

$$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}\text{'s}$$

left-multiplication algebra

"$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$"

encode particles?

$$\overbrace{w(x(yz))}$$

$$w, x, y, z \in \mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$$

$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$'s left-multiplication algebra "$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$"

$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$'s left-multiplication algebra "$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$"

$$w(x(yz))$$

$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$'s

left-multiplication algebra

"$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$"

operator

$\overbrace{}$

$w(x(yz))$

$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$'s left-multiplication algebra "$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$"

operator

$$\overbrace{w(x(yz))}$$

state

$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$'s left-multiplication algebra "$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$"

$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$'s left-multiplication algebra "$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$"

multiply

yz $\qquad$ $y, z \in \mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$

$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$'s left-multiplication algebra "$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$"

multiply

yz $\qquad$ $y, z \in \mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$

operator ↑

$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$'s left-multiplication algebra "$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$"

multiply

yz $\qquad$ $y, z \in \mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$

operator $\quad$ state

$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$'s left-multiplication algebra "$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$"

yz $\qquad$ $y, z \in \mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$

$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$'s left-multiplication algebra "$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$"

yz $\qquad\qquad$ $y, z \in \mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$

$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$'s left-multiplication algebra "$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$"

$$L_y(z) := yz \qquad\qquad y, z \in \mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$$

$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$'s left-multiplication algebra "$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$"

$$L_y(z) := yz \qquad\qquad y, z \in \mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$$

$$L_y \in End_{\mathbb{C}}(\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O})$$

$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$'s left-multiplication algebra "$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$"

$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$'s left-multiplication algebra "$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$"

$$(L_x \circ L_y)(z)$$

$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$'s left-multiplication algebra "$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$"

$$(L_x \circ L_y)(z) = L_x(L_y(z))$$

$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$'s left-multiplication algebra "$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$"

$$\left(L_x \circ L_y\right)(z) = L_x\left(L_y(z)\right) = x(yz)$$

$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$'s left-multiplication algebra "$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$"

Multiplication:

$$(L_x \circ L_y)(z) = L_x(L_y(z)) = x(yz)$$

$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$'s left-multiplication algebra "$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$"

$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$'s left-multiplication algebra "$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$"

$$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} := \text{subalgebra of } End_{\mathbb{C}}(\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O})$$
$$\text{generated by } \{L_y \mid y \in \mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}\}$$

$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$'s left-multiplication algebra "$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$"

$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$'s left-multiplication algebra "$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$"

$$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \;\simeq\; M_{16\times 16}(\mathbb{C})$$

$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$'s left-multiplication algebra "$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$"

$$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq M_{16\times 16}(\mathbb{C})$$

$$\simeq \mathbb{C}l(8)$$

$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$'s left-multiplication algebra "$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$"

$$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq M_{16\times 16}(\mathbb{C})$$

$$\simeq \mathbb{C}l(8)$$

8 γ_j
generate $\mathbb{C}l(8)$

$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$'s left-multiplication algebra "$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$"

$$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq M_{16\times 16}(\mathbb{C})$$

$$\simeq \mathbb{C}l(8)$$

8 γ_j $\longleftrightarrow$ A, C, T, G

generate $\mathbb{C}l(8)$

Standard Model Irreps
$(SU(3)_C, SU(2)_L, U(1)_Y)$

LH fermions

$$
\begin{array}{ll}
(u,\ d)_L & \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \tfrac{1}{6} \right)_2 \\
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(t,\ b)_L & \left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \tfrac{1}{6} \right)_2
\end{array}
$$

$$
\begin{array}{ll}
(\nu_e,\ e)_L & \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\tfrac{1}{2} \right)_2 \\
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(\nu_\tau,\ \tau)_L & \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\tfrac{1}{2} \right)_2
\end{array}
$$

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$$
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\end{array}
$$

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\begin{array}{ll}
u_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \tfrac{2}{3} \right)_2 \\
c_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \tfrac{2}{3} \right)_2 \\
t_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \tfrac{2}{3} \right)_2
\end{array}
$$

$$
\begin{array}{ll}
d_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\tfrac{1}{3} \right)_2 \\
s_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\tfrac{1}{3} \right)_2 \\
b_R & \left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\tfrac{1}{3} \right)_2
\end{array}
$$

$$
\begin{array}{ll}
e_R & \left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1 \right)_2 \\
\mu_R & \left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1 \right)_2 \\
\tau_R & \left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1 \right)_2
\end{array}
$$

Higgs

$$
H \quad \left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\tfrac{1}{2} \right)_1
$$

$$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O} \quad \longleftrightarrow \quad 1 \text{ generation}$$

$$64\ \mathbb{R}$$

N.F., M.J. Hughes,

One generation of standard model Weyl representations as a single copy of $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$,

Phys.Lett.B, 827 (2022) https://pirsa.org/21030013


sugar phosphate
backbone
cytosine
C
T
thymine
ribose
deoxyribose
G
guanine
adenine
A
weak hydrogen bonds

sequence

$$\overbrace{w(x(yz))}$$

$$\overbrace{w(x(yz))}^{\text{sequence}}$$

$$w, x, y, z \ \in \ \mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$$

$\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$'s left-multiplication algebra "$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$"

sequence

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$$L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq \mathbb{C}l(8)$$

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$8\ \gamma_j \qquad \longleftrightarrow \qquad$ A, C, T, G

generate $\mathbb{C}l(8)$

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256 $\mathbb{C}$

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Need

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Need

256 $\mathbb{C}$

244 $\mathbb{R}$

$$\dagger : \quad L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \;\rightarrow\; L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$$

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($\mathbb{C}$ imaginary) $\qquad i \mapsto -i$

$$\dagger: \quad L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \;\to\; L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$$

($\mathbb{C}$ imaginary)	$i \mapsto -i$	
($\mathbb{H}$ imaginaries)	$\epsilon_j \mapsto -\epsilon_j$	$j \in \{1,2,3\}$

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($\mathbb{O}$ imaginaries)	$e_k \mapsto -e_k$	$k \in \{1, 2, \ldots 7\}$

"Hermitian conjugate"

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($\mathbb{C}$ imaginary)	$i \mapsto -i$	
($\mathbb{H}$ imaginaries)	$\epsilon_j \mapsto -\epsilon_j$	$j \in \{1,2,3\}$
($\mathbb{O}$ imaginaries)	$e_k \mapsto -e_k$	$k \in \{1,2,\ldots 7\}$

Consider hermitian subspace
of $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq \mathcal{H}_{16}(\mathbb{C})$

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- $256\,\mathbb{R} \gtrsim 244\,\mathbb{R}$

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$\mathbb{O}$ multiplication algebra

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Idempotents

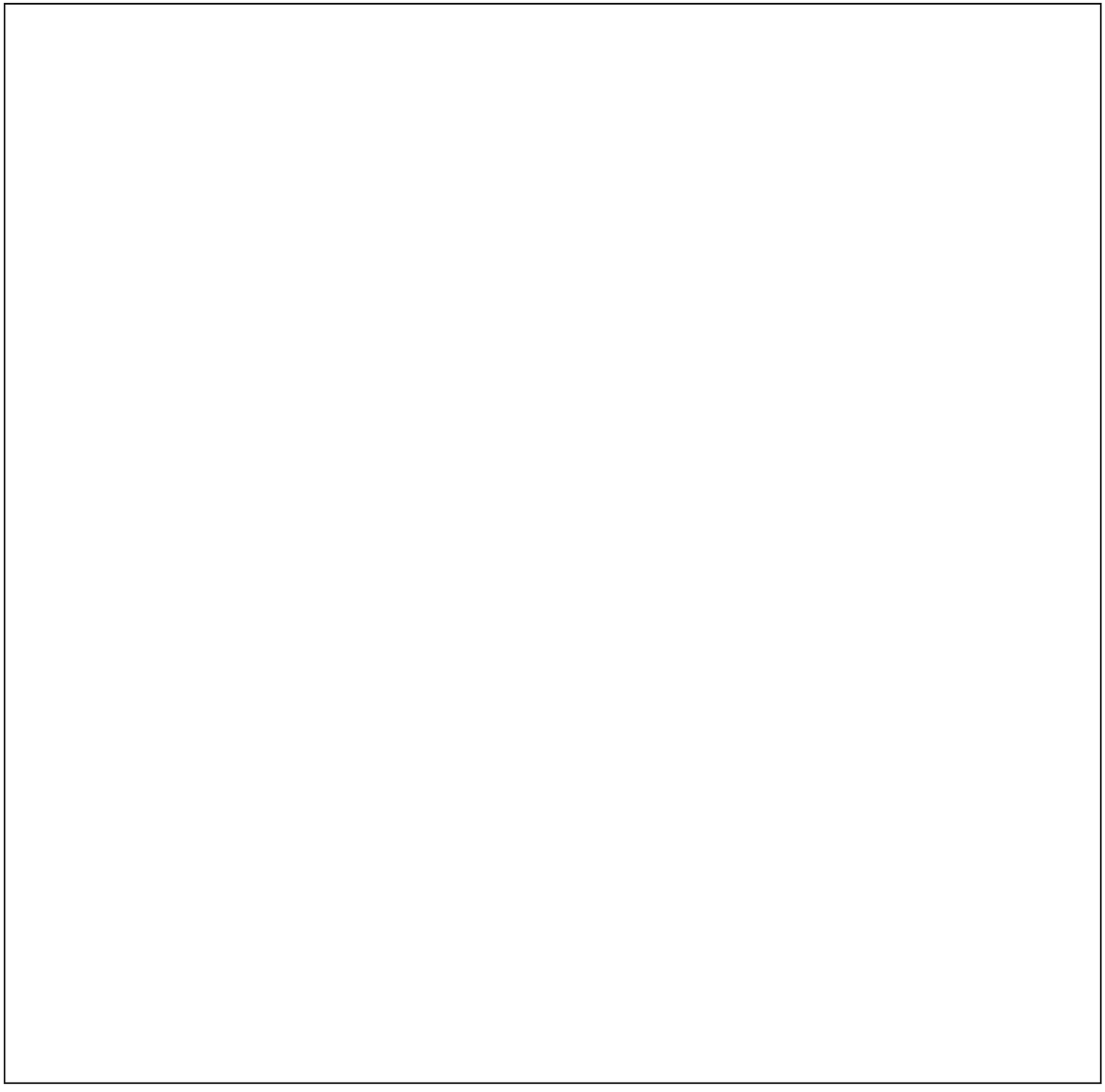
$\mathbb{O}$ multiplication algebra

Idempotents

$$s := \tfrac{1}{2}(1 + iL_{e_7}) \qquad S := \tfrac{1}{2}(1 + iR_{e_7})$$

$$s^* := \tfrac{1}{2}(1 - iL_{e_7}) \qquad S^* := \tfrac{1}{2}(1 - iR_{e_7})$$

$\mathcal{H}_{16}(\mathbb{C})$



$\mathcal{H}_{16}(\mathbb{C})$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

$\mathcal{H}_{16}(\mathbb{C})$

"Peirce decomposition"

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

$\mathcal{H}_{16}(\mathbb{C})$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$$\mathcal{H}_{16}(\mathbb{C})$$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(u,\ d)_L \quad (\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(c,\ s)_L \quad (\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(t,\ b)_L \quad (\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$

$(\nu_e,\ e)_L \quad (\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
$(\nu_\mu,\ \mu)_L \quad (\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
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$\nu_{eR} \quad (\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
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$p_\mu \quad (\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$
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$\mathcal{H}_{16}(\mathbb{C})$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
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Generic element of G_{sm}'s Lie algebra

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$\mathfrak{su}(3)_C$ $\mathfrak{su}(2)_L$ $\mathfrak{u}(1)_Y$

$$\ell_{sm} := \; ir'_n \Lambda_n s \; + \; r_k L_{\epsilon_k} s^* S \; + \; \frac{r}{2}\left(\frac{i}{3} s S^* - i s S - L_{\epsilon_3} s^* S^*\right)$$

$n \in \{1, 2, \ldots 8\}$

$k \in \{1, 2, 3\}$

Generic element of G_{sm}'s Lie algebra

$$\begin{array}{ccccc} & \mathfrak{su}(3)_C & \mathfrak{su}(2)_L & & \mathfrak{u}(1)_Y \\ \ell_{sm} := & ir'_n \Lambda_n s & + \; r_k L_{\epsilon_k} s^* S & + & \frac{r}{2}\left(\frac{i}{3} s S^* - i s S - L_{\epsilon_3} s^* S^*\right) \end{array}$$

$$\in \mathfrak{der}(\mathbb{O})$$

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Action of ℓ_{sm} on $\mathcal{H}_{16}(\mathbb{C})$

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$$\delta b = \ell_{sm} b + b\, \ell_{sm}^{\dagger}$$

diagonal

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$$\delta b = \ell_{sm} b + b\,\ell_{sm}^{\dagger} \qquad \text{diagonal}$$

$$\delta f_0 = \ell_{sm} s f_0 s^* + s f_0 s^* \ell_{sm} + h.c. \qquad \text{outer off-diagonal}$$

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$$\delta f_0 = \ell_{sm} s f_0 s^* + s f_0 s^* \ell_{sm} + h.c. \qquad \text{outer off-diagonal}$$

$$\delta f_+ = \ell_{sm}\big(sS^* f_+ sS + s^* S^* f_+ s^* S\big)$$

$$+\big(sS^* f_+ sS + s^* S^* f_+ s^* S\big)\ell_{sm}^{\dagger *} + h.c. \qquad \text{inner off-diagonal}$$

Physical content

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$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(\,u,\,d\,)_L$	$(\,\underline{\mathbf{3}},\,\underline{\mathbf{2}},\,\frac{1}{6}\,)_2$
$(\,c,\,s\,)_L$	$(\,\underline{\mathbf{3}},\,\underline{\mathbf{2}},\,\frac{1}{6}\,)_2$
$(\,t,\,b\,)_L$	$(\,\underline{\mathbf{3}},\,\underline{\mathbf{2}},\,\frac{1}{6}\,)_2$
$(\,\nu_e,\,e\,)_L$	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{2}},\,-\frac{1}{2}\,)_2$
$(\,\nu_\mu,\,\mu\,)_L$	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{2}},\,-\frac{1}{2}\,)_2$
$(\,\nu_\tau,\,\tau\,)_L$	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{2}},\,-\frac{1}{2}\,)_2$
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ν_{eR}	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_2$
$\nu_{\mu R}$	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_2$
$\nu_{\tau R}$	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_2$
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μ_R	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,-1\,)_2$
τ_R	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,-1\,)_2$
p_μ	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_4$
p'_μ	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_4$
G_μ	$(\,\underline{\mathbf{8}},\,\underline{\mathbf{1}},\,0\,)_4$
W_μ	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{3}},\,0\,)_4$
B_μ	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_4$

$$\mathcal{H}_{16}(\mathbb{C})$$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(\, u,\, d\,)_L$	$\left(\, \underline{\mathbf{3}},\, \underline{\mathbf{2}},\, \frac{1}{6}\,\right)_2$
$(\, c,\, s\,)_L$	$\left(\, \underline{\mathbf{3}},\, \underline{\mathbf{2}},\, \frac{1}{6}\,\right)_2$
$(\, t,\, b\,)_L$	$\left(\, \underline{\mathbf{3}},\, \underline{\mathbf{2}},\, \frac{1}{6}\,\right)_2$
$(\, \nu_e,\, e\,)_L$	$\left(\, \underline{\mathbf{1}},\, \underline{\mathbf{2}},\, -\frac{1}{2}\,\right)_2$
$(\, \nu_\mu,\, \mu\,)_L$	$\left(\, \underline{\mathbf{1}},\, \underline{\mathbf{2}},\, -\frac{1}{2}\right)_2$
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u_R	$\left(\, \underline{\mathbf{3}},\, \underline{\mathbf{1}},\, \frac{2}{3}\,\right)_2$
c_R	$\left(\, \underline{\mathbf{3}},\, \underline{\mathbf{1}},\, \frac{2}{3}\,\right)_2$
t_R	$\left(\, \underline{\mathbf{3}},\, \underline{\mathbf{1}},\, \frac{2}{3}\,\right)_2$
d_R	$\left(\, \underline{\mathbf{3}},\, \underline{\mathbf{1}},\, -\frac{1}{3}\,\right)_2$
s_R	$\left(\, \underline{\mathbf{3}},\, \underline{\mathbf{1}},\, -\frac{1}{3}\,\right)_2$
b_R	$\left(\, \underline{\mathbf{3}},\, \underline{\mathbf{1}},\, -\frac{1}{3}\,\right)_2$
ν_{eR}	$(\, \underline{\mathbf{1}},\, \underline{\mathbf{1}},\, 0\,)_2$
$\nu_{\mu R}$	$(\, \underline{\mathbf{1}},\, \underline{\mathbf{1}},\, 0\,)_2$
$\nu_{\tau R}$	$(\, \underline{\mathbf{1}},\, \underline{\mathbf{1}},\, 0\,)_2$
e_R	$(\, \underline{\mathbf{1}},\, \underline{\mathbf{1}},\, -1\,)_2$
μ_R	$(\, \underline{\mathbf{1}},\, \underline{\mathbf{1}},\, -1\,)_2$
τ_R	$(\, \underline{\mathbf{1}},\, \underline{\mathbf{1}},\, -1\,)_2$
p_μ	$(\, \underline{\mathbf{1}},\, \underline{\mathbf{1}},\, 0\,)_4$
p'_μ	$(\, \underline{\mathbf{1}},\, \underline{\mathbf{1}},\, 0\,)_4$
G_μ	$(\, \underline{\mathbf{8}},\, \underline{\mathbf{1}},\, 0\,)_4$
W_μ	$(\, \underline{\mathbf{1}},\, \underline{\mathbf{3}},\, 0\,)_4$
B_μ	$(\, \underline{\mathbf{1}},\, \underline{\mathbf{1}},\, 0\,)_4$

$\mathcal{H}_{16}(\mathbb{C})$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(u,\ d)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(c,\ s)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(t,\ b)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(\nu_e,\ e)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
$(\nu_\mu,\ \mu)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
$(\nu_\tau,\ \tau)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
u_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
c_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
t_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
d_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
s_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
b_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
ν_{eR}	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
$\nu_{\mu R}$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
$\nu_{\tau R}$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
e_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
μ_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
τ_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
p_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$
p'_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$
G_μ	$\left(\underline{\mathbf{8}},\ \underline{\mathbf{1}},\ 0\right)_4$
W_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{3}},\ 0\right)_4$
B_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$

$\mathcal{H}_{16}(\mathbb{C})$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(u,\ d)_L$	$(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(c,\ s)_L$	$(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(t,\ b)_L$	$(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(\nu_e,\ e)_L$	$(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
$(\nu_\mu,\ \mu)_L$	$(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
$(\nu_\tau,\ \tau)_L$	$(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
u_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
c_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
t_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
d_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
s_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
b_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
ν_{eR}	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
$\nu_{\mu R}$	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
$\nu_{\tau R}$	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
e_R	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
μ_R	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
τ_R	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
p_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$
p'_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$
G_μ	$(\underline{\mathbf{8}},\ \underline{\mathbf{1}},\ 0)_4$
W_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{3}},\ 0)_4$
B_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$

$\mathcal{H}_{16}(\mathbb{C})$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(u,\ d)_L$	$(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(c,\ s)_L$	$(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(t,\ b)_L$	$(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(\nu_e,\ e)_L$	$(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
$(\nu_\mu,\ \mu)_L$	$(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
$(\nu_\tau,\ \tau)_L$	$(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
u_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
c_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
t_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
d_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
s_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
b_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
ν_{eR}	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
$\nu_{\mu R}$	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
$\nu_{\tau R}$	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
e_R	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
μ_R	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
τ_R	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
p_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$
p'_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$
G_μ	$(\underline{\mathbf{8}},\ \underline{\mathbf{1}},\ 0)_4$
W_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{3}},\ 0)_4$
B_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$

$\mathcal{H}_{16}(\mathbb{C})$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(u,\ d)_L$	$(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(c,\ s)_L$	$(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(t,\ b)_L$	$(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(\nu_e,\ e)_L$	$(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
$(\nu_\mu,\ \mu)_L$	$(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
$(\nu_\tau,\ \tau)_L$	$(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
u_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
c_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
t_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
d_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
s_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
b_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
ν_{eR}	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
$\nu_{\mu R}$	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
$\nu_{\tau R}$	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
e_R	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
μ_R	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
τ_R	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
p_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$
p'_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$
G_μ	$(\underline{\mathbf{8}},\ \underline{\mathbf{1}},\ 0)_4$
W_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{3}},\ 0)_4$
B_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$

$$\mathcal{H}_{16}(\mathbb{C})$$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(u,\ d)_L$	$(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(c,\ s)_L$	$(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(t,\ b)_L$	$(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(\nu_e,\ e)_L$	$(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
$(\nu_\mu,\ \mu)_L$	$(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
$(\nu_\tau,\ \tau)_L$	$(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
u_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
c_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
t_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
d_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
s_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
b_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
ν_{eR}	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
$\nu_{\mu R}$	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
$\nu_{\tau R}$	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
e_R	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
μ_R	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
τ_R	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
p_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$
p'_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$
G_μ	$(\underline{\mathbf{8}},\ \underline{\mathbf{1}},\ 0)_4$
W_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{3}},\ 0)_4$
B_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$

$\mathcal{H}_{16}(\mathbb{C})$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(u,\ d)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(c,\ s)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(t,\ b)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(\nu_e,\ e)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
$(\nu_\mu,\ \mu)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
$(\nu_\tau,\ \tau)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
u_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
c_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
t_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
d_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
s_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
b_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
ν_{eR}	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
$\nu_{\mu R}$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
$\nu_{\tau R}$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
e_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
μ_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
τ_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
p_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$
p'_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$
G_μ	$\left(\underline{\mathbf{8}},\ \underline{\mathbf{1}},\ 0\right)_4$
W_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{3}},\ 0\right)_4$
B_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$

$\mathcal{H}_{16}(\mathbb{C})$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(u,\ d)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(c,\ s)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(t,\ b)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(\nu_e,\ e)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
$(\nu_\mu,\ \mu)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
$(\nu_\tau,\ \tau)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
u_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
c_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
t_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
d_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
s_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
b_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
ν_{eR}	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
$\nu_{\mu R}$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
$\nu_{\tau R}$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
e_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
μ_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
τ_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
p_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$
p'_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$
G_μ	$\left(\underline{\mathbf{8}},\ \underline{\mathbf{1}},\ 0\right)_4$
W_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{3}},\ 0\right)_4$
B_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$

$\mathcal{H}_{16}(\mathbb{C})$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(u,\ d)_L$	$(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(c,\ s)_L$	$(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(t,\ b)_L$	$(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(\nu_e,\ e)_L$	$(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
$(\nu_\mu,\ \mu)_L$	$(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
$(\nu_\tau,\ \tau)_L$	$(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
u_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
c_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
t_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
d_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
s_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
b_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
ν_{eR}	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
$\nu_{\mu R}$	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
$\nu_{\tau R}$	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
e_R	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
μ_R	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
τ_R	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
p_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$
p'_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$
G_μ	$(\underline{\mathbf{8}},\ \underline{\mathbf{1}},\ 0)_4$
W_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{3}},\ 0)_4$
B_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$

$\mathcal{H}_{16}(\mathbb{C})$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(u,\ d)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(c,\ s)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(t,\ b)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(\nu_e,\ e)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
$(\nu_\mu,\ \mu)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
$(\nu_\tau,\ \tau)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
u_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
c_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
t_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
d_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
s_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
b_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
ν_{eR}	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
$\nu_{\mu R}$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
$\nu_{\tau R}$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
e_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
μ_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
τ_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
p_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$
p'_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$
G_μ	$\left(\underline{\mathbf{8}},\ \underline{\mathbf{1}},\ 0\right)_4$
W_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{3}},\ 0\right)_4$
B_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$

$\mathcal{H}_{16}(\mathbb{C})$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(u,\ d)_L$	$(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(c,\ s)_L$	$(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(t,\ b)_L$	$(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(\nu_e,\ e)_L$	$(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
$(\nu_\mu,\ \mu)_L$	$(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
$(\nu_\tau,\ \tau)_L$	$(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
u_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
c_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
t_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
d_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
s_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
b_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
ν_{eR}	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
$\nu_{\mu R}$	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
$\nu_{\tau R}$	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
e_R	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
μ_R	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
τ_R	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
p_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$
p'_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$
G_μ	$(\underline{\mathbf{8}},\ \underline{\mathbf{1}},\ 0)_4$
W_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{3}},\ 0)_4$
B_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$

$\mathcal{H}_{16}(\mathbb{C})$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$ too many here	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(u,\ d)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(c,\ s)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(t,\ b)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(\nu_e,\ e)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
$(\nu_\mu,\ \mu)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
$(\nu_\tau,\ \tau)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
u_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
c_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
t_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
d_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
s_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
b_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
ν_{eR}	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
$\nu_{\mu R}$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
$\nu_{\tau R}$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
e_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
μ_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
τ_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
p_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$
p'_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$
G_μ	$\left(\underline{\mathbf{8}},\ \underline{\mathbf{1}},\ 0\right)_4$
W_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{3}},\ 0\right)_4$
B_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$

$\mathcal{H}_{16}(\mathbb{C})$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(u,\ d)_L$	$(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(c,\ s)_L$	$(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(t,\ b)_L$	$(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(\nu_e,\ e)_L$	$(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
$(\nu_\mu,\ \mu)_L$	$(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
$(\nu_\tau,\ \tau)_L$	$(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
u_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
c_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
t_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
d_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
s_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
b_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
ν_{eR}	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
$\nu_{\mu R}$	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
$\nu_{\tau R}$	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
e_R	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
μ_R	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
τ_R	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
p_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$
p'_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$
G_μ	$(\underline{\mathbf{8}},\ \underline{\mathbf{1}},\ 0)_4$
W_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{3}},\ 0)_4$
B_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$

