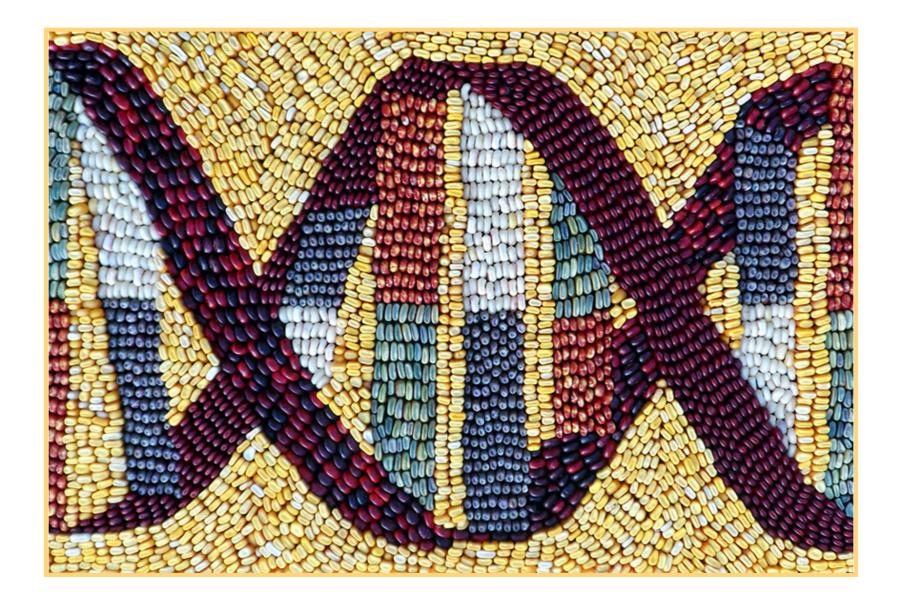
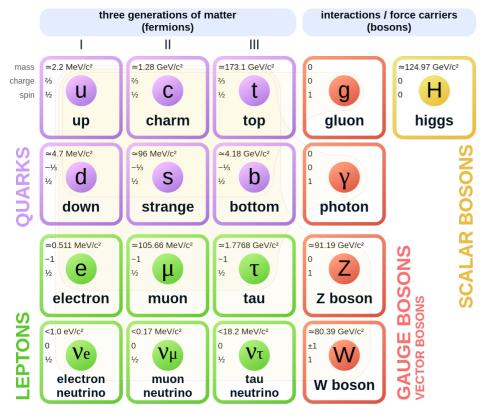
### Standard model irreps as an extension of 4-momentum: our first attempt



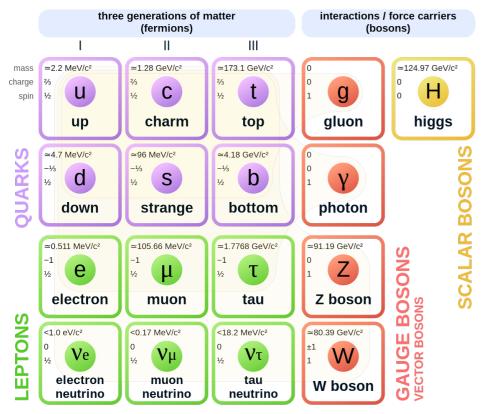
N. Furey and B. Romano

Humboldt-Universität zu Berlin University of Oxford

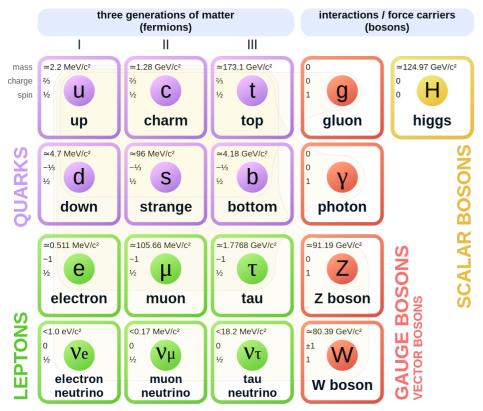
#### **Standard Model of Elementary Particles**



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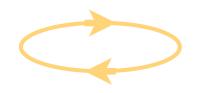


#### **Standard Model of Elementary Particles**

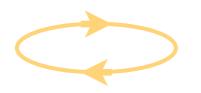


 $\mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O}$ 

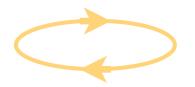
**—** 



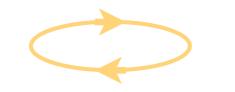
### Symmetry:



# Symmetry: SU(3)×SU(2)×U(1) / $\mathbb{Z}_6$

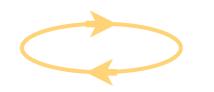


# $G_{sm} \coloneqq$ SU(3)×SU(2)×U(1) / Z<sub>6</sub>





# $G_{sm} :=$ Particles: SU(3)×SU(2)×U(1) / Z<sub>6</sub> Irreps





Which?

### $G_{sm} := \qquad Particles:$ SU(3)×SU(2)×U(1) / Z<sub>6</sub> $\underbrace{Irreps}$

TTT	•
	tormione
	fermions

$(\ u,\ d\ )_L$	$\left( \underline{3}, \ \underline{2}, \ \frac{1}{6} \ \right)_2$
$(\ c,\ s\ )_L$	$\left( \underline{3}, \ \underline{2}, \ \frac{1}{6} \ \right)_2$
$(t, b)_L$	$\left( \underline{3}, \ \underline{2}, \ \underline{1}_{6} \ \right)_{2}$

$$(\nu_{e}, e)_{L} (\underline{1}, \underline{2}, -\frac{1}{2})_{2} (\nu_{\mu}, \mu)_{L} (\underline{1}, \underline{2}, -\frac{1}{2})_{2} (\nu_{\tau}, \tau)_{L} (\underline{1}, \underline{2}, -\frac{1}{2})_{2}$$

### **Gauge bosons**

$$G_{\mu} \quad (\underline{\mathbf{8}}, \underline{\mathbf{1}}, 0)_{4}$$
$$W_{\mu} \quad (\underline{\mathbf{1}}, \underline{\mathbf{3}}, 0)_{4}$$
$$B_{\mu} \quad (\underline{\mathbf{1}}, \underline{\mathbf{1}}, 0)_{4}$$

#### **RH fermions**

$u_R$	$\left( \ \underline{3}, \ \underline{1}, \ \underline{2} \ \right)_2$
$c_R$	$\left( \ \underline{3}, \ \underline{1}, \ \frac{2}{3} \ \right)_2$
$t_R$	$\left( \ \underline{3}, \ \underline{1}, \ \frac{2}{3} \ \right)_2$

$$d_{R} \quad \left(\underline{\mathbf{3}}, \underline{\mathbf{1}}, -\frac{1}{3}\right)_{2}$$

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#### LH fermions

$(\ u,\ d\ )_L$	$\left( \underline{3}, \ \underline{2}, \ \frac{1}{6} \right)_2$
$(\ c,\ s\ )_L$	$\left( \underline{3}, \ \underline{2}, \ \frac{1}{6} \right)_2$
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$$(\nu_e, e)_L \qquad \left(\underline{1}, \underline{2}, -\frac{1}{2}\right)_2 \\ (\nu_\mu, \mu)_L \qquad \left(\underline{1}, \underline{2}, -\frac{1}{2}\right)_2 \\ (\nu_\tau, \tau)_L \qquad \left(\underline{1}, \underline{2}, -\frac{1}{2}\right)_2$$

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	fermions

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 $(u, d)_L \quad \left(\underline{\mathbf{3}}, \underline{\mathbf{2}}, \frac{1}{6}\right)_2$ 

#### LH fermions

 $(u, d)_L \quad \left(\underline{\mathbf{3}}, \underline{\mathbf{2}}, \frac{1}{6}\right)_2$ 

#### **RH fermions**

 $u_R \quad \left(\underline{\mathbf{3}}, \underline{\mathbf{1}}, \frac{2}{3}\right)_2$ 

LH fermions	<b>RH fermions</b>
$(u, d)_L$ $\left(\underline{3}, \underline{2}, \frac{1}{6}\right)_2$	$u_R  \left( \underline{3}, \underline{1}, \frac{2}{3} \right)_2$
$(\nu_e, e)_L  \left(  \underline{1},  \underline{2},  -\frac{1}{2}  \right)_2$	$d_R  \left( \underline{3}, \ \underline{1}, \ -\frac{1}{3} \right)_2$
	$e_R$ $(\underline{1}, \underline{1}, -1)_2$

LH fermions	<b>RH fermions</b>
$(u, d)_L$ $\left(\underline{3}, \underline{2}, \frac{1}{6}\right)_2$	$u_R  \left( \underline{3}, \ \underline{1}, \ \frac{2}{3} \right)_2$
$(\nu_e, e)_L (\underline{1}, \underline{2}, -\frac{1}{2})_2$	$d_R  \left( \underline{3}, \ \underline{1}, \ -\frac{1}{3} \right)_2$
	$e_R$ ( <u>1</u> , <u>1</u> , -1) <sub>2</sub>

### **1** generation

LH f	ermions	RH	fermions
	$\left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_2 \\ \left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_2 \end{array}$		$\left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_2 \\ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_2 \end{array}$
	$\left( \begin{array}{ccc} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array}  ight)_2 \ \left( \begin{array}{ccc} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array}  ight)_2 \end{array}$		$\left( \begin{array}{cc} \underline{3}, \ \underline{1}, \ -\frac{1}{3} \end{array} \right)_2 \\ \left( \begin{array}{cc} \underline{3}, \ \underline{1}, \ -\frac{1}{3} \end{array} \right)_2 \end{array}$
			$\left( \underline{1}, \underline{1}, -1 \right)_2$ $\left( \underline{1}, \underline{1}, -1 \right)_2$

#### **2** generations

LH fe	ermions	
$(\ c,\ s\ )_L$	$egin{array}{llllllllllllllllllllllllllllllllllll$	
$ u_{\mu},\ \mu\)_L$	$ \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_{2} \\ \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_{2} \\ \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_{2} \end{array} $	

#### **RH** fermions

$u_R$	$\left( \ \underline{3}, \ \underline{1}, \ \frac{2}{3} \ \right)_2$
$c_R$	$\left( \underline{3}, \ \underline{1}, \ \frac{2}{3} \ \right)_2$
$t_R$	$\left( \ \underline{3}, \ \underline{1}, \ \frac{2}{3} \ \right)_2$

$$d_{R} \quad \left(\underline{\mathbf{3}}, \underline{\mathbf{1}}, -\frac{1}{3}\right)_{2}$$

$$s_{R} \quad \left(\underline{\mathbf{3}}, \underline{\mathbf{1}}, -\frac{1}{3}\right)_{2}$$

$$b_{R} \quad \left(\underline{\mathbf{3}}, \underline{\mathbf{1}}, -\frac{1}{3}\right)_{2}$$

$$e_R \qquad (\underline{1}, \underline{1}, -1)_2$$
  

$$\mu_R \qquad (\underline{1}, \underline{1}, -1)_2$$
  

$$\tau_R \qquad (\underline{1}, \underline{1}, -1)_2$$

### **3** generations

LH fe	ermions	
$(\ c,\ s\ )_L$	$egin{array}{llllllllllllllllllllllllllllllllllll$	
$ u_{\mu},\ \mu\)_L$	$ \begin{pmatrix} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{pmatrix}_2 \\ ( \ \underline{1}, \ \underline{2}, \ -\frac{1}{2} )_2 \\ ( \ \underline{1}, \ \underline{2}, \ -\frac{1}{2} )_2 \end{cases} $	

#### **RH** fermions

$u_R$	$\left( \ \underline{3}, \ \underline{1}, \ \frac{2}{3} \ \right)_2$
$c_R$	$\left( \underline{3}, \ \underline{1}, \ \frac{2}{3} \ \right)_2$
$t_R$	$\left( \underline{3}, \ \underline{1}, \ \frac{2}{3} \ \right)_2$

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$$\mu_R \qquad (\underline{1}, \underline{1}, -1)_2$$
  

$$\tau_R \qquad (\underline{1}, \underline{1}, -1)_2$$

#### **Fermion content**

### LH fermions

$(\ u,\ d\ )_L$	$\left( \underline{3}, \ \underline{2}, \ \frac{1}{6} \right)_2$
$(\ c,\ s\ )_L$	$\left( \underline{3}, \ \underline{2}, \ \frac{1}{6} \right)_2$
$(t, b)_L$	$\left( \underline{3}, \ \underline{2}, \ \frac{1}{6} \right)_2$

$$(\nu_e, e)_L \qquad \left(\underline{1}, \underline{2}, -\frac{1}{2}\right)_2 \\ (\nu_\mu, \mu)_L \qquad \left(\underline{1}, \underline{2}, -\frac{1}{2}\right)_2 \\ (\nu_\tau, \tau)_L \qquad \left(\underline{1}, \underline{2}, -\frac{1}{2}\right)_2$$

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$$\tau_R \qquad (\underline{1}, \underline{1}, -1)_2$$

#### LH fermions

$(\ u,\ d\ )_L$	$(\underline{3}, \underline{2},$	$\left(\frac{1}{6}\right)_2$
$(\ c,\ s\ )_L$	$\left( \underline{3}, \ \underline{2}, \right.$	$\left(\frac{1}{6}\right)_2$
$(t, b)_L$	$\left( \underline{3}, \ \underline{2}, \right.$	$\left(\frac{1}{6}\right)_2$

$$(\nu_e, e)_L \qquad (\underline{1}, \underline{2}, -\underline{1}_2)_2 (\nu_\mu, \mu)_L \qquad (\underline{1}, \underline{2}, -\underline{1}_2)_2 (\nu_\tau, \tau)_L \qquad (\underline{1}, \underline{2}, -\underline{1}_2)_2$$

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TTT	•
	tormione
	fermions

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LH fermions		
$(\ c,\ s\ )_L$	$egin{array}{llllllllllllllllllllllllllllllllllll$	
$( u_\mu,\mu)_L$	$ig( egin{array}{cccccccccccccccccccccccccccccccccccc$	
Gauge bosons		
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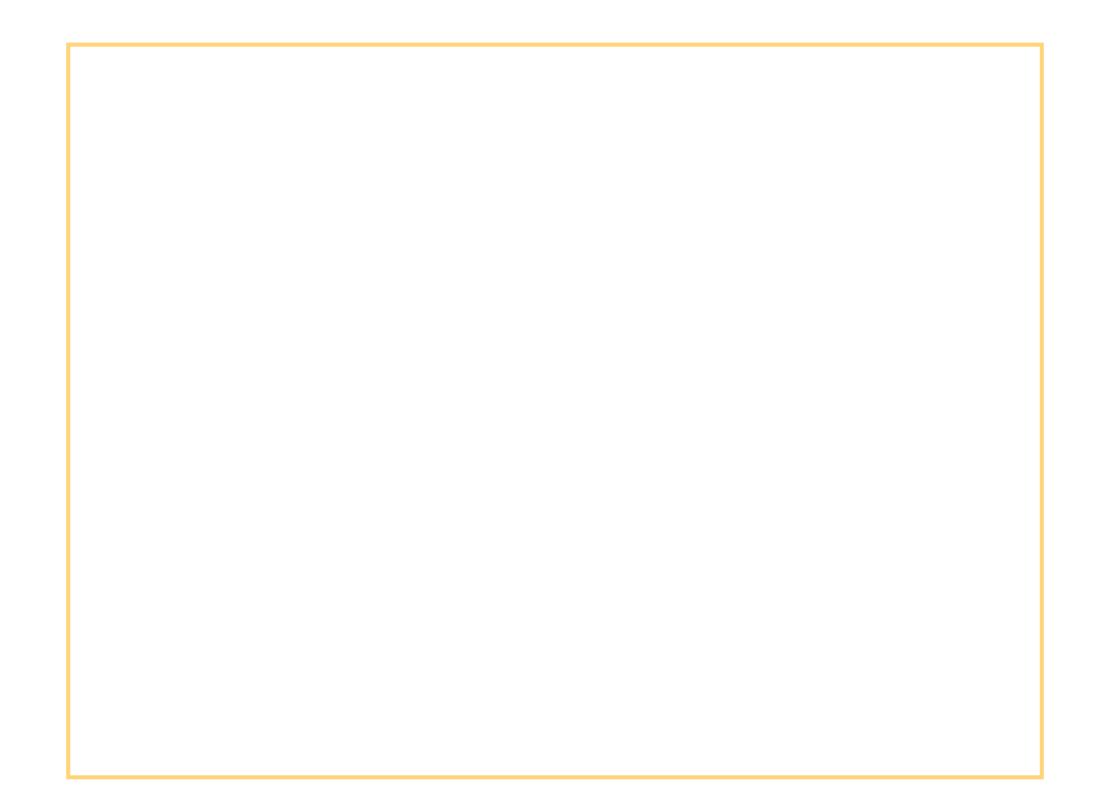
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Higgs  $+ 3\nu_R$  $H \quad (\underline{1}, \underline{2}, -\frac{1}{2})_1 \implies 244 \mathbb{R}$ 