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$\mathcal{H}_{16}(\mathbb{C})$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(\,u,\,d\,)_L \quad (\,\underline{\mathbf{3}},\,\underline{\mathbf{2}},\,\frac{1}{6}\,)_2$
$(\,c,\,s\,)_L \quad (\,\underline{\mathbf{3}},\,\underline{\mathbf{2}},\,\frac{1}{6}\,)_2$
$(\,t,\,b\,)_L \quad (\,\underline{\mathbf{3}},\,\underline{\mathbf{2}},\,\frac{1}{6}\,)_2$

$(\,\nu_e,\,e\,)_L \quad (\,\underline{\mathbf{1}},\,\underline{\mathbf{2}},\,-\frac{1}{2}\,)_2$
$(\,\nu_\mu,\,\mu\,)_L \quad (\,\underline{\mathbf{1}},\,\underline{\mathbf{2}},\,-\frac{1}{2})_2$
$(\,\nu_\tau,\,\tau\,)_L \quad (\,\underline{\mathbf{1}},\,\underline{\mathbf{2}},\,-\frac{1}{2}\,)_2$

$u_R \quad (\,\underline{\mathbf{3}},\,\underline{\mathbf{1}},\,\frac{2}{3}\,)_2$
$c_R \quad (\,\underline{\mathbf{3}},\,\underline{\mathbf{1}},\,\frac{2}{3}\,)_2$
$t_R \quad (\,\underline{\mathbf{3}},\,\underline{\mathbf{1}},\,\frac{2}{3}\,)_2$

$d_R \quad (\,\underline{\mathbf{3}},\,\underline{\mathbf{1}},\,-\frac{1}{3}\,)_2$
$s_R \quad (\,\underline{\mathbf{3}},\,\underline{\mathbf{1}},\,-\frac{1}{3}\,)_2$
$b_R \quad (\,\underline{\mathbf{3}},\,\underline{\mathbf{1}},\,-\frac{1}{3}\,)_2$

$\nu_{eR} \quad (\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_2$
$\nu_{\mu R} \quad (\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_2$
$\nu_{\tau R} \quad (\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_2$

$e_R \quad (\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,-1\,)_2$
$\mu_R \quad (\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,-1\,)_2$
$\tau_R \quad (\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,-1\,)_2$

$p_\mu \quad (\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_4$
$p'_\mu \quad (\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_4$

$G_\mu \quad (\,\underline{\mathbf{8}},\,\underline{\mathbf{1}},\,0\,)_4$
$W_\mu \quad (\,\underline{\mathbf{1}},\,\underline{\mathbf{3}},\,0\,)_4$
$B_\mu \quad (\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_4$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(u,\ d)_L \quad (\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(c,\ s)_L \quad (\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(t,\ b)_L \quad (\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$

$(\nu_e,\ e)_L \quad (\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
$(\nu_\mu,\ \mu)_L \quad (\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
$(\nu_\tau,\ \tau)_L \quad (\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$

$u_R \quad (\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
$c_R \quad (\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
$t_R \quad (\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$

$d_R \quad (\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
$s_R \quad (\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
$b_R \quad (\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$

$\nu_{eR} \quad (\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
$\nu_{\mu R} \quad (\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
$\nu_{\tau R} \quad (\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$

$e_R \quad (\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
$\mu_R \quad (\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
$\tau_R \quad (\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$

$p_\mu \quad (\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$
$p'_\mu \quad (\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$

$G_\mu \quad (\underline{\mathbf{8}},\ \underline{\mathbf{1}},\ 0)_4$
$W_\mu \quad (\underline{\mathbf{1}},\ \underline{\mathbf{3}},\ 0)_4$
$B_\mu \quad (\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$

Covariant derivative

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(u,\ d)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(c,\ s)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(t,\ b)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(\nu_e,\ e)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
$(\nu_\mu,\ \mu)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
$(\nu_\tau,\ \tau)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
u_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
c_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
t_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
d_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
s_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
b_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
ν_{eR}	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
$\nu_{\mu R}$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
$\nu_{\tau R}$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
e_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
μ_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
τ_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
p_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$
p'_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$
G_μ	$\left(\underline{\mathbf{8}},\ \underline{\mathbf{1}},\ 0\right)_4$
W_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{3}},\ 0\right)_4$
B_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$

$\mathcal{H}_{16}(\mathbb{C})$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(u,\ d)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(c,\ s)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(t,\ b)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(\nu_e,\ e)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
$(\nu_\mu,\ \mu)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
$(\nu_\tau,\ \tau)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
u_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
c_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
t_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
d_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
s_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
b_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
ν_{eR}	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
$\nu_{\mu R}$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
$\nu_{\tau R}$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
e_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
μ_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
τ_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
p_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$
p'_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$
G_μ	$\left(\underline{\mathbf{8}},\ \underline{\mathbf{1}},\ 0\right)_4$
W_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{3}},\ 0\right)_4$
B_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(u,\ d)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(c,\ s)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(t,\ b)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(\nu_e,\ e)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
$(\nu_\mu,\ \mu)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
$(\nu_\tau,\ \tau)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
u_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
c_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
t_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
d_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
s_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
b_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
ν_{eR}	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
$\nu_{\mu R}$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
$\nu_{\tau R}$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
e_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
μ_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
τ_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
p_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$
p'_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$
G_μ	$\left(\underline{\mathbf{8}},\ \underline{\mathbf{1}},\ 0\right)_4$
W_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{3}},\ 0\right)_4$
B_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$

Fermions

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(u,\ d)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(c,\ s)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(t,\ b)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(\nu_e,\ e)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
$(\nu_\mu,\ \mu)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
$(\nu_\tau,\ \tau)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
u_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
c_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
t_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
d_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
s_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
b_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
ν_{eR}	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
$\nu_{\mu R}$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
$\nu_{\tau R}$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
e_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
μ_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
τ_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
p_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$
p'_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$
G_μ	$\left(\underline{\mathbf{8}},\ \underline{\mathbf{1}},\ 0\right)_4$
W_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{3}},\ 0\right)_4$
B_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$

$\mathcal{H}_{16}(\mathbb{C})$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(\,u,\,d\,)_L$	$(\,\underline{\mathbf{3}},\,\underline{\mathbf{2}},\,\frac{1}{6}\,)_2$
$(\,c,\,s\,)_L$	$(\,\underline{\mathbf{3}},\,\underline{\mathbf{2}},\,\frac{1}{6}\,)_2$
$(\,t,\,b\,)_L$	$(\,\underline{\mathbf{3}},\,\underline{\mathbf{2}},\,\frac{1}{6}\,)_2$
$(\,\nu_e,\,e\,)_L$	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{2}},\,-\frac{1}{2}\,)_2$
$(\,\nu_\mu,\,\mu\,)_L$	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{2}},\,-\frac{1}{2}\,)_2$
$(\,\nu_\tau,\,\tau\,)_L$	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{2}},\,-\frac{1}{2}\,)_2$
u_R	$(\,\underline{\mathbf{3}},\,\underline{\mathbf{1}},\,\frac{2}{3}\,)_2$
c_R	$(\,\underline{\mathbf{3}},\,\underline{\mathbf{1}},\,\frac{2}{3}\,)_2$
t_R	$(\,\underline{\mathbf{3}},\,\underline{\mathbf{1}},\,\frac{2}{3}\,)_2$
d_R	$(\,\underline{\mathbf{3}},\,\underline{\mathbf{1}},\,-\frac{1}{3}\,)_2$
s_R	$(\,\underline{\mathbf{3}},\,\underline{\mathbf{1}},\,-\frac{1}{3}\,)_2$
b_R	$(\,\underline{\mathbf{3}},\,\underline{\mathbf{1}},\,-\frac{1}{3}\,)_2$
ν_{eR}	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_2$
$\nu_{\mu R}$	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_2$
$\nu_{\tau R}$	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_2$
e_R	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,-1\,)_2$
μ_R	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,-1\,)_2$
τ_R	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,-1\,)_2$
p_μ	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_4$
p'_μ	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_4$
G_μ	$(\,\underline{\mathbf{8}},\,\underline{\mathbf{1}},\,0\,)_4$
W_μ	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{3}},\,0\,)_4$
B_μ	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_4$

$\mathcal{H}_{16}(\mathbb{C})$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(u,\ d)_L$	$(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(c,\ s)_L$	$(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(t,\ b)_L$	$(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(\nu_e,\ e)_L$	$(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
$(\nu_\mu,\ \mu)_L$	$(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
$(\nu_\tau,\ \tau)_L$	$(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
u_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
c_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
t_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
d_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
s_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
b_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
ν_{eR}	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
$\nu_{\mu R}$	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
$\nu_{\tau R}$	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
e_R	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
μ_R	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
τ_R	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
p_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$
p'_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$
G_μ	$(\underline{\mathbf{8}},\ \underline{\mathbf{1}},\ 0)_4$
W_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{3}},\ 0)_4$
B_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$

$\mathcal{H}_{16}(\mathbb{C})$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$ too many here	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

End of the line?

End of the line?

Maybe, maybe not.

Physical content

$(u,\ d)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(c,\ s)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(t,\ b)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(\nu_e,\ e)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
$(\nu_\mu,\ \mu)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
$(\nu_\tau,\ \tau)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
u_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
c_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
t_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
d_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
s_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
b_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
ν_{eR}	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
$\nu_{\mu R}$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
$\nu_{\tau R}$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
e_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
μ_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
τ_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
p_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$
p'_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$
G_μ	$\left(\underline{\mathbf{8}},\ \underline{\mathbf{1}},\ 0\right)_4$
W_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{3}},\ 0\right)_4$
B_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$

$\mathcal{H}_{16}(\mathbb{C})$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(\,u,\,d\,)_L$	$(\,\underline{\mathbf{3}},\,\underline{\mathbf{2}},\,\frac{1}{6}\,)_2$
$(\,c,\,s\,)_L$	$(\,\underline{\mathbf{3}},\,\underline{\mathbf{2}},\,\frac{1}{6}\,)_2$
$(\,t,\,b\,)_L$	$(\,\underline{\mathbf{3}},\,\underline{\mathbf{2}},\,\frac{1}{6}\,)_2$
$(\,\nu_e,\,e\,)_L$	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{2}},\,-\frac{1}{2}\,)_2$
$(\,\nu_\mu,\,\mu\,)_L$	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{2}},\,-\frac{1}{2}\,)_2$
$(\,\nu_\tau,\,\tau\,)_L$	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{2}},\,-\frac{1}{2}\,)_2$
u_R	$(\,\underline{\mathbf{3}},\,\underline{\mathbf{1}},\,\frac{2}{3}\,)_2$
c_R	$(\,\underline{\mathbf{3}},\,\underline{\mathbf{1}},\,\frac{2}{3}\,)_2$
t_R	$(\,\underline{\mathbf{3}},\,\underline{\mathbf{1}},\,\frac{2}{3}\,)_2$
d_R	$(\,\underline{\mathbf{3}},\,\underline{\mathbf{1}},\,-\frac{1}{3}\,)_2$
s_R	$(\,\underline{\mathbf{3}},\,\underline{\mathbf{1}},\,-\frac{1}{3}\,)_2$
b_R	$(\,\underline{\mathbf{3}},\,\underline{\mathbf{1}},\,-\frac{1}{3}\,)_2$
ν_{eR}	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_2$
$\nu_{\mu R}$	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_2$
$\nu_{\tau R}$	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_2$
e_R	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,-1\,)_2$
μ_R	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,-1\,)_2$
τ_R	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,-1\,)_2$
p_μ	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_4$
p'_μ	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_4$
G_μ	$(\,\underline{\mathbf{8}},\,\underline{\mathbf{1}},\,0\,)_4$
W_μ	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{3}},\,0\,)_4$
B_μ	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_4$

$$\mathcal{H}_{16}(\mathbb{C})$$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(\,u,\,d\,)_L$	$(\,\underline{\mathbf{3}},\,\underline{\mathbf{2}},\,\frac{1}{6}\,)_2$
$(\,c,\,s\,)_L$	$(\,\underline{\mathbf{3}},\,\underline{\mathbf{2}},\,\frac{1}{6}\,)_2$
$(\,t,\,b\,)_L$	$(\,\underline{\mathbf{3}},\,\underline{\mathbf{2}},\,\frac{1}{6}\,)_2$
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ν_{eR}	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_2$
$\nu_{\mu R}$	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_2$
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e_R	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,-1\,)_2$
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$$\mathcal{H}_{16}(\mathbb{C})$$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

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$(\,u,\,d\,)_L$	$(\,\underline{\mathbf{3}},\,\underline{\mathbf{2}},\,\frac{1}{6}\,)_2$
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$$\mathcal{H}_{16}(\mathbb{C})$$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

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$$\mathcal{H}_{16}(\mathbb{C})$$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(\,u,\,d\,)_L$	$(\,\underline{\mathbf{3}},\,\underline{\mathbf{2}},\,\frac{1}{6}\,)_2$
$(\,c,\,s\,)_L$	$(\,\underline{\mathbf{3}},\,\underline{\mathbf{2}},\,\frac{1}{6}\,)_2$
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$\nu_{\tau R}$	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_2$
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p_μ	$(\,\underline{\mathbf{1}},\,\underline{\mathbf{1}},\,0\,)_4$
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$$\mathcal{H}_{16}(\mathbb{C})$$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