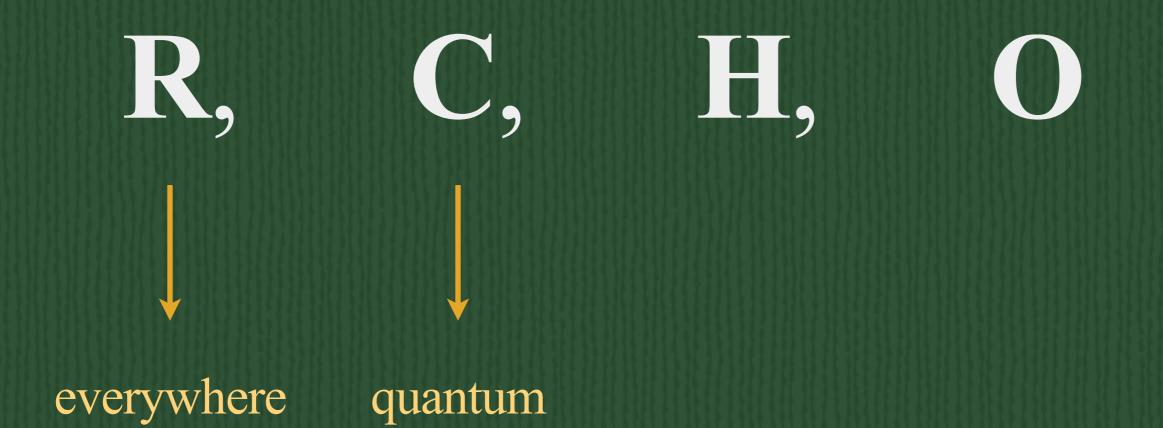


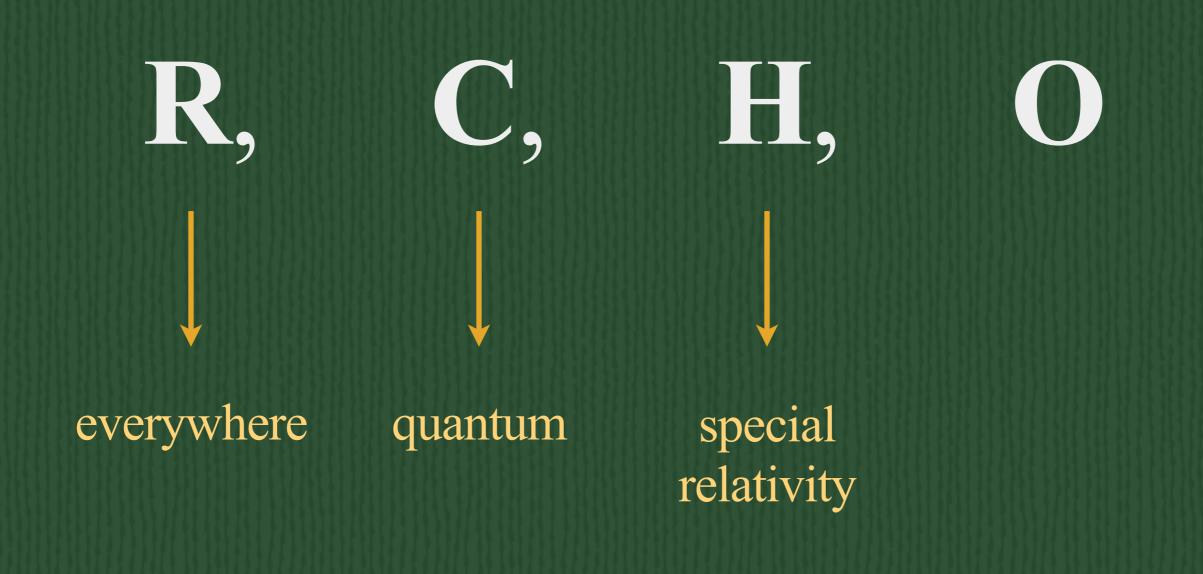


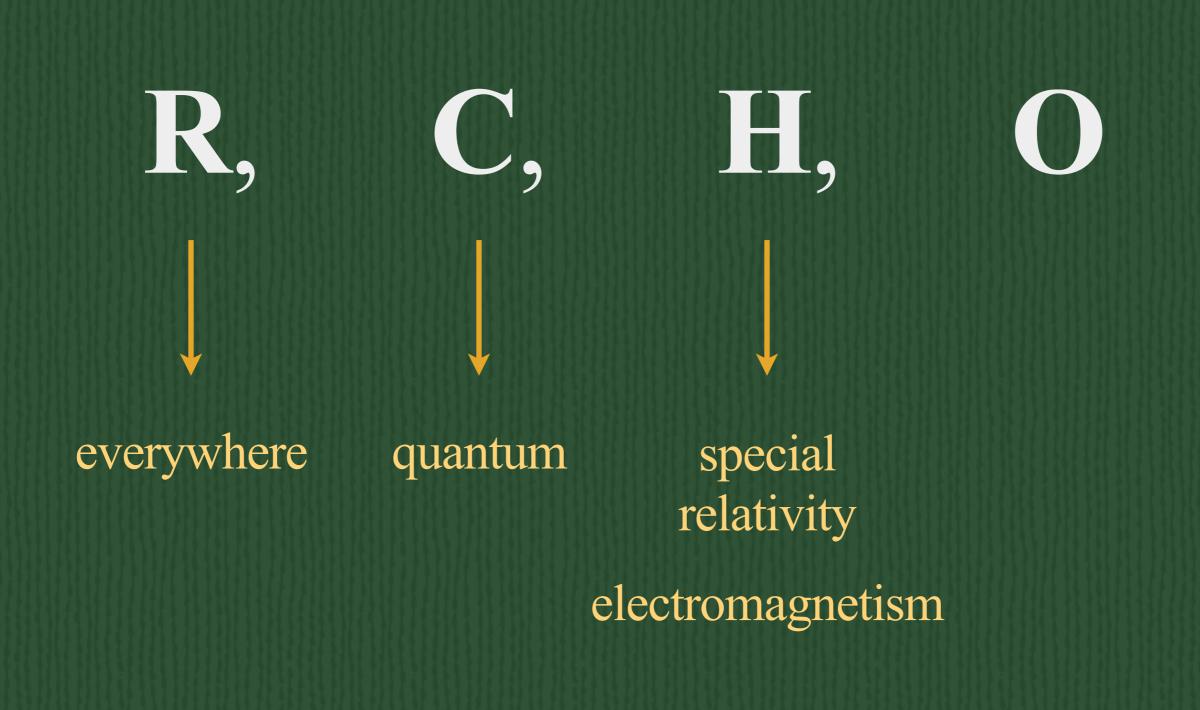


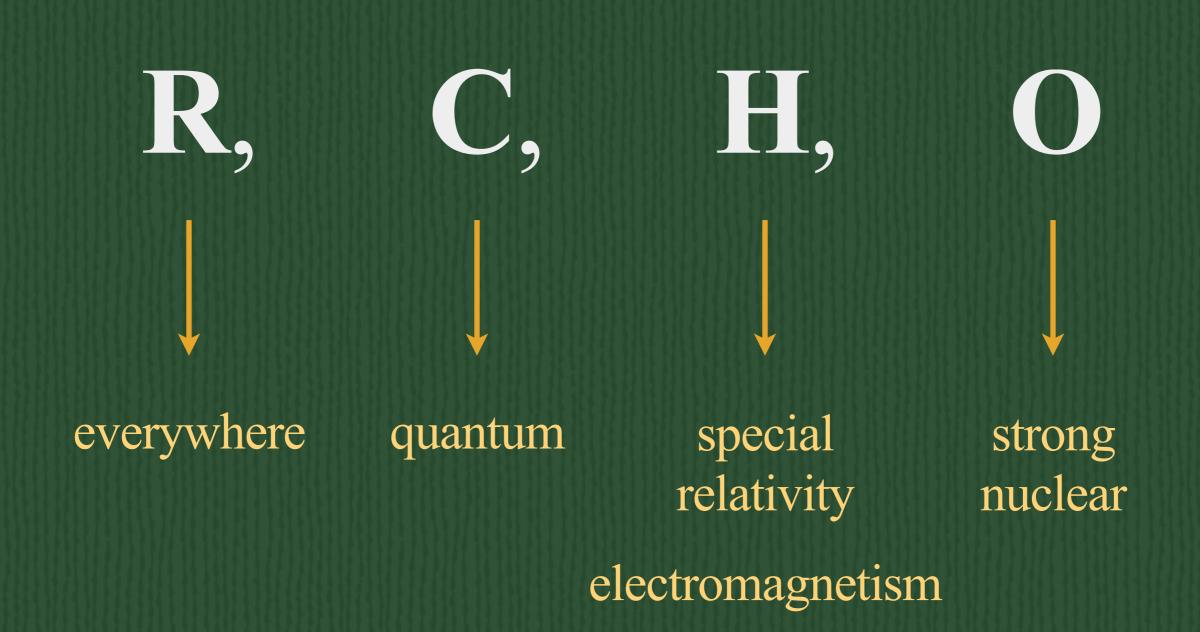
# R, C, H, O

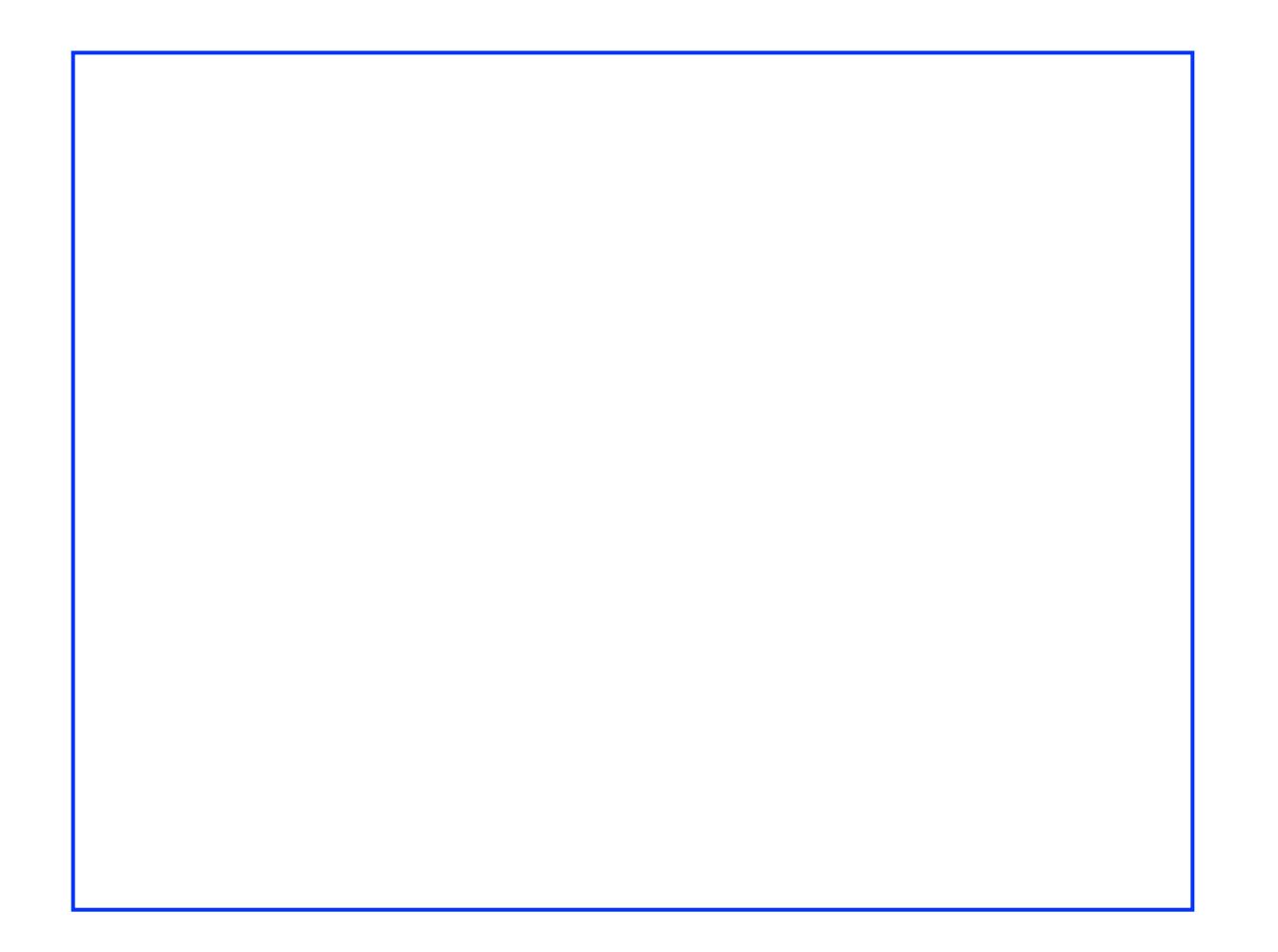
everywhere











N.F., M.J. Hughes,

One generation of standard model Weyl representations as a single copy of  $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ ,

#### $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$

N.F., M.J. Hughes,

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## $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O} = \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ G. Dixon

N.F., M.J. Hughes,

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N.F., M.J. Hughes,

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## $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ $64 \ \mathbb{R}$

#### 1 generation

N.F., M.J. Hughes,

One generation of standard model Weyl representations as a single copy of  $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ ,

## $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O} \quad \longleftrightarrow \quad 1 \text{ generation}$ $64 \mathbb{R}$

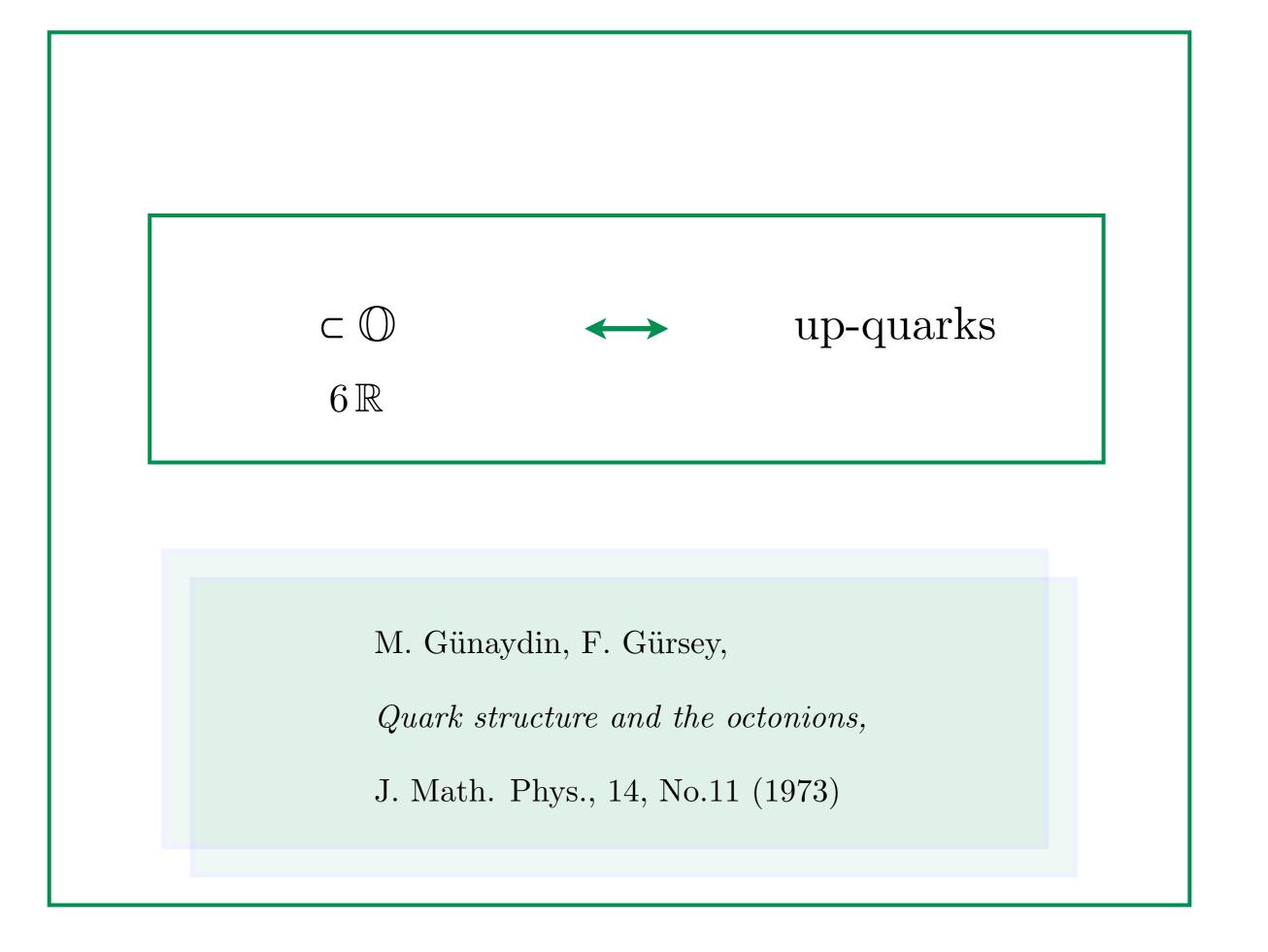
N.F., M.J. Hughes,

One generation of standard model Weyl representations as a single copy of  $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ ,

# $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O} \quad \longleftrightarrow \quad 1 \text{ generation}$ $64 \mathbb{R}$

N.F., M.J. Hughes,

One generation of standard model Weyl representations as a single copy of  $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ ,



#### Standard Model Irreps ( $SU(3)_C$ , $SU(2)_L$ , $U(1)_Y$ )

LH fermions	<b>RH fermions</b>
$(u, d)_L$ $\left(\underline{3}, \underline{2}, \frac{1}{6}\right)_2$	$u_R  \left( \underline{3}, \ \underline{1}, \ \frac{2}{3} \right)_2$
$(\nu_e, e)_L (\underline{1}, \underline{2}, -\frac{1}{2})_2$	$d_R  \left( \underline{3}, \ \underline{1}, \ -\frac{1}{3} \right)_2$
	$e_R$ $(\underline{1}, \underline{1}, -1)_2$

#### **1** generation

## Standard Model Irreps ( $SU(3)_C$ , $SU(2)_L$ , $U(1)_Y$ )

LH fermions		
$(\ c,\ s\ )_L$	$egin{array}{llllllllllllllllllllllllllllllllllll$	
$ u_{\mu},\ \mu\)_L$	$ \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_{2} \\ \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_{2} \\ \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_{2} \end{array} $	

#### **RH** fermions

$u_R$	$\left( \ \underline{3}, \ \underline{1}, \ \frac{2}{3} \ \right)_2$
$c_R$	$\left( \underline{3}, \ \underline{1}, \ \frac{2}{3} \ \right)_2$
$t_R$	$\left( \ \underline{3}, \ \underline{1}, \ \frac{2}{3} \ \right)_2$

$$d_{R} \quad \left(\underline{\mathbf{3}}, \underline{\mathbf{1}}, -\frac{1}{3}\right)_{2}$$

$$s_{R} \quad \left(\underline{\mathbf{3}}, \underline{\mathbf{1}}, -\frac{1}{3}\right)_{2}$$

$$b_{R} \quad \left(\underline{\mathbf{3}}, \underline{\mathbf{1}}, -\frac{1}{3}\right)_{2}$$

$$e_R \qquad (\underline{1}, \underline{1}, -1)_2$$
  

$$\mu_R \qquad (\underline{1}, \underline{1}, -1)_2$$
  

$$\tau_R \qquad (\underline{1}, \underline{1}, -1)_2$$

#### **3** generations

## Standard Model Irreps ( $SU(3)_C$ , $SU(2)_L$ , $U(1)_Y$ )

TTT	•
	tormione
	fermions

$(\ u,\ d\ )_L$	$\left( \underline{3}, \ \underline{2}, \ \frac{1}{6} \ \right)_2$
$(\ c,\ s\ )_L$	$\left( \underline{3}, \ \underline{2}, \ \frac{1}{6} \ \right)_2$
$(t, b)_L$	$\left( \underline{3}, \ \underline{2}, \ \underline{1}_{6} \ \right)_{2}$

$$(\nu_{e}, e)_{L} (\underline{1}, \underline{2}, -\frac{1}{2})_{2} (\nu_{\mu}, \mu)_{L} (\underline{1}, \underline{2}, -\frac{1}{2})_{2} (\nu_{\tau}, \tau)_{L} (\underline{1}, \underline{2}, -\frac{1}{2})_{2}$$

#### **Gauge bosons**

$$G_{\mu} \quad (\underline{\mathbf{8}}, \underline{\mathbf{1}}, 0)_{4}$$
$$W_{\mu} \quad (\underline{\mathbf{1}}, \underline{\mathbf{3}}, 0)_{4}$$
$$B_{\mu} \quad (\underline{\mathbf{1}}, \underline{\mathbf{1}}, 0)_{4}$$

#### **RH fermions**

$u_R$	$\left( \ \underline{3}, \ \underline{1}, \ \underline{2} \ \right)_2$
$c_R$	$\left( \ \underline{3}, \ \underline{1}, \ \frac{2}{3} \ \right)_2$
$t_R$	$\left( \ \underline{3}, \ \underline{1}, \ \frac{2}{3} \ \right)_2$

$$d_{R} \quad \left(\underline{\mathbf{3}}, \underline{\mathbf{1}}, -\frac{1}{3}\right)_{2}$$

$$s_{R} \quad \left(\underline{\mathbf{3}}, \underline{\mathbf{1}}, -\frac{1}{3}\right)_{2}$$

$$b_{R} \quad \left(\underline{\mathbf{3}}, \underline{\mathbf{1}}, -\frac{1}{3}\right)_{2}$$

$$e_{R} \quad (\underline{1}, \underline{1}, -1)_{2}$$

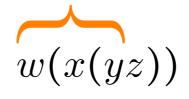
$$\mu_{R} \quad (\underline{1}, \underline{1}, -1)_{2}$$

$$\tau_{R} \quad (\underline{1}, \underline{1}, -1)_{2}$$

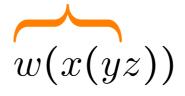
#### **Higgs** $H \quad \left(\underline{\mathbf{1}}, \ \underline{\mathbf{2}}, \ -\frac{1}{2} \right)_1$



#### sequence

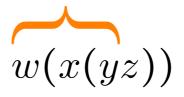






#### $w,x,y,z \ \in \ \mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$

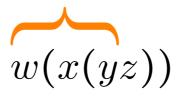
#### encode particles?



#### $w, x, y, z \ \in \ \mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$

 $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}'s$  left-multiplication algebra

encode particles?

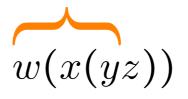


 $w, x, y, z \in \mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ 

 $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ 's left-multiplication algebra

 $``L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}"$ 

encode particles?



 $w, x, y, z \in \mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ 

 $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}\text{'s}$  left-multiplication algebra

" $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$ "

 $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ 's left-multiplication algebra

" $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$ "

w(x(yz))

 $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ 's left-multiplication algebra

" $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$ "

operator w(x(yz))  $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ 's left-multiplication algebra

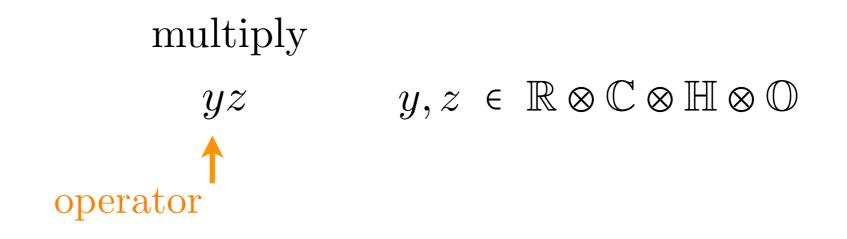
 $``L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}"$ 

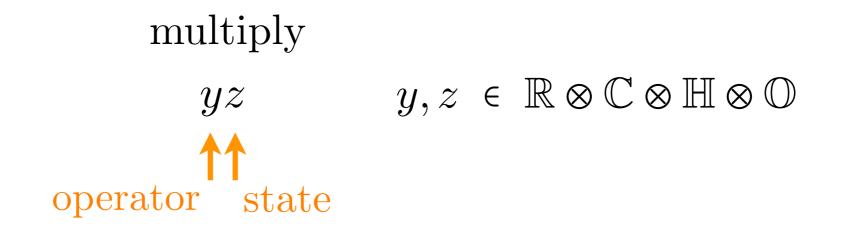
operator w(x(yz))tate  $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}\text{'s}$  left-multiplication algebra

" $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$ "

 $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ 's left-multiplication algebra " $L_{\mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}}$ "

 $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ 's left-multiplication algebra " $L_{\mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}}$ "





yz  $y,z \in \mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ operator state

## $yz \qquad \qquad y,z \in \mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$

 $L_y(z) \coloneqq yz \qquad \qquad y,z \in \mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ 

 $L_y(z) \coloneqq yz \qquad \qquad y,z \in \mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ 

 $L_y \in End_{\mathbb{C}}(\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O})$ 

 $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}\text{'s}$  left-multiplication algebra

" $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$ "

 $(L_x \circ L_y)(z)$ 

 $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ 's left-multiplication algebra

 $``L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}"$ 

 $(L_x \circ L_y)(z) = L_x(L_y(z))$ 

 $(L_x \circ L_y)(z) = L_x(L_y(z)) = x(yz)$ 

Multiplication:

 $(L_x \circ L_y)(z) = L_x(L_y(z)) = x(yz)$ 

 $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}\text{'s}$  left-multiplication algebra

" $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$ "

 $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} := \text{ subalgebra of } End_{\mathbb{C}}(\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O})$ generated by  $\{L_y \mid y \in \mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}\}$   $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}\text{'s}$  left-multiplication algebra

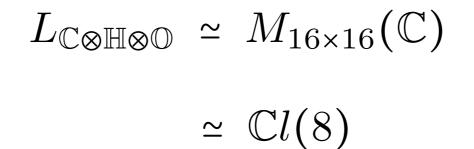
" $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$ "

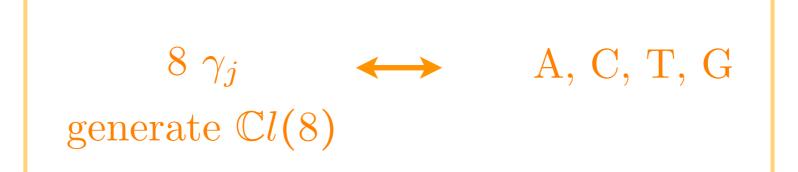
 $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq M_{16\times 16}(\mathbb{C})$ 

> $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq M_{16\times16}(\mathbb{C})$  $\simeq \mathbb{C}l(8)$

> $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq M_{16\times16}(\mathbb{C})$  $\simeq \mathbb{C}l(8)$

8  $\gamma_j$ generate  $\mathbb{C}l(8)$ 





# Standard Model Irreps ( $SU(3)_C$ , $SU(2)_L$ , $U(1)_Y$ )

TTT	•
	tormione
	fermions

$(\ u,\ d\ )_L$	$\left( \underline{3}, \ \underline{2}, \ \frac{1}{6} \ \right)_2$
$(\ c,\ s\ )_L$	$\left( \underline{3}, \ \underline{2}, \ \frac{1}{6} \ \right)_2$
$(t, b)_L$	$\left( \underline{3}, \ \underline{2}, \ \underline{1}_{6} \ \right)_{2}$

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### **Gauge bosons**

$$G_{\mu} \quad (\underline{\mathbf{8}}, \underline{\mathbf{1}}, 0)_{4}$$
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$$d_{R} \quad \left(\underline{\mathbf{3}}, \underline{\mathbf{1}}, -\frac{1}{3}\right)_{2}$$

$$s_{R} \quad \left(\underline{\mathbf{3}}, \underline{\mathbf{1}}, -\frac{1}{3}\right)_{2}$$

$$b_{R} \quad \left(\underline{\mathbf{3}}, \underline{\mathbf{1}}, -\frac{1}{3}\right)_{2}$$

$$e_{R} \quad (\underline{1}, \underline{1}, -1)_{2}$$

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$$\tau_{R} \quad (\underline{1}, \underline{1}, -1)_{2}$$

## **Higgs** $H \quad \left(\underline{\mathbf{1}}, \ \underline{\mathbf{2}}, \ -\frac{1}{2} \right)_1$

# $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O} \quad \longleftrightarrow \quad 1 \text{ generation}$ $64 \mathbb{R}$

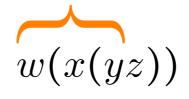
N.F., M.J. Hughes,

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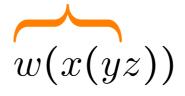
Phys.Lett.B, 827 (2022) https://pirsa.org/21030013



## sequence





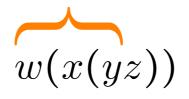


## $w,x,y,z \ \in \ \mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$

 $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ 's left-multiplication algebra

" $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$ "

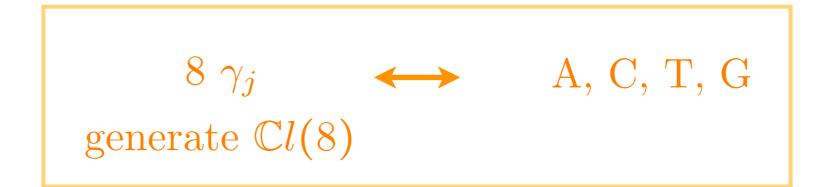
sequence



 $w, x, y, z \ \in \ \mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ 

# $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}\simeq\mathbb{C}l(8)$

# $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}\simeq\mathbb{C}l(8)$



# $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq \mathbb{C}l(8)$

# $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq \mathbb{C}l(8)$ $256 \mathbb{C}$

# $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}\simeq\mathbb{C}l(8)$ $256\mathbb{C}$

Need

# $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq \mathbb{C}l(8) \qquad \text{Need}$ $256 \mathbb{C} \qquad 244 \mathbb{R}$

 $(\mathbb{C} \text{ imaginary }) \quad i \mapsto -i$ 

 $(\mathbb{C} \text{ imaginary }) \qquad i \mapsto -i$  $(\mathbb{H} \text{ imaginaries }) \qquad \epsilon_j \mapsto -\epsilon_j \qquad j \in \{1, 2, 3\}$ 

 $(\mathbb{C} \text{ imaginary }) \qquad i \mapsto -i$  $(\mathbb{H} \text{ imaginaries }) \qquad \epsilon_j \mapsto -\epsilon_j \qquad j \in \{1, 2, 3\}$  $(\mathbb{O} \text{ imaginaries }) \qquad e_k \mapsto -e_k \qquad k \in \{1, 2, \dots, 7\}$ 

"Hermitian conjugate"

 $\dagger: \quad L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \rightarrow L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}}$ 

 $(\mathbb{C} \text{ imaginary }) \qquad i \mapsto -i$  $(\mathbb{H} \text{ imaginaries }) \qquad \epsilon_j \mapsto -\epsilon_j \qquad j \in \{1, 2, 3\}$  $(\mathbb{O} \text{ imaginaries }) \qquad e_k \mapsto -e_k \qquad k \in \{1, 2, \dots, 7\}$ 

• Important applications in physics

• Important applications in physics  $p \in \mathcal{H}_2(\mathbb{C}) \to \mathcal{H}_{16}(\mathbb{C})$ 

- Important applications in physics  $p \in \mathcal{H}_2(\mathbb{C}) \to \mathcal{H}_{16}(\mathbb{C})$
- $256 \mathbb{R} \gtrsim 244 \mathbb{R}$

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- $256 \mathbb{R} \gtrsim 244 \mathbb{R}$

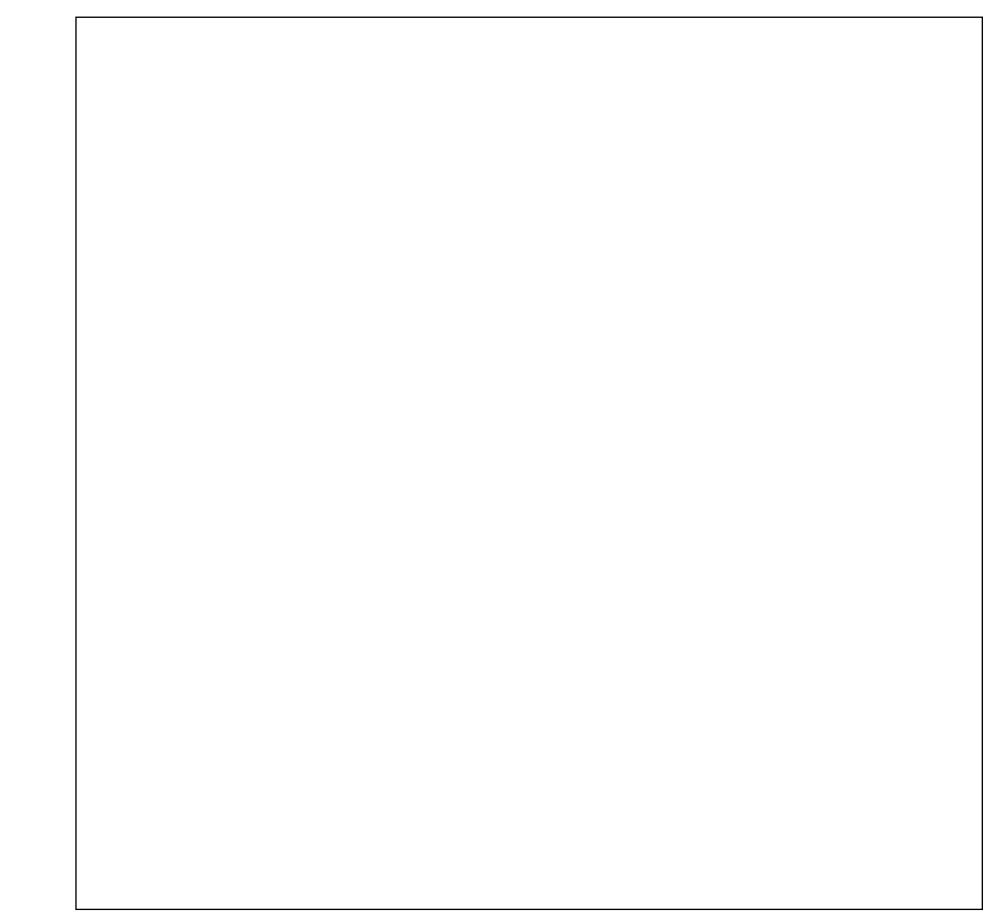


- Important applications in physics  $p \in \mathcal{H}_2(\mathbb{C}) \to \mathcal{H}_{16}(\mathbb{C})$
- $256 \mathbb{R} \gtrsim 244 \mathbb{R}$