To be continued …

Colour and projective measurements

$\mathbb{O}$ multiplication algebra

Idempotents

$$s := \tfrac{1}{2}(1 + iL_{e_7}) \qquad S := \tfrac{1}{2}(1 + iR_{e_7})$$

$$s^* := \tfrac{1}{2}(1 - iL_{e_7}) \qquad S^* := \tfrac{1}{2}(1 - iR_{e_7})$$

Octonion

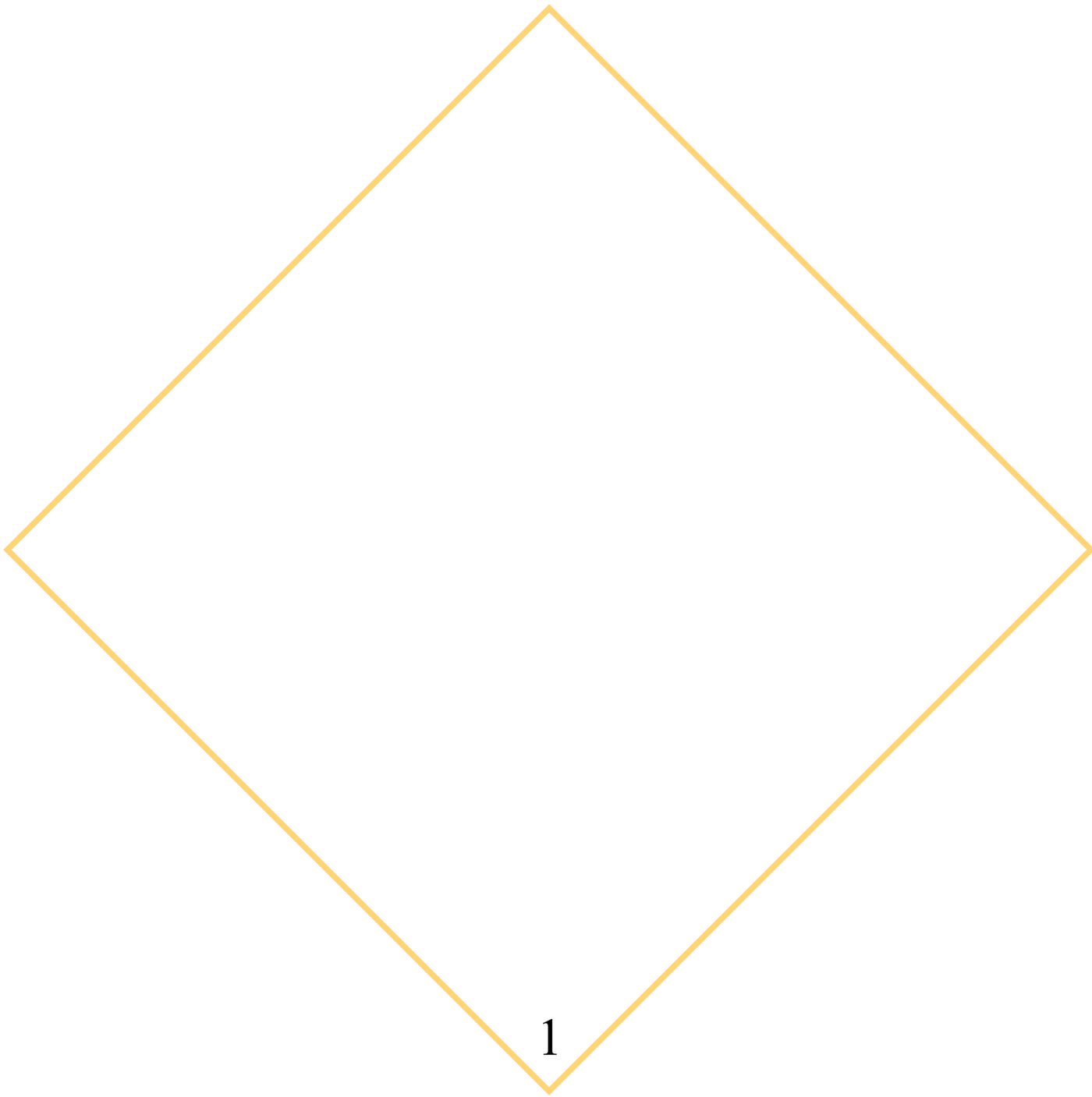
Left multiplication

Octonion

Left multiplication

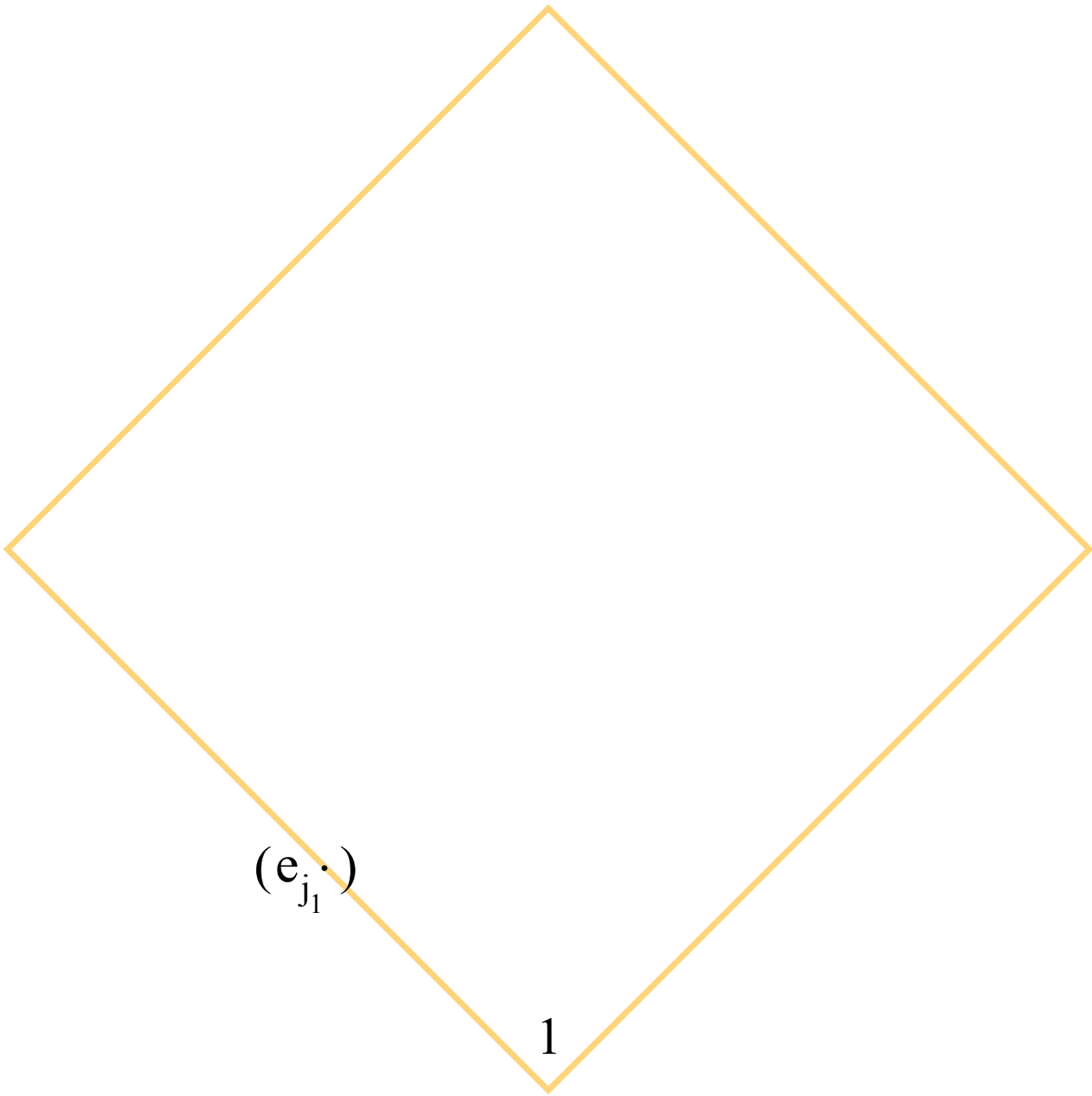
Octonion

Left multiplication



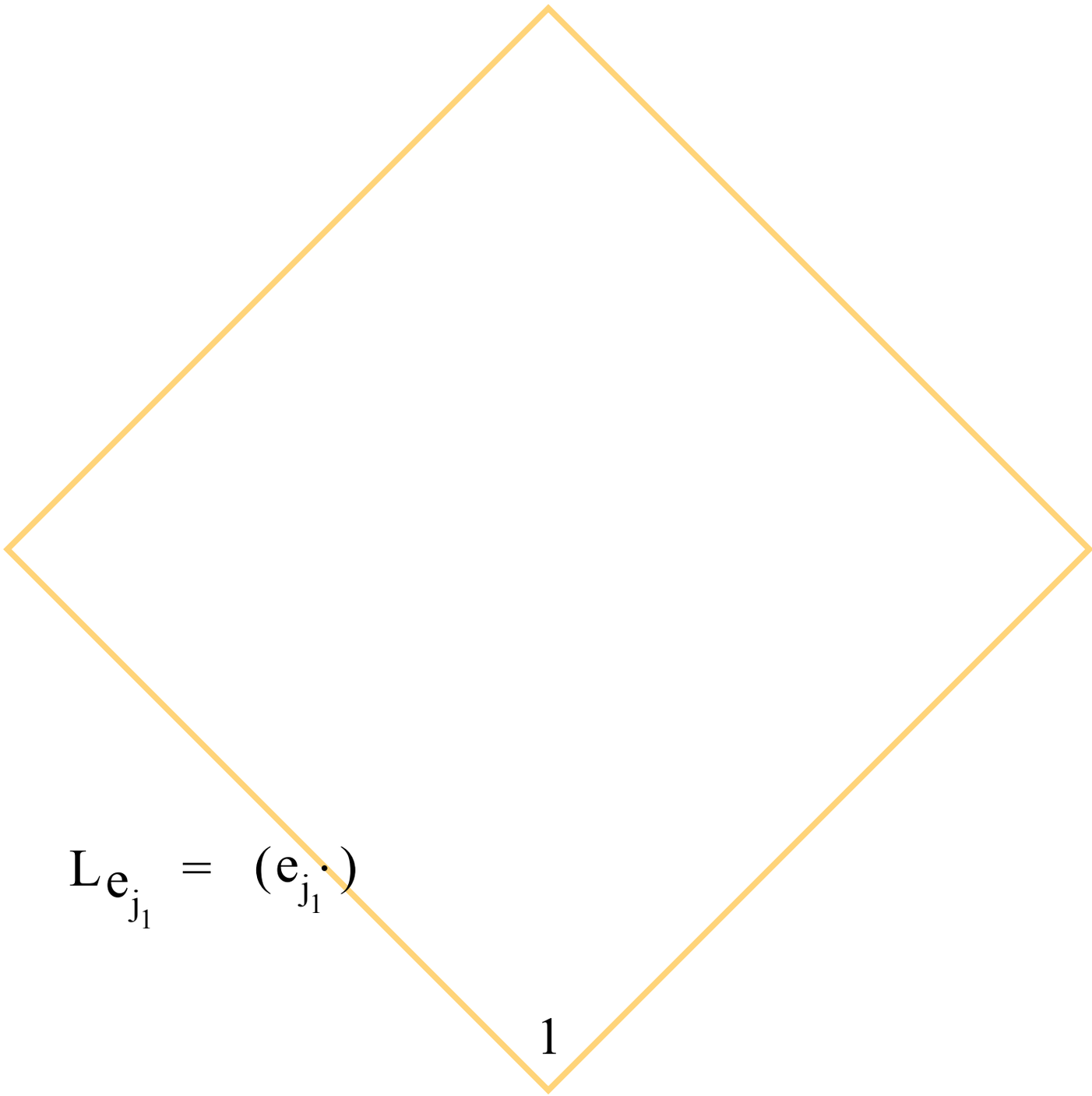
Octonion

Left multiplication



Octonion

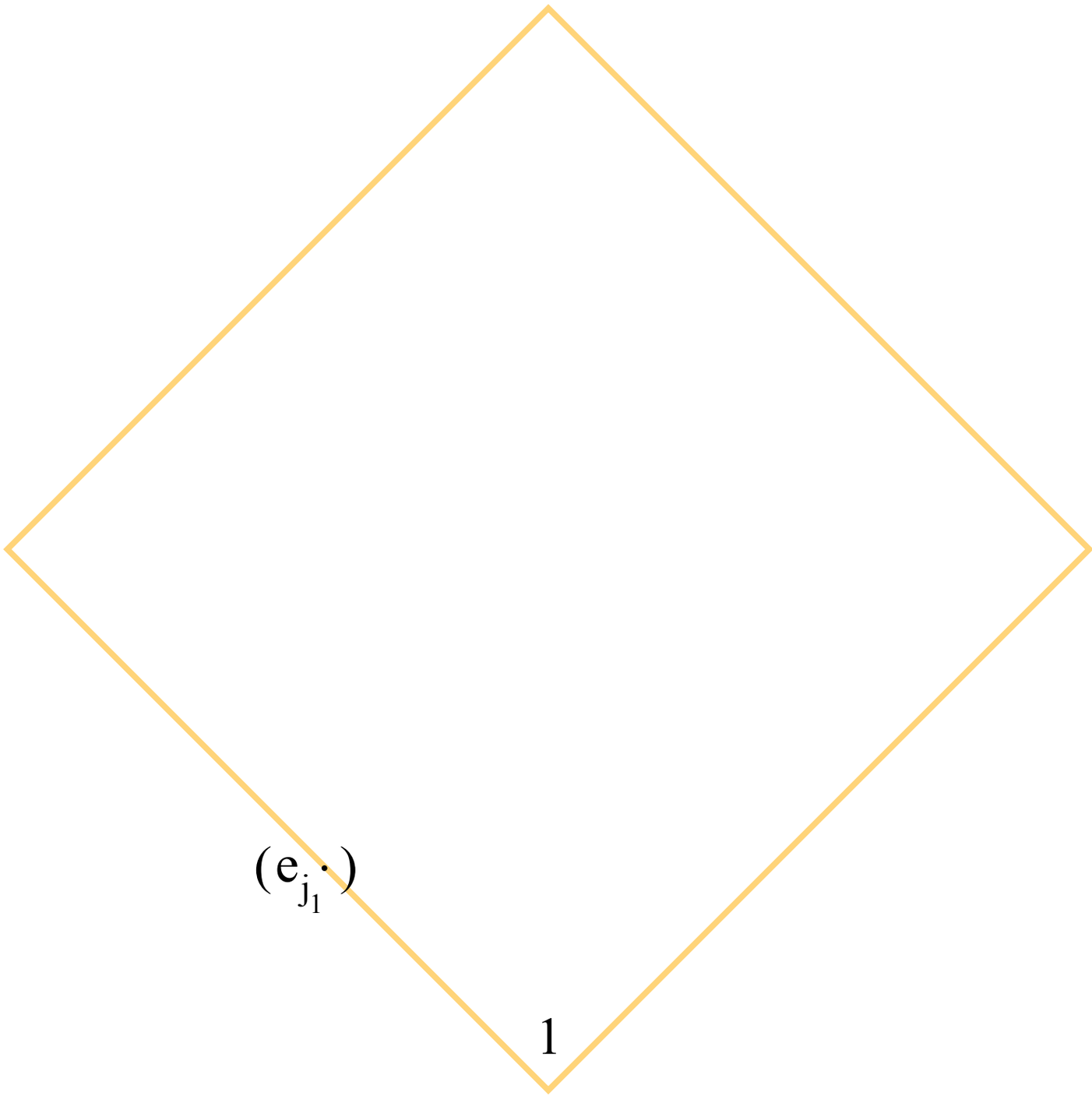
Left multiplication



$$L_{e_{j_1}} = (e_{j_1} \cdot)$$

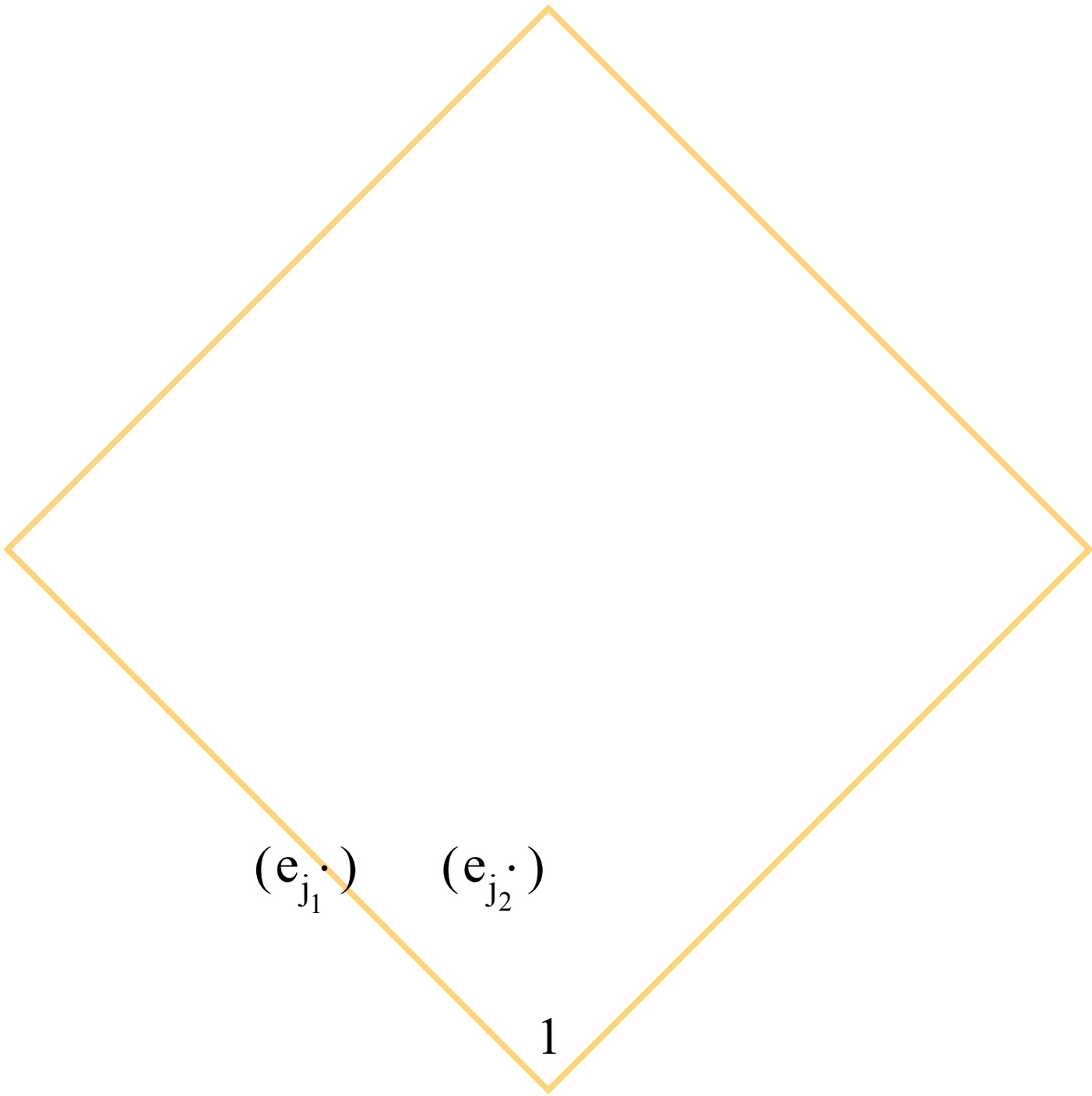
Octonion

Left multiplication



Octonion

Left multiplication



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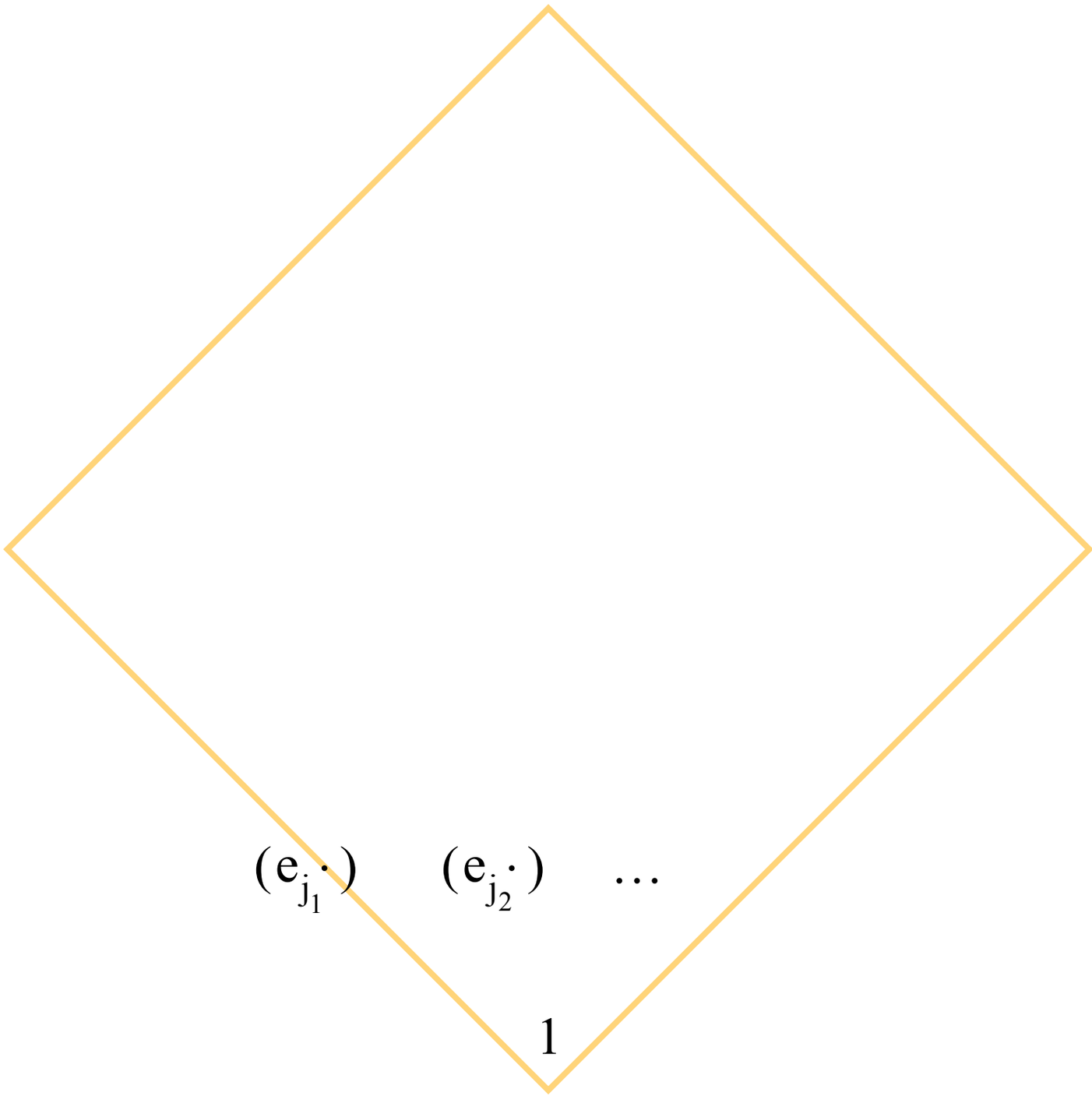
Left multiplication



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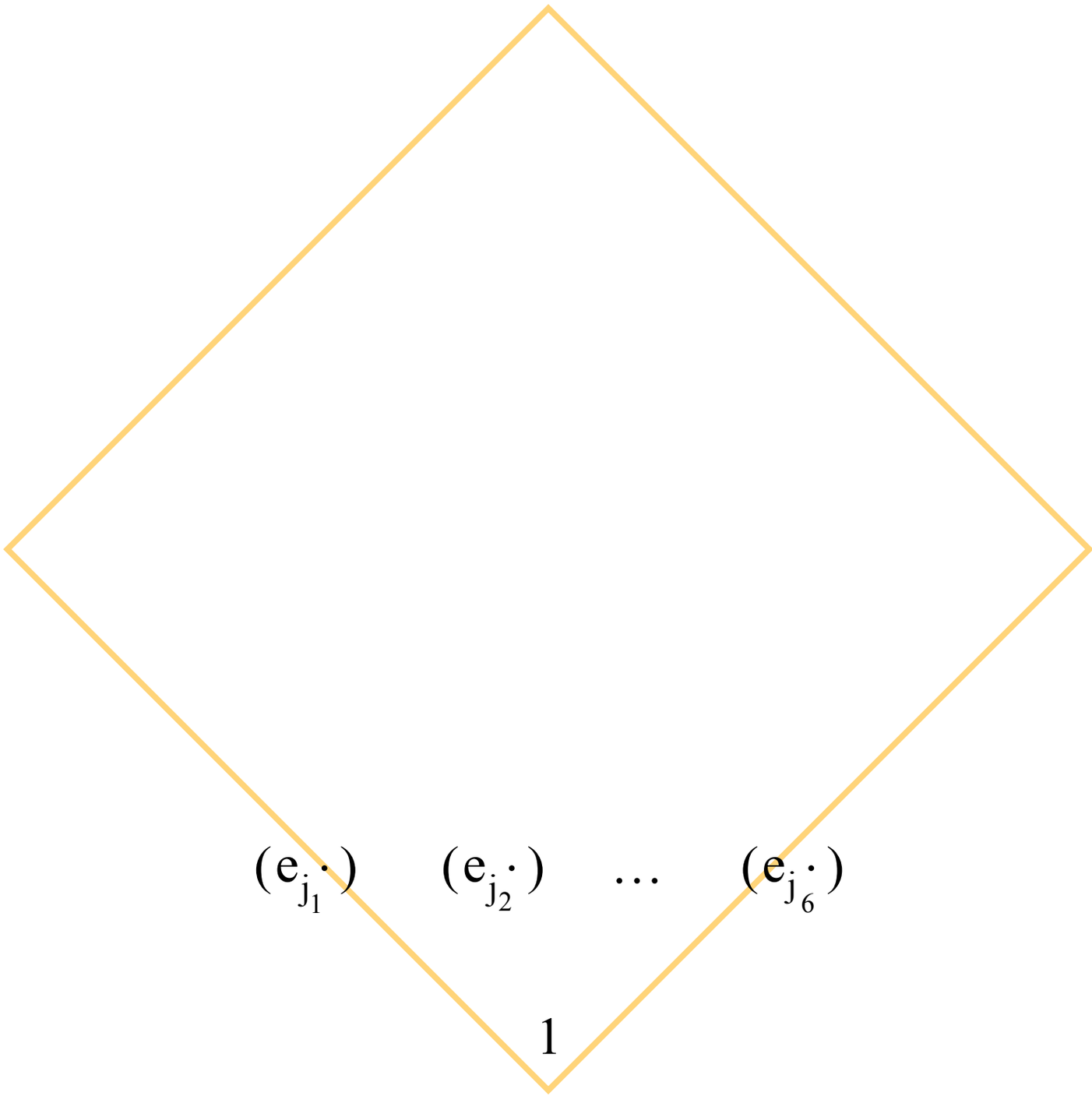
Left multiplication



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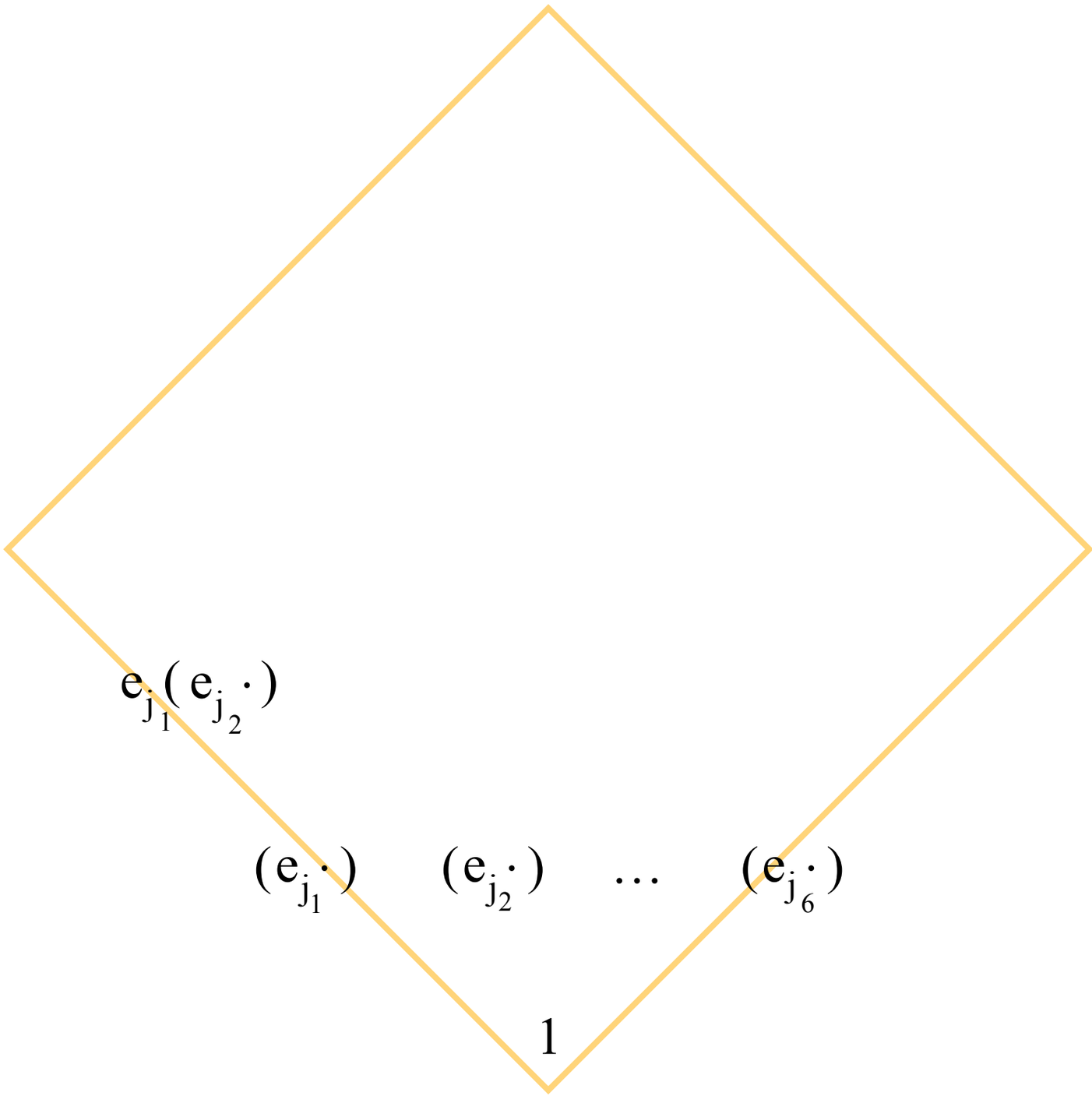
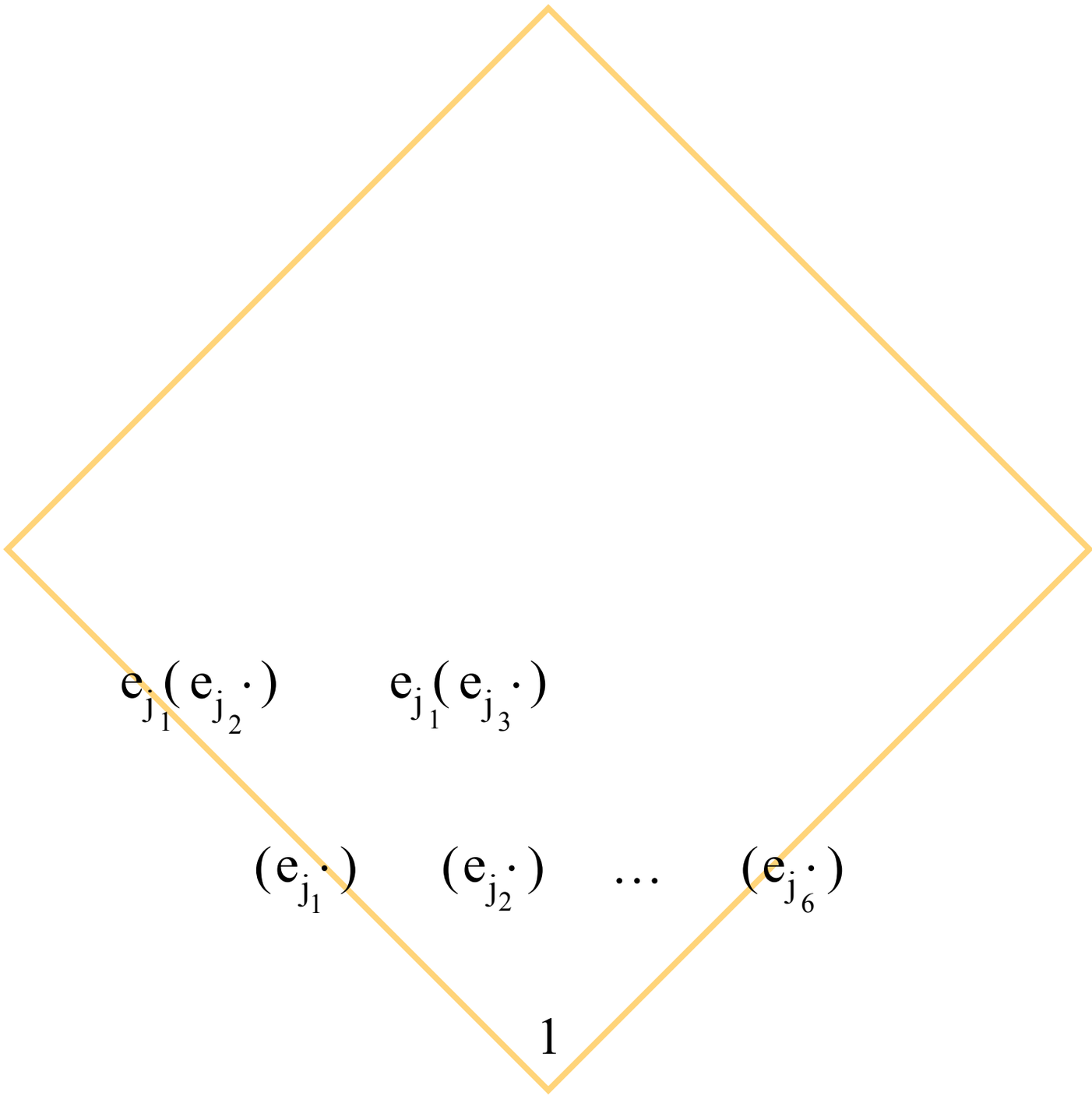
Left multiplication