Idempotents

## Idempotents

$$s \coloneqq \frac{1}{2} (1 + iL_{e_7}) \qquad S \coloneqq \frac{1}{2} (1 + iR_{e_7})$$
$$s^* \coloneqq \frac{1}{2} (1 - iL_{e_7}) \qquad S^* \coloneqq \frac{1}{2} (1 - iR_{e_7})$$



$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$

#### "Peirce decomposition"

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$

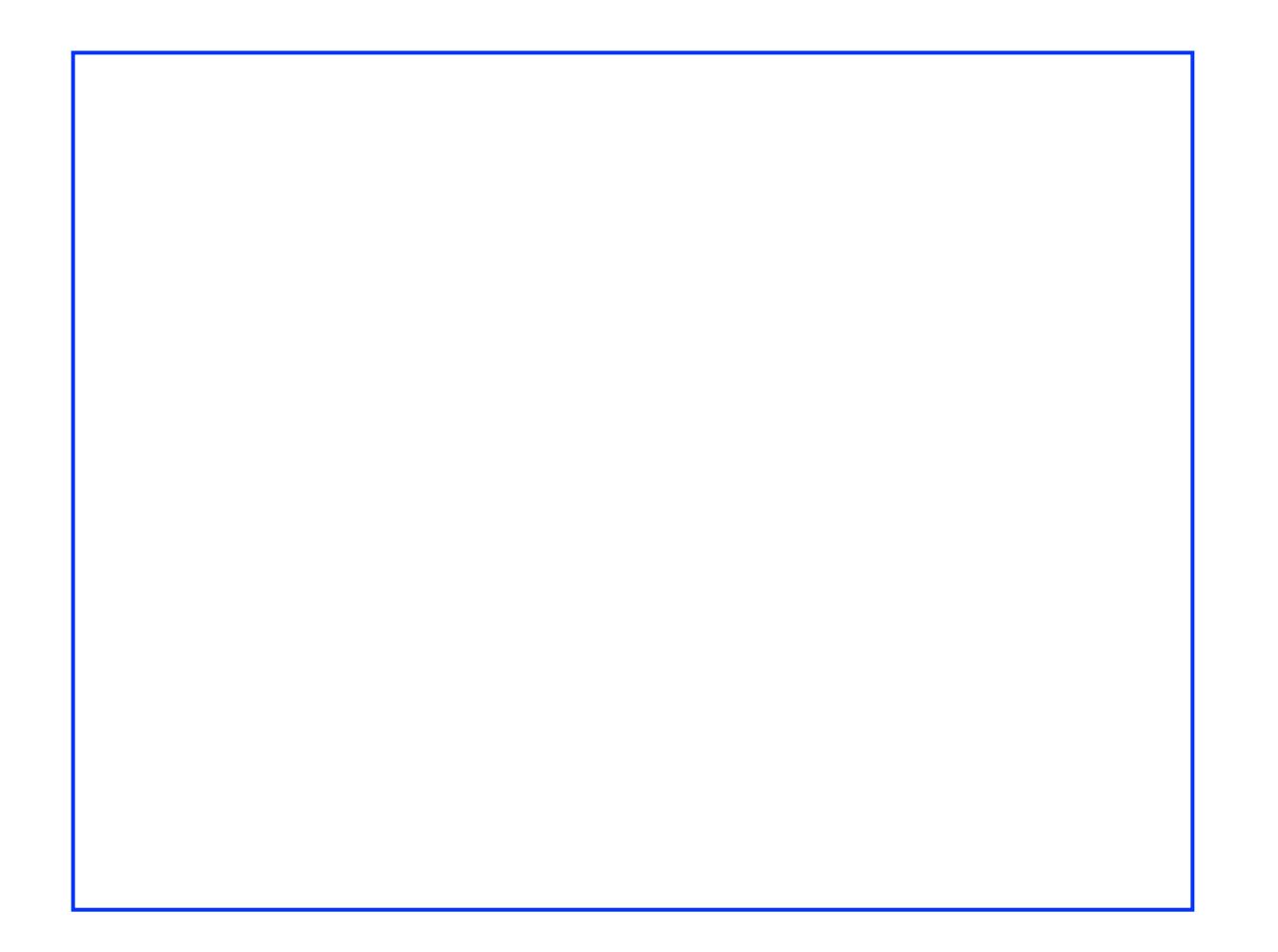
 $\mathcal{H}_{16}(\mathbb{C})$ 

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S \; \mathcal{H}_{16}(\mathbb{C}) \; s^{\star}S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2$
$( \  u_e, \ e \ )_L \ ( \  u_\mu, \ \mu \ )_L \ ( \  u_ au, \  au \ )_L$	$ig( egin{array}{cccccccccccccccccccccccccccccccccccc$
$egin{array}{l} u_R \ c_R \ t_R \end{array}$	$ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \end{array} $
$d_R$ $s_R$ $b_R$	$ig( \ \underline{3}, \ \underline{1}, \ -\frac{1}{3} \ ig)_2 \ ig( \ \underline{3}, \ \underline{1}, \ -\frac{1}{3} \ ig)_2 \ ig( \ \underline{3}, \ \underline{1}, \ -\frac{1}{3} \ ig)_2 \ ig( \ \underline{3}, \ \underline{1}, \ -\frac{1}{3} \ ig)_2$
$ u_{eR} $ $ u_{\mu R} $ $ u_{ au R}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$e_R \ \mu_R \  au_R$	$\begin{array}{l}(\underline{1}, \underline{1}, -1)_2\\(\underline{1}, \underline{1}, -1)_2\\(\underline{1}, \underline{1}, -1)_2\end{array}$
$p_{\mu} \ p_{\mu}^{\prime}$	$( \underline{1}, \ \underline{1}, \ 0 )_4$ $( \underline{1}, \ \underline{1}, \ 0 )_4$
$egin{array}{c} G_\mu \ W_\mu \ B_\mu \end{array}$	$(\underline{8}, \underline{1}, 0)_4$ $(\underline{1}, \underline{3}, 0)_4$ $(\underline{1}, \underline{1}, 0)_4$

$\mathcal{H}_{16}$	$(\mathbb{C})$
1010	$( \cup )$

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S \; \mathcal{H}_{16}(\mathbb{C}) \; s^{\star}S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$



# Generic element of $G_{sm}$ 's Lie algebra

Generic element of  $G_{sm}$ 's Lie algebra

$$\mathfrak{su}(3)_{C} \quad \mathfrak{su}(2)_{L} \quad \mathfrak{u}(1)_{Y} \\
\ell_{sm} \coloneqq ir'_{n}\Lambda_{n}s + r_{k}L_{\epsilon_{k}}s^{*}S + \frac{r}{2}(\frac{i}{3}sS^{*} - isS - L_{\epsilon_{3}}s^{*}S^{*}) \\
n \in \{1, 2, \dots 8\} \\
k \in \{1, 2, 3\}$$

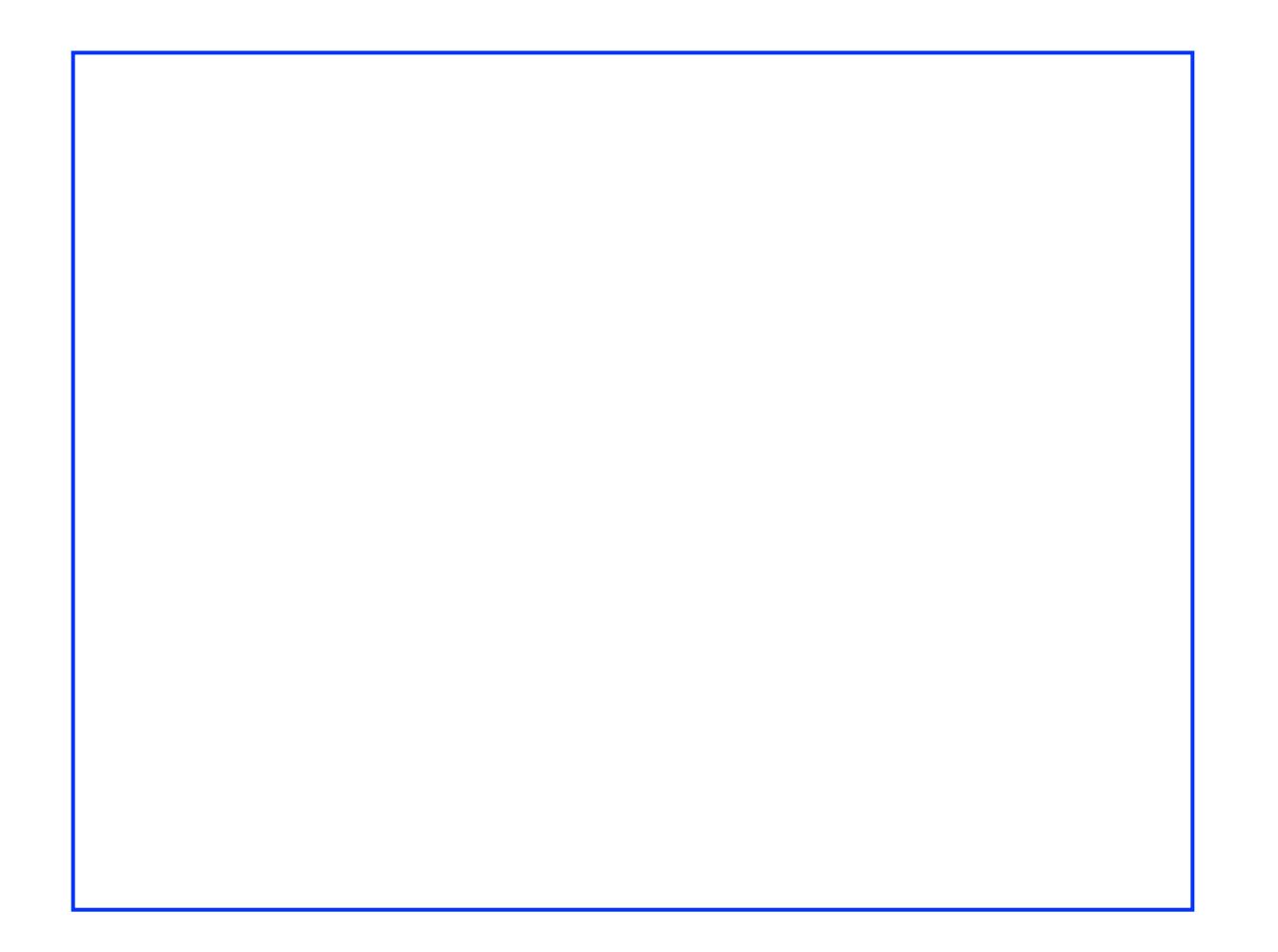
Generic element of  $G_{sm}$ 's Lie algebra

$$\mathfrak{su}(3)_C \quad \mathfrak{su}(2)_L \quad \mathfrak{u}(1)_Y$$

$$\ell_{sm} \coloneqq ir'_n \Lambda_n s + r_k L_{\epsilon_k} s^* S + \frac{r}{2} (\frac{i}{3} s S^* - is S - L_{\epsilon_3} s^* S^*)$$

$$\stackrel{\uparrow}{\in} \mathfrak{der}(\mathbb{O}) \quad n \in \{1, 2, \dots 8\}$$

$$k \in \{1, 2, 3\}$$



$$\delta b = \ell_{sm} b + b \, \ell_{sm}^{\dagger}$$

#### diagonal

$$\delta b = \ell_{sm} b + b \,\ell_{sm}^{\dagger}$$
$$\delta f_0 = \ell_{sm} s f_0 s^* + s f_0 s^* \ell_{sm} + h.c.$$

diagonal

outer off-diagonal

$$\begin{split} \delta b &= \ell_{sm} b + b \, \ell_{sm}^{\dagger} & \text{diagonal} \\ \delta f_0 &= \ell_{sm} s f_0 s^* + s f_0 s^* \ell_{sm} + h.c. & \text{outer off-diagonal} \\ \delta f_+ &= \ell_{sm} (s S^* f_+ s S + s^* S^* f_+ s^* S) \\ &+ (s S^* f_+ s S + s^* S^* f_+ s^* S) \ell_{sm}^{\dagger *} + h.c. & \text{inner off-diagonal} \end{split}$$

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2$
$( \  u_e, \ e \ )_L \ ( \  u_\mu, \ \mu \ )_L \ ( \  u_ au, \  au \ )_L$	$ig( egin{array}{cccccccccccccccccccccccccccccccccccc$
$egin{array}{l} u_R \ c_R \ t_R \end{array}$	$ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \end{array} $
$d_R$ $s_R$ $b_R$	$ig( \ \underline{3}, \ \underline{1}, \ -\frac{1}{3} \ ig)_2 \ ig( \ \underline{3}, \ \underline{1}, \ -\frac{1}{3} \ ig)_2 \ ig( \ \underline{3}, \ \underline{1}, \ -\frac{1}{3} \ ig)_2 \ ig( \ \underline{3}, \ \underline{1}, \ -\frac{1}{3} \ ig)_2$
$ u_{eR} $ $ u_{\mu R} $ $ u_{ au R}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$e_R \ \mu_R \  au_R$	$\begin{array}{l}(\underline{1}, \underline{1}, -1)_2\\(\underline{1}, \underline{1}, -1)_2\\(\underline{1}, \underline{1}, -1)_2\end{array}$
$p_{\mu} \ p_{\mu}^{\prime}$	$( \underline{1}, \ \underline{1}, \ 0 )_4$ $( \underline{1}, \ \underline{1}, \ 0 )_4$
$egin{array}{c} G_\mu \ W_\mu \ B_\mu \end{array}$	$(\underline{8}, \underline{1}, 0)_4$ $(\underline{1}, \underline{3}, 0)_4$ $(\underline{1}, \underline{1}, 0)_4$

$\mathcal{H}_{16}$	$(\mathbb{C})$
1010	$( \cup )$

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S \; \mathcal{H}_{16}(\mathbb{C}) \; s^{\star}S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2$
$( \  u_e, \ e \ )_L \ ( \  u_\mu, \ \mu \ )_L \ ( \  u_\tau, \  au \ )_L$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$egin{array}{l} u_R \ c_R \ t_R \end{array}$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$d_R \ s_R \ b_R$	$ig( \ \underline{3}, \ \underline{1}, \ -\frac{1}{3} \ ig)_2 \ ig( \ \underline{3}, \ \underline{1}, \ -\frac{1}{3} \ ig)_2 \ ig( \ \underline{3}, \ \underline{1}, \ -\frac{1}{3} \ ig)_2 \ ig( \ \underline{3}, \ \underline{1}, \ -\frac{1}{3} \ ig)_2$
$ u_{eR} $ $ u_{\mu R} $ $ u_{ au R}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$e_R \ \mu_R \  au_R$	$(\underline{1}, \underline{1}, -1)_2$ $(\underline{1}, \underline{1}, -1)_2$ $(\underline{1}, \underline{1}, -1)_2$
$p_{\mu} \ p'_{\mu}$	$( \underline{1}, \ \underline{1}, \ 0 )_4$ $( \underline{1}, \ \underline{1}, \ 0 )_4$
$egin{array}{c} G_\mu \ W_\mu \ B_\mu \end{array}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$( \  u_e, \ e \ )_L \ ( \  u_\mu, \ \mu \ )_L \ ( \  u_\tau, \  au \ )_L$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$egin{array}{c} u_R \ c_R \ t_R \end{array}$	$ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \end{array} $
$d_R \ s_R \ b_R$	$ \left( \begin{array}{ccc} \underline{3}, \ \underline{1}, \ -\frac{1}{3} \end{array} \right)_{2} \\ \left( \begin{array}{ccc} \underline{3}, \ \underline{1}, \ -\frac{1}{3} \end{array} \right)_{2} \\ \left( \begin{array}{ccc} \underline{3}, \ \underline{1}, \ -\frac{1}{3} \end{array} \right)_{2} \end{array} $
$ u_{eR} $ $ u_{\mu R} $ $ u_{ au R}$	$(\underline{1}, \underline{1}, 0)_2$ $(\underline{1}, \underline{1}, 0)_2$ $(\underline{1}, \underline{1}, 0)_2$
$e_R \ \mu_R \  au_R$	$(\underline{1}, \underline{1}, -1)_2$ $(\underline{1}, \underline{1}, -1)_2$ $(\underline{1}, \underline{1}, -1)_2$
$p_\mu \ p'_\mu$	$\left(\begin{array}{ccc} \underline{1}, \ \underline{1}, \ 0 \end{array}\right)_4$ $\left(\begin{array}{ccc} \underline{1}, \ \underline{1}, \ 0 \end{array}\right)_4$
$G_{\mu} \ W_{\mu} \ B_{\mu}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^*S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$\left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_2 \\ \left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_2 \\ \left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_2 \end{array}$
$( \  u_e, \ e \ )_L \ ( \  u_\mu, \ \mu \ )_L \ ( \  u_ au, \  au \ )_L$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$egin{array}{c} u_R \ c_R \ t_R \end{array}$	$ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_2 \\ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_2 \\ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_2 \\ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_2 \end{array} $
$d_R \ s_R \ b_R$	$ig( \ \underline{3}, \ \underline{1}, \ -\frac{1}{3} \ ig)_2 \ ig( \ \underline{3}, \ \underline{1}, \ -\frac{1}{3} \ ig)_2 \ ig( \ \underline{3}, \ \underline{1}, \ -\frac{1}{3} \ ig)_2 \ ig( \ \underline{3}, \ \underline{1}, \ -\frac{1}{3} \ ig)_2$
$ u_{eR} $ $ u_{\mu R} $ $ u_{ au R}$	$( \underline{1}, \underline{1}, 0 )_{2} ( \underline{1}, \underline{1}, 0 )_{2} ( \underline{1}, \underline{1}, 0 )_{2} $
$e_R \ \mu_R \  au_R$	$\begin{array}{cccc}(\underline{1},\ \underline{1},\ -1\ )_2\\(\underline{1},\ \underline{1},\ -1\ )_2\\(\underline{1},\ \underline{1},\ -1\ )_2\end{array}$
$p_\mu \ p'_\mu$	$\begin{array}{cccccccccc} (\ \underline{1},\ \underline{1},\ 0\ )_{4} \\ (\ \underline{1},\ \underline{1},\ 0\ )_{4} \end{array}$
$egin{array}{c} G_\mu \ W_\mu \ B_\mu \end{array}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^*S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$ig( { { { { { 3 } } } , { { { 2 } } , { { { 1 \over 6 } } } } }_2 } } )_2 \ ( { { { { 3 } , { { 2 } , { { 1 \over 6 } } } } }_2 } )_2 \ ( { { { 3 } , { { 2 } , { { 1 \over 6 } } } } }_2 \ )_2 \ ( { { { 3 } , { 2 } , { { 1 \over 6 } } } } )_2 \ )_2 \ ( { { 3 } , { 2 } , { { 1 \over 6 } } } )_2 \ )_2 \ )_2 \ ( { { 3 } , { 2 } , { 1 \over 6 } } )_2 \ )_2 \ )_2 \ )_2 \ ( { 3 } , { 2 } , { 1 \over 6 } )_2 \ )_2$
$( \  u_e, \ e \ )_L \ ( \  u_\mu, \ \mu \ )_L \ ( \  u_ au, \  au \ )_L$	$ \begin{pmatrix} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{pmatrix}_2 \\ ( \ \underline{1}, \ \underline{2}, \ -\frac{1}{2} )_2 \\ ( \ \underline{1}, \ \underline{2}, \ -\frac{1}{2} )_2 \end{cases} $
$egin{array}{c} u_R \ c_R \ t_R \end{array}$	$ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_2 \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_2 \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_2 \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_2 \end{array} $
$d_R \ s_R \ b_R$	$ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ -\frac{1}{3} \end{array} \right)_2 \\ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ -\frac{1}{3} \end{array} \right)_2 \\ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ -\frac{1}{3} \end{array} \right)_2 \end{array} \\ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ -\frac{1}{3} \end{array} \right)_2 \end{array} $
$egin{array}{l}  u_{eR} \  u_{\mu R} \  u_{ au R} \end{array}$	$\begin{array}{ccccccccc} (\underline{1}, \underline{1}, 0)_2 \\ (\underline{1}, \underline{1}, 0)_2 \\ (\underline{1}, \underline{1}, 0)_2 \end{array}$
$e_R \ \mu_R \  au_R$	$ \begin{array}{c} (\underline{1}, \underline{1}, -1)_2 \\ (\underline{1}, \underline{1}, -1)_2 \\ (\underline{1}, \underline{1}, -1)_2 \end{array} \\ \end{array} $
$p_{\mu} \ p'_{\mu}$	$\begin{array}{ccccccccc} (\ \underline{1},\ \underline{1},\ 0\ )_{4} \\ (\ \underline{1},\ \underline{1},\ 0\ )_{4} \end{array}$
$egin{array}{c} G_\mu \ W_\mu \ B_\mu \end{array}$	$( \underline{8}, \ \underline{1}, \ 0 )_4$ $( \underline{1}, \ \underline{3}, \ 0 )_4$ $( \underline{1}, \ \underline{1}, \ 0 )_4$

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^*S \mathcal{H}_{16}(\mathbb{C}) sS^*$	$s^{\star}S \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$

$( \ u, \ d \ )_L \ ( \ c, \ s \ )_L \ ( \ t, \ b \ )_L$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$( \  u_e, \ e \ )_L \ ( \  u_{\mu}, \ \mu \ )_L \ ( \  u_{\tau}, \  au \ )_L$	$ig( \ \underline{1}, \ \underline{2}, \ -\frac{1}{2} \ ig)_2 \ ig( \ \underline{1}, \ \underline{2}, \ -\frac{1}{2} \ ig)_2 \ ig( \ \underline{1}, \ \underline{2}, \ -\frac{1}{2} \ ig)_2 \ ig( \ \underline{1}, \ \underline{2}, \ -\frac{1}{2} \ ig)_2$
$egin{array}{c} u_R \ c_R \ t_R \end{array}$	$ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \end{array} $
$d_R \ s_R \ b_R$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$egin{array}{l}  u_{eR} \  u_{\mu R} \  u_{ au R} \end{array}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$e_R \ \mu_R \  au_R$	$\begin{array}{cccc} (\underline{1}, \underline{1}, -1)_2 \\ (\underline{1}, \underline{1}, -1)_2 \\ (\underline{1}, \underline{1}, -1)_2 \end{array}$
$p_\mu \ p'_\mu$	$\left(\begin{array}{ccc} \underline{1}, \ \underline{1}, \ 0 \end{array}\right)_4 \\ \left(\begin{array}{ccc} \underline{1}, \ \underline{1}, \ 0 \end{array}\right)_4 \end{array}$
$egin{array}{c} G_\mu \ W_\mu \ B_\mu \end{array}$	$\left(\begin{array}{ccc} \underline{8}, \ \underline{1}, \ 0 \end{array}\right)_4 \\ \left(\begin{array}{ccc} \underline{1}, \ \underline{3}, \ 0 \end{array}\right)_4 \\ \left(\begin{array}{ccc} \underline{1}, \ \underline{1}, \ 0 \end{array}\right)_4 \end{array}$

	$\mathcal{H}_{16}(\mathbb{C})$			$\downarrow$ $\downarrow$
	$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
<b>* *</b>	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2$
$( \  u_e, \ e \ )_L \ ( \  u_\mu, \ \mu \ )_L \ ( \  u_ au, \  au \ )_L$	$ \begin{pmatrix} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{pmatrix}_2 \\ ( \ \underline{1}, \ \underline{2}, \ -\frac{1}{2} )_2 \\ ( \ \underline{1}, \ \underline{2}, \ -\frac{1}{2} )_2 \end{cases} $
$egin{array}{c} u_R \ c_R \ t_R \end{array}$	$ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \end{array} $
$d_R$ $s_R$ $b_R$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$ u_{eR} $ $ u_{\mu R}$	$\left( \begin{array}{ccc} \underline{1}, \ \underline{1}, \ 0 \end{array} \right)_2 \ \left( \begin{array}{ccc} \underline{1}, \ \underline{1}, \ 0 \end{array} \right)_2 \end{array}$
$ u_{ au R}$	$(\underline{1}, \underline{1}, 0)_2$
$ \begin{array}{c} \nu_{\tau R} \\ e_R \\ \mu_R \\ \tau_R \end{array} $	$(\underline{1}, \underline{1}, 0)_2$ $(\underline{1}, \underline{1}, -1)_2$ $(\underline{1}, \underline{1}, -1)_2$ $(\underline{1}, \underline{1}, -1)_2$
$e_R \ \mu_R$	$ (\underline{1}, \underline{1}, -1)_2  (\underline{1}, \underline{1}, -1)_2 $

	$\mathcal{H}_{16}(\mathbb{C})$			$\downarrow\downarrow\downarrow$
	$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
	$s^*S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^*S \mathcal{H}_{16}(\mathbb{C}) s^*S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
<b>* *</b>	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$

(	$egin{array}{llllllllllllllllllllllllllllllllllll$	$ \left(\begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array}\right)_{2} \\ \left(\begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array}\right)_{2} \\ \left(\begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array}\right)_{2} \end{array} $	
(1	$( egin{array}{c} \psi_{e}, \; e \; )_{L} \ ( eta_{\mu}, \; \mu \; )_{L} \ ( eta_{ au}, \;  au \; )_{L} \end{array}$	$ \begin{array}{c} \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_2 \\ \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_2 \\ \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_2 \end{array} \\ \end{array} $	
	$egin{array}{c} u_R \ c_R \ t_R \end{array}$	$egin{array}{llllllllllllllllllllllllllllllllllll$	
	$d_R \ s_R \ b_R$	$ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ -\frac{1}{3} \end{array} \right)_{2} \\ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ -\frac{1}{3} \end{array} \right)_{2} \\ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ -\frac{1}{3} \end{array} \right)_{2} \end{array} \right. $	
	$egin{array}{c}  u_{eR}  u_{\mu R}  u_{ au R}  u_{ au R}  \end{array}$	$\begin{array}{cccc} \left( \begin{array}{ccc} \underline{1}, \ \underline{1}, \ 0 \end{array} \right)_2 \\ \left( \begin{array}{cccc} \underline{1}, \ \underline{1}, \ 0 \end{array} \right)_2 \\ \left( \begin{array}{ccccc} \underline{1}, \ \underline{1}, \ 0 \end{array} \right)_2 \end{array}$	++
	$e_R \ \mu_R \  au_R$		++
	$p_\mu \ p'_\mu$	$( \underline{1}, \ \underline{1}, \ 0 )_4$ $( \underline{1}, \ \underline{1}, \ 0 )_4$	
	$W_{\mu}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	