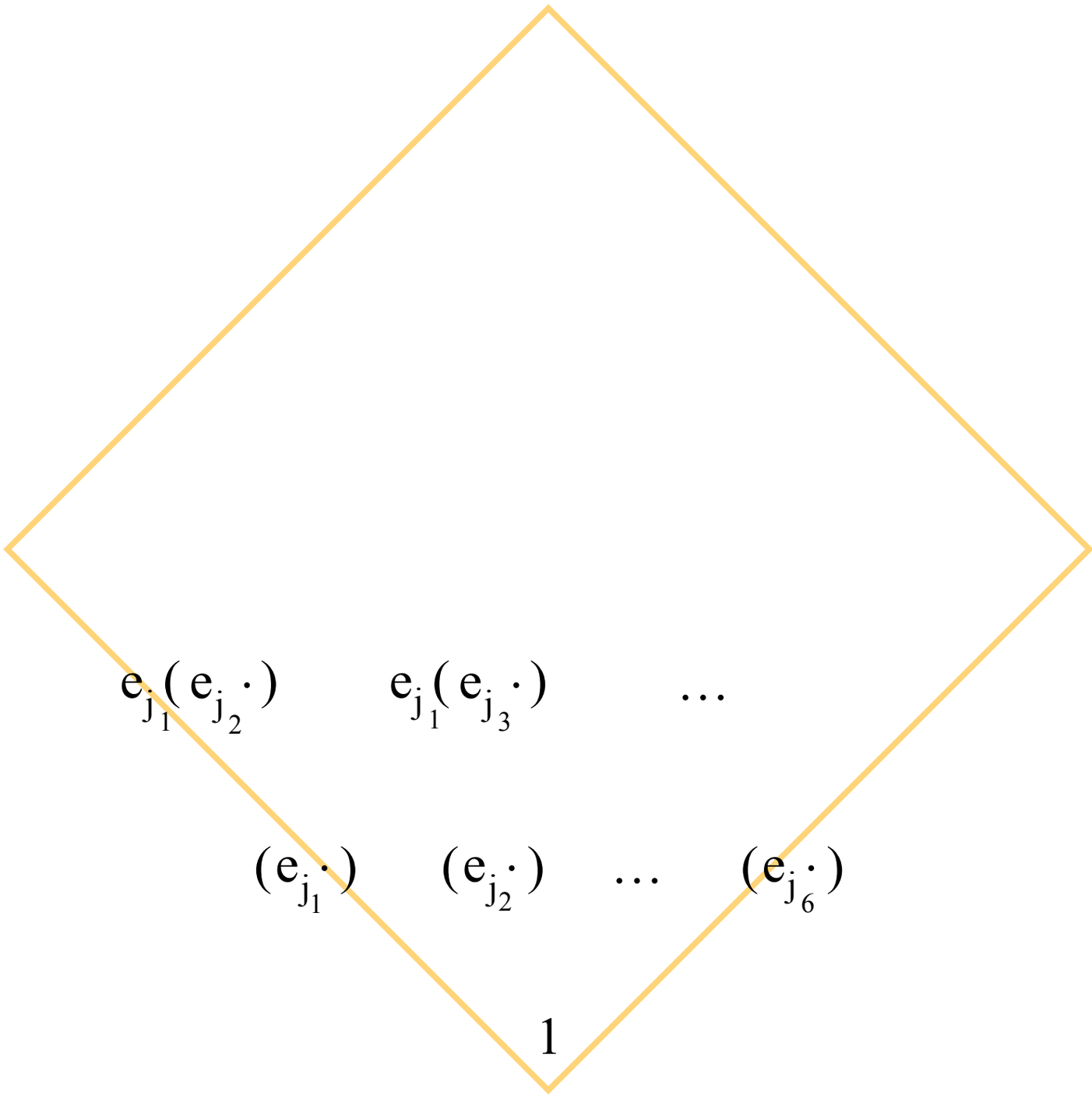
Octonion

Left multiplication



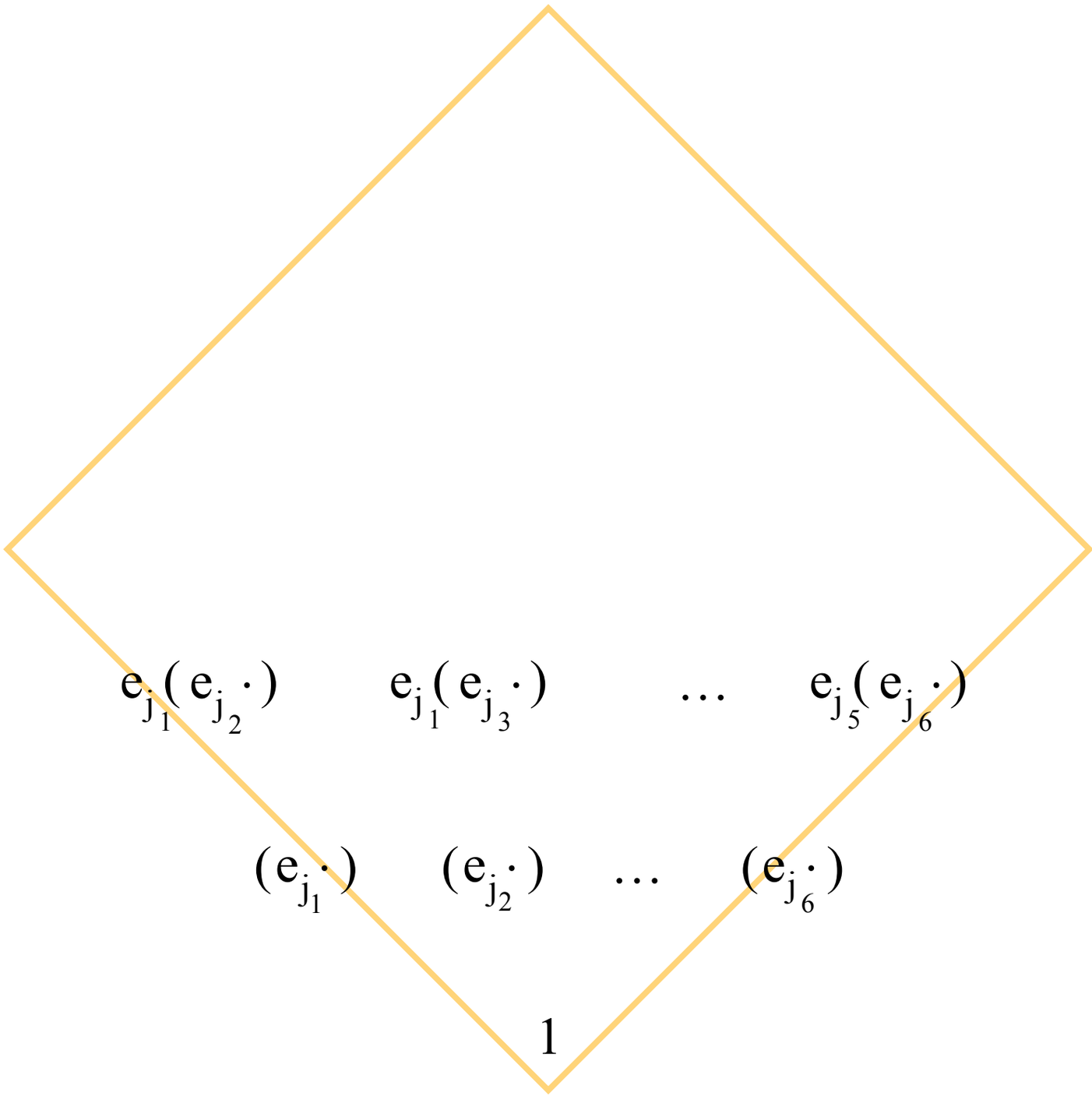
Octonion

Left multiplication



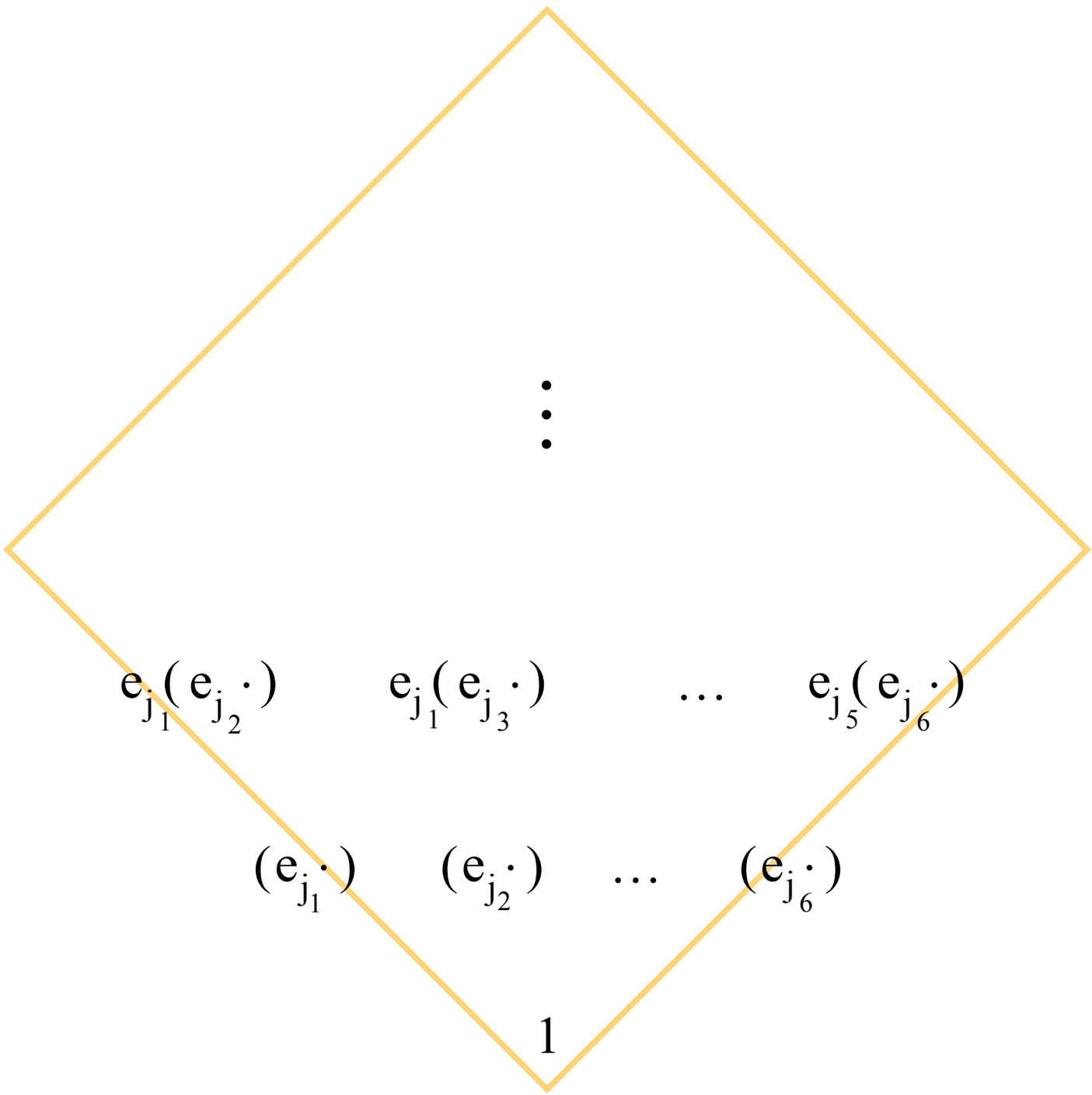
Octonion

Left multiplication



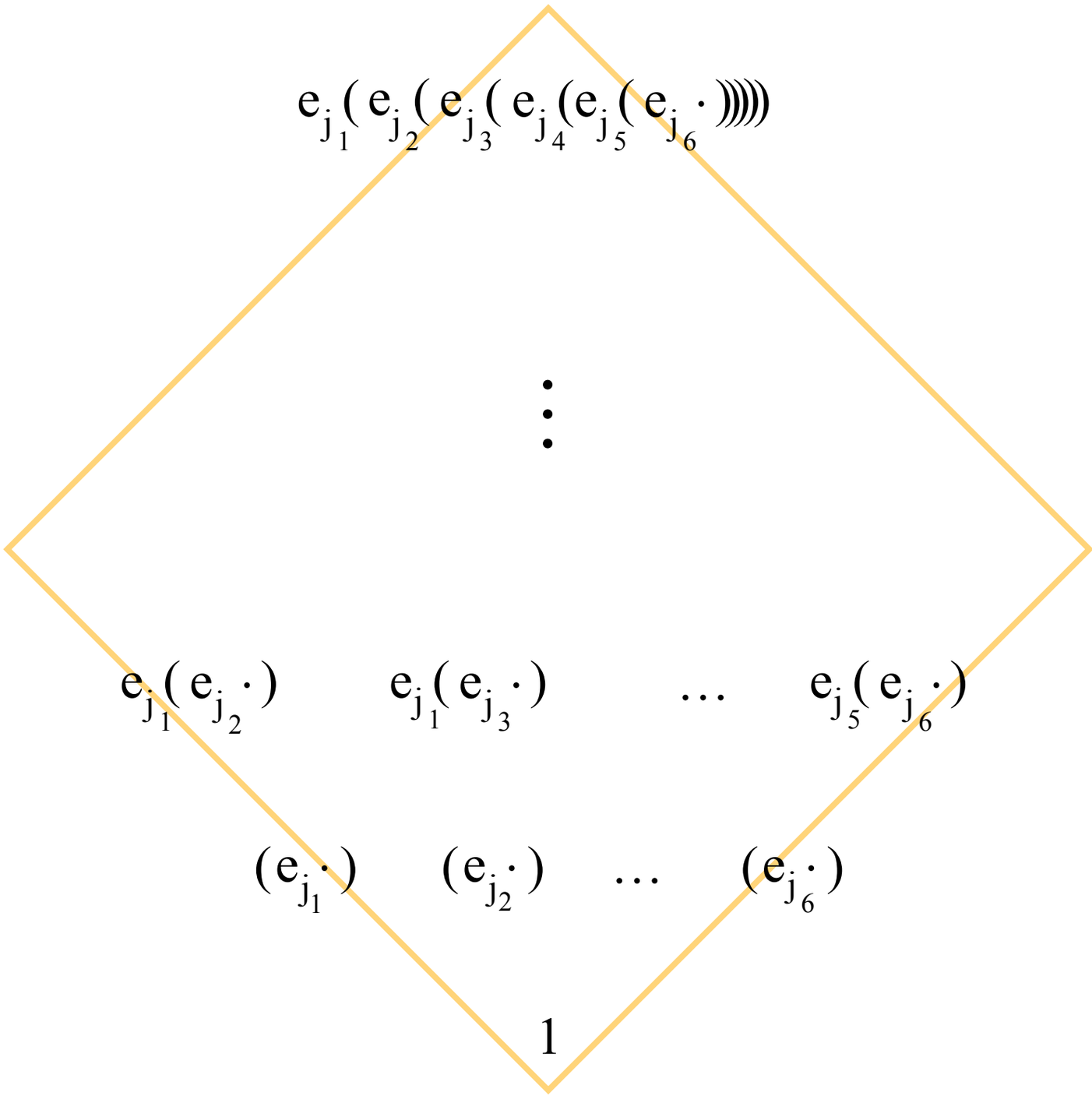
Octonion

Left multiplication



Octonion

Left multiplication



Octonion

Left multiplication

$$e_{j_1}(e_{j_2}(e_{j_3}(e_{j_4}(e_{j_5}(e_{j_6}\cdot))))) \;=\; \pm(e_{j_7}\cdot)$$

$$\vdots$$

$$e_{j_1}(e_{j_2}\cdot) \qquad e_{j_1}(e_{j_3}\cdot) \qquad \dots \qquad e_{j_5}(e_{j_6}\cdot)$$

$$(e_{j_1}\cdot) \qquad (e_{j_2}\cdot) \qquad \dots \qquad (e_{j_6}\cdot)$$

$$1$$

Octonion

Left multiplication

isolate

$$e_{j_1}(e_{j_2}(e_{j_3}(e_{j_4}(e_{j_5}(e_{j_6}\cdot))))) \;=\; \pm(e_{j_7}\cdot)$$

$$\vdots$$

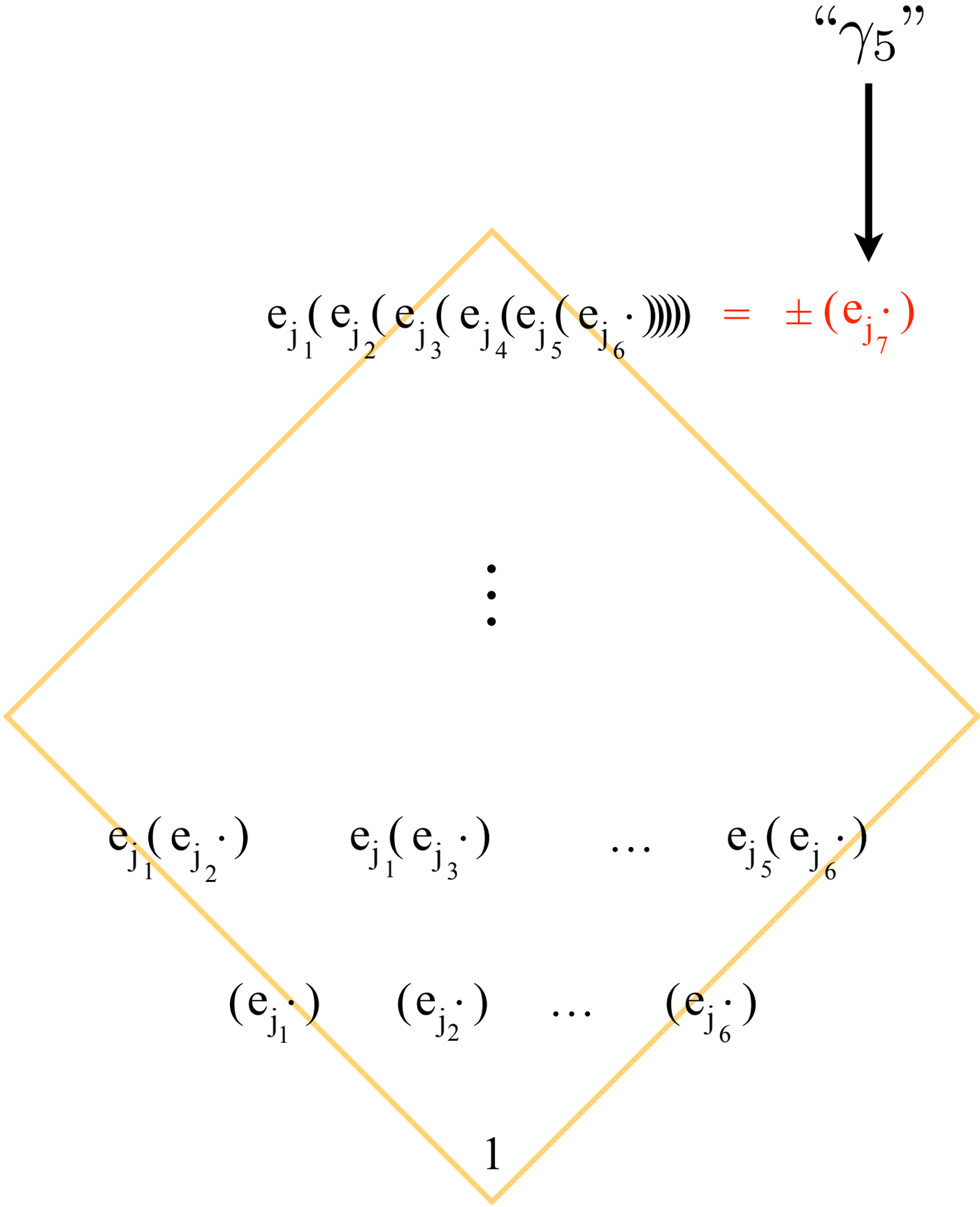
$$e_{j_1}(e_{j_2}\cdot) \qquad e_{j_1}(e_{j_3}\cdot) \qquad \dots \qquad e_{j_5}(e_{j_6}\cdot)$$

$$(e_{j_1}\cdot) \qquad (e_{j_2}\cdot) \qquad \dots \qquad (e_{j_6}\cdot)$$

$$1$$

Octonion

Left multiplication



$\mathbb{O}$ multiplication algebra

Idempotents

$$s := \tfrac{1}{2}(1 + iL_{e_7}) \qquad S := \tfrac{1}{2}(1 + iR_{e_7})$$

$$s^* := \tfrac{1}{2}(1 - iL_{e_7}) \qquad S^* := \tfrac{1}{2}(1 - iR_{e_7})$$

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Octonion

Left multiplication

$$e_{j_1}(e_{j_2}(e_{j_3}(e_{j_4}(e_{j_5}(e_{j_6}\cdot))))) \;=\; \pm(e_{j_7}\cdot)$$

$$\vdots$$

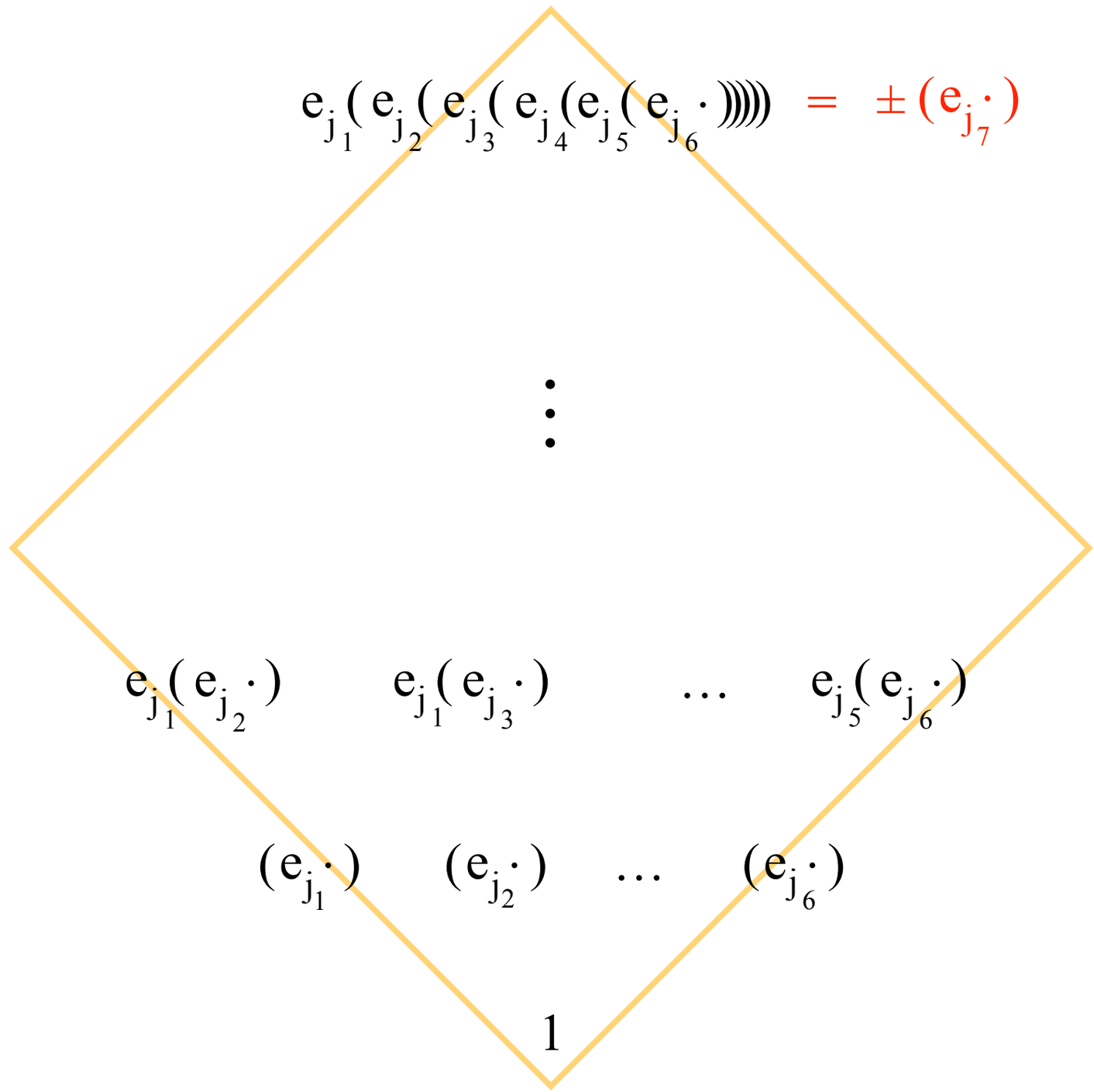
$$e_{j_1}(e_{j_2}\cdot) \qquad e_{j_1}(e_{j_3}\cdot) \qquad \dots \qquad e_{j_5}(e_{j_6}\cdot)$$

$$(e_{j_1}\cdot) \qquad (e_{j_2}\cdot) \qquad \dots \qquad (e_{j_6}\cdot)$$

$$1$$

Octonion

Left multiplication



Octonion

Left multiplication

⇒ Cl(0,6)

$$e_{j_1}(e_{j_2}(e_{j_3}(e_{j_4}(e_{j_5}(e_{j_6}\cdot))))) = \pm(e_{j_7}\cdot)$$

$$\vdots$$

$$e_{j_1}(e_{j_2}\cdot) \qquad e_{j_1}(e_{j_3}\cdot) \qquad \dots \qquad e_{j_5}(e_{j_6}\cdot)$$

$$(e_{j_1}\cdot) \qquad (e_{j_2}\cdot) \qquad \dots \qquad (e_{j_6}\cdot)$$

$$1$$

Octonion

Octonion

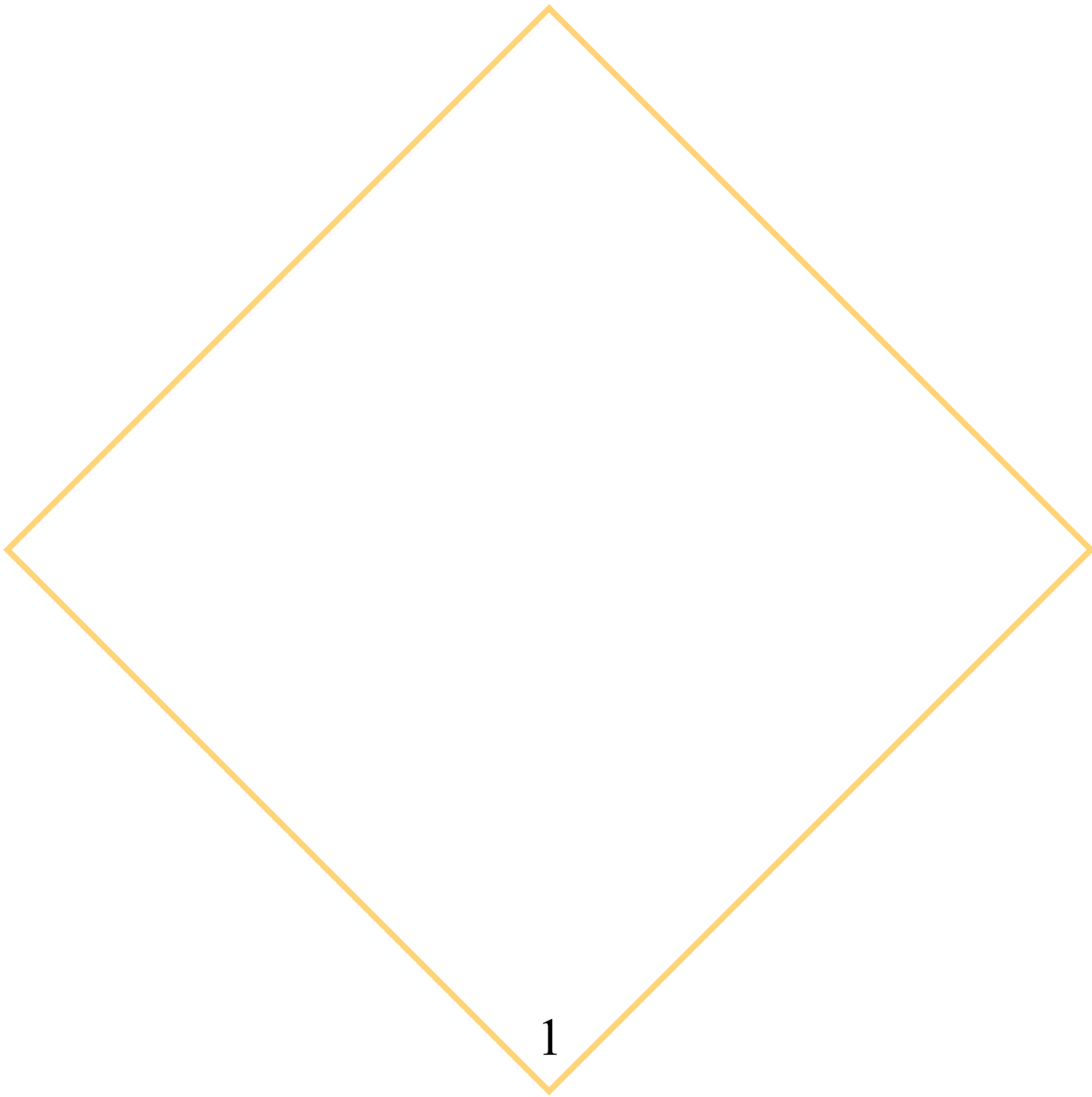
Right multiplication

Octonion

Right multiplication

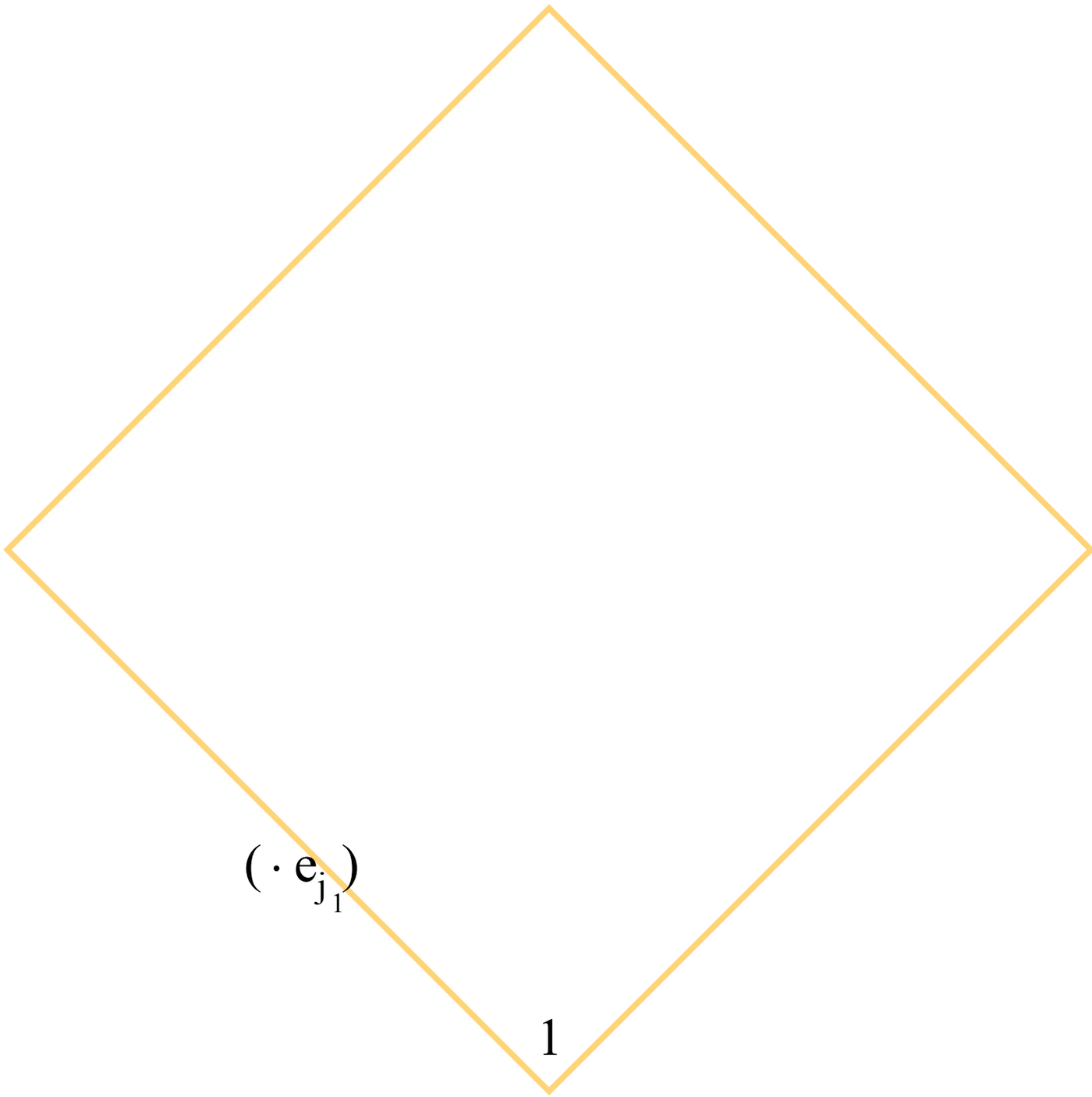
Octonion

Right multiplication



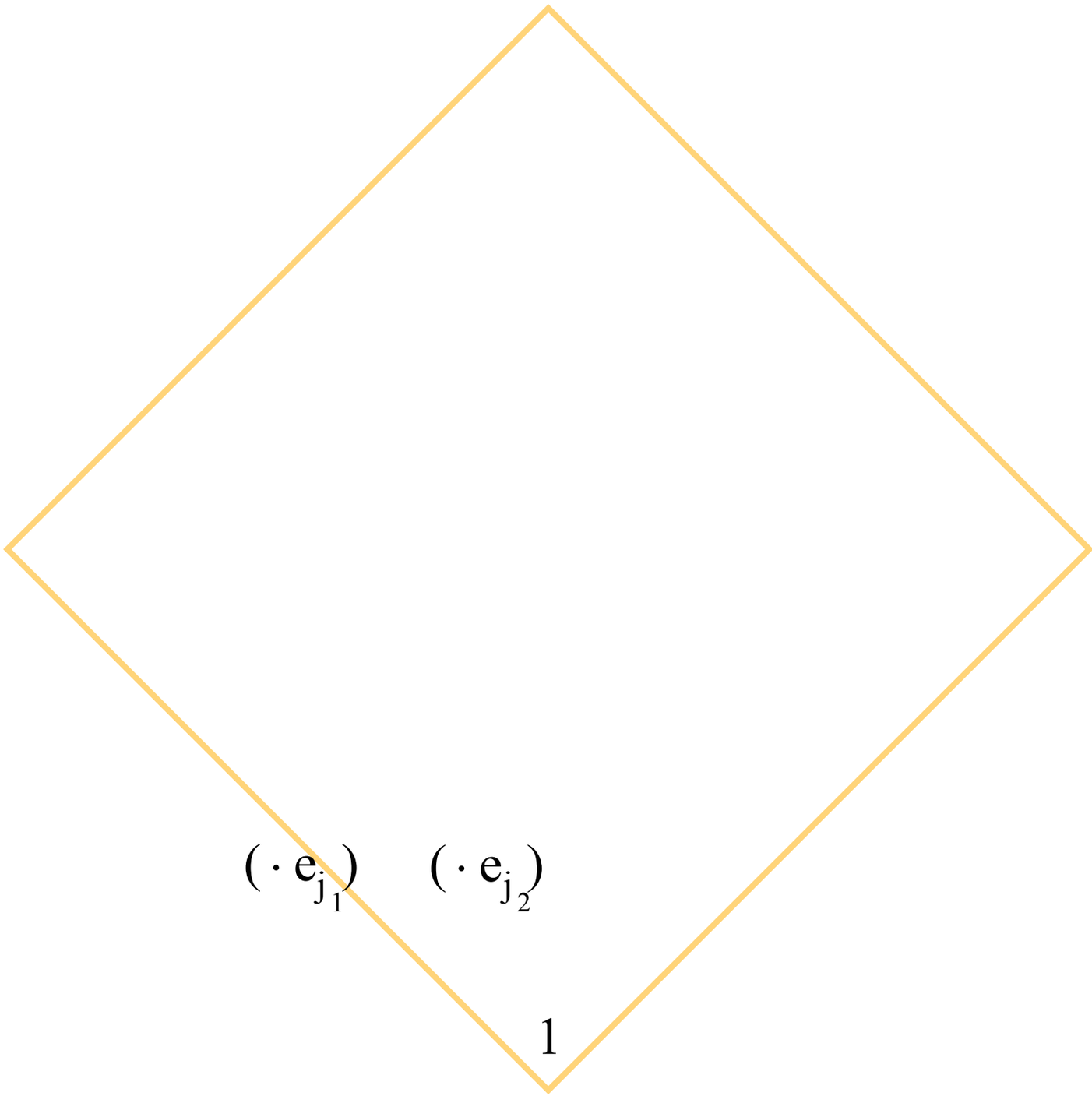
Octonion

Right multiplication



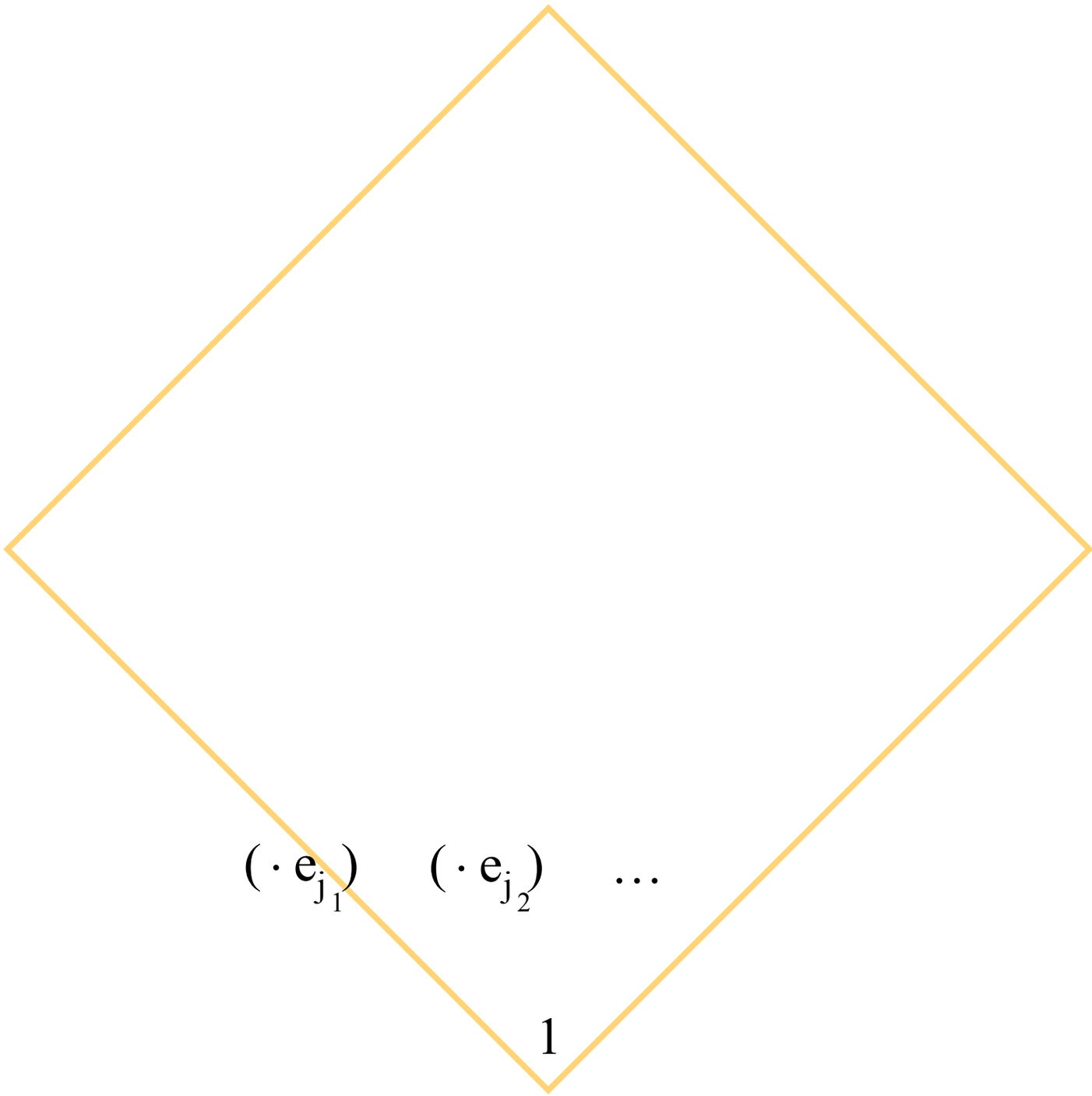
Octonion

Right multiplication



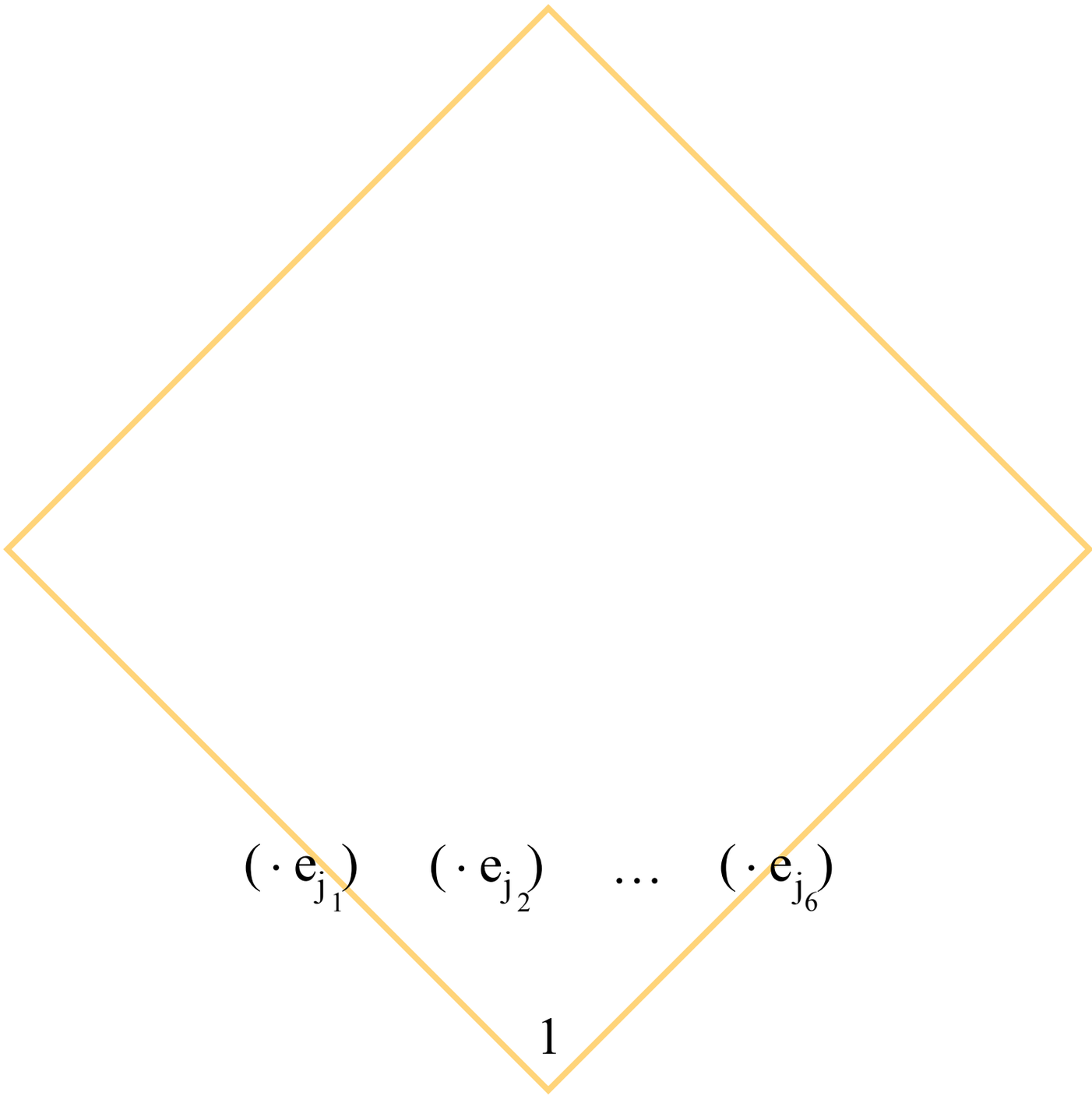
Octonion

Right multiplication



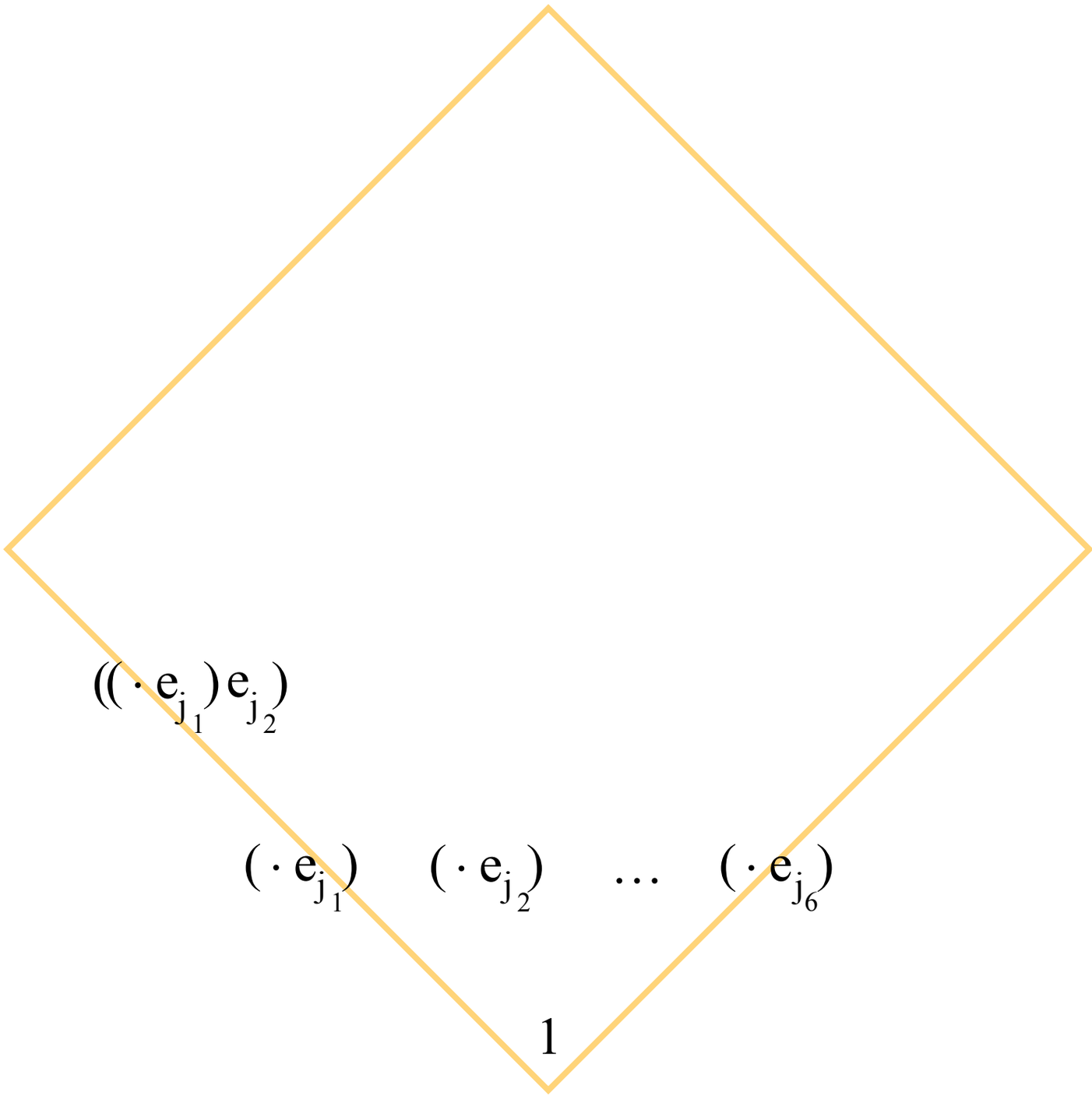
Octonion

Right multiplication



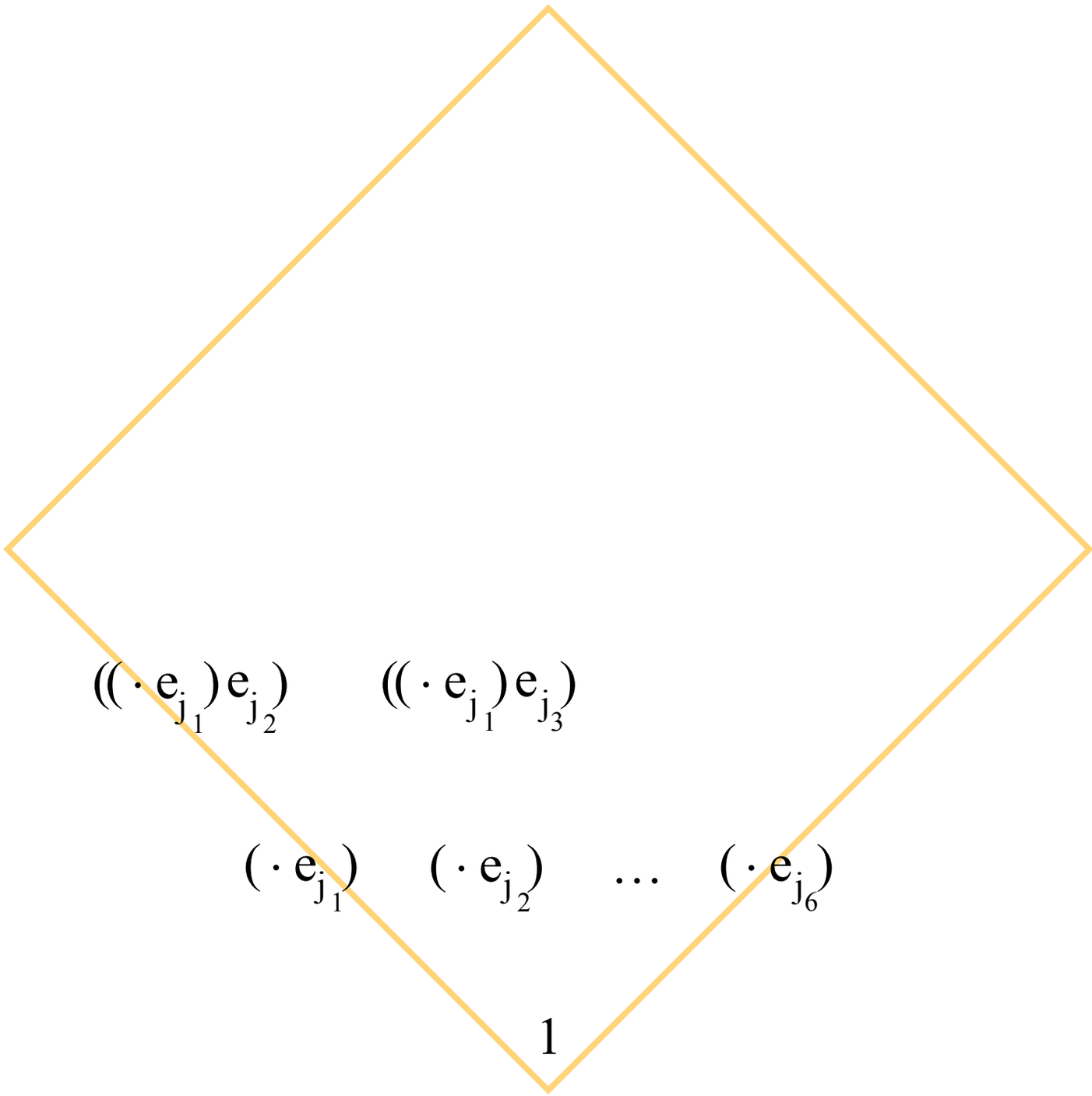
Octonion

Right multiplication