 $\mathcal{H}_{16}(\mathbb{C})$ 

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$ig( \ {\underline{3}}, \ {\underline{2}}, \ {\frac{1}{6}} \ ig)_2 \ (\ {\underline{3}}, \ {\underline{2}}, \ {\frac{1}{6}} \ ig)_2 \ (\ {\underline{3}}, \ {\underline{2}}, \ {\frac{1}{6}} \ ig)_2$
$( \  u_e, \ e \ )_L \ ( \  u_\mu, \ \mu \ )_L \ ( \  u_ au, \  au \ )_L$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$egin{array}{c} u_R \ c_R \ t_R \end{array}$	$ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_2 \\ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_2 \\ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_2 \\ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_2 \end{array} $
$d_R \ s_R \ b_R$	$ig( \ {\underline{3}}, \ {\underline{1}}, \ -{1\over 3} \ ig)_2 \ ig( \ {\underline{3}}, \ {\underline{1}}, \ -{1\over 3} \ ig)_2 \ ig( \ {\underline{3}}, \ {\underline{1}}, \ -{1\over 3} \ ig)_2 \ ig( \ {\underline{3}}, \ {\underline{1}}, \ -{1\over 3} \ ig)_2$
$ u_{eR}$	$(\underline{1}, \underline{1}, 0)_2$
$ u_{\mu R}  u_{ au R}$	$(\underline{1}, \underline{1}, 0)_2 (\underline{1}, \underline{1}, 0)_2$
$e_R$	$(\underline{1}, \underline{1}, -1)_2$
$\mu_R \  au_R$	$\left( \begin{array}{cc} \underline{1}, \ \underline{1}, \ -1 \end{array} \right)_2 \\ \left( \begin{array}{cc} \underline{1}, \ \underline{1}, \ -1 \end{array} \right)_2 \end{array}$
$p_\mu \ p'_\mu$	$( \underline{1}, \ \underline{1}, \ 0 )_4$ $( \underline{1}, \ \underline{1}, \ 0 )_4$
$egin{array}{c} G_\mu \ W_\mu \ B_\mu \end{array}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$ \left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_{2} \\ \left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_{2} \\ \left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_{2} \end{array} $
$( \  u_e, \ e \ )_L \ ( \  u_\mu, \ \mu \ )_L \ ( \  u_\tau, \  au \ )_L$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$egin{array}{c} u_R \ c_R \ t_R \end{array}$	$ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \end{array} $
$d_R \ s_R \ b_R$	$ig( \ \underline{3}, \ \underline{1}, \ -\frac{1}{3} \ ig)_2 \ ig( \ \underline{3}, \ \underline{1}, \ -\frac{1}{3} \ ig)_2 \ ig( \ \underline{3}, \ \underline{1}, \ -\frac{1}{3} \ ig)_2 \ ig( \ \underline{3}, \ \underline{1}, \ -\frac{1}{3} \ ig)_2$
$ u_{eR} $ $ u_{\mu R}$	$(\underline{1}, \underline{1}, 0)_2$ $(\underline{1}, \underline{1}, 0)_2$
$ u_{ au R} $ $ e_R $ $ \mu_R $	$(\underline{1}, \underline{1}, 0)_2$ $(\underline{1}, \underline{1}, -1)_2$ $(\underline{1}, \underline{1}, -1)_2$
$ au_R$	$ (\underline{1}, \underline{1}, -1)_2 $ $ (\underline{1}, \underline{1}, 0)_4 $
$p'_{\mu} \ G_{\mu} \ W_{\mu} \ B_{\mu}$	$(\underline{1}, \underline{1}, 0)_{4}$ $(\underline{8}, \underline{1}, 0)_{4}$ $(\underline{1}, \underline{3}, 0)_{4}$ $(\underline{1}, \underline{1}, 0)_{4}$

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^*S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2$
$( \nu_e, e )_L \ ( \nu_\mu, \mu )_L \ ( \nu_\tau, \tau )_L $	$ig( {f 1}, {f 2}, -{f 1}_2 ig)_2 \ ig( {f 1}, {f 2}, -{f 1}_2 ig)_2 \ ig( {f 1}, {f 2}, -{f 1}_2 ig)_2 \ ig( {f 1}, {f 2}, -{f 1}_2 ig)_2 \ ig)_2$
$egin{array}{c} u_R \ c_R \ t_R \end{array}$	$ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \end{array} $
$d_R \ s_R \ b_R$	$ig( \ {\underline{3}}, \ {\underline{1}}, \ -{1\over 3} \ ig)_2 \ ig( \ {\underline{3}}, \ {\underline{1}}, \ -{1\over 3} \ ig)_2 \ ig( \ {\underline{3}}, \ {\underline{1}}, \ -{1\over 3} \ ig)_2 \ ig( \ {\underline{3}}, \ {\underline{1}}, \ -{1\over 3} \ ig)_2$
$ u_{eR}$	$(\underline{1}, \underline{1}, 0)_2$
$ u_{\mu R}  u_{ au R}$	$( \underline{1}, \underline{1}, 0 )_2$ $( \underline{1}, \underline{1}, 0 )_2$
$e_R$	$(\underline{1}, \underline{1}, -1)_2$
$\mu_R \  au_R$	$\left( \begin{array}{ccc} \underline{1}, \ \underline{1}, \ -1 \end{array} \right)_2$ $\left( \begin{array}{ccc} \underline{1}, \ \underline{1}, \ -1 \end{array} \right)_2$
$p_{\mu} \ p'_{\mu}$	$\begin{array}{cccc} (\underline{1}, \ \underline{1}, \ 0 \ )_4 \\ (\underline{1}, \ \underline{1}, \ 0 \ )_4 \end{array}$
$G_{\mu}$	$(\underline{8}, \underline{1}, 0)_4$
$W_{\mu} \ B_{\mu}$	$( \underline{1}, \underline{3}, 0 )_4$ $( \underline{1}, \underline{1}, 0 )_4$

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$ig( { { { { { 3 } } } , { { { 2 } } , { { { 1 \over 6 } } } } } } ig)_2 \ ig( { { { 3 } , { { 2 } , { { 1 \over 6 } } } } ig)_2 \ ig( { { { 3 } , { 2 } , { { 1 \over 6 } } } ig)_2 \ ig( { { 3 , { 2 } , { 1 \over 6 } } ig)_2 \ ig)_2 \ ig)_2$
$( \  u_e, \ e \ )_L \ ( \  u_\mu, \ \mu \ )_L \ ( \  u_\tau, \  au \ )_L$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$egin{array}{c} u_R \ c_R \ t_R \end{array}$	$ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array} \right)_{2} \\ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array} \right)_{2} \\ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array} \right)_{2} \\ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array} \right)_{2} \end{array} $
$d_R \ s_R \ b_R$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$ u_{eR}$	$(\underline{1}, \underline{1}, 0)_2$
$ u_{\mu R}  u_{ au R}$	$\begin{array}{cccc}(\underline{1},\ \underline{1},\ 0\ )_{2}\\(\underline{1},\ \underline{1},\ 0\ )_{2}\end{array}$
$e_R$	$(\underline{1}, \underline{1}, -1)_2$
$\mu_R \  au_R$	$\left(\begin{array}{ccc} \underline{1}, \ \underline{1}, \ -1 \end{array}\right)_2$ $\left(\begin{array}{ccc} \underline{1}, \ \underline{1}, \ -1 \end{array}\right)_2$
$p_{\mu} \ p'_{\mu}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$egin{array}{c} G_\mu \ W_\mu \ B_\mu \end{array}$	$\begin{array}{c} (\underline{8},  \underline{1},  0 )_4 \\ (\underline{1},  \underline{3},  0 )_4 \\ (\underline{1},  \underline{1},  0 )_4 \end{array}$

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^*S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^*S \mathcal{H}_{16}(\mathbb{C}) s^*S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$ig( { { { { { 3 } } } , { { { 2 } } , { { { 1 \over 6 } } } } } } ig)_2 \ {ig( { { { 3 } , { { 2 } , { { 1 \over 6 } } } } ig)_2 \ {ig( { { 3 , { 2 } , { { 1 \over 6 } } } ig)_2 \ {ig( { { 3 , { 2 } , { 1 \over 6 } } ig)_2 \ {ig)_2 } } ig)_2 }$
$( \  u_e, \ e \ )_L \ ( \  u_\mu, \ \mu \ )_L \ ( \  u_ au, \  au \ )_L$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$egin{array}{c} u_R \ c_R \ t_R \end{array}$	$ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \end{array} $
$d_R \ s_R \ b_R$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$ u_{eR} $ $ u_{\mu R} $ $ u_{ au R}$	$\begin{array}{c} (\underline{1}, \underline{1}, 0)_{2} \\ (\underline{1}, \underline{1}, 0)_{2} \\ (\underline{1}, \underline{1}, 0)_{2} \end{array}$
$e_R \ \mu_R \  au_R$	$\begin{array}{c}(\underline{1}, \underline{1}, -1)_2\\(\underline{1}, \underline{1}, -1)_2\\(\underline{1}, \underline{1}, -1)_2\end{array}$
$p_{\mu} \ p'_{\mu} \ G_{\mu}$	$(\underline{1}, \underline{1}, 0)_4$ $(\underline{1}, \underline{1}, 0)_4$ $(\underline{8}, \underline{1}, 0)_4$
$W_{\mu} \ B_{\mu}$	$(\underline{1}, \underline{3}, 0)_4$ $(\underline{1}, \underline{1}, 0)_4$

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	s*S H₁6(ℂ) s*S too many here	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$ig( { { { { { 3 } } } , { { { 2 } } , { { { 1 \over 6 } } } } } } ig)_2 \ ig( { { { 3 } , { { 2 } , { { 1 \over 6 } } } } ig)_2 \ ig( { { { 3 } , { 2 } , { { 1 \over 6 } } } ig)_2 \ ig( { { 3 , { 2 } , { 1 \over 6 } } ig)_2 \ ig)_2 \ ig)_2$
$( \  u_e, \ e \ )_L \ ( \  u_\mu, \ \mu \ )_L \ ( \  u_\tau, \  au \ )_L$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$egin{array}{c} u_R \ c_R \ t_R \end{array}$	$ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array} \right)_{2} \\ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array} \right)_{2} \\ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array} \right)_{2} \\ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array} \right)_{2} \end{array} $
$d_R \ s_R \ b_R$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$ u_{eR}$	$(\underline{1}, \underline{1}, 0)_2$
$ u_{\mu R}  u_{ au R}$	$\begin{array}{cccc}(\underline{1},\ \underline{1},\ 0\ )_{2}\\(\underline{1},\ \underline{1},\ 0\ )_{2}\end{array}$
$e_R$	$(\underline{1}, \underline{1}, -1)_2$
$\mu_R \  au_R$	$\left(\begin{array}{ccc} \underline{1}, \ \underline{1}, \ -1 \end{array}\right)_2$ $\left(\begin{array}{ccc} \underline{1}, \ \underline{1}, \ -1 \end{array}\right)_2$
$p_{\mu} \ p'_{\mu}$	$( \underline{1}, \underline{1}, 0 )_4$ $( \underline{1}, \underline{1}, 0 )_4$
$egin{array}{c} G_\mu \ W_\mu \ B_\mu \end{array}$	$\begin{array}{c} (\underline{8},  \underline{1},  0 )_4 \\ (\underline{1},  \underline{3},  0 )_4 \\ (\underline{1},  \underline{1},  0 )_4 \end{array}$

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^*S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^*S \mathcal{H}_{16}(\mathbb{C}) s^*S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$\left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_2 \\ \left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_2 \\ \left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_2 \end{array}$
$( \  u_e, \ e \ )_L \ ( \  u_\mu, \ \mu \ )_L \ ( \  u_\tau, \  au \ )_L$	$ \begin{array}{c} \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_2 \\ \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_2 \\ \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_2 \end{array} \\ \end{array} $
$egin{array}{c} u_R \ c_R \ t_R \end{array}$	$ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \end{array} $
$d_R \ s_R \ b_R$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$ u_{eR} $ $ u_{\mu R} $ $ u_{ au R}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$e_R \ \mu_R \  au_R$	$( \underline{1}, \underline{1}, -1 )_2$ $( \underline{1}, \underline{1}, -1 )_2$ $( \underline{1}, \underline{1}, -1 )_2$
$p_{\mu} \ p_{\mu}^{\prime}$	$(\underline{1}, \underline{1}, 0)_4 \\ (\underline{1}, \underline{1}, 0)_4$
$egin{array}{c} G_\mu \ W_\mu \ B_\mu \end{array}$	$(\underline{8}, \underline{1}, 0)_4$ $(\underline{1}, \underline{3}, 0)_4$ $(\underline{1}, \underline{1}, 0)_4$

$232~\mathbb{R}$	$\mathcal{H}_{16}$	$_{\mathrm{S}}(\mathbb{C})$	
$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^*S \mathcal{H}_{16}(\mathbb{C}) sS^*$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$

 $\left( \underline{\mathbf{3}}, \ \underline{\mathbf{2}}, \ \frac{1}{6} \ \right)_2$  $(\ u,\ d\ )_L$  $(c, s)_L (\underline{3}, \underline{2}, \frac{1}{6})_2$  $(t, b)_L$  $\left( \underline{\mathbf{3}}, \ \underline{\mathbf{2}}, \ \frac{1}{6} \right)_2$  $\left( \ \underline{\mathbf{1}}, \ \underline{\mathbf{2}}, \ -\frac{1}{2} \ 
ight)_2$  $(\nu_e, e)_L$  $(\nu_{\mu}, \mu)_L \quad \left(\underline{1}, \underline{2}, -\frac{1}{2}\right)_2$  $(\nu_{\tau}, \tau)_L \quad \left(\underline{\mathbf{1}}, \underline{\mathbf{2}}, -\frac{1}{2}\right)_2$  $\left(\underline{\mathbf{3}}, \underline{\mathbf{1}}, \frac{2}{3}\right)_2$  $u_R$  $\left(\underline{\mathbf{3}}, \underline{\mathbf{1}}, \frac{2}{3}\right)_2$  $c_R$  $\left(\underline{\mathbf{3}}, \underline{\mathbf{1}}, \frac{2}{3}\right)_2$  $t_R$  $\left( \underline{\mathbf{3}}, \ \underline{\mathbf{1}}, \ -\frac{1}{3} \right)_2$  $d_R$  $\left(\underline{\mathbf{3}}, \underline{\mathbf{1}}, -\frac{1}{3}\right)_2$  $s_R$  $b_R$  $\left(\underline{\mathbf{3}}, \underline{\mathbf{1}}, -\frac{1}{3}\right)_2$  $(\underline{\mathbf{1}}, \underline{\mathbf{1}}, 0)_2$  $\nu_{eR}$  $(\underline{\mathbf{1}}, \underline{\mathbf{1}}, 0)_2$  $u_{\mu R}$  $(\underline{\mathbf{1}}, \underline{\mathbf{1}}, 0)_2$  $\nu_{\tau R}$  $(\underline{\mathbf{1}}, \underline{\mathbf{1}}, -1)_2$  $e_R$  $(\underline{\mathbf{1}}, \underline{\mathbf{1}}, -1)_2$  $\mu_R$  $(\underline{\mathbf{1}}, \underline{\mathbf{1}}, -1)_2$  $au_R$  $(\underline{\mathbf{1}}, \underline{\mathbf{1}}, 0)_4$  $p_{\mu}$  $p'_{\mu}$  $(\underline{\mathbf{1}}, \underline{\mathbf{1}}, 0)_4$  $G_{\mu}$  $(\underline{\mathbf{8}}, \underline{\mathbf{1}}, 0)_4$  $W_{\mu}$  $(\underline{\mathbf{1}}, \underline{\mathbf{3}}, 0)_4$  $B_{\mu}$  $(\underline{\mathbf{1}}, \underline{\mathbf{1}}, 0)_4$ 

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^*S \mathcal{H}_{16}(\mathbb{C}) s^*S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$ \left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_{2} \\ \left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_{2} \\ \left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_{2} \end{array} $
$( \  u_e, \ e \ )_L \ ( \  u_\mu, \ \mu \ )_L \ ( \  u_\tau, \  au \ )_L$	$ \begin{array}{c} \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_2 \\ \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_2 \\ \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_2 \end{array} \end{array} $
$egin{array}{l} u_R \ c_R \ t_R \end{array}$	$ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \end{array} $
$d_R \ s_R \ b_R$	$ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ -\frac{1}{3} \end{array} \right)_{2} \\ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ -\frac{1}{3} \end{array} \right)_{2} \\ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ -\frac{1}{3} \end{array} \right)_{2} \end{array} \right. $
$ u_{eR} $ $ u_{\mu R} $ $ u_{ au R}$	$(\underline{1}, \underline{1}, 0)_2$ $(\underline{1}, \underline{1}, 0)_2$ $(\underline{1}, \underline{1}, 0)_2$
$e_R \ \mu_R \  au_R$	$(\underline{1}, \underline{1}, -1)_2 (\underline{1}, \underline{1}, -1)_2 (\underline{1}, \underline{1}, -1)_2$
$p_{\mu} \ p'_{\mu}$	$(\underline{1}, \underline{1}, 0)_4 \\ (\underline{1}, \underline{1}, 0)_4$
$egin{array}{c} G_\mu \ W_\mu \ B_\mu \end{array}$	$(\underline{8}, \underline{1}, 0)_4$ $(\underline{1}, \underline{3}, 0)_4$ $(\underline{1}, \underline{1}, 0)_4$

### Covariant derivative

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^*S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$ig( { { { { { 3 } } } , { { { 2 } } , { { { 1 \over 6 } } } } } } ig)_2 \ {ig( { { { 3 } , { { 2 } , { { 1 \over 6 } } } } ig)_2 \ {ig( { { 3 , { 2 } , { 1 \over 6 } } } ig)_2 \ {ig( { { 3 , { 2 } , { 1 \over 6 } } } ig)_2 \ {ig)_2 \ ig)_2 \$
$( \  u_e, \ e \ )_L \ ( \  u_\mu, \ \mu \ )_L \ ( \  u_\tau, \  au \ )_L$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$egin{array}{c} u_R \ c_R \ t_R \end{array}$	$ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \end{array} $
$d_R \ s_R \ b_R$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$ u_{eR}$	$(\underline{1}, \underline{1}, 0)_2$
$ u_{\mu R}  u_{ au R}$	$\begin{array}{cccc}(\underline{1},\ \underline{1},\ 0\ )_{2}\\(\underline{1},\ \underline{1},\ 0\ )_{2}\end{array}$
$e_R$	$(\underline{1}, \underline{1}, -1)_2$
$\mu_R \  au_R$	$\left(\begin{array}{ccc} \underline{1}, \ \underline{1}, \ -1 \end{array}\right)_2$ $\left(\begin{array}{ccc} \underline{1}, \ \underline{1}, \ -1 \end{array}\right)_2$
$p_{\mu} \ p'_{\mu}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$egin{array}{c} G_\mu \ W_\mu \ B_\mu \end{array}$	$\begin{array}{c} (\underline{8},  \underline{1},  0 )_4 \\ (\underline{1},  \underline{3},  0 )_4 \\ (\underline{1},  \underline{1},  0 )_4 \end{array}$

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^*S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^*S \mathcal{H}_{16}(\mathbb{C}) s^*S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$ \left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_{2} \\ \left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_{2} \\ \left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_{2} \end{array} $
$( \  u_e, \ e \ )_L \ ( \  u_{\mu}, \ \mu \ )_L \ ( \  u_{\tau}, \  au \ )_L$	$ \begin{array}{c} \left( \underline{1}, \ \underline{2}, \ -\frac{1}{2} \ \right)_2 \\ \left( \ \underline{1}, \ \underline{2}, \ -\frac{1}{2} \right)_2 \\ \left( \ \underline{1}, \ \underline{2}, \ -\frac{1}{2} \right)_2 \end{array} $
$egin{array}{c} u_R \ c_R \ t_R \end{array}$	$ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \end{array} $
$d_R \ s_R \ b_R$	$ig( \ \underline{3}, \ \underline{1}, \ -\frac{1}{3} \ ig)_2 \ ig( \ \underline{3}, \ \underline{1}, \ -\frac{1}{3} \ ig)_2 \ ig( \ \underline{3}, \ \underline{1}, \ -\frac{1}{3} \ ig)_2 \ ig( \ \underline{3}, \ \underline{1}, \ -\frac{1}{3} \ ig)_2$
$ u_{eR} $ $ u_{\mu R} $ $ u_{ au R}$	$( \underline{1}, \underline{1}, 0 )_2$ $( \underline{1}, \underline{1}, 0 )_2$ $( \underline{1}, \underline{1}, 0 )_2$
$e_R \ \mu_R \  au_R$	$( \underline{1}, \underline{1}, -1 )_2$ $( \underline{1}, \underline{1}, -1 )_2$ $( \underline{1}, \underline{1}, -1 )_2$
$p_{\mu} \ p_{\mu}^{\prime}$	$( \underline{1}, \ \underline{1}, \ 0 )_4$ $( \underline{1}, \ \underline{1}, \ 0 )_4$
$egin{array}{c} G_\mu \ W_\mu \ B_\mu \end{array}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^*S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$

 $sS \mathcal{H}_{16}(\mathbb{C}) sS$ 

 $sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$ 

 $s^*S \mathcal{H}_{16}(\mathbb{C}) sS$ 

 $s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$ 

 $s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$ 

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2$
$( \  u_e, \ e \ )_L \ ( \  u_{\mu}, \ \mu \ )_L \ ( \  u_{\tau}, \  au \ )_L$	$ \begin{array}{c} \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_2 \\ \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_2 \\ \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_2 \end{array} \\ \end{array} $
$egin{array}{c} u_R \ c_R \ t_R \end{array}$	$ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \end{array} $
$d_R \ s_R \ b_R$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$ u_{eR} $ $ u_{\mu R} $ $ u_{ au R}$	$\begin{array}{c} (\underline{1}, \underline{1}, 0)_{2} \\ (\underline{1}, \underline{1}, 0)_{2} \\ (\underline{1}, \underline{1}, 0)_{2} \end{array}$
$e_R \ \mu_R \  au_R$	$(\underline{1}, \underline{1}, -1)_2$ $(\underline{1}, \underline{1}, -1)_2$ $(\underline{1}, \underline{1}, -1)_2$
$p_{\mu} \ p'_{\mu}$	$( \underline{1}, \underline{1}, 0 )_4$ $( \underline{1}, \underline{1}, 0 )_4$
$egin{array}{c} G_\mu \ W_\mu \ B_\mu \end{array}$	$( \underline{8}, \ \underline{1}, \ 0 \ )_4$ $( \ \underline{1}, \ \underline{3}, \ 0 \ )_4$ $( \ \underline{1}, \ \underline{1}, \ 0 \ )_4$

# $sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$ $sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$ $sS \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$ $sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$ $sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$ $sS^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$ $s^{\star}S \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$ $s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$ $s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$

 $s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$ 

### Fermions

 $s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$ 

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$ig( { { { { { 3 } } } , { { { 2 } } , { { { 1 \over 6 } } } } } } ig)_2 \ ig( { { { 3 } , { { 2 } , { { 1 \over 6 } } } } ig)_2 \ ig( { { { 3 } , { 2 } , { { 1 \over 6 } } } ig)_2 \ ig( { { 3 , { 2 } , { 1 \over 6 } } ig)_2 \ ig)_2 \ ig)_2$
$( \  u_e, \ e \ )_L \ ( \  u_\mu, \ \mu \ )_L \ ( \  u_\tau, \  au \ )_L$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$egin{array}{c} u_R \ c_R \ t_R \end{array}$	$ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array} \right)_{2} \\ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array} \right)_{2} \\ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array} \right)_{2} \\ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array} \right)_{2} \end{array} $
$d_R \ s_R \ b_R$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$ u_{eR}$	$(\underline{1}, \underline{1}, 0)_2$
$ u_{\mu R}  u_{ au R}$	$\begin{array}{cccc}(\underline{1},\ \underline{1},\ 0\ )_{2}\\(\underline{1},\ \underline{1},\ 0\ )_{2}\end{array}$
$e_R$	$(\underline{1}, \underline{1}, -1)_2$
$\mu_R \  au_R$	$\left(\begin{array}{ccc} \underline{1}, \ \underline{1}, \ -1 \end{array}\right)_2$ $\left(\begin{array}{ccc} \underline{1}, \ \underline{1}, \ -1 \end{array}\right)_2$
$p_{\mu} \ p'_{\mu}$	$( \underline{1}, \underline{1}, 0 )_4$ $( \underline{1}, \underline{1}, 0 )_4$
$egin{array}{c} G_\mu \ W_\mu \ B_\mu \end{array}$	$\begin{array}{c} (\underline{8},  \underline{1},  0 )_4 \\ (\underline{1},  \underline{3},  0 )_4 \\ (\underline{1},  \underline{1},  0 )_4 \end{array}$

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^*S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^*S \mathcal{H}_{16}(\mathbb{C}) s^*S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$\left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_2 \\ \left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_2 \\ \left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_2 \end{array}$	
$( \  u_e, \ e \ )_L \ ( \  u_\mu, \ \mu \ )_L \ ( \  u_ au, \  au \ )_L$	$ \begin{pmatrix} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{pmatrix}_2 \\ \left( \ \underline{1}, \ \underline{2}, \ -\frac{1}{2} \right)_2 \\ \left( \ \underline{1}, \ \underline{2}, \ -\frac{1}{2} \right)_2 $	
$egin{array}{c} u_R \ c_R \ t_R \end{array}$	$ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\  \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\  \left(\begin{array}{cccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \end{array} $	
$d_R$ $s_R$ $b_R$	$egin{array}{llllllllllllllllllllllllllllllllllll$	
$egin{array}{c}  u_{eR}  u_{\mu R}  u_{ au R}  u_{ au R}  \end{array}$	$\left(\underline{1}, \underline{1}, 0\right)_2$	++
$e_R$ $\mu_R$ $ au_R$	$\begin{array}{c}(\underline{1}, \underline{1}, -1)_2\\(\underline{1}, \underline{1}, -1)_2\\(\underline{1}, \underline{1}, -1)_2\end{array}$	++
$p_{\mu} \ p'_{\mu}$	$\begin{array}{cccc} (\underline{1}, \underline{1}, 0)_4 \\ (\underline{1}, \underline{1}, 0)_4 \end{array}$	
$W_{\mu}$	$(\underline{8}, \underline{1}, 0)_4$ $(\underline{1}, \underline{3}, 0)_4$ $(\underline{1}, \underline{1}, 0)_4$	

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^*S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^*S \mathcal{H}_{16}(\mathbb{C}) s^*S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$ig( { { { { { 3 } } } , { { { 2 } } , { { { 1 \over 6 } } } } } } ig)_2 \ {ig( { { { 3 } , { { 2 } , { { 1 \over 6 } } } } ig)_2 \ {ig( { { 3 , { 2 } , { { 1 \over 6 } } } ig)_2 \ {ig( { { 3 , { 2 } , { 1 \over 6 } } ig)_2 \ {ig)_2 } } ig)_2 }$
$( \  u_e, \ e \ )_L \ ( \  u_\mu, \ \mu \ )_L \ ( \  u_ au, \  au \ )_L$	$ \begin{array}{c} \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_2 \\ \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_2 \\ \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_2 \end{array} \end{array} $
$egin{array}{c} u_R \ c_R \ t_R \end{array}$	$ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \end{array} $
$d_R \ s_R \ b_R$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$ u_{eR} $ $ u_{\mu R} $ $ u_{ au R}$	$\begin{array}{c} (\underline{1}, \underline{1}, 0)_{2} \\ (\underline{1}, \underline{1}, 0)_{2} \\ (\underline{1}, \underline{1}, 0)_{2} \end{array}$
$e_R \ \mu_R \  au_R$	$\begin{array}{c}(\underline{1}, \underline{1}, -1)_2\\(\underline{1}, \underline{1}, -1)_2\\(\underline{1}, \underline{1}, -1)_2\end{array}$
$p_{\mu} \ p'_{\mu} \ G_{\mu}$	$(\underline{1}, \underline{1}, 0)_4$ $(\underline{1}, \underline{1}, 0)_4$ $(\underline{8}, \underline{1}, 0)_4$
$W_{\mu} \ B_{\mu}$	$(\underline{1}, \underline{3}, 0)_4$ $(\underline{1}, \underline{1}, 0)_4$

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	s*S H₁6(ℂ) s*S too many here	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$

### End of the line?

End of the line? Maybe, maybe not.

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$ig( { { { { { 3 } } } , { { { 2 } } , { { { 1 \over 6 } } } } } } ig)_2 \ {ig( { { { 3 } , { { 2 } , { { 1 \over 6 } } } } ig)_2 \ {ig( { { 3 , { 2 } , { 1 \over 6 } } } ig)_2 \ {ig( { { 3 , { 2 } , { 1 \over 6 } } } ig)_2 \ {ig)_2 \ ig)_2 \$
$( \  u_e, \ e \ )_L \ ( \  u_\mu, \ \mu \ )_L \ ( \  u_\tau, \  au \ )_L$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$egin{array}{c} u_R \ c_R \ t_R \end{array}$	$ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \end{array} $
$d_R \ s_R \ b_R$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$ u_{eR}$	$(\underline{1}, \underline{1}, 0)_2$
$ u_{\mu R}  u_{ au R}$	$\begin{array}{cccc}(\underline{1},\ \underline{1},\ 0\ )_{2}\\(\underline{1},\ \underline{1},\ 0\ )_{2}\end{array}$
$e_R$	$(\underline{1}, \underline{1}, -1)_2$
$\mu_R \  au_R$	$\left(\begin{array}{ccc} \underline{1}, \ \underline{1}, \ -1 \end{array}\right)_2$ $\left(\begin{array}{ccc} \underline{1}, \ \underline{1}, \ -1 \end{array}\right)_2$
$p_{\mu} \ p'_{\mu}$	$( \underline{1}, \underline{1}, 0 )_4$ $( \underline{1}, \underline{1}, 0 )_4$
$egin{array}{c} G_\mu \ W_\mu \ B_\mu \end{array}$	$\begin{array}{c} (\underline{8},  \underline{1},  0 )_4 \\ (\underline{1},  \underline{3},  0 )_4 \\ (\underline{1},  \underline{1},  0 )_4 \end{array}$

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^*S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^*S \mathcal{H}_{16}(\mathbb{C}) s^*S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2$
$( \  u_e, \ e \ )_L \ ( \  u_\mu, \ \mu \ )_L \ ( \  u_ au, \  au \ )_L$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$egin{array}{c} u_R \ c_R \ t_R \ d_R \ s_R \ b_R \end{array}$	$ \begin{pmatrix} \underline{3}, \underline{1}, \frac{2}{3} \end{pmatrix}_{2} \\ (\underline{3}, \underline{1}, \frac{2}{3} )_{2} \\ (\underline{3}, \underline{1}, \frac{2}{3} )_{2} \\ (\underline{3}, \underline{1}, -\frac{1}{3} )_{2} \end{cases} $
$egin{aligned}  u_{eR} \  u_{\mu R} \  u_{ au R} \  u_{ a$	$(\underline{1}, \underline{1}, 0)_{2}$ $(\underline{1}, \underline{1}, 0)_{2}$ $(\underline{1}, \underline{1}, 0)_{2}$ $(\underline{1}, \underline{1}, 0)_{2}$ $(\underline{1}, \underline{1}, -1)_{2}$ $(\underline{1}, \underline{1}, -1)_{2}$ $(\underline{1}, \underline{1}, -1)_{2}$
$p_\mu \ p'_\mu \ G_\mu \ W_\mu \ B_\mu$	$(\underline{1}, \underline{1}, 0)_{4}$ $(\underline{1}, \underline{1}, 0)_{4}$ $(\underline{8}, \underline{1}, 0)_{4}$ $(\underline{1}, \underline{3}, 0)_{4}$ $(\underline{1}, \underline{1}, 0)_{4}$

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^*S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S \; \mathcal{H}_{16}(\mathbb{C}) \; s^{\star}S$	$s^*S \mathcal{H}_{16}(\mathbb{C}) s^*S^*$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$ig( { { { { { 3 } } } , { { { 2 } } , { { { 1 \over 6 } } } } } } ig)_2 \ {ig( { { { 3 } , { 2 } , { { 1 \over 6 } } } ig)_2 \ {ig( { { 3 } , { 2 } , { { 1 \over 6 } } } ig)_2 \ {ig( { { 3 } , { 2 } , { { 1 \over 6 } } } ig)_2 \ {ig)_2 \ {i$
$( \  u_e, \ e \ )_L \ ( \  u_\mu, \ \mu \ )_L \ ( \  u_\tau, \  au \ )_L$	$ \begin{pmatrix} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{pmatrix}_{2} \\ ( \ \underline{1}, \ \underline{2}, \ -\frac{1}{2} )_{2} \\ ( \ \underline{1}, \ \underline{2}, \ -\frac{1}{2} )_{2} \end{cases} $
$egin{array}{c} u_R \ c_R \ t_R \ d_R \ s_R \ b_R \end{array}$	$ \begin{array}{c} \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array} \right)_{2} \\ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array} \right)_{2} \\ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array} \right)_{2} \\ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ -\frac{1}{3} \end{array} \right)_{2} \\ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ -\frac{1}{3} \end{array} \right)_{2} \\ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ -\frac{1}{3} \end{array} \right)_{2} \\ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ -\frac{1}{3} \end{array} \right)_{2} \end{array} $
$egin{array}{c}  u_{eR} \  u_{\mu R} \  u_{ au R} \  u_{ $	$(\underline{1}, \underline{1}, 0)_{2}$ $(\underline{1}, \underline{1}, 0)_{2}$ $(\underline{1}, \underline{1}, 0)_{2}$ $(\underline{1}, \underline{1}, 0)_{2}$ $(\underline{1}, \underline{1}, -1)_{2}$ $(\underline{1}, \underline{1}, -1)_{2}$ $(\underline{1}, \underline{1}, -1)_{2}$
$p_{\mu} \ p_{\mu}^{\prime} \ G_{\mu}$	$(\underline{1}, \underline{1}, 0)_4$ $(\underline{1}, \underline{1}, 0)_4$ $(\underline{8}, \underline{1}, 0)_4$
$W_{\mu} \ B_{\mu}$	$\left(\begin{array}{ccc} \underline{1}, \ \underline{3}, \ 0 \end{array}\right)_4 \\ \left(\begin{array}{ccc} \underline{1}, \ \underline{1}, \ 0 \end{array}\right)_4 \end{array}$

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$s^{\star}S \; \mathcal{H}_{16}(\mathbb{C}) \; s^{\star}S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2$
$( \  u_e, \ e \ )_L \ ( \  u_\mu, \ \mu \ )_L \ ( \  u_ au, \  au \ )_L$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$egin{array}{c} u_R \ c_R \ t_R \ d_R \ s_R \ b_R \end{array}$	$ \begin{pmatrix} \underline{3}, \underline{1}, \frac{2}{3} \end{pmatrix}_{2} \\ (\underline{3}, \underline{1}, \frac{2}{3} )_{2} \\ (\underline{3}, \underline{1}, \frac{2}{3} )_{2} \\ (\underline{3}, \underline{1}, -\frac{1}{3} )_{2} \end{cases} $
$egin{aligned}  u_{eR} \  u_{\mu R} \  u_{ au R} \  u_{ a$	$(\underline{1}, \underline{1}, 0)_{2}$ $(\underline{1}, \underline{1}, 0)_{2}$ $(\underline{1}, \underline{1}, 0)_{2}$ $(\underline{1}, \underline{1}, 0)_{2}$ $(\underline{1}, \underline{1}, -1)_{2}$ $(\underline{1}, \underline{1}, -1)_{2}$ $(\underline{1}, \underline{1}, -1)_{2}$
$p_\mu \ p'_\mu \ G_\mu \ W_\mu \ B_\mu$	$(\underline{1}, \underline{1}, 0)_{4} \\ (\underline{1}, \underline{1}, 0)_{4} \\ (\underline{8}, \underline{1}, 0)_{4} \\ (\underline{1}, \underline{3}, 0)_{4} \\ (\underline{1}, \underline{1}, 0)_{4} \\ (\underline{1}, \underline{1}, 0)_{4} $

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^*S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S \; \mathcal{H}_{16}(\mathbb{C}) \; s^{\star}S$	$s^*S \mathcal{H}_{16}(\mathbb{C}) s^*S^*$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$ig( { { { { { 3 } } } , { { { 2 } } , { { { 1 \over 6 } } } } } } ig)_2 \ {ig( { { { 3 } , { { 2 } , { { 1 \over 6 } } } } ig)_2 \ {ig( { { { 3 } , { 2 } , { { 1 \over 6 } } } ig)_2 \ {ig( { { 3 } , { 2 } , { { 1 \over 6 } } ig)_2 \ {ig)_2 } } ig)_2 }$
$( \  u_e, \ e \ )_L \ ( \  u_\mu, \ \mu \ )_L \ ( \  u_ au, \  au \ )_L$	$ \begin{pmatrix} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{pmatrix}_{2} \\ ( \ \underline{1}, \ \underline{2}, \ -\frac{1}{2} )_{2} \\ ( \ \underline{1}, \ \underline{2}, \ -\frac{1}{2} )_{2} \end{cases} $
$egin{array}{c} u_R \ c_R \ t_R \end{array}$	$ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \end{array} $
$d_R \ s_R \ b_R$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$ u_{eR}$	$(\underline{1}, \underline{1}, 0)_2$
$ u_{\mu R} $ $ u_{ au R}$	$( \underline{1}, \underline{1}, 0 )_2$ $( \underline{1}, \underline{1}, 0 )_2$
$e_R$	$(\underline{1}, \underline{1}, -1)_2$
$\mu_R \  au_R$	$\begin{array}{c} (\underline{1}, \underline{1}, -1)_2 \\ (\underline{1}, \underline{1}, -1)_2 \end{array}$
$p_\mu \ p'_\mu$	$( \underline{1}, \underline{1}, 0 )_4$ $( \underline{1}, \underline{1}, 0 )_4$
$egin{array}{c} G_\mu \ W_\mu \ B_\mu \end{array}$	$\begin{array}{c}(\underline{8},\underline{1},0)_{4}\\(\underline{1},\underline{3},0)_{4}\\(\underline{1},\underline{1},0)_{4}\end{array}$

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S \; \mathcal{H}_{16}(\mathbb{C}) \; s^{\star} \; S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2$
$( \  u_e, \ e \ )_L \ ( \  u_\mu, \ \mu \ )_L \ ( \  u_ au, \  au \ )_L$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$egin{array}{c} u_R \ c_R \ t_R \ d_R \ s_R \ b_R \end{array}$	$ \begin{pmatrix} \underline{3}, \underline{1}, \frac{2}{3} \end{pmatrix}_{2} \\ (\underline{3}, \underline{1}, \frac{2}{3} )_{2} \\ (\underline{3}, \underline{1}, \frac{2}{3} )_{2} \\ (\underline{3}, \underline{1}, -\frac{1}{3} )_{2} \end{cases} $
$egin{aligned}  u_{eR} \  u_{\mu R} \  u_{ au R} \  u_{ a$	$(\underline{1}, \underline{1}, 0)_{2}$ $(\underline{1}, \underline{1}, 0)_{2}$ $(\underline{1}, \underline{1}, 0)_{2}$ $(\underline{1}, \underline{1}, 0)_{2}$ $(\underline{1}, \underline{1}, -1)_{2}$ $(\underline{1}, \underline{1}, -1)_{2}$ $(\underline{1}, \underline{1}, -1)_{2}$
$p_\mu \ p'_\mu \ G_\mu \ W_\mu \ B_\mu$	$(\underline{1}, \underline{1}, 0)_{4} \\ (\underline{1}, \underline{1}, 0)_{4} \\ (\underline{8}, \underline{1}, 0)_{4} \\ (\underline{1}, \underline{3}, 0)_{4} \\ (\underline{1}, \underline{1}, 0)_{4} \\ (\underline{1}, \underline{1}, 0)_{4} $

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^*S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S \; \mathcal{H}_{16}(\mathbb{C}) \; s^{\star}S$	$s^*S \mathcal{H}_{16}(\mathbb{C}) s^*S^*$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$

To be continued ...

### Colour and projective measurements

### $\mathbb O$ multiplication algebra

### Idempotents

$$s \coloneqq \frac{1}{2} (1 + iL_{e_7}) \qquad S \coloneqq \frac{1}{2} (1 + iR_{e_7})$$
$$s^* \coloneqq \frac{1}{2} (1 - iL_{e_7}) \qquad S^* \coloneqq \frac{1}{2} (1 - iR_{e_7})$$