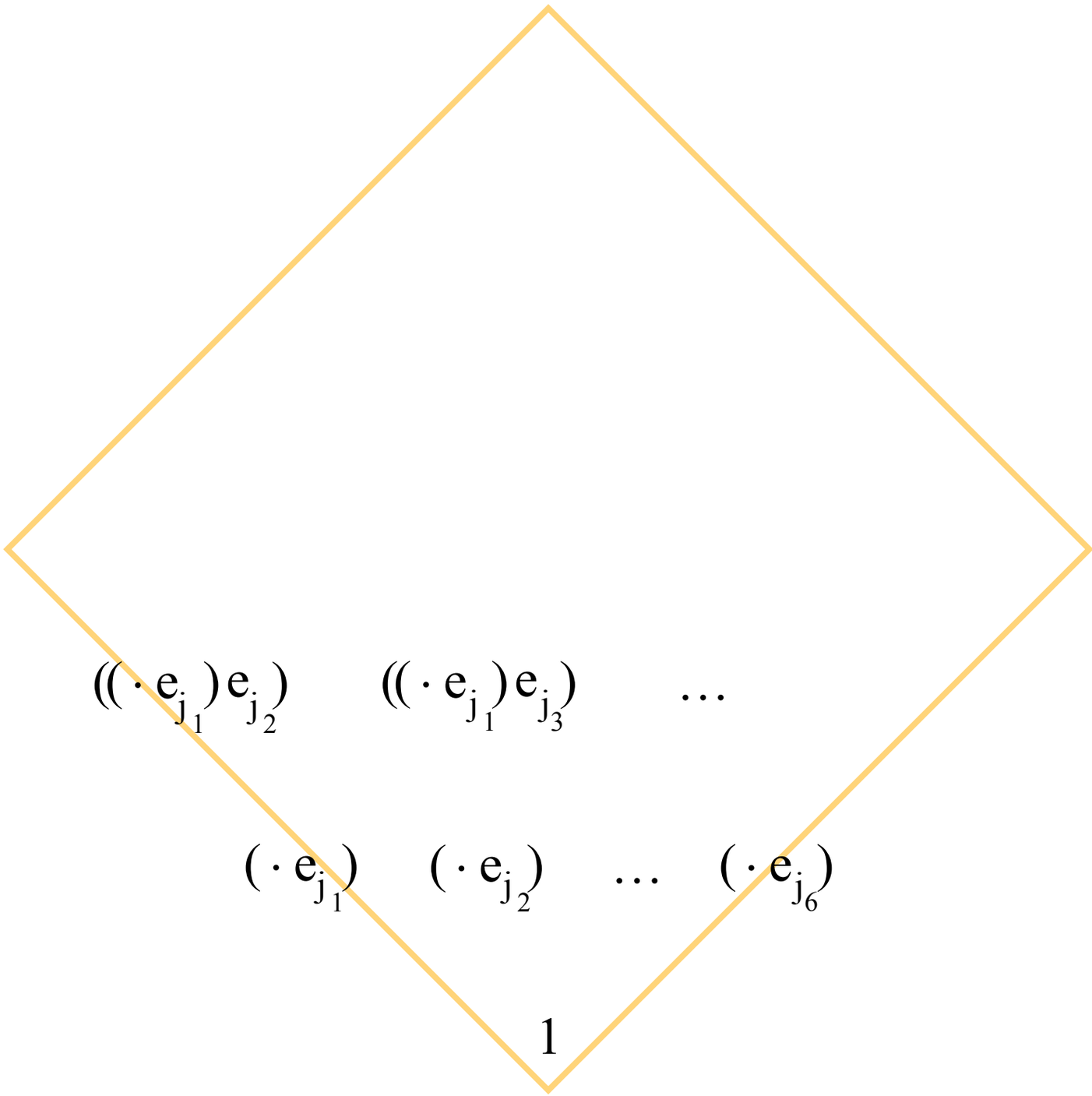
Octonion

Right multiplication



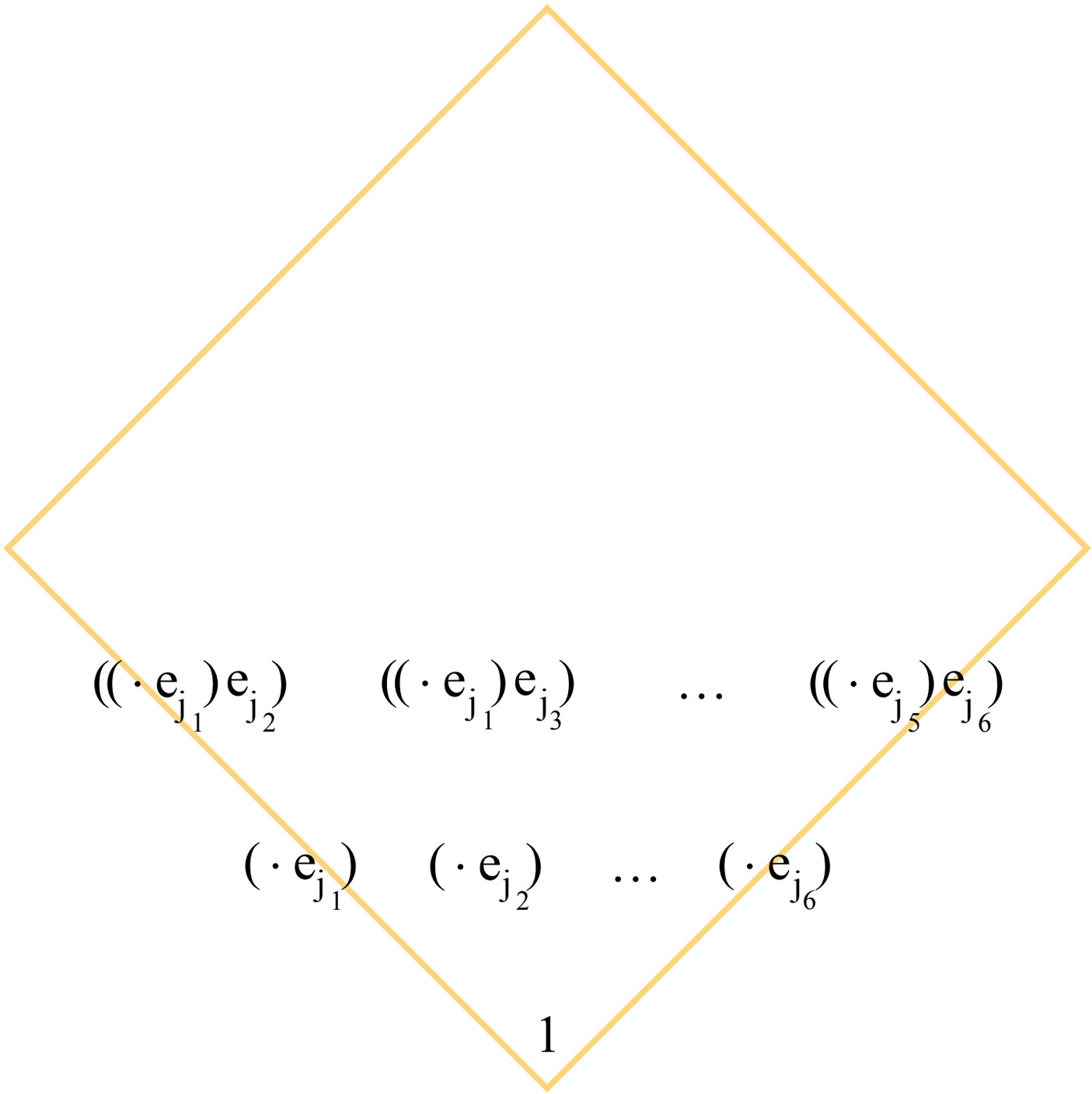
Octonion

Right multiplication



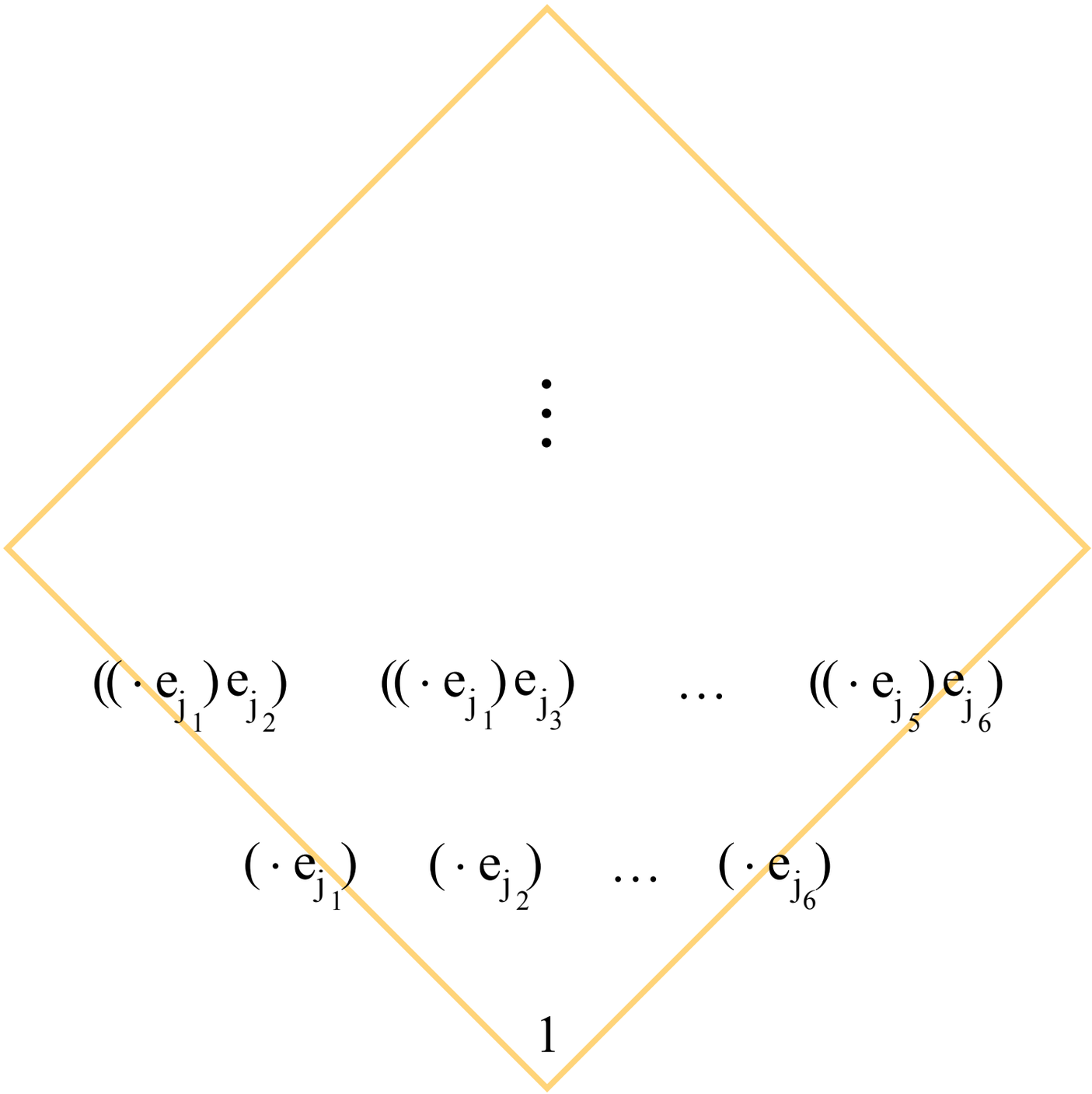
Octonion

Right multiplication



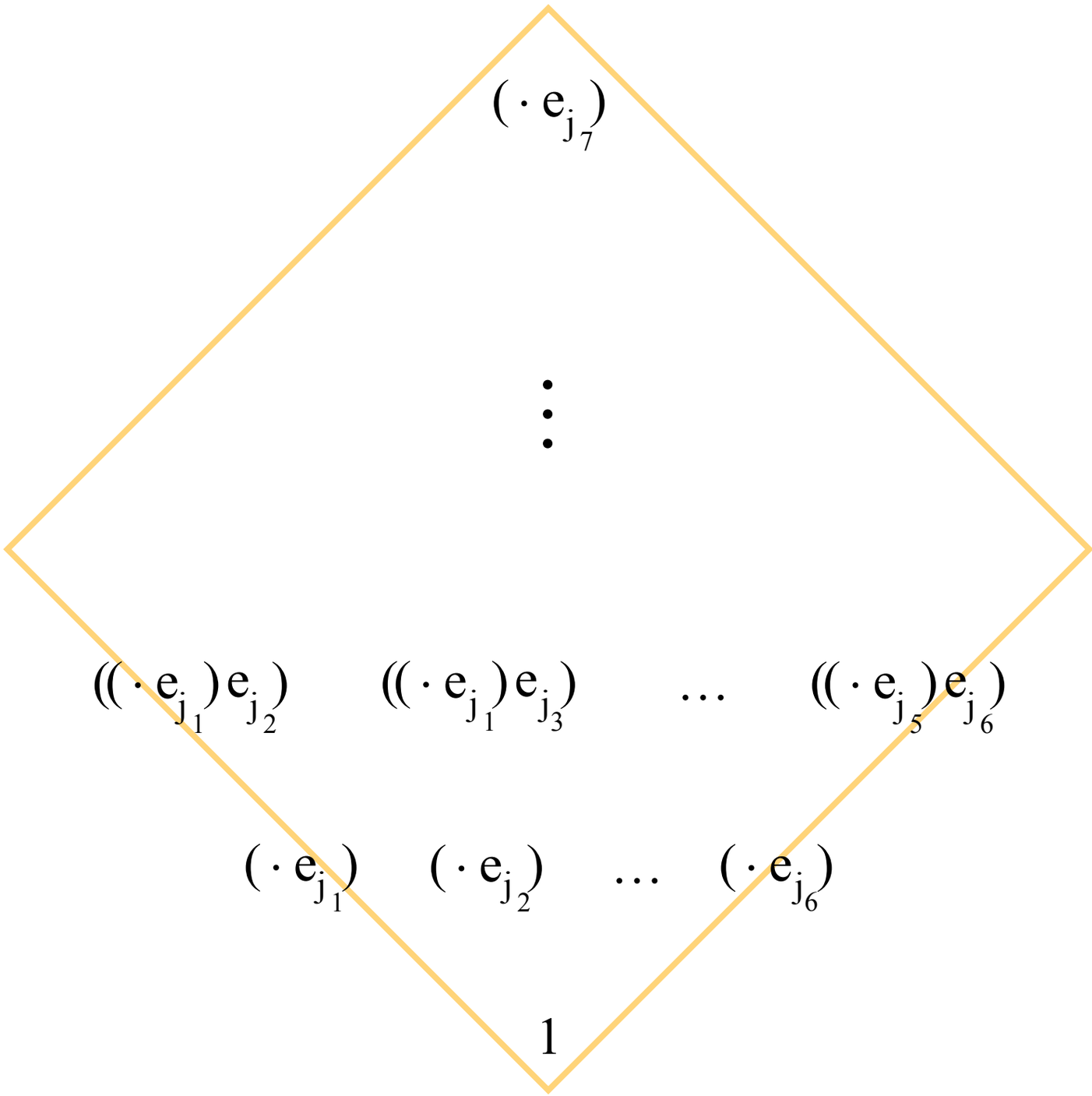
Octonion

Right multiplication



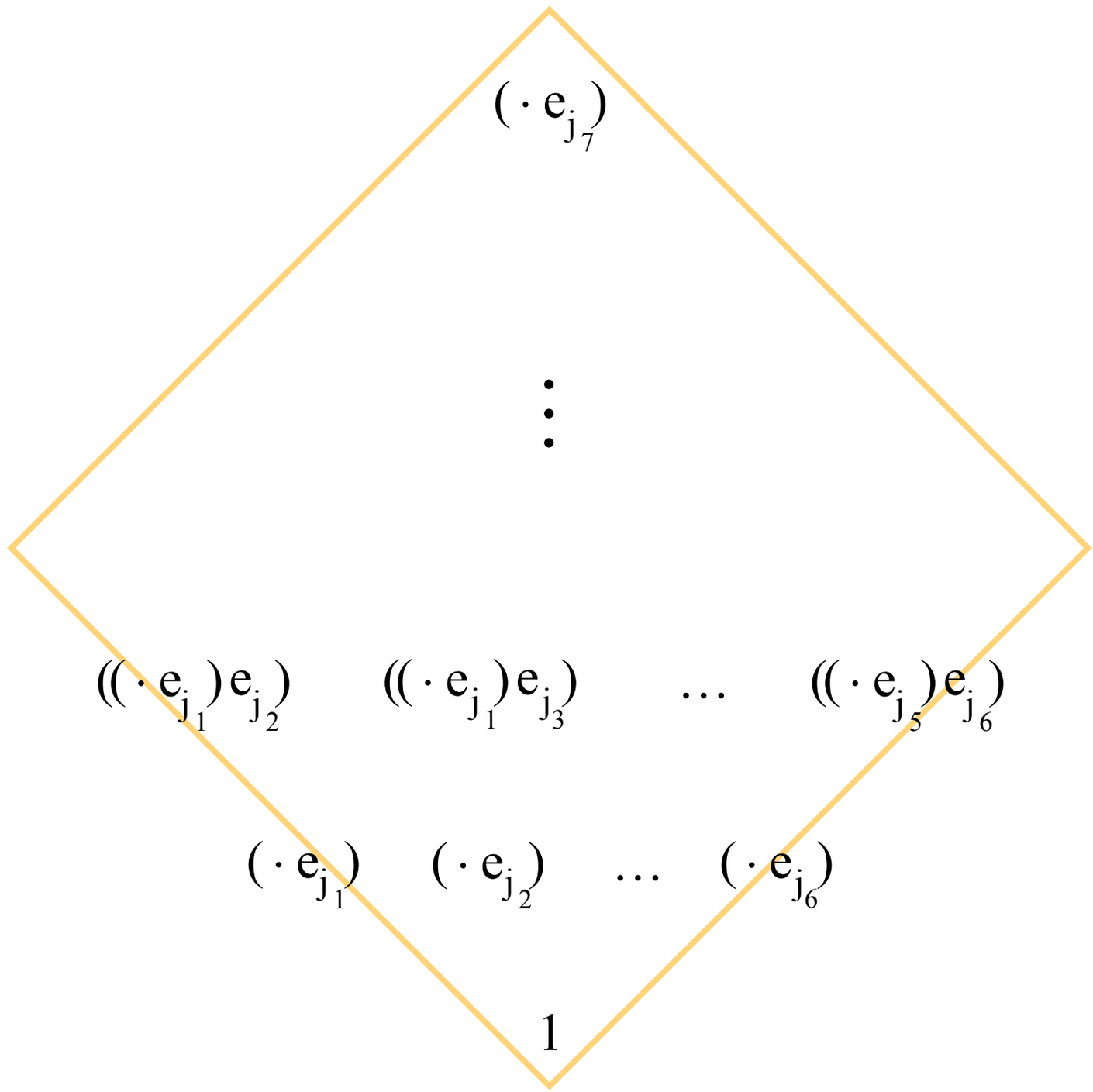
Octonion

Right multiplication



Octonion

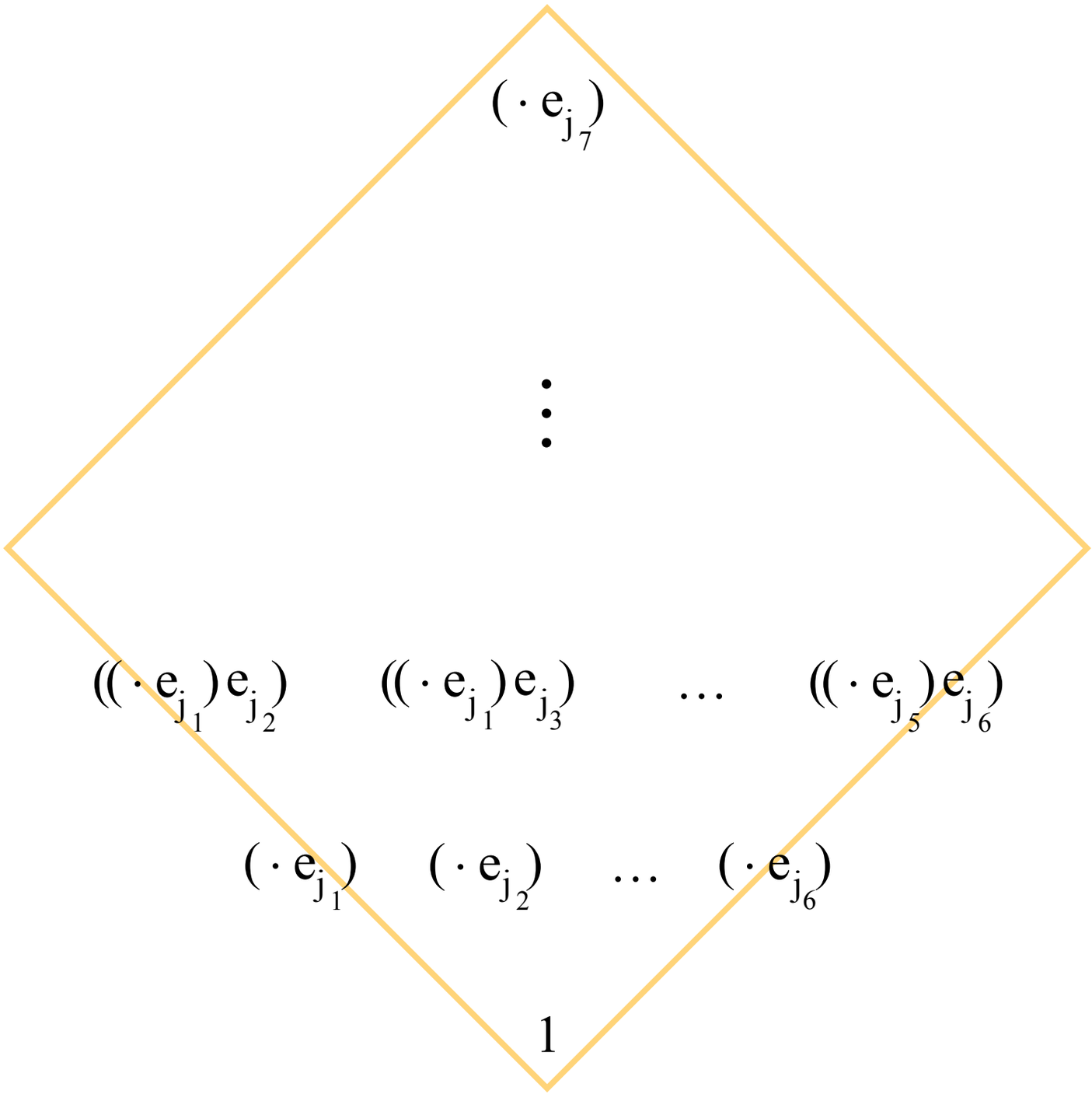
Right multiplication



Octonion

Right multiplication

⇒ Cl(0,6)

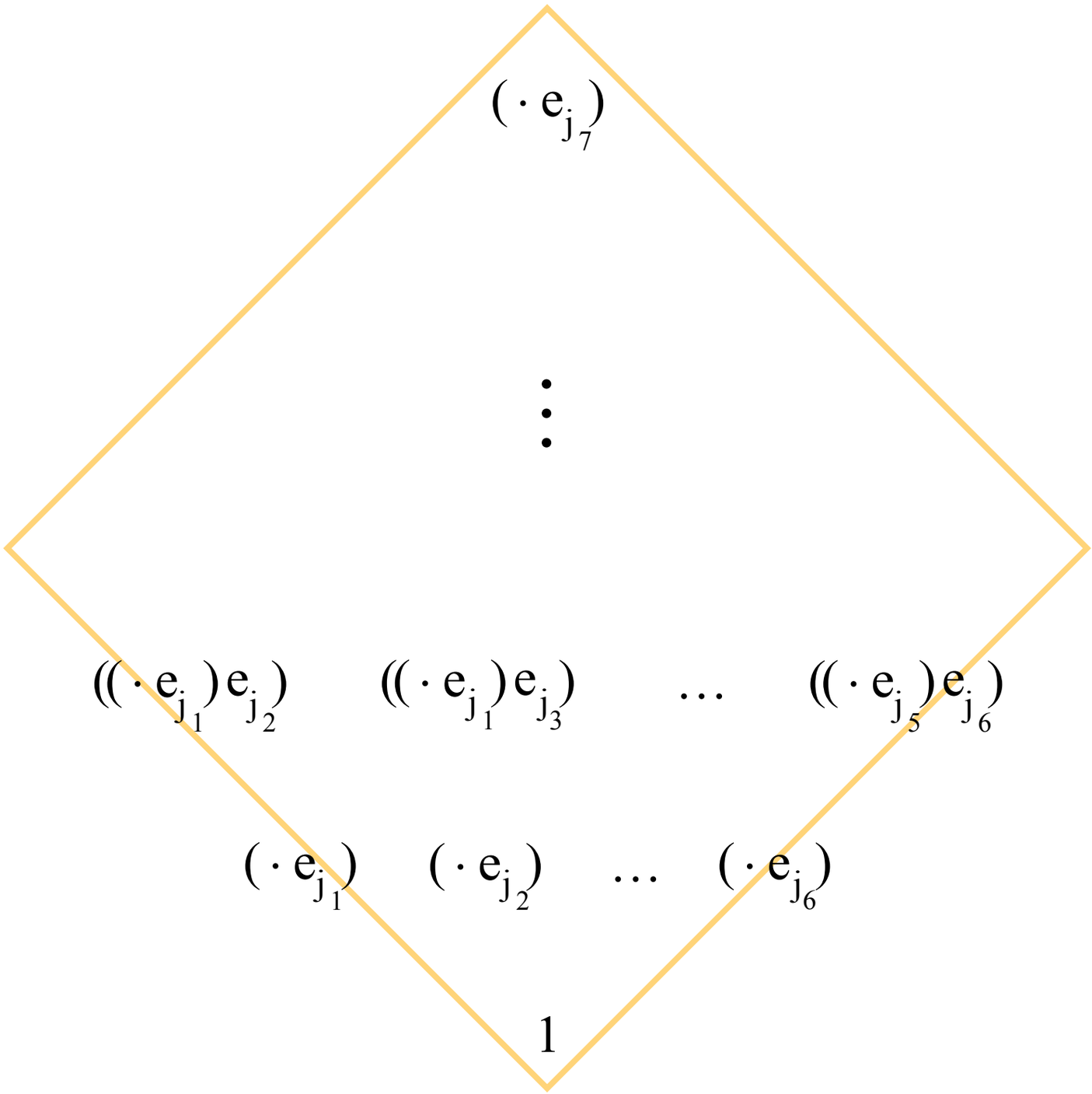


Octonion

Right multiplication

$$(\cdot\, e_7) = 1/2\left(e_1(e_3\,\cdot) + e_2(e_6\,\cdot) + e_4(e_5\,\cdot) - (e_7\,\cdot)\right)$$

⇒ Cl(0,6)



$\mathbb{O}$ multiplication algebra

$\downarrow$

Idempotents

$$s := \tfrac{1}{2}(1 + iL_{e_7}) \qquad S := \tfrac{1}{2}(1 + iR_{e_7})$$

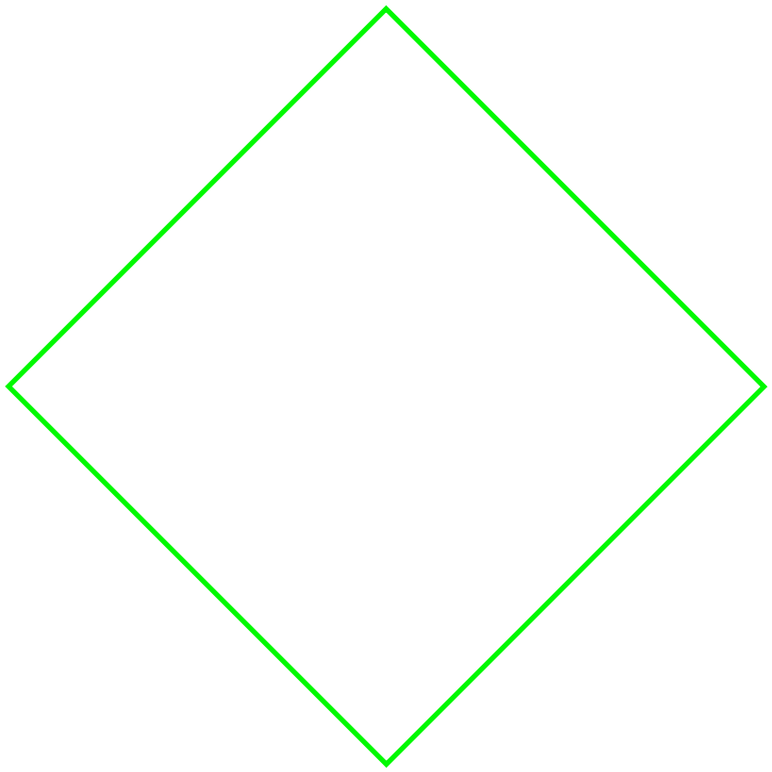
$$s^* := \tfrac{1}{2}(1 - iL_{e_7}) \qquad S^* := \tfrac{1}{2}(1 - iR_{e_7})$$

Quaternion

Left and **Right** multiplication

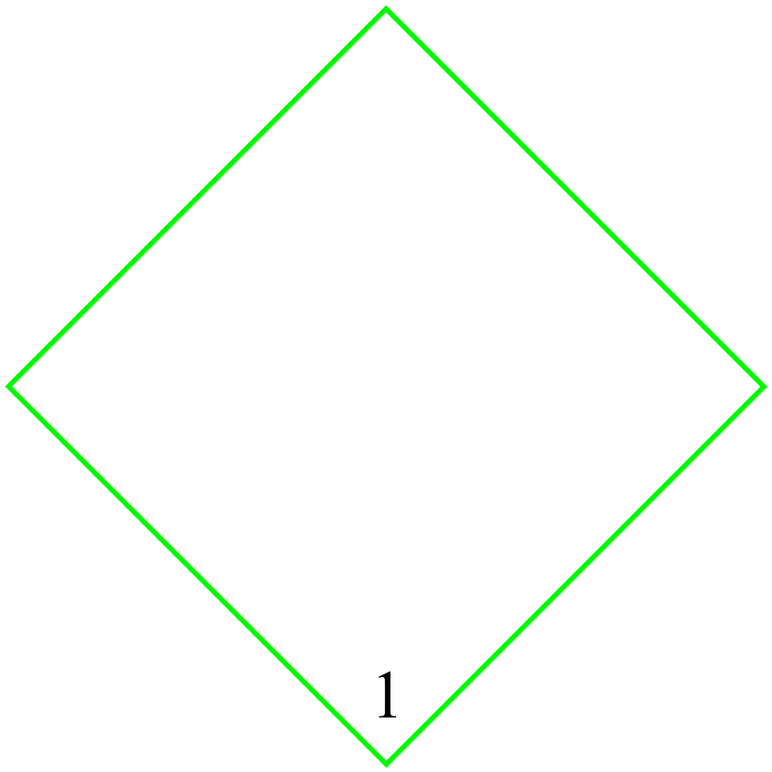
Quaternion

Left and **Right** multiplication



Quaternion

Left and **Right** multiplication



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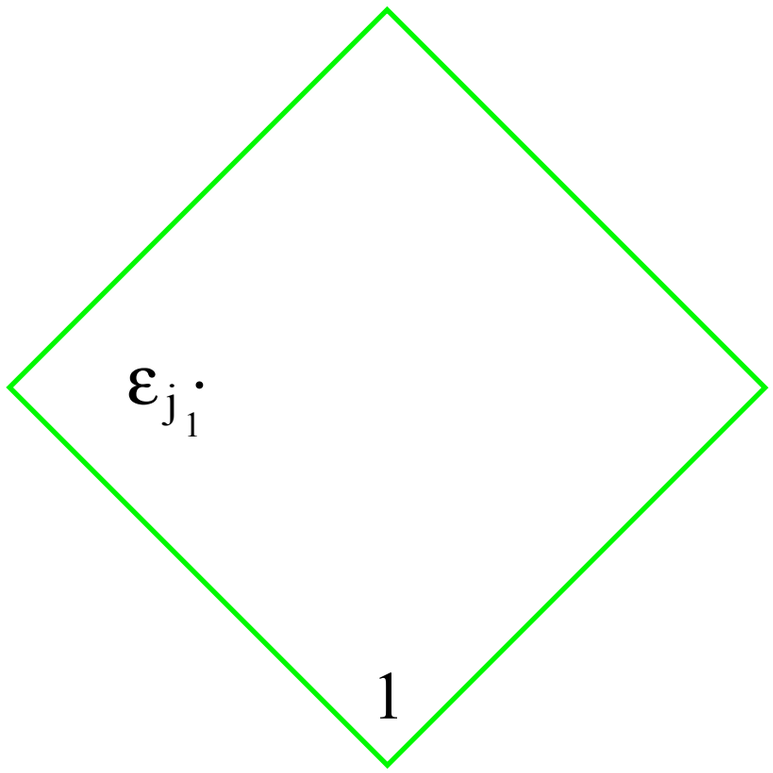
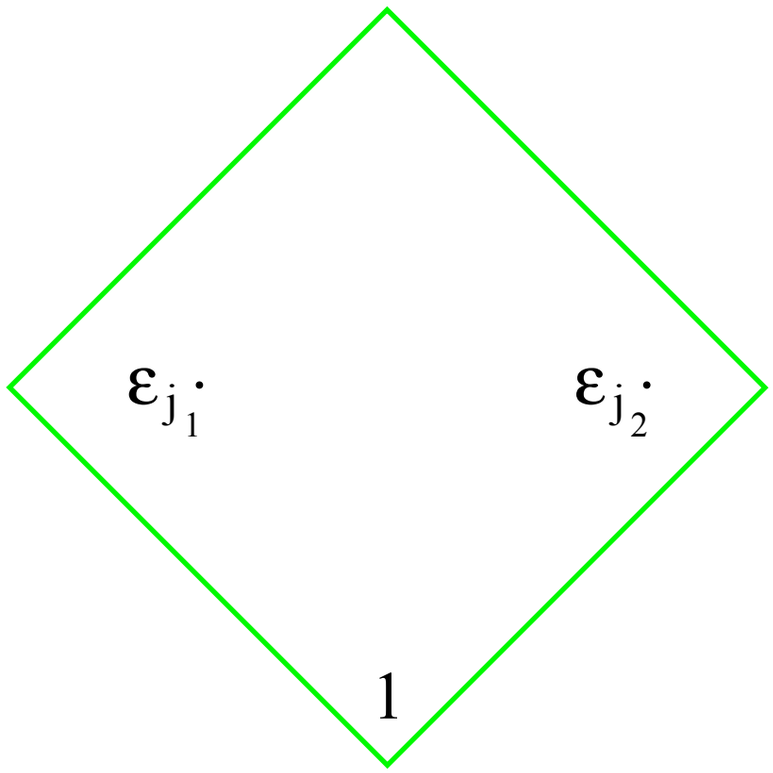


Quaternion

Left and **Right** multiplication



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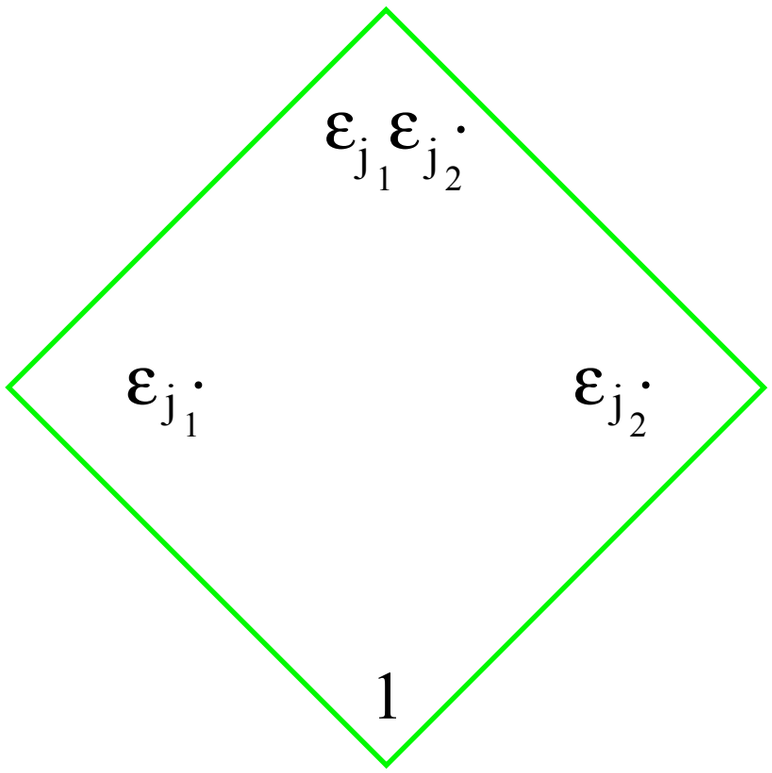

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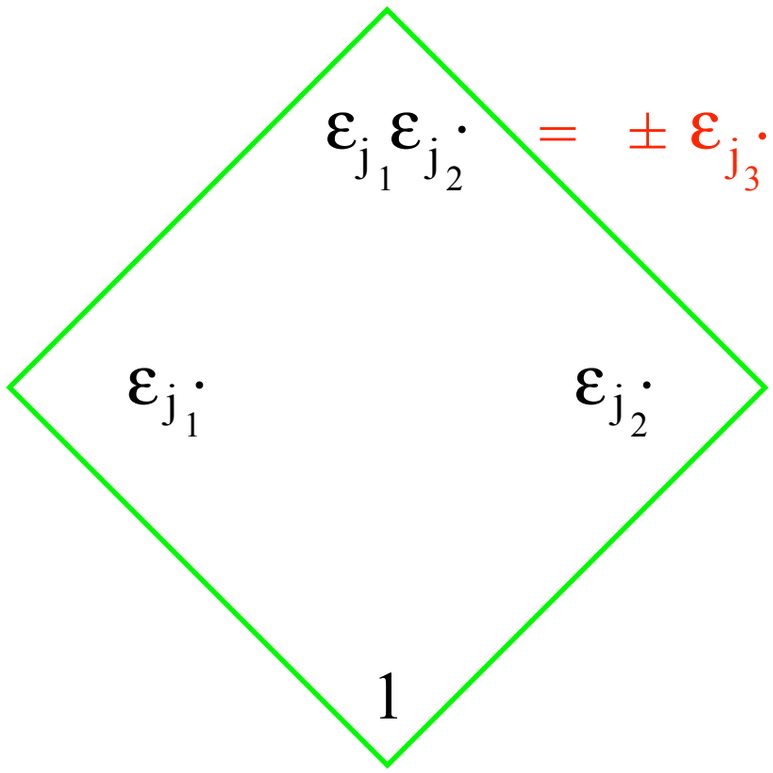

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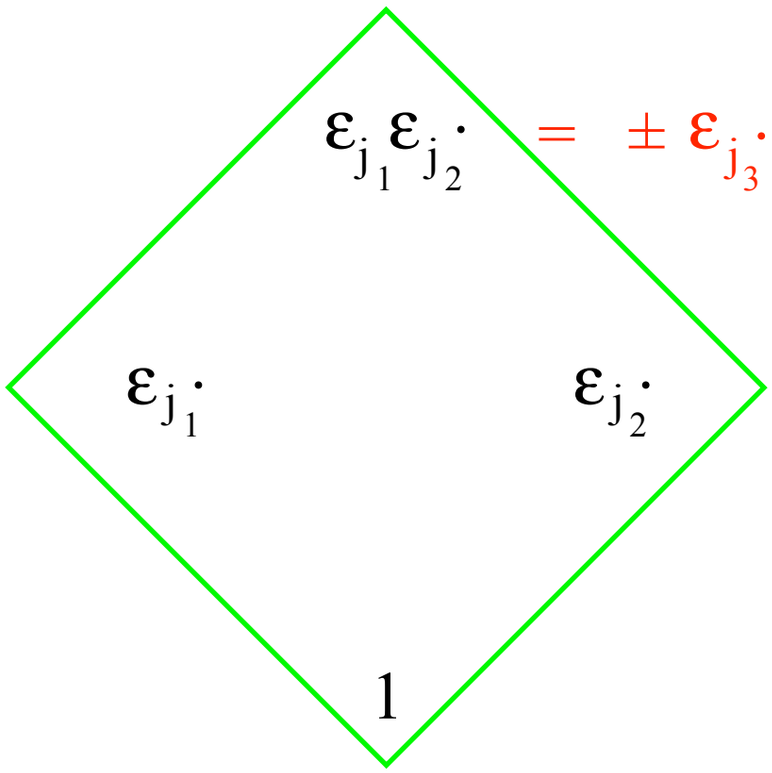
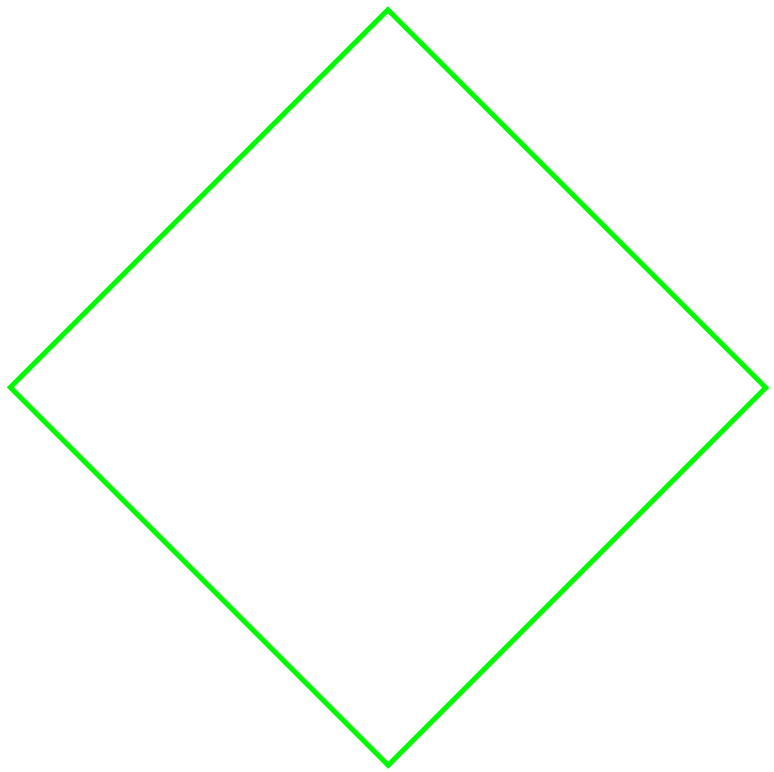
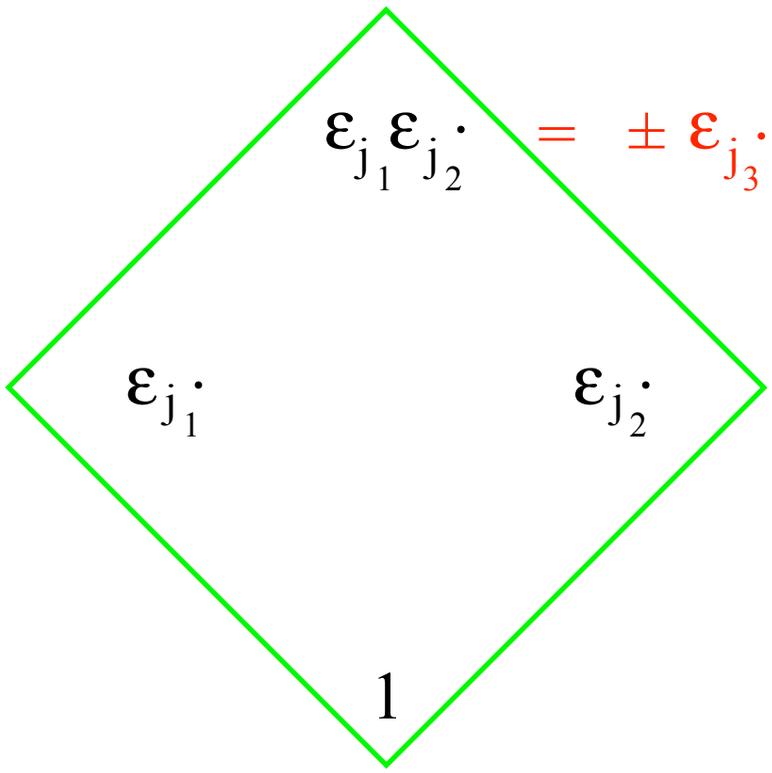
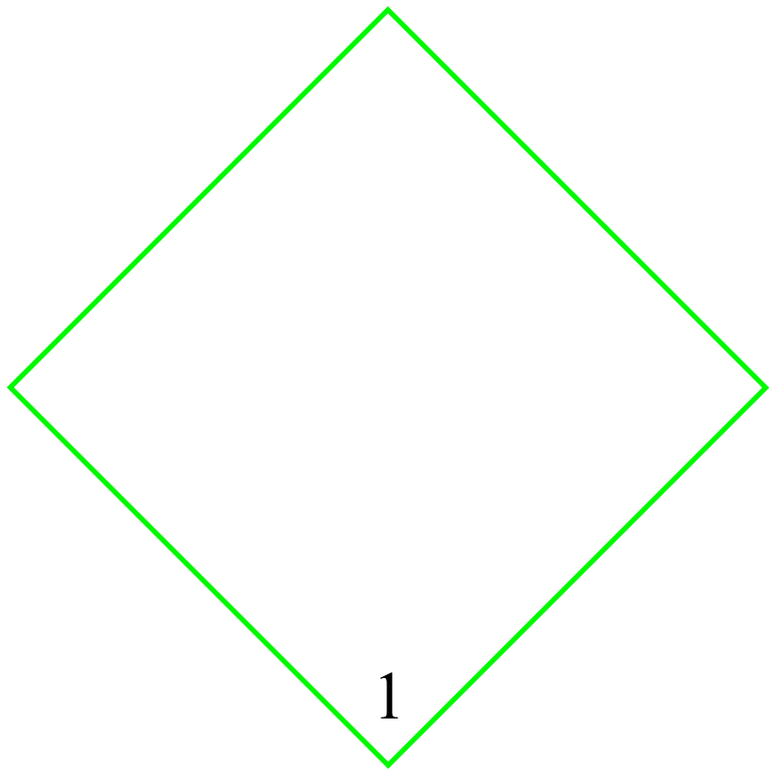




Quaternion

Left and **Right** multiplication





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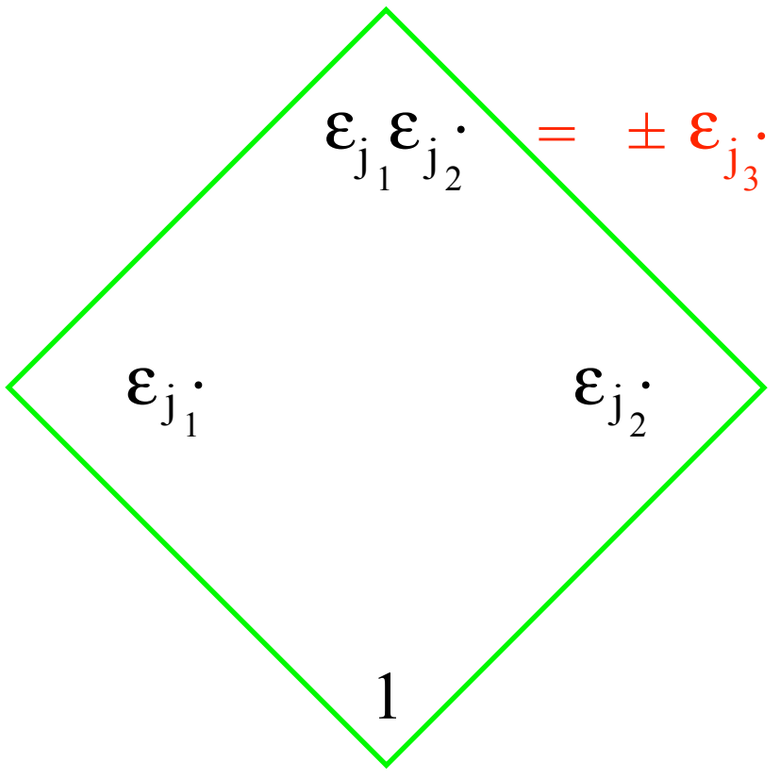

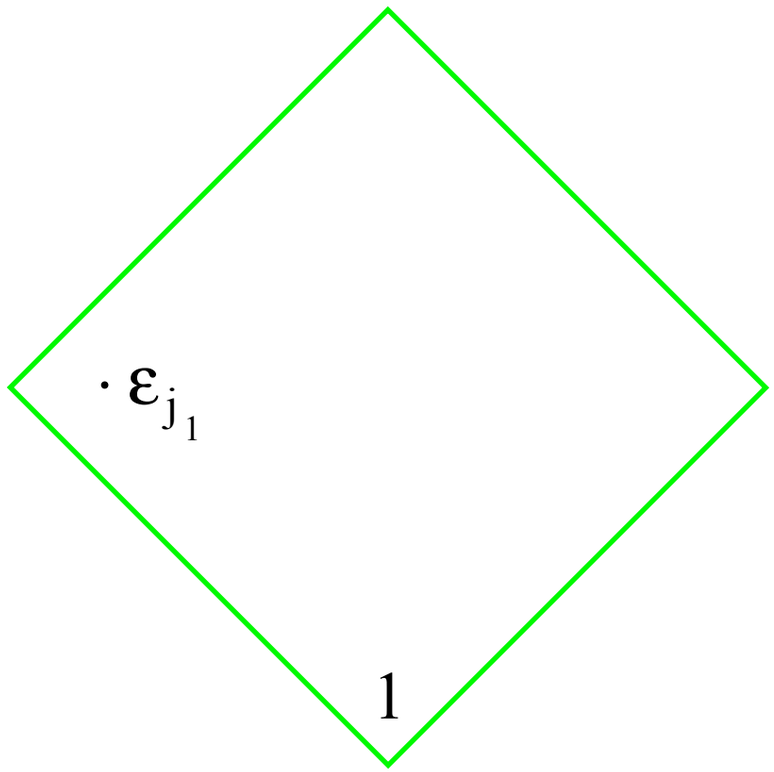

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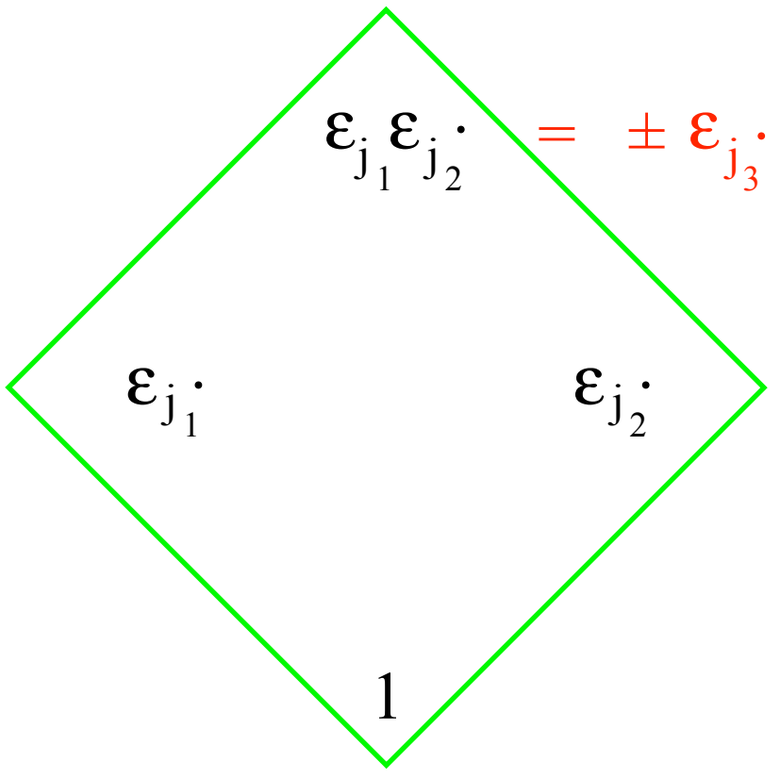