Octonion Left multiplication

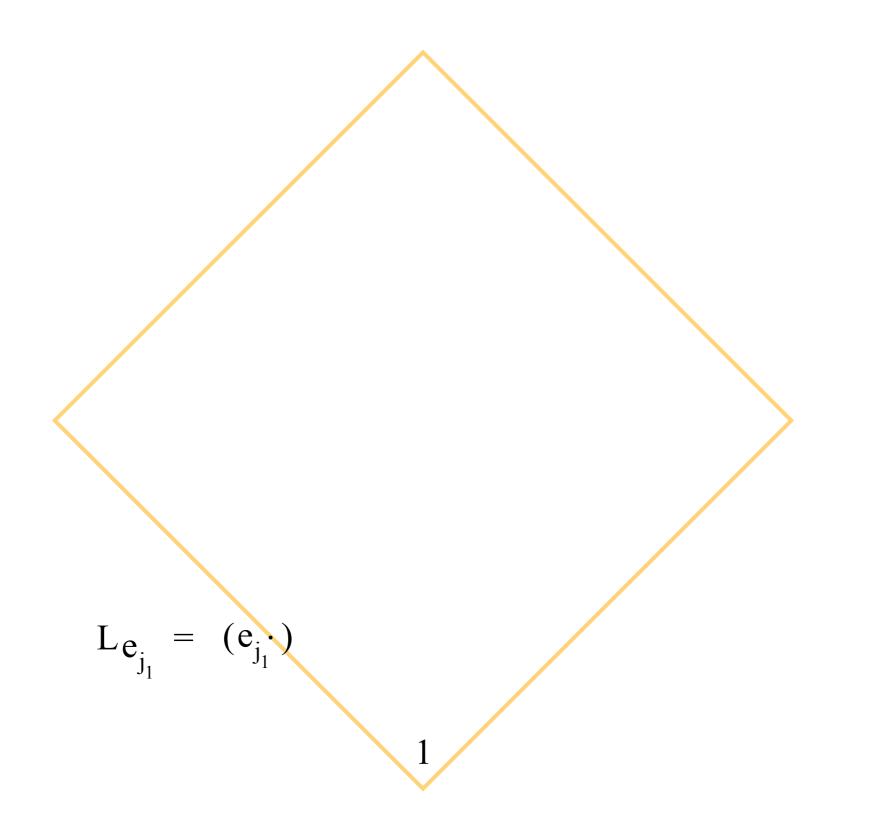






 $(e_{j_1})$ 1







 $(e_{j_1})$ 1

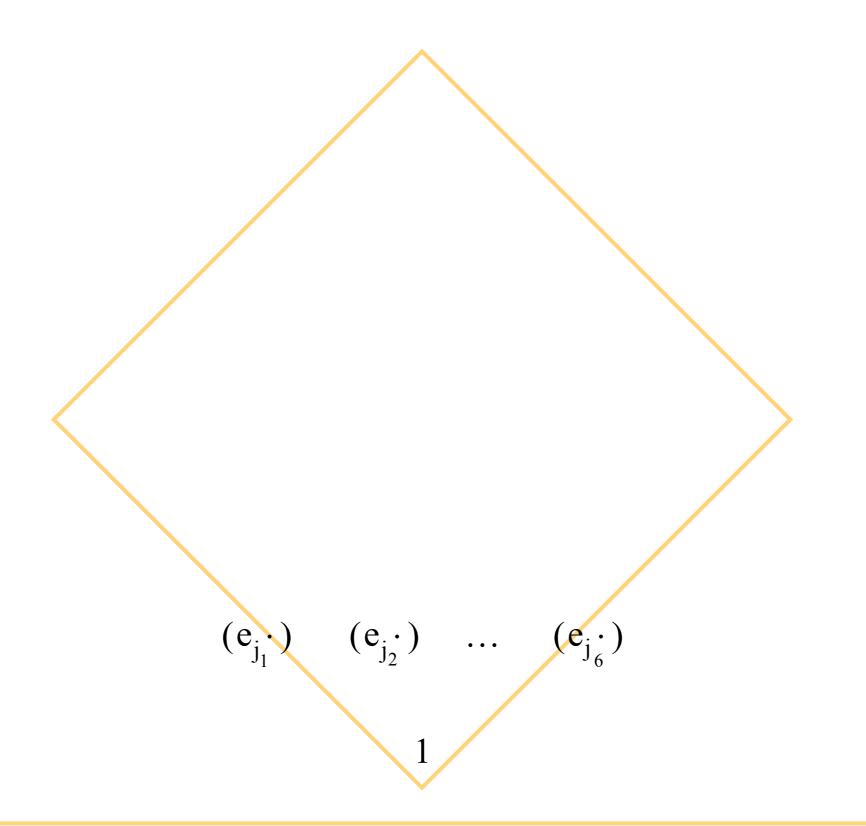


 $(e_{j_1})$  $(e_{j_2} \cdot)$ 1

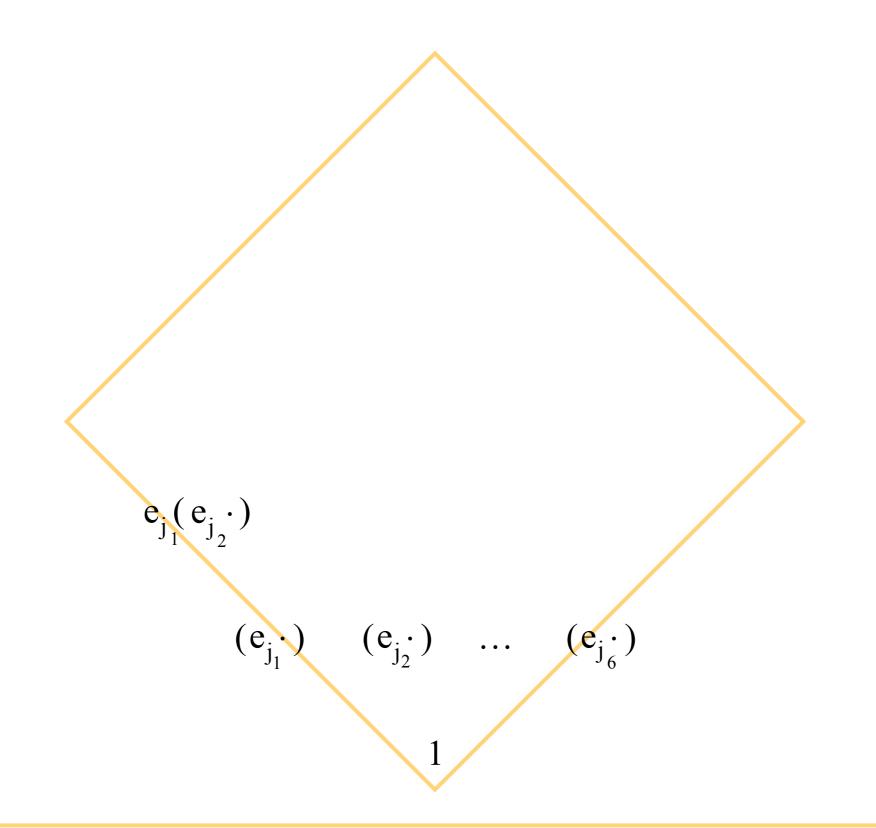


 $(e_{j_1})$  $(e_{j_2})$ ••• 1

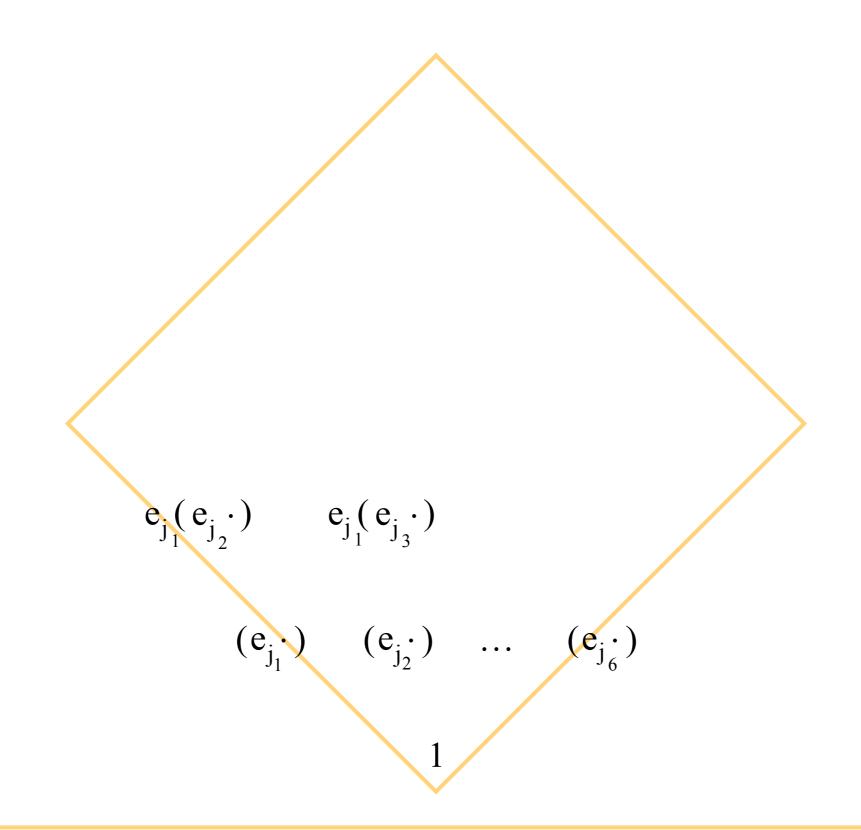




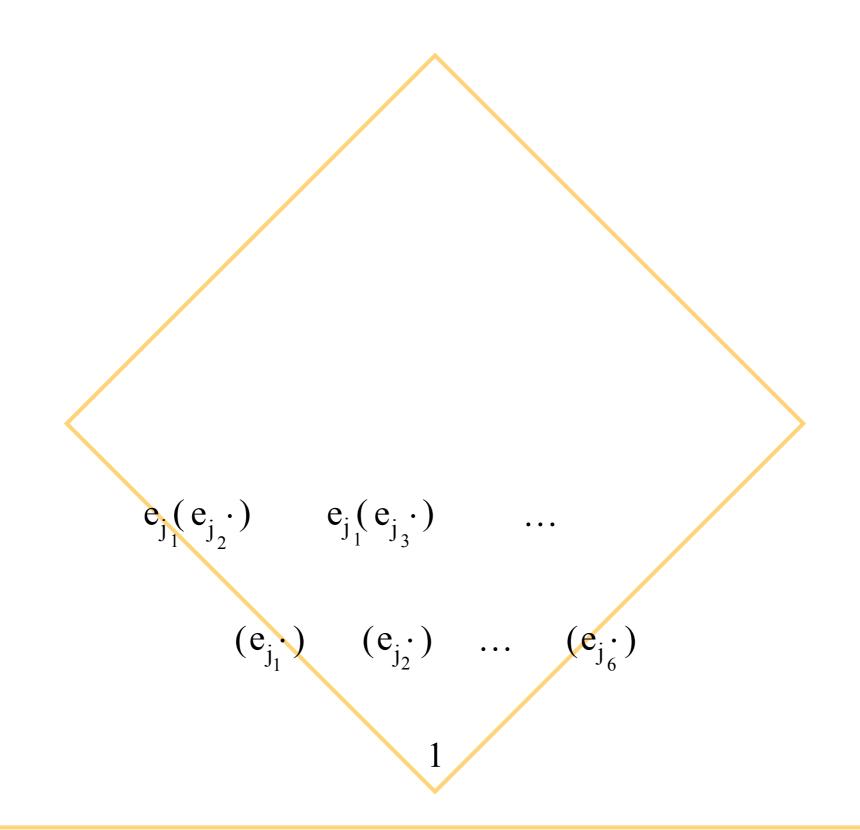




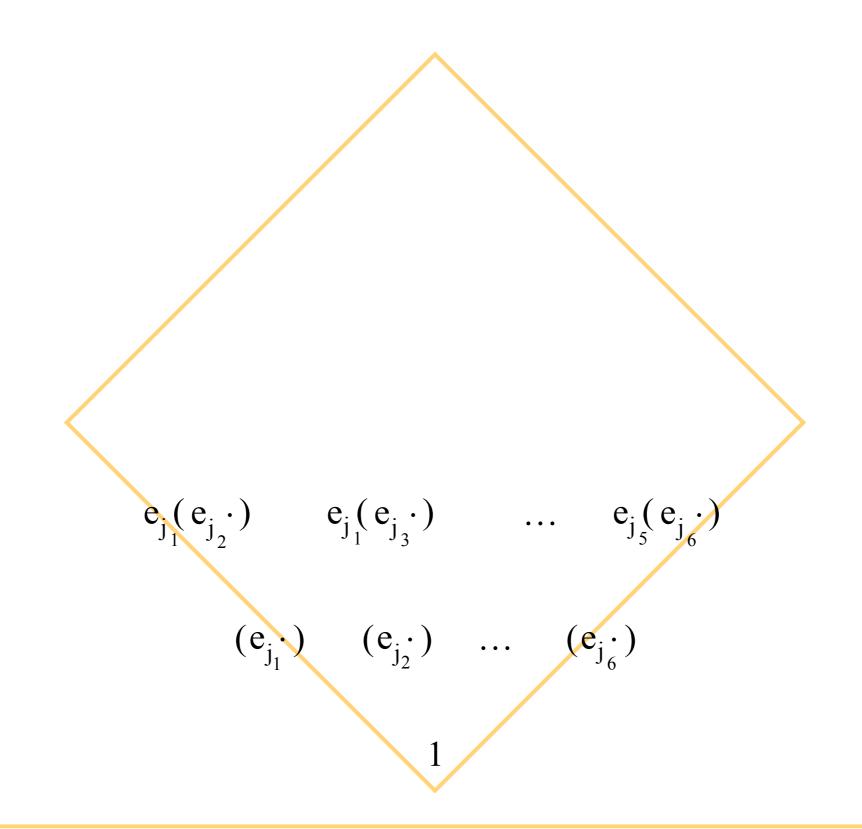




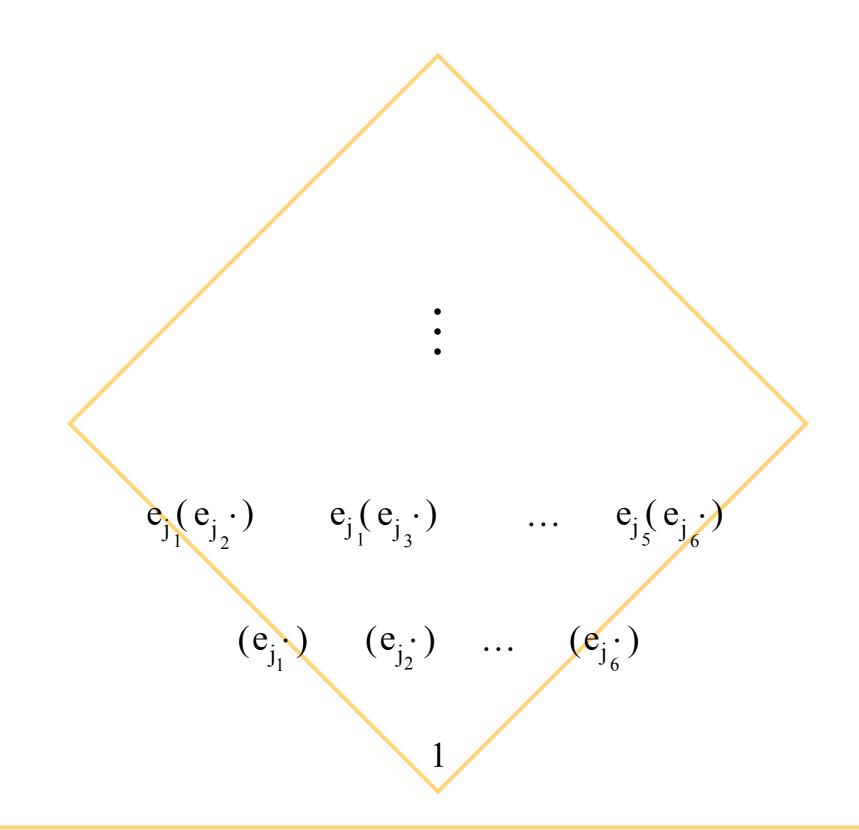




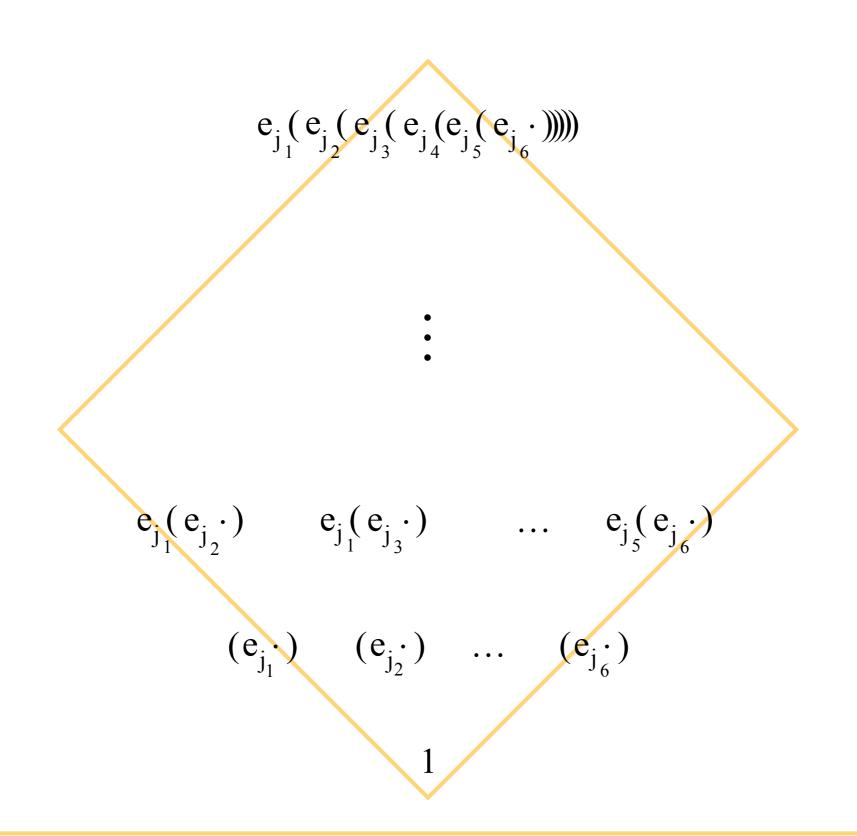




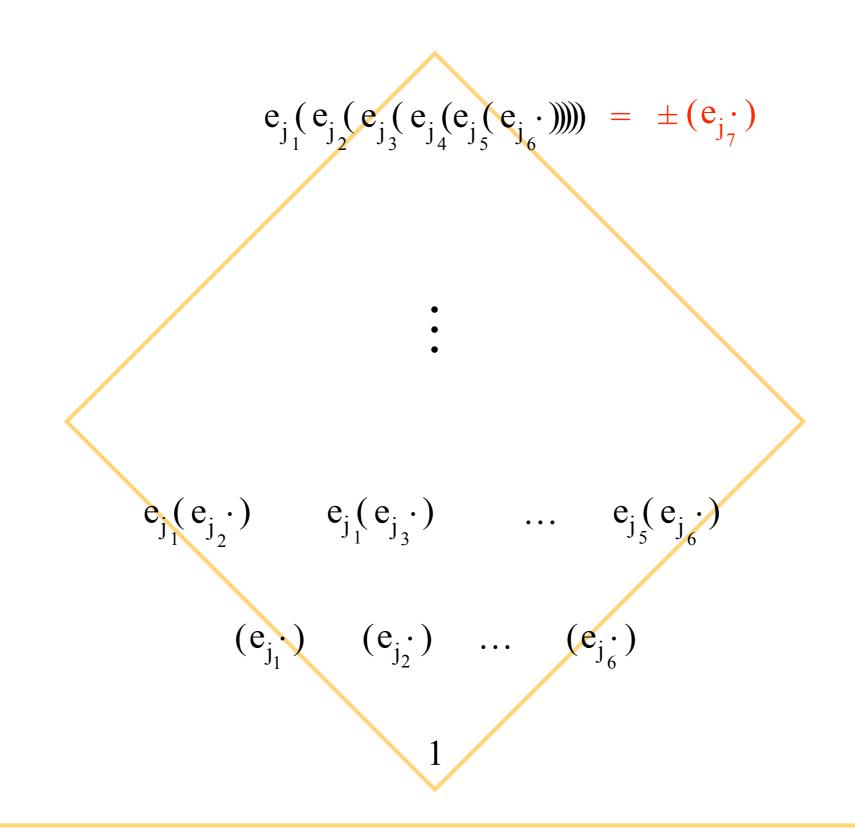


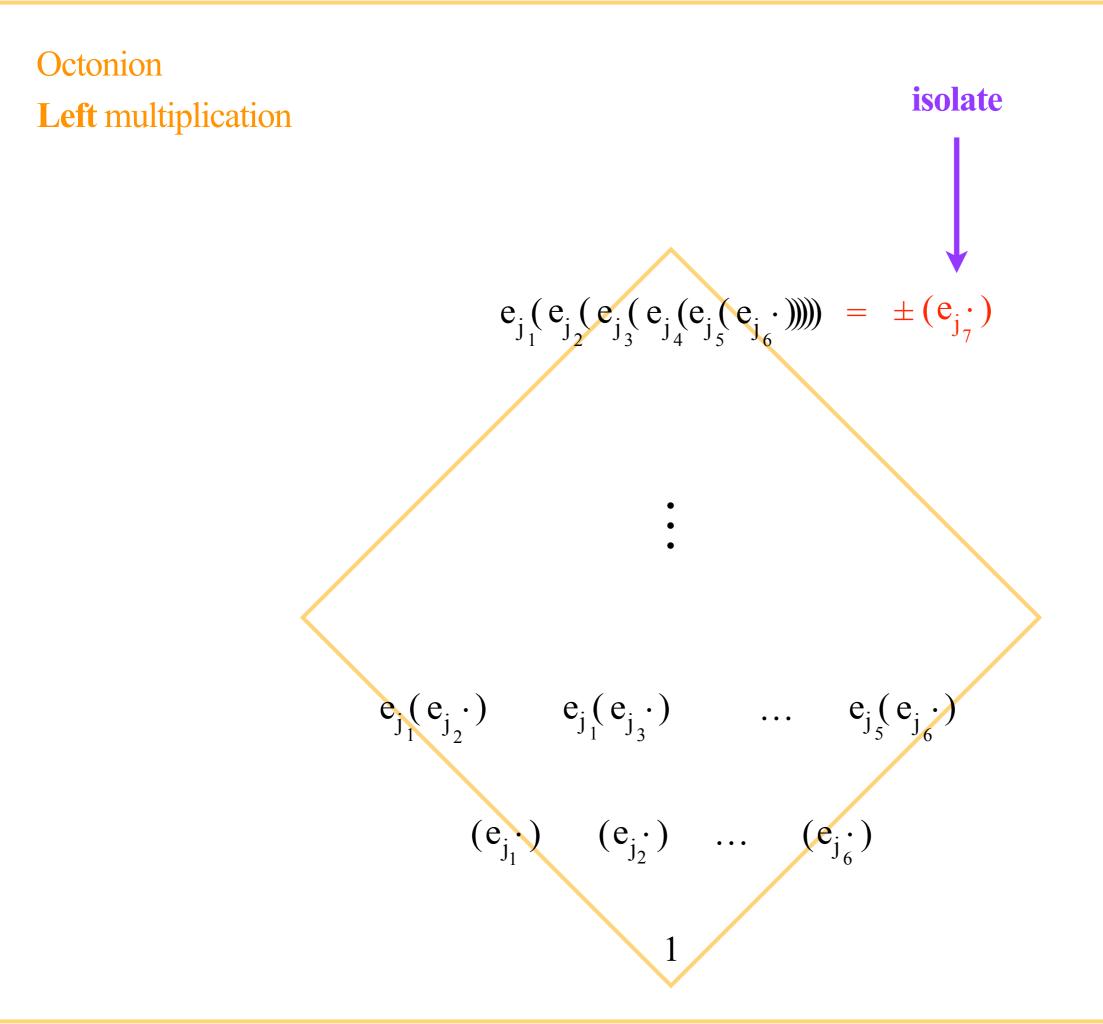


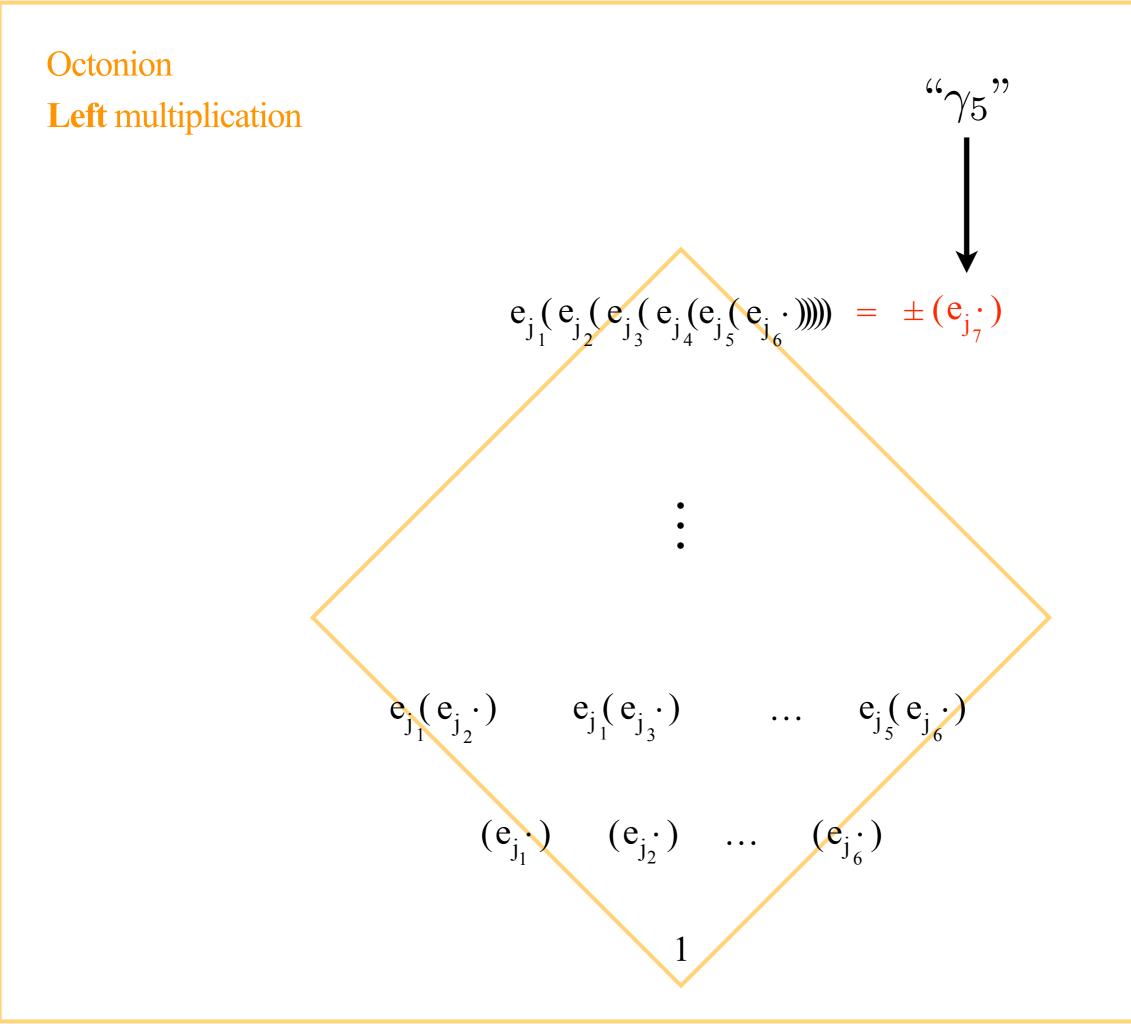












#### $\mathbb O$ multiplication algebra

#### Idempotents

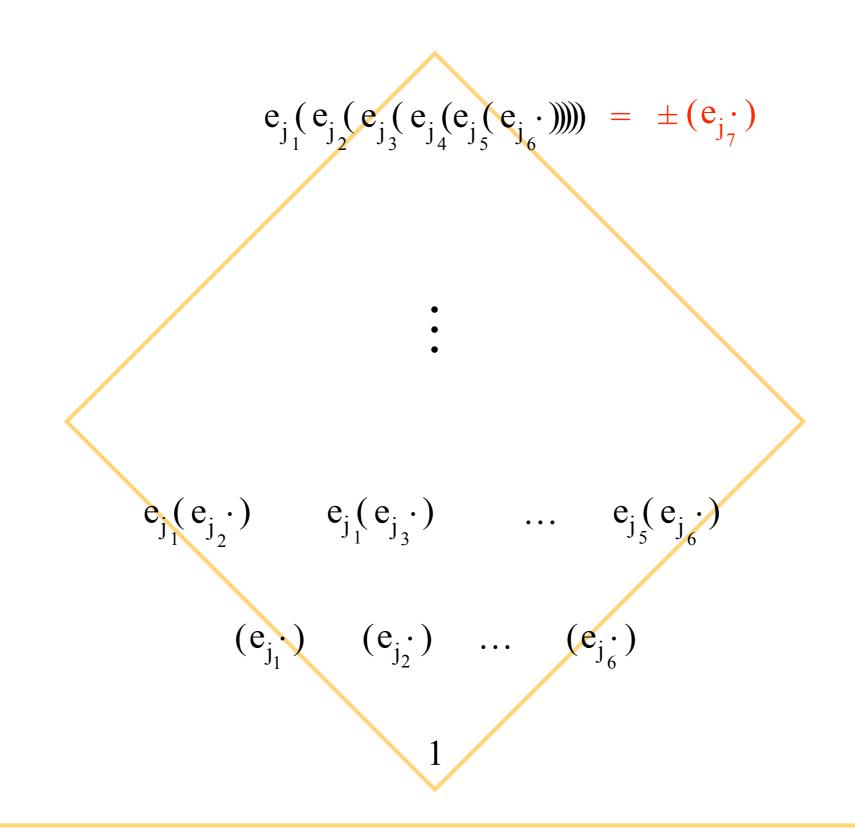
$$s \coloneqq \frac{1}{2} (1 + iL_{e_7}) \qquad S \coloneqq \frac{1}{2} (1 + iR_{e_7})$$
$$s^* \coloneqq \frac{1}{2} (1 - iL_{e_7}) \qquad S^* \coloneqq \frac{1}{2} (1 - iR_{e_7})$$

#### $\mathbb O$ multiplication algebra

#### Idempotents

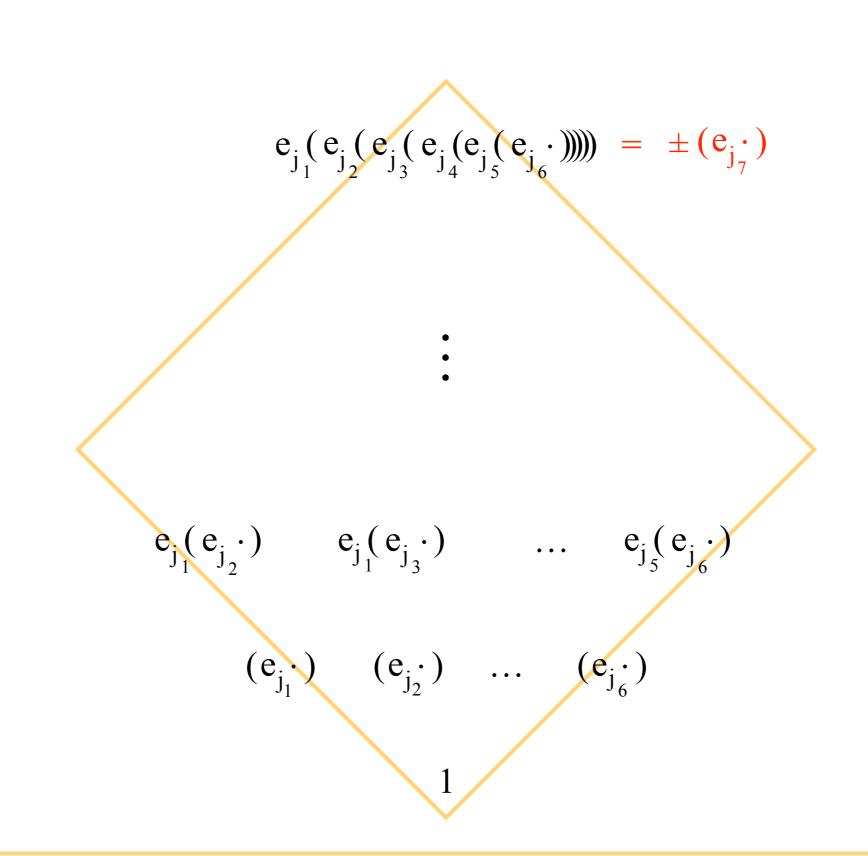
$$s \coloneqq \frac{1}{2}(1 + iL_{e_7}) \qquad S \coloneqq \frac{1}{2}(1 + iR_{e_7})$$
$$s^* \coloneqq \frac{1}{2}(1 - iL_{e_7}) \qquad S^* \coloneqq \frac{1}{2}(1 - iR_{e_7})$$