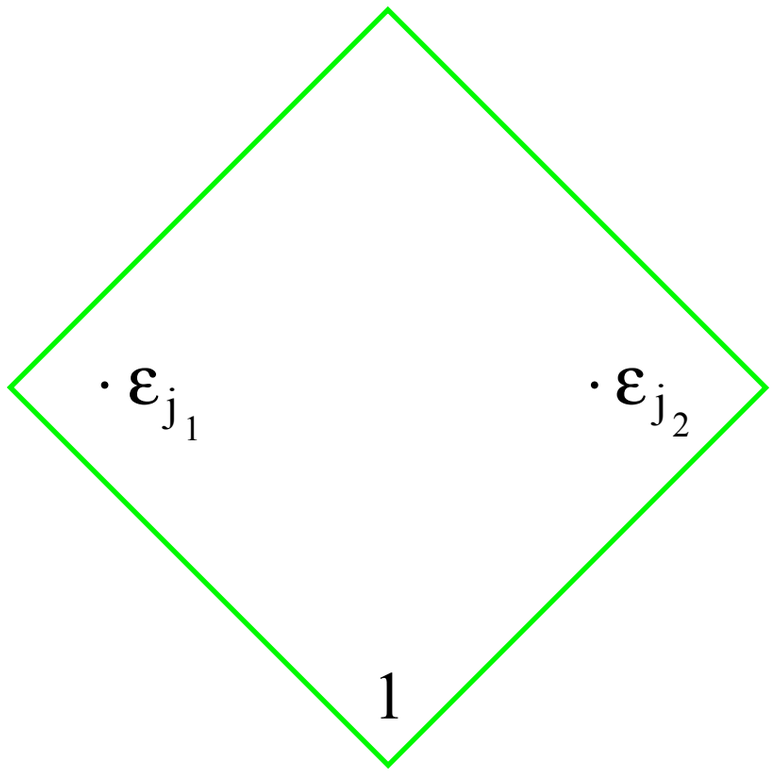

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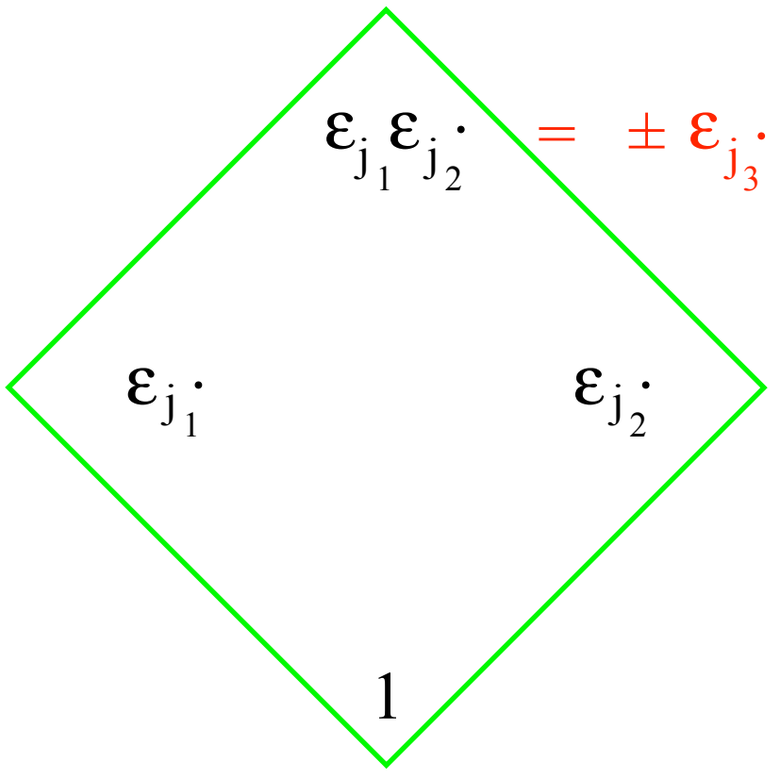

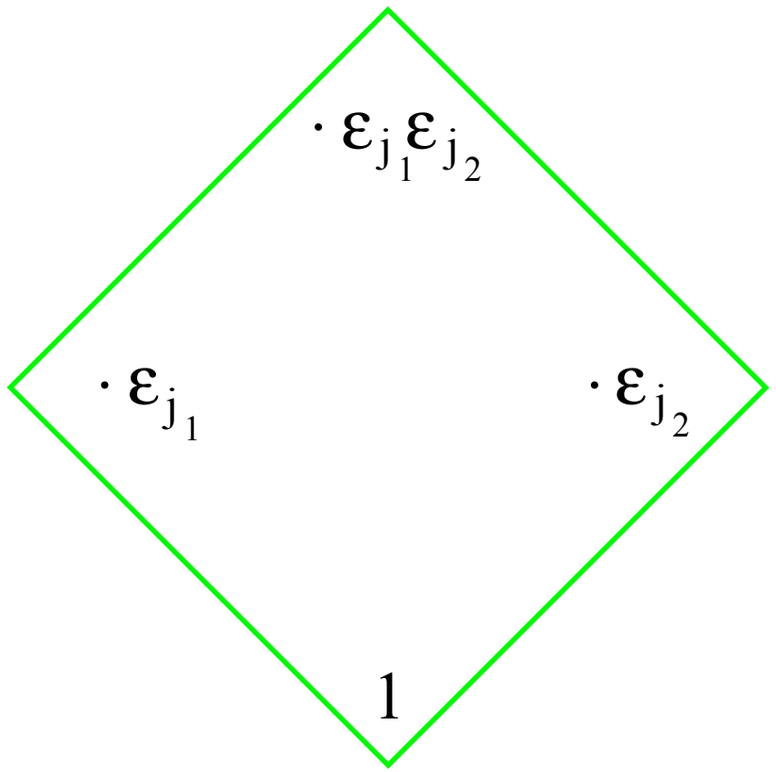

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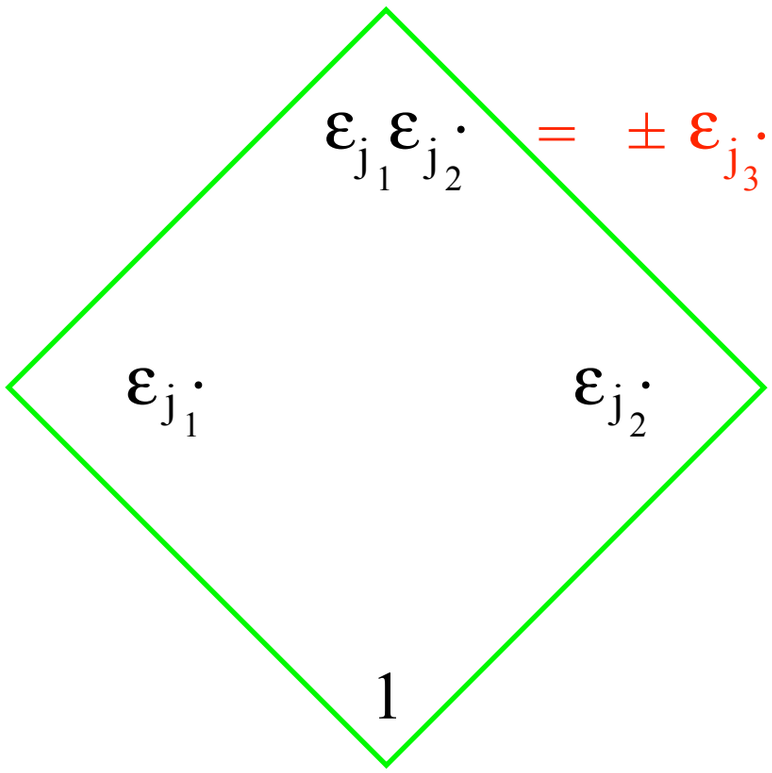

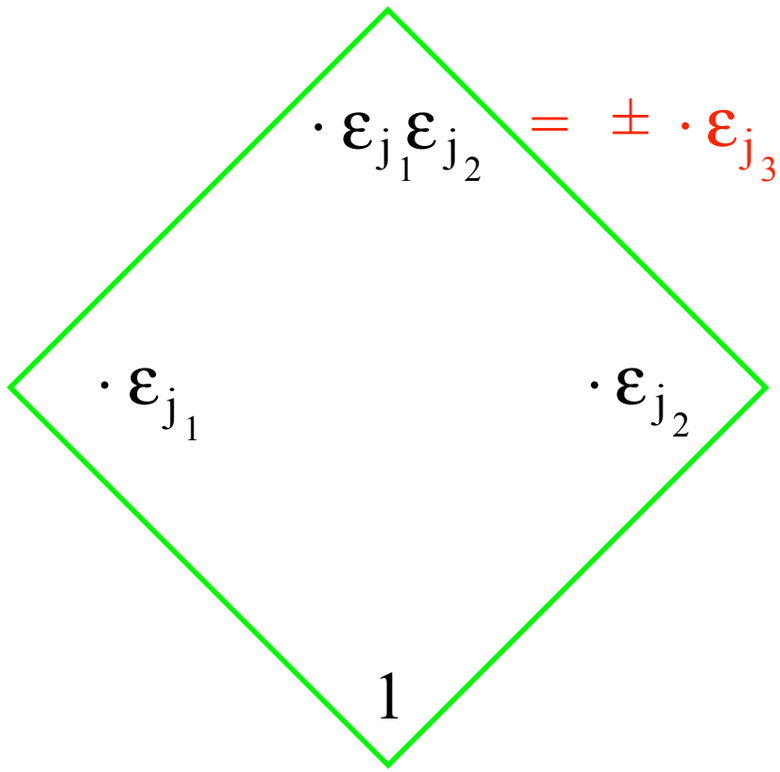

Quaternion

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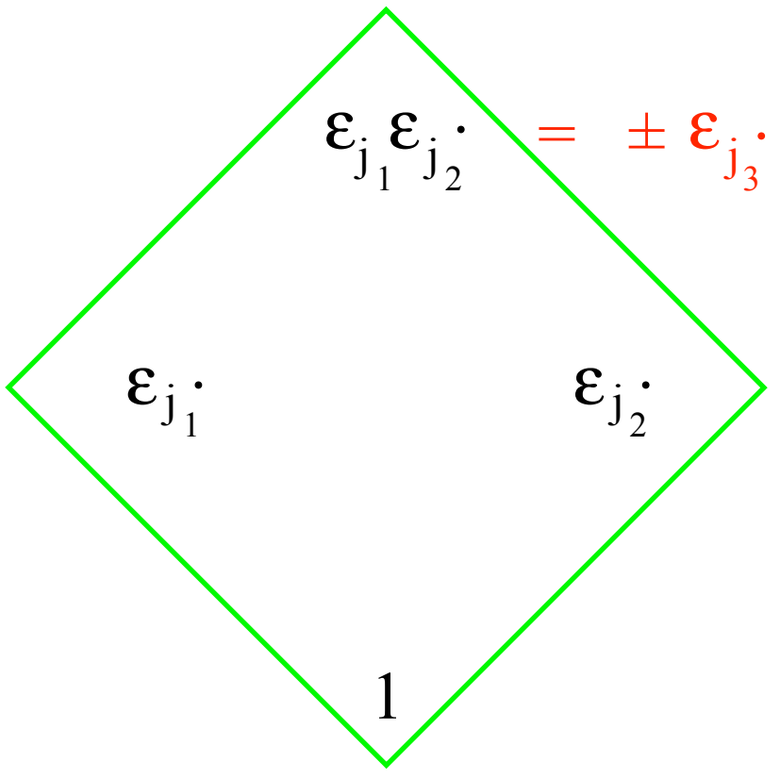
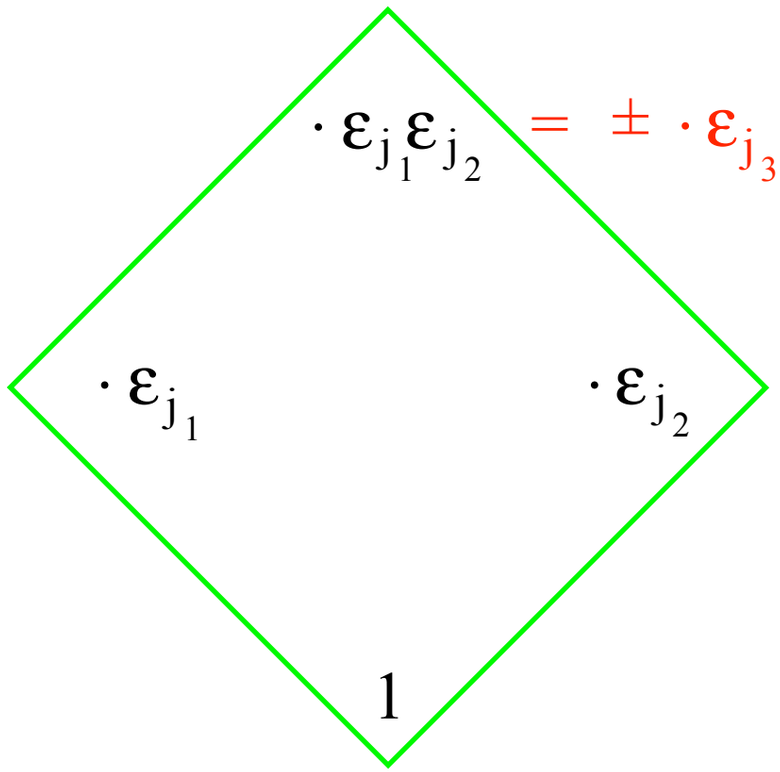
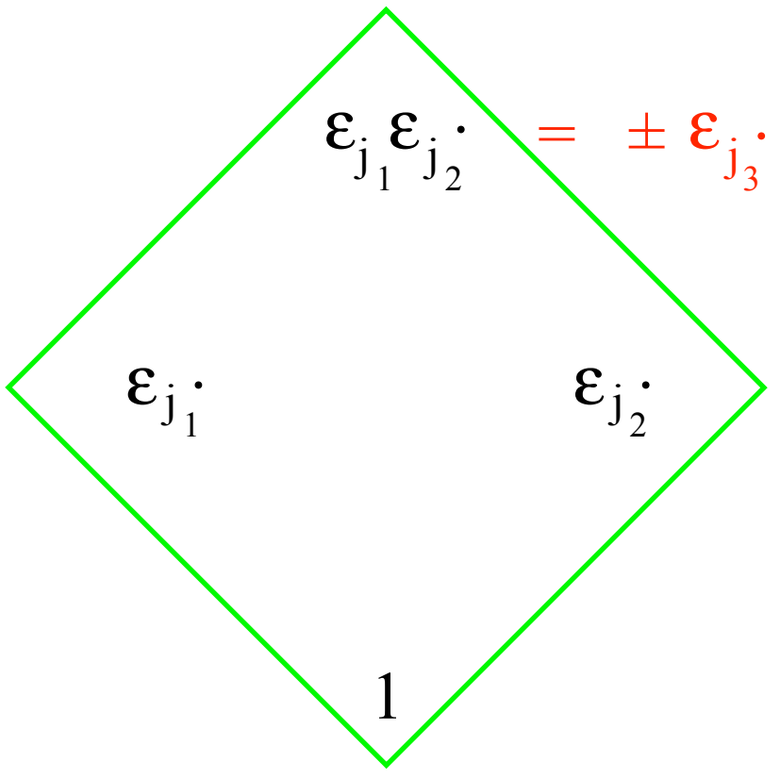




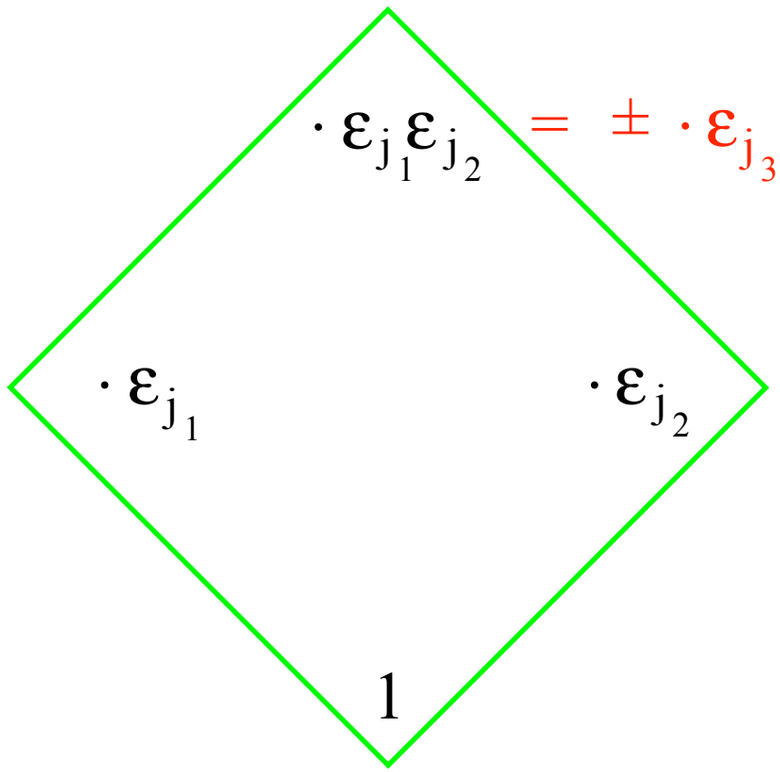
Cl(0, 2)

Quaternion

Left and **Right** multiplication



Cl(0, 2)



Cl(0, 2)

Idempotents

$$s := \tfrac{1}{2}(1 + iL_{e_7}) \qquad S := \tfrac{1}{2}(1 + iR_{e_7})$$

$$s^* := \tfrac{1}{2}(1 - iL_{e_7}) \qquad S^* := \tfrac{1}{2}(1 - iR_{e_7})$$

Idempotents

$$s := \tfrac{1}{2}(1 + iL_{e_7}) \qquad S := \tfrac{1}{2}(1 + iR_{e_7})$$

$$s^* := \tfrac{1}{2}(1 - iL_{e_7}) \qquad S^* := \tfrac{1}{2}(1 - iR_{e_7})$$

$$t := \tfrac{1}{2}(1 + iL_{\epsilon_3}) \qquad T := \tfrac{1}{2}(1 + iR_{\epsilon_3})$$

$$t^* := \tfrac{1}{2}(1 - iL_{\epsilon_3}) \qquad T^* := \tfrac{1}{2}(1 - iR_{\epsilon_3})$$

Idempotents

$$s := \tfrac{1}{2}(1 + iL_{e_7})$$

$$s^* := \tfrac{1}{2}(1 - iL_{e_7})$$

$$S := \tfrac{1}{2}(1 + iR_{e_7})$$

$$S^* := \tfrac{1}{2}(1 - iR_{e_7})$$

$$t := \tfrac{1}{2}(1 + iL_{\epsilon_3})$$

$$t^* := \tfrac{1}{2}(1 - iL_{\epsilon_3})$$

$$T := \tfrac{1}{2}(1 + iR_{\epsilon_3})$$

$$T^* := \tfrac{1}{2}(1 - iR_{\epsilon_3})$$

Idempotents

Idempotents

sSt

sSt^*

sS^*t

sS^*t^*

s^*St

s^*St^*

s^*S^*t

$s^*S^*t^*$

Idempotents

sSt	helicity	$\uparrow$	lepton
sSt^*	helicity	$\downarrow$	lepton
sS^*t	helicity	$\uparrow$	baryon
sS^*t^*	helicity	$\downarrow$	baryon
s^*St	isospin	$\uparrow$	LH
s^*St^*	isospin	$\downarrow$	LH
s^*S^*t	isospin	$\uparrow$	RH
$s^*S^*t^*$	isospin	$\downarrow$	RH

Idempotents

sSt	helicity	$\uparrow$	lepton
sSt^*	helicity	$\downarrow$	lepton
sS^*t	helicity	$\uparrow$	baryon
sS^*t^*	helicity	$\downarrow$	baryon
s^*St	isospin	$\uparrow$	LH
s^*St^*	isospin	$\downarrow$	LH
s^*S^*t	isospin	$\uparrow$	RH
$s^*S^*t^*$	isospin	$\downarrow$	RH

Observation: no projections on colour.

Unfinished business

Unfinished business

Consider hermitian parts of

Unfinished business

Consider hermitian parts of

Left:

Unfinished business

Consider hermitian parts of

Left: $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq \mathbb{C}l(8)$

Unfinished business

Consider hermitian parts of

Left: $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq \mathbb{C}l(8)$

Left and right:

Unfinished business

Consider hermitian parts of

Left: $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq \mathbb{C}l(8)$

Left and right: $M_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq \mathbb{C}l(8) \otimes_{\mathbb{C}} \mathbb{C}l(2)$

Unfinished business

Consider hermitian parts of

Left: $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq \mathbb{C}l(8)$

Left and right: $M_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq \mathbb{C}l(8) \otimes_{\mathbb{C}} \mathbb{C}l(2) \simeq \mathbb{C}l(10)$

Unfinished business

Consider hermitian parts of

Left: $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq \mathbb{C}l(8)$

Left and right: $M_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq \mathbb{C}l(8) \otimes_{\mathbb{C}} \mathbb{C}l(2) \simeq \mathbb{C}l(10)$

$\sim$ vielbein

Unfinished business

Consider hermitian parts of

Left: $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq \mathbb{C}l(8)$

Left and right: $M_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq \mathbb{C}l(8) \otimes_{\mathbb{C}} \mathbb{C}l(2) \simeq \mathbb{C}l(10)$

covariant derivative (spin connection)

$\sim$ vielbein

Unfinished business

Consider hermitian parts of

Left: $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq \mathbb{C}l(8)$

Left and right: $M_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq \mathbb{C}l(8) \otimes_{\mathbb{C}} \mathbb{C}l(2) \simeq \mathbb{C}l(10)$

covariant derivative (spin connection)

$\sim$ vielbein

→ **Gravity...**

Summary

Action of ℓ_{sm} on $\mathcal{H}_{16}(\mathbb{C})$

$$\delta b = \ell_{sm} b + b\,\ell_{sm}^{\dagger} \qquad \text{diagonal}$$

$$\delta f_0 = \ell_{sm} s f_0 s^* + s f_0 s^* \ell_{sm} + h.c. \qquad \text{outer off-diagonal}$$

$$\delta f_+ = \ell_{sm}\big(sS^* f_+ sS + s^* S^* f_+ s^* S\big)$$

$$+\big(sS^* f_+ sS + s^* S^* f_+ s^* S\big)\ell_{sm}^{\dagger *} + h.c. \qquad \text{inner off-diagonal}$$

Physical content

$(u,\ d)_L$	$(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(c,\ s)_L$	$(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(t,\ b)_L$	$(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(\nu_e,\ e)_L$	$(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
$(\nu_\mu,\ \mu)_L$	$(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
$(\nu_\tau,\ \tau)_L$	$(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
u_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
c_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
t_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
d_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
s_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
b_R	$(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
ν_{eR}	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
$\nu_{\mu R}$	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
$\nu_{\tau R}$	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
e_R	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
μ_R	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
τ_R	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
p_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$
p'_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$
G_μ	$(\underline{\mathbf{8}},\ \underline{\mathbf{1}},\ 0)_4$
W_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{3}},\ 0)_4$
B_μ	$(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$

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$\mathcal{H}_{16}(\mathbb{C})$

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

$\mathcal{H}_{16}(\mathbb{C})$

"Peirce decomposition"

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(u,\ d)_L \quad (\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(c,\ s)_L \quad (\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(t,\ b)_L \quad (\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$

$(\nu_e,\ e)_L \quad (\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
$(\nu_\mu,\ \mu)_L \quad (\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$
$(\nu_\tau,\ \tau)_L \quad (\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$

$u_R \quad (\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
$c_R \quad (\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
$t_R \quad (\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$

$d_R \quad (\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
$s_R \quad (\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
$b_R \quad (\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$

$\nu_{eR} \quad (\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
$\nu_{\mu R} \quad (\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$
$\nu_{\tau R} \quad (\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_2$

$e_R \quad (\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
$\mu_R \quad (\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$
$\tau_R \quad (\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1)_2$

$p_\mu \quad (\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$
$p'_\mu \quad (\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$

$G_\mu \quad (\underline{\mathbf{8}},\ \underline{\mathbf{1}},\ 0)_4$
$W_\mu \quad (\underline{\mathbf{1}},\ \underline{\mathbf{3}},\ 0)_4$
$B_\mu \quad (\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$

Covariant derivative

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(u,\ d)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(c,\ s)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(t,\ b)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(\nu_e,\ e)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
$(\nu_\mu,\ \mu)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
$(\nu_\tau,\ \tau)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
u_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
c_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
t_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
d_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
s_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
b_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
ν_{eR}	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
$\nu_{\mu R}$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
$\nu_{\tau R}$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
e_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
μ_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
τ_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
p_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$
p'_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$
G_μ	$\left(\underline{\mathbf{8}},\ \underline{\mathbf{1}},\ 0\right)_4$
W_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{3}},\ 0\right)_4$
B_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$

Fermions

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(u,\ d)_L \quad (\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
$(c,\ s)_L \quad (\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6})_2$
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$(\nu_\tau,\ \tau)_L \quad (\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2})_2$

$u_R \quad (\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$
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$t_R \quad (\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3})_2$

$d_R \quad (\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
$s_R \quad (\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3})_2$
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$G_\mu \quad (\underline{\mathbf{8}},\ \underline{\mathbf{1}},\ 0)_4$
$W_\mu \quad (\underline{\mathbf{1}},\ \underline{\mathbf{3}},\ 0)_4$
$B_\mu \quad (\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0)_4$

Covariant derivative

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Physical content

$(u,\ d)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(c,\ s)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(t,\ b)_L$	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{2}},\ \frac{1}{6}\right)_2$
$(\nu_e,\ e)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
$(\nu_\mu,\ \mu)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
$(\nu_\tau,\ \tau)_L$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{2}},\ -\frac{1}{2}\right)_2$
u_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
c_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
t_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ \frac{2}{3}\right)_2$
d_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
s_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
b_R	$\left(\underline{\mathbf{3}},\ \underline{\mathbf{1}},\ -\frac{1}{3}\right)_2$
ν_{eR}	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
$\nu_{\mu R}$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
$\nu_{\tau R}$	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_2$
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μ_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
τ_R	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ -1\right)_2$
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W_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{3}},\ 0\right)_4$
B_μ	$\left(\underline{\mathbf{1}},\ \underline{\mathbf{1}},\ 0\right)_4$

Fermions

$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$sS^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$
$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ sS^\star$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S$	$s^\star S^\star\ \mathcal{H}_{16}(\mathbb{C})\ s^\star S^\star$

Standard model irreps as an extension of 4-momentum: our first attempt

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