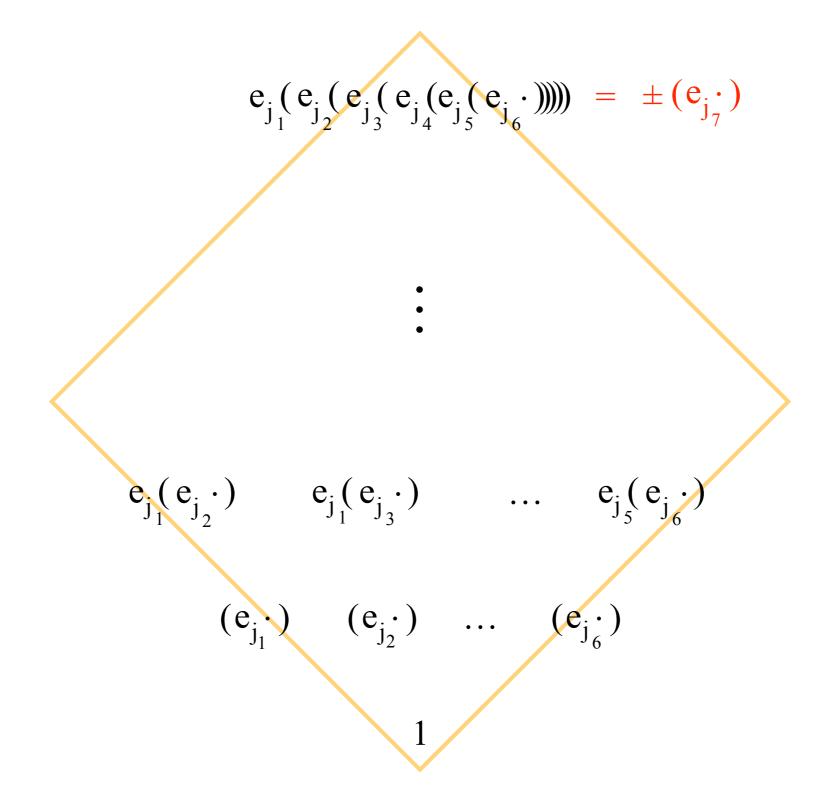
Left multiplication





Left multiplication

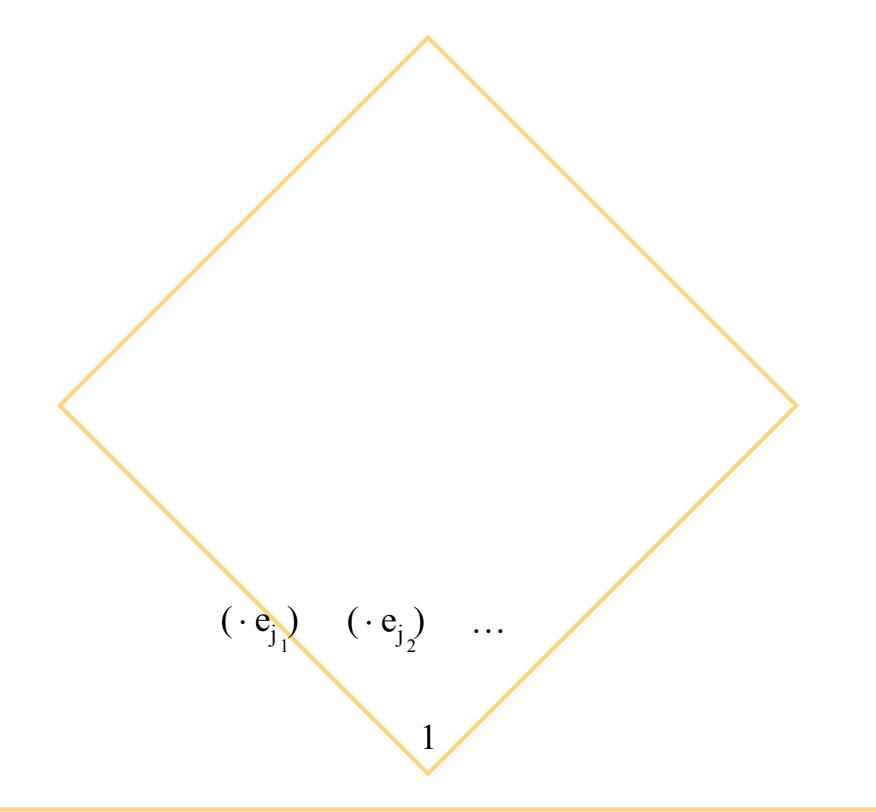


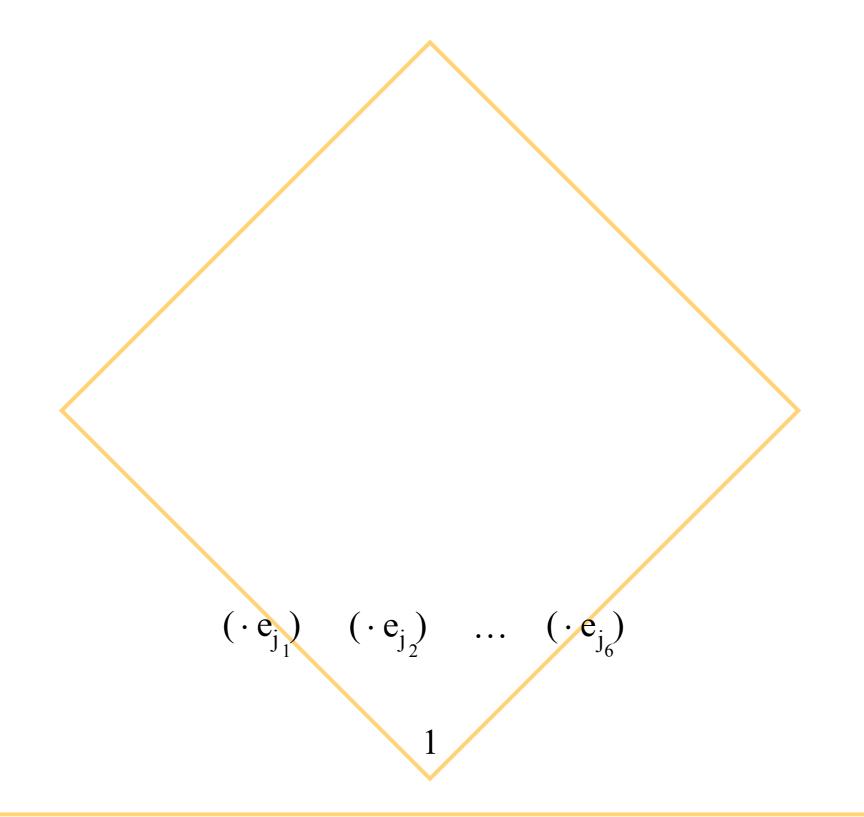


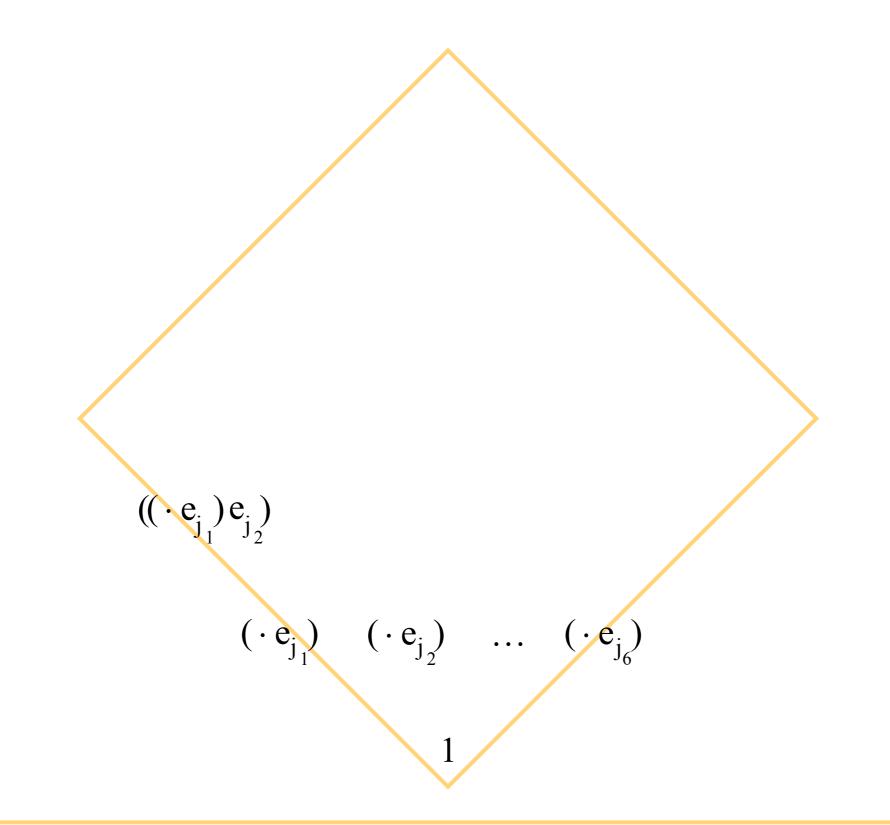
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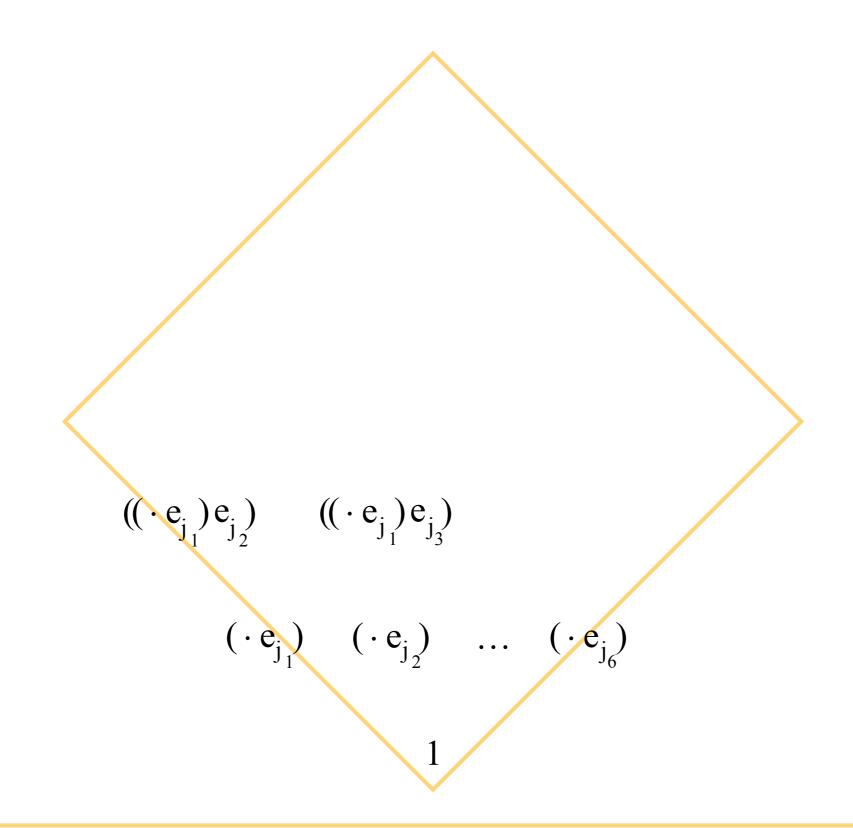
 $(\cdot e_{j_1})$ 1

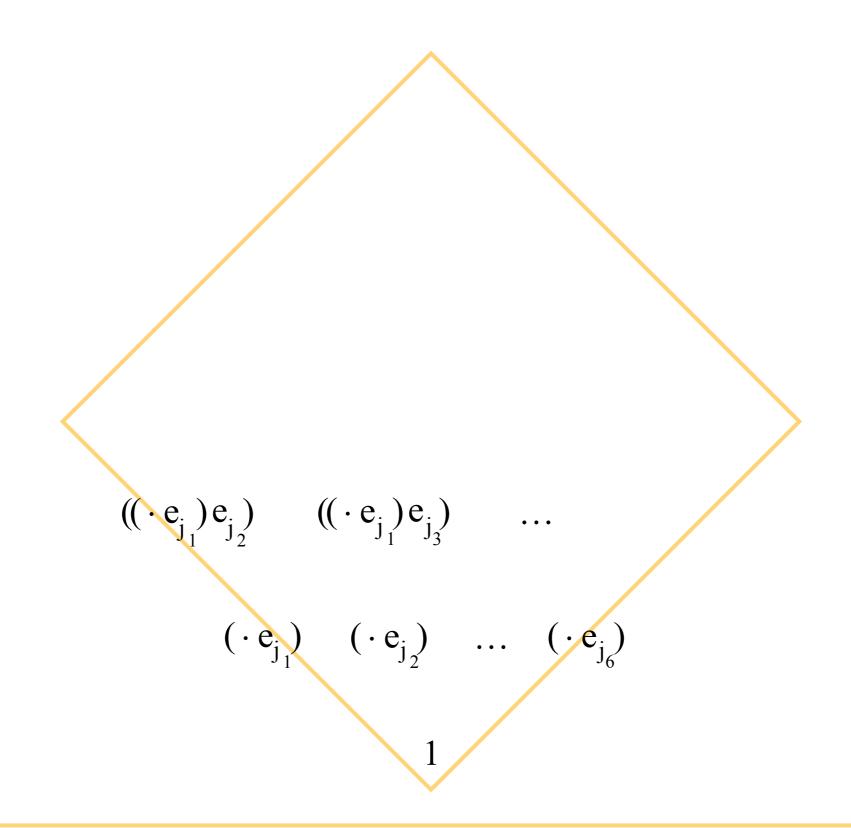
 $(\cdot e_{j_1})$  $(\cdot e_{j_2})$ 1

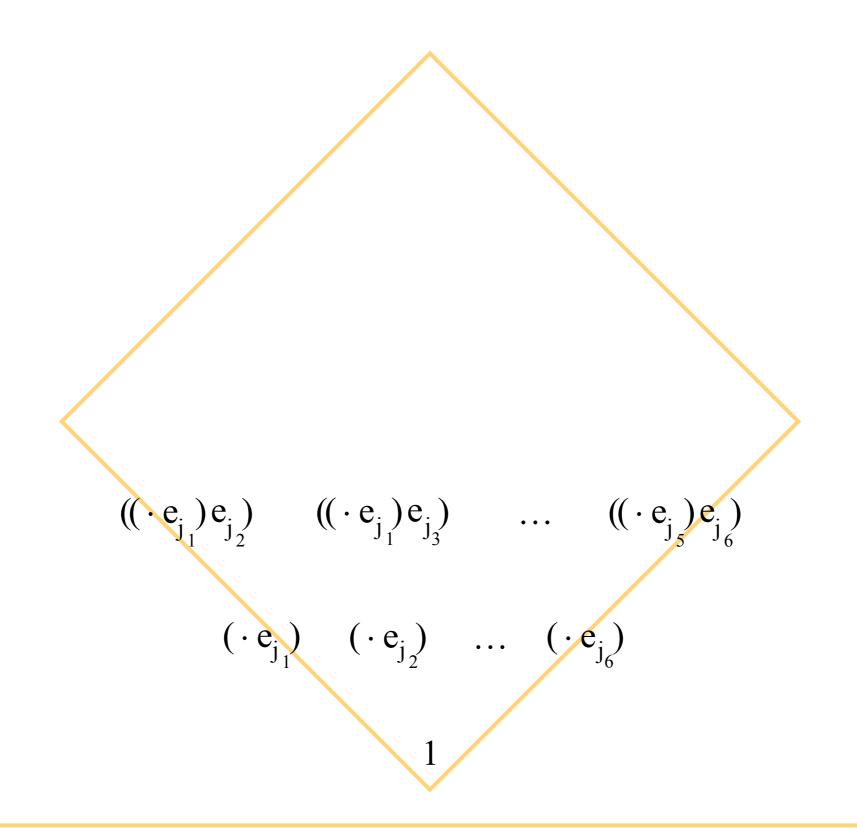


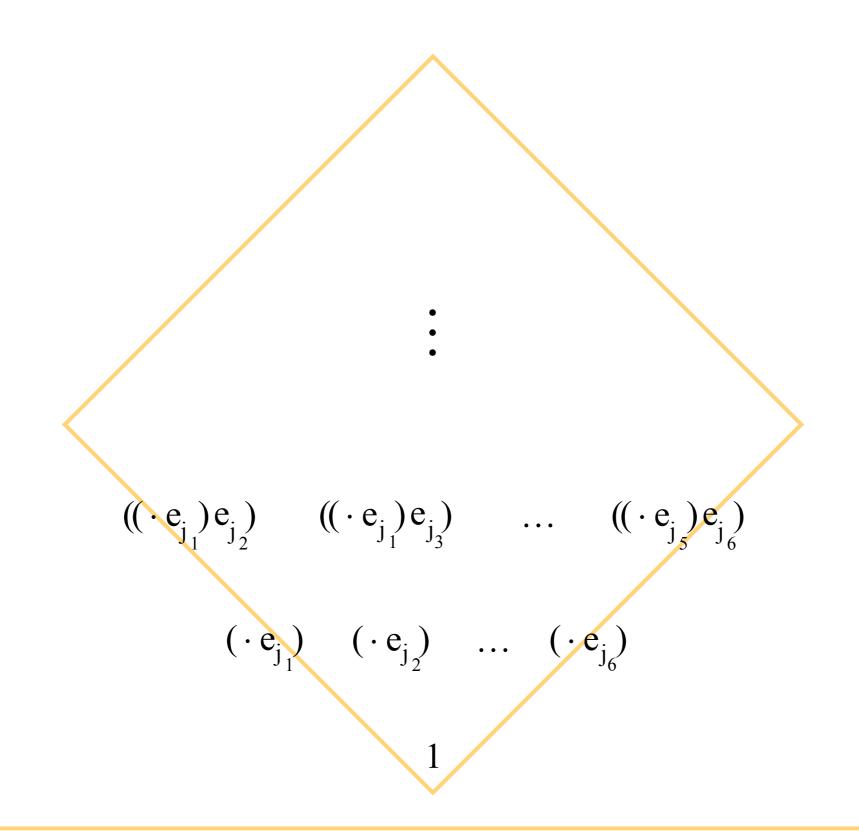


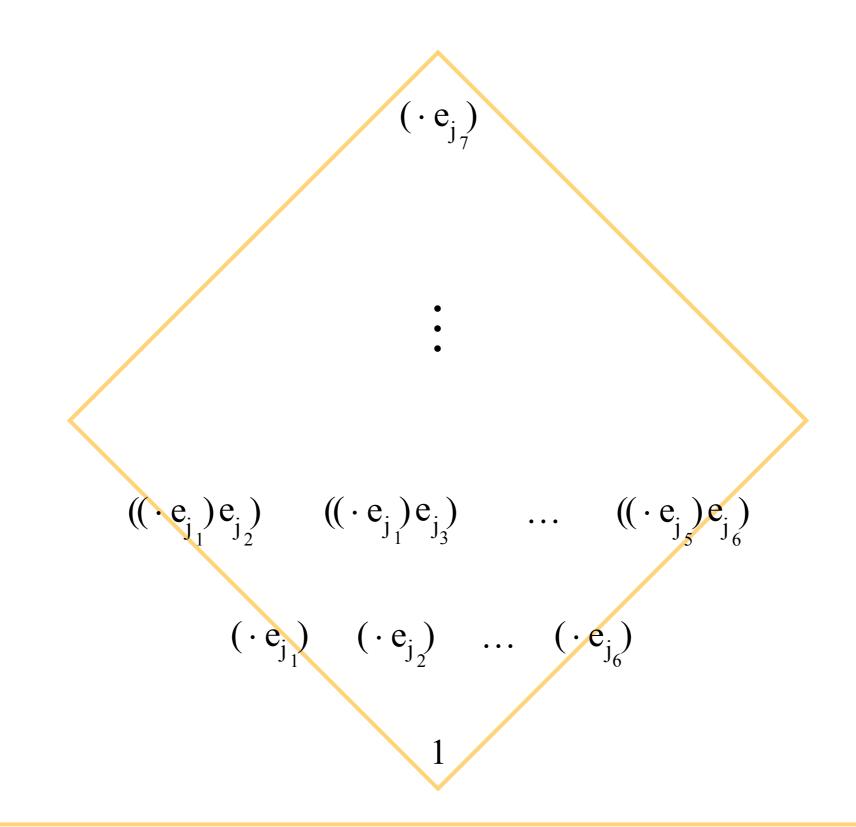




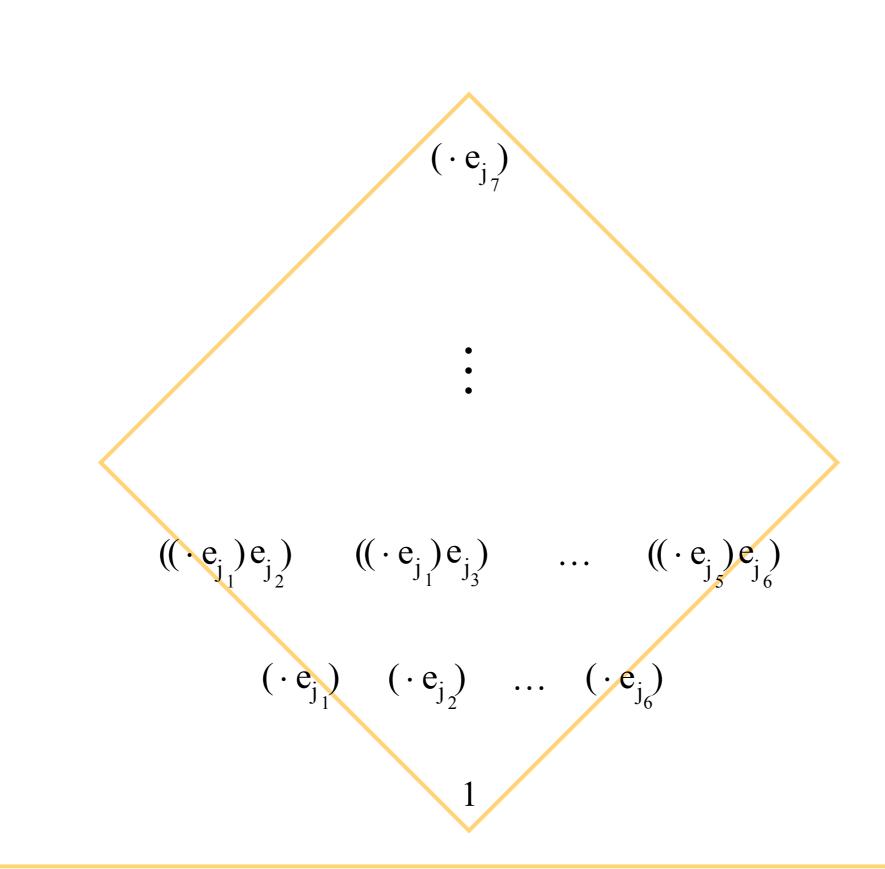






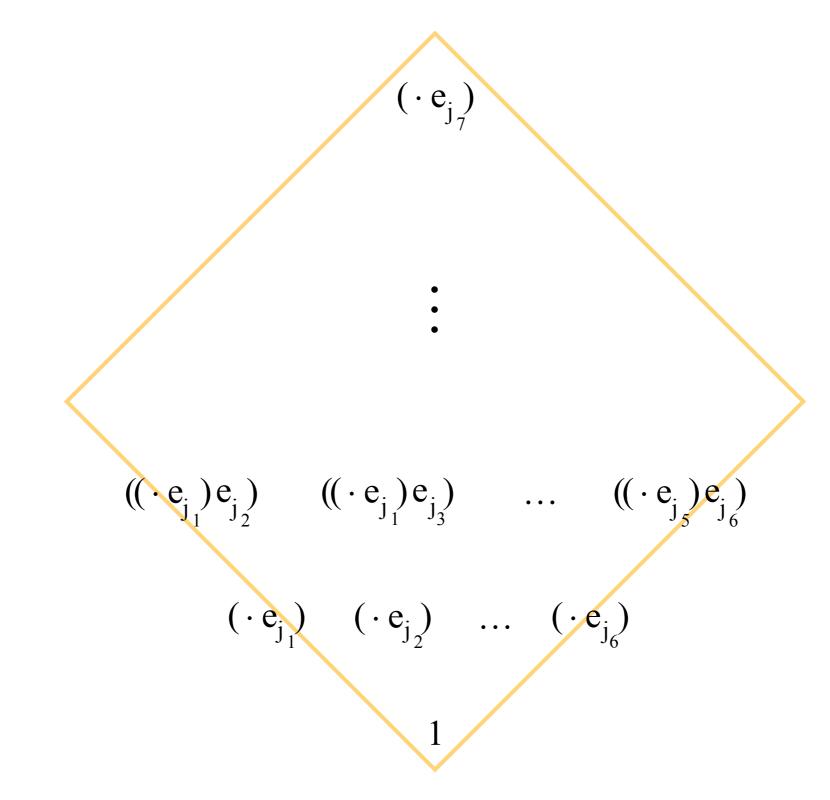






**Right** multiplication

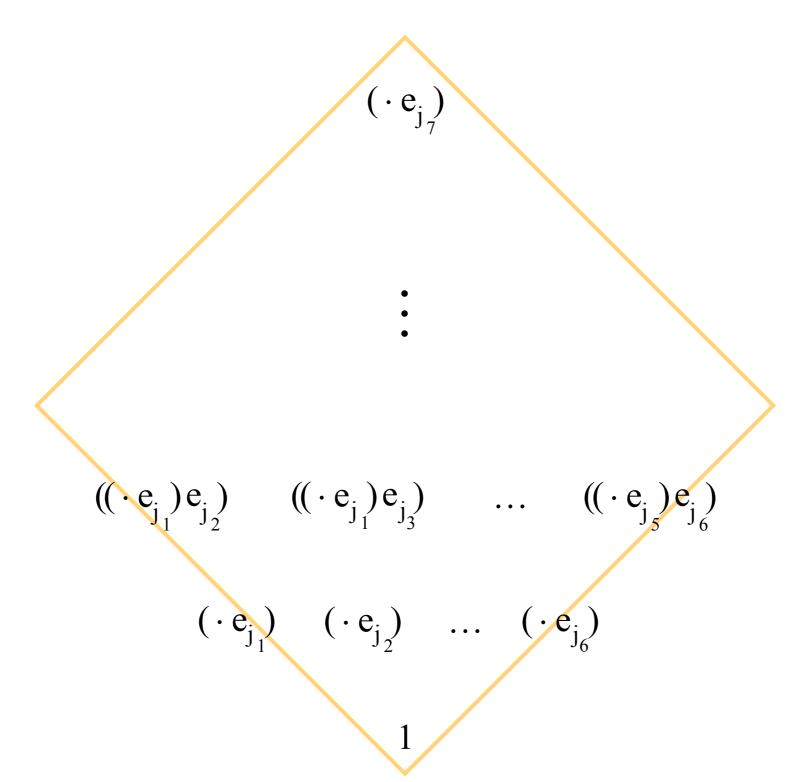




**Right** multiplication

$$(\cdot e_7) = 1/2 \left( e_1(e_3 \cdot) + e_2(e_6 \cdot) + e_4(e_5 \cdot) - (e_7 \cdot) \right)$$



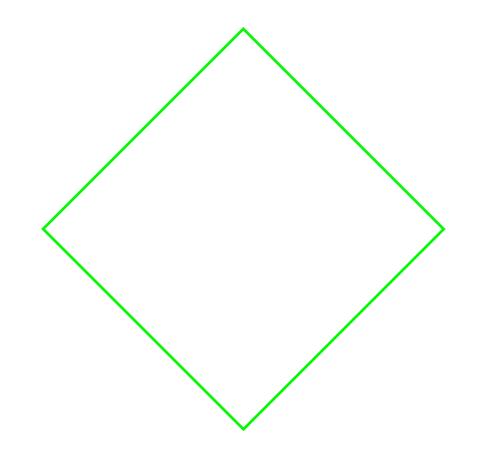


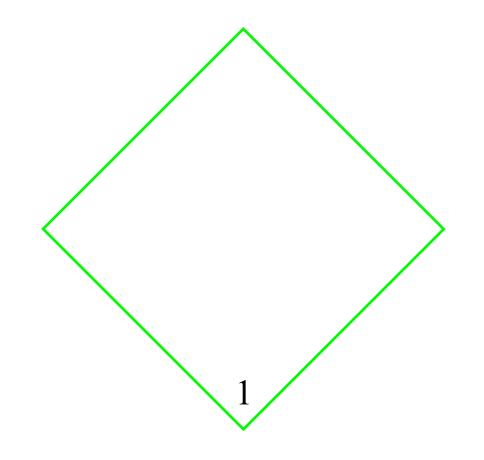
#### $\mathbb O$ multiplication algebra

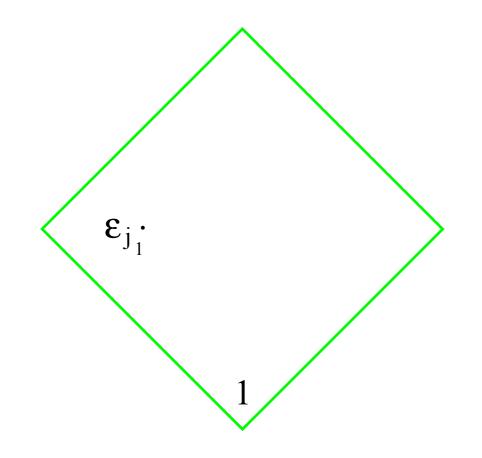
#### Idempotents

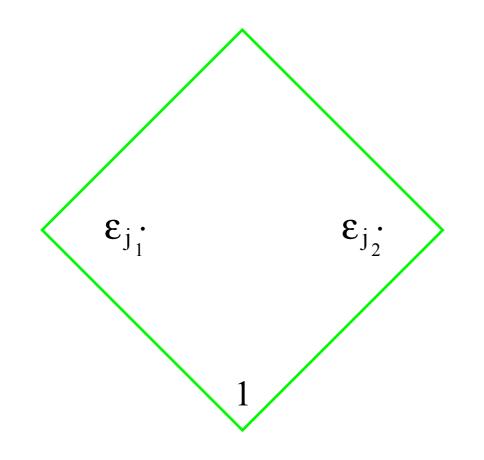
$$s \coloneqq \frac{1}{2} (1 + iL_{e_7}) \qquad S \coloneqq \frac{1}{2} (1 + iR_{e_7})$$
$$s^* \coloneqq \frac{1}{2} (1 - iL_{e_7}) \qquad S^* \coloneqq \frac{1}{2} (1 - iR_{e_7})$$

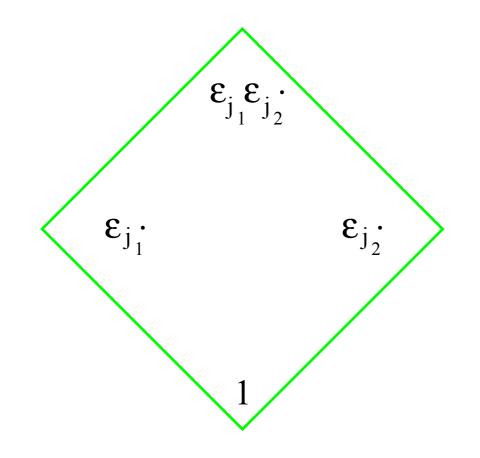
Quaternion Left and Right multiplication

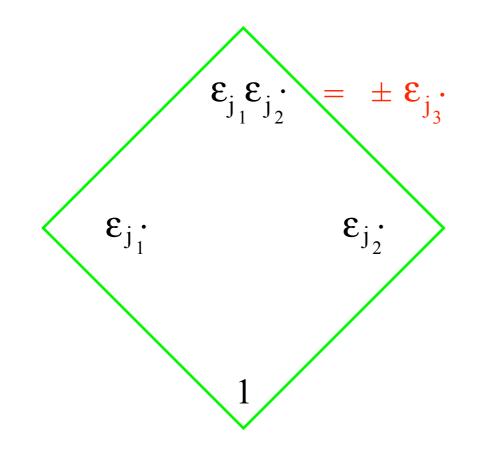


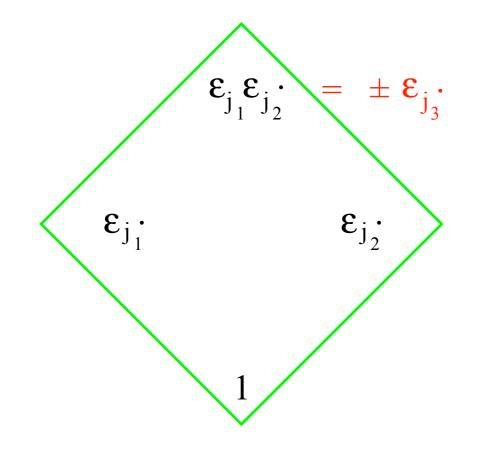


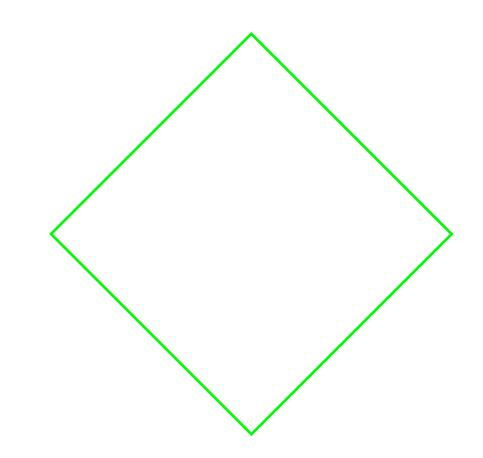


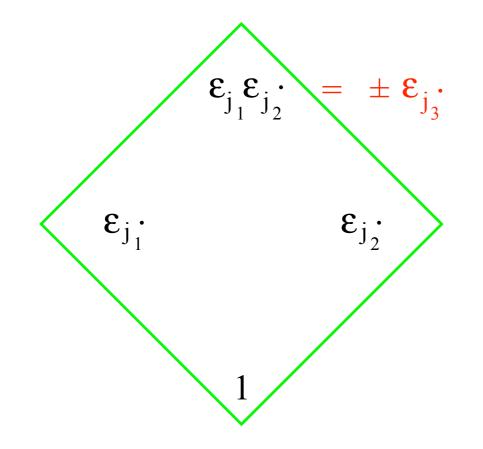


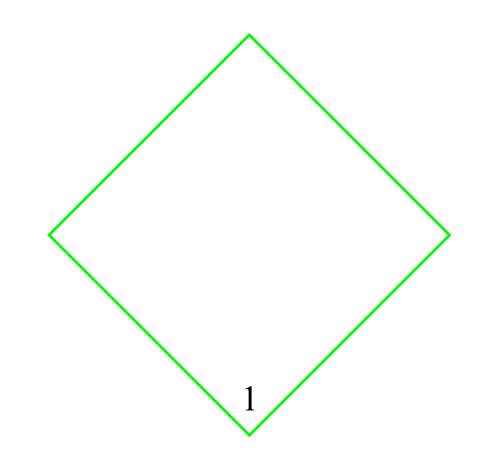


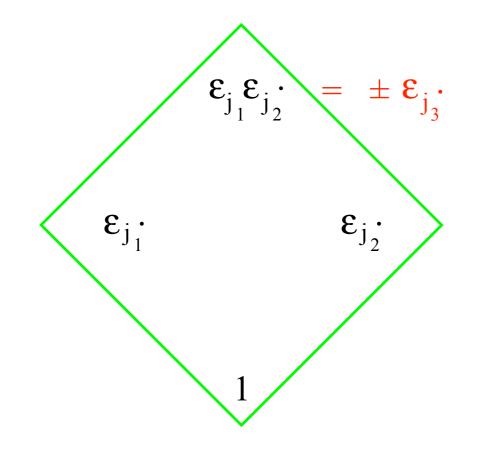


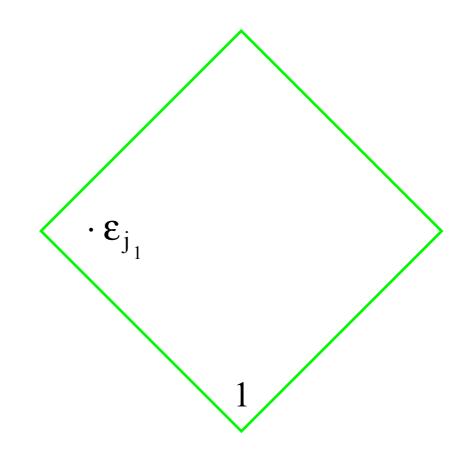


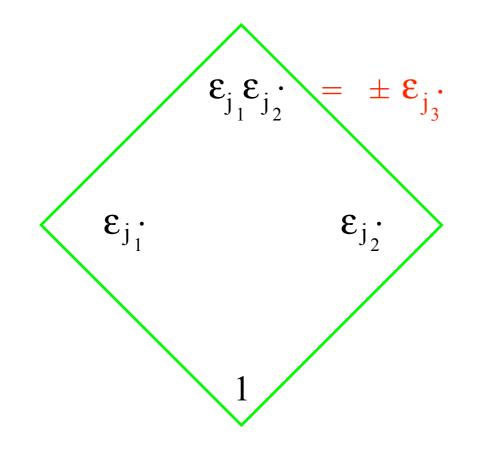


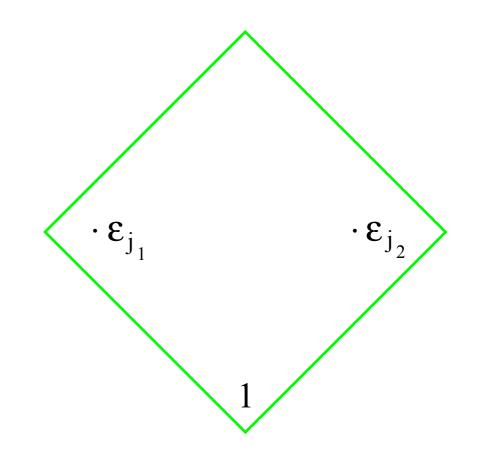


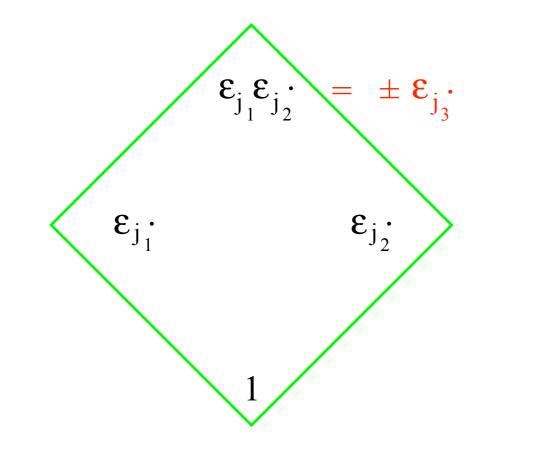


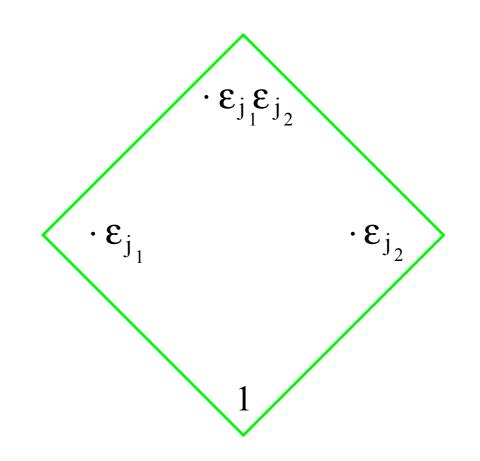


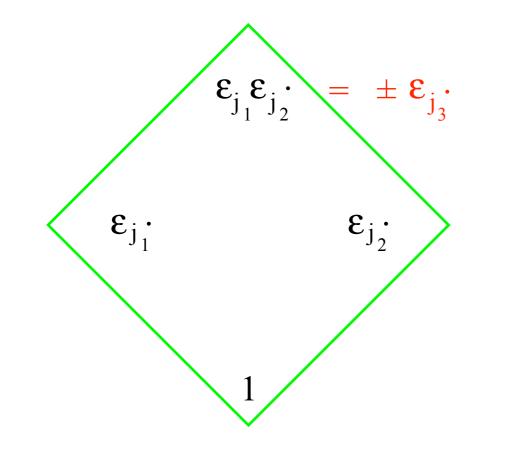


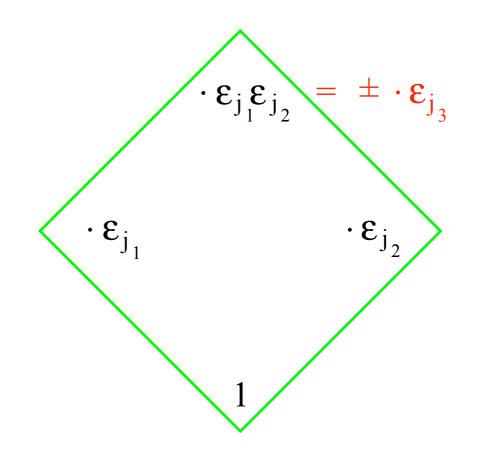


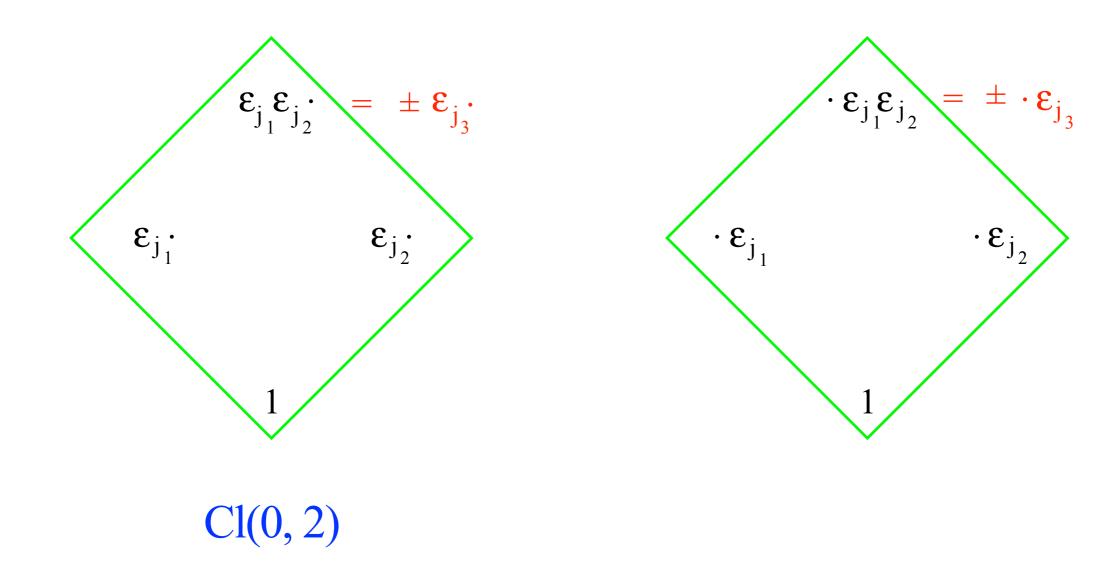


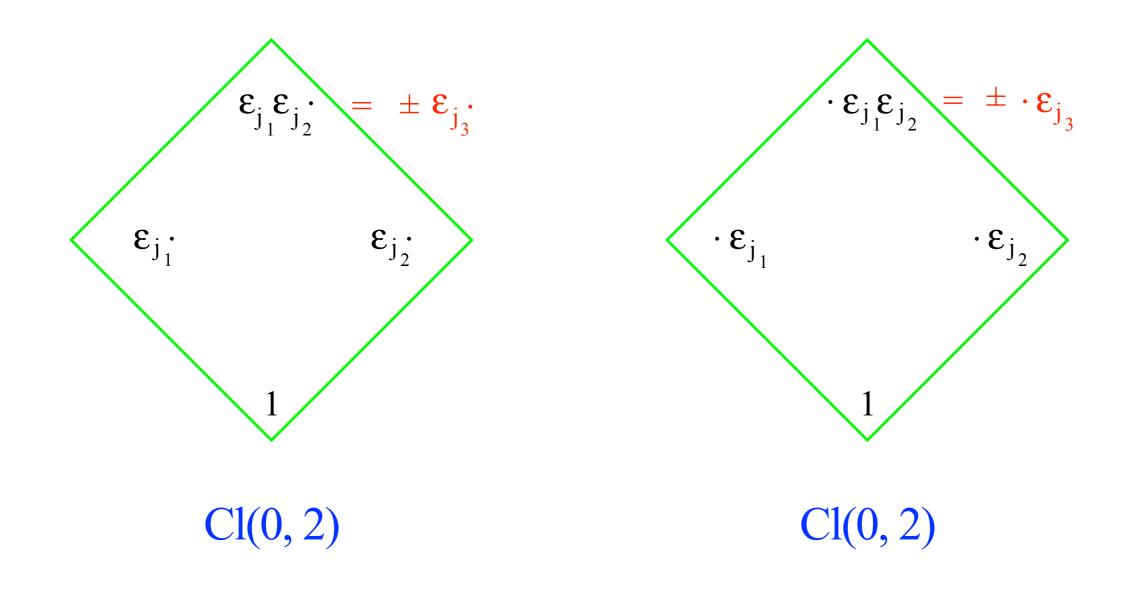












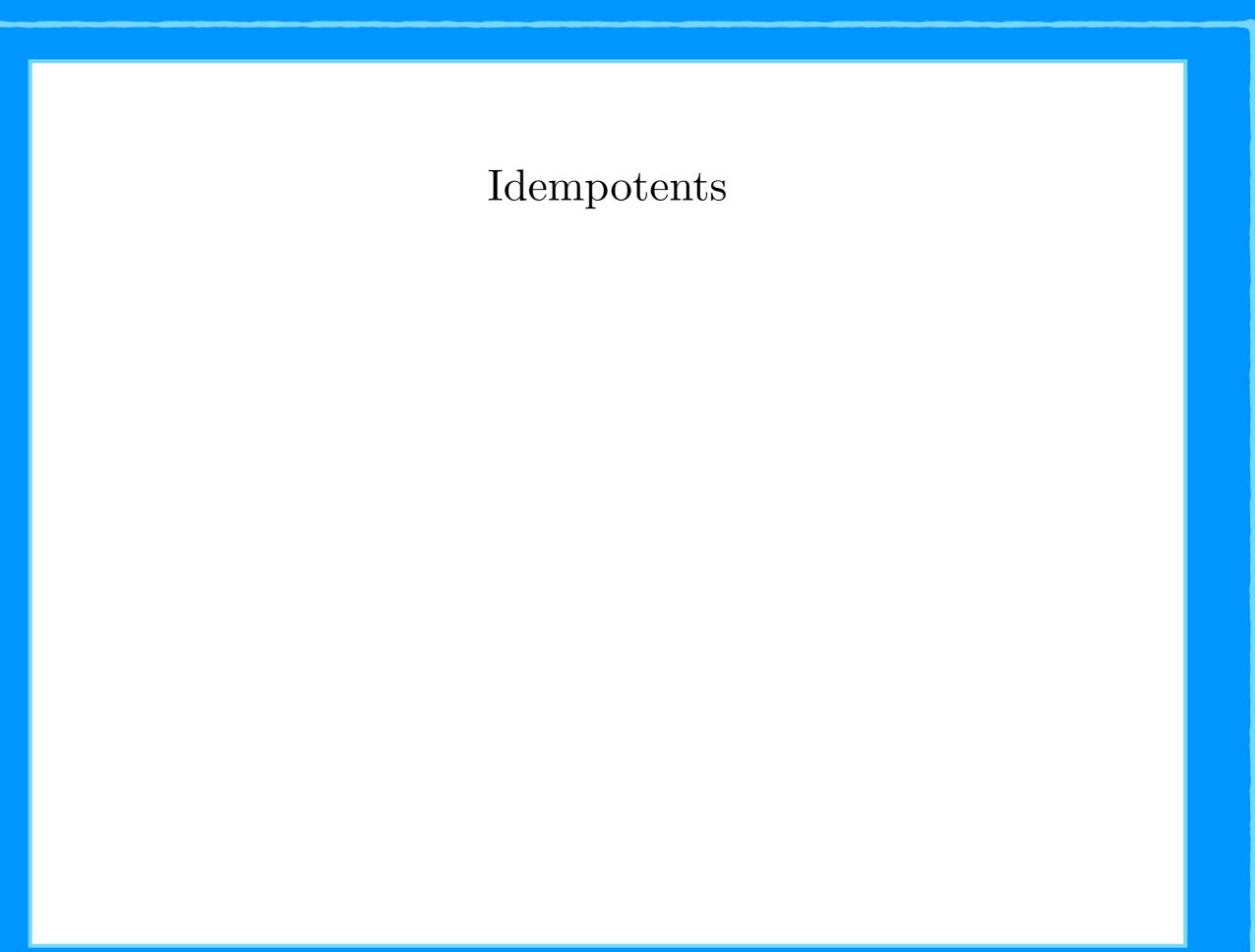
$$s \coloneqq \frac{1}{2} (1 + iL_{e_7}) \qquad S \coloneqq \frac{1}{2} (1 + iR_{e_7})$$
$$s^* \coloneqq \frac{1}{2} (1 - iL_{e_7}) \qquad S^* \coloneqq \frac{1}{2} (1 - iR_{e_7})$$

$$s \coloneqq \frac{1}{2} (1 + iL_{e_7}) \qquad S \coloneqq \frac{1}{2} (1 + iR_{e_7})$$
$$s^* \coloneqq \frac{1}{2} (1 - iL_{e_7}) \qquad S^* \coloneqq \frac{1}{2} (1 - iR_{e_7})$$

$$t := \frac{1}{2} (1 + iL_{\epsilon_3}) \qquad T := \frac{1}{2} (1 + iR_{\epsilon_3})$$
$$t^* := \frac{1}{2} (1 - iL_{\epsilon_3}) \qquad T^* := \frac{1}{2} (1 - iR_{\epsilon_3})$$

$$s \coloneqq \frac{1}{2}(1 + iL_{e_7}) \qquad S \coloneqq \frac{1}{2}(1 + iR_{e_7})$$
$$s^* \coloneqq \frac{1}{2}(1 - iL_{e_7}) \qquad S^* \coloneqq \frac{1}{2}(1 - iR_{e_7})$$

$$t := \frac{1}{2} (1 + iL_{\epsilon_3}) \qquad T := \frac{1}{2} (1 + iR_{\epsilon_3})$$
$$t^* := \frac{1}{2} (1 - iL_{\epsilon_3}) \qquad T^* := \frac{1}{2} (1 - iR_{\epsilon_3})$$

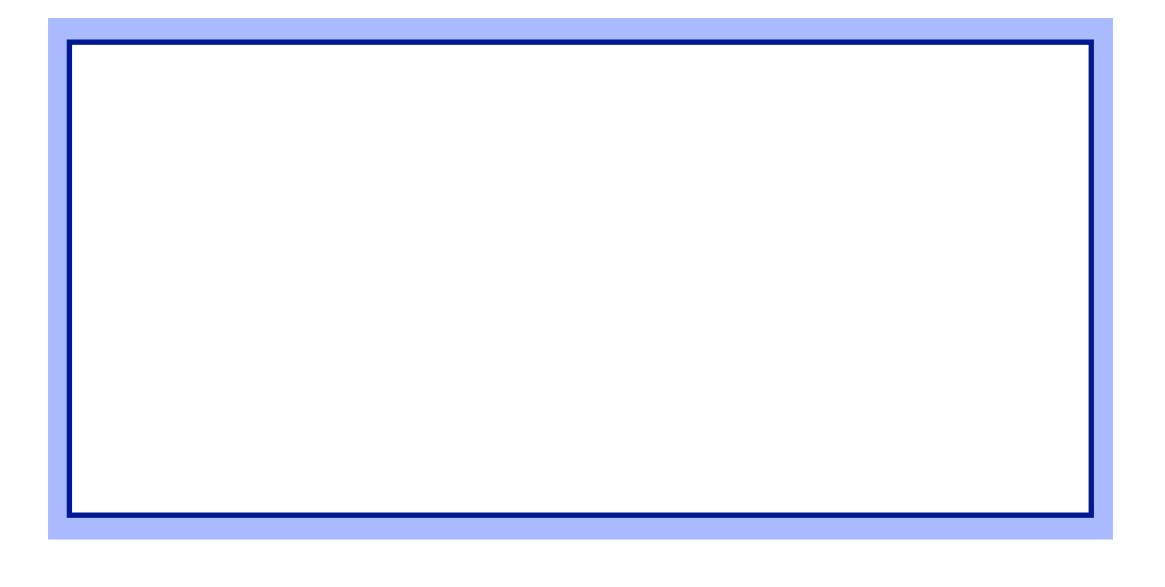


sSt
$sSt^*$
$sS^{*}t$
$sS^{*}t^{*}$
$s^*St$
$s^*St^*$
$s^*S^*t$
$s^*S^*t^*$

sSt	helicity $\uparrow$	lepton
$sSt^*$	helicity $\downarrow$	lepton
$sS^{*}t$	helicity $\uparrow$	baryon
$sS^{*}t^{*}$	helicity $\downarrow$	baryon
$s^*St$	isospin $\uparrow$	LH
$s^*St^*$	isospin $\downarrow$	LH
$s^*S^*t$	isospin $\uparrow$	RH
$s^*S^*t^*$	isospin $\downarrow$	RH

sSt	helicity $\uparrow$	lepton
$sSt^*$	helicity $\downarrow$	lepton
$sS^{*}t$	helicity $\uparrow$	baryon
$sS^{*}t^{*}$	helicity $\downarrow$	baryon
$s^*St$	isospin $\uparrow$	LH
$s^*St^*$	isospin $\downarrow$	LH
$s^*S^*t$	isospin $\uparrow$	RH
$s^*S^*t^*$	isospin $\downarrow$	RH

### Observation: no projections on colour.



Consider hermitian parts of

Consider hermitian parts of

Left:

Consider hermitian parts of

Left:  $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq \mathbb{C}l(8)$ 

Consider hermitian parts of

Left:  $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq \mathbb{C}l(8)$ 

Left and right:

Consider hermitian parts of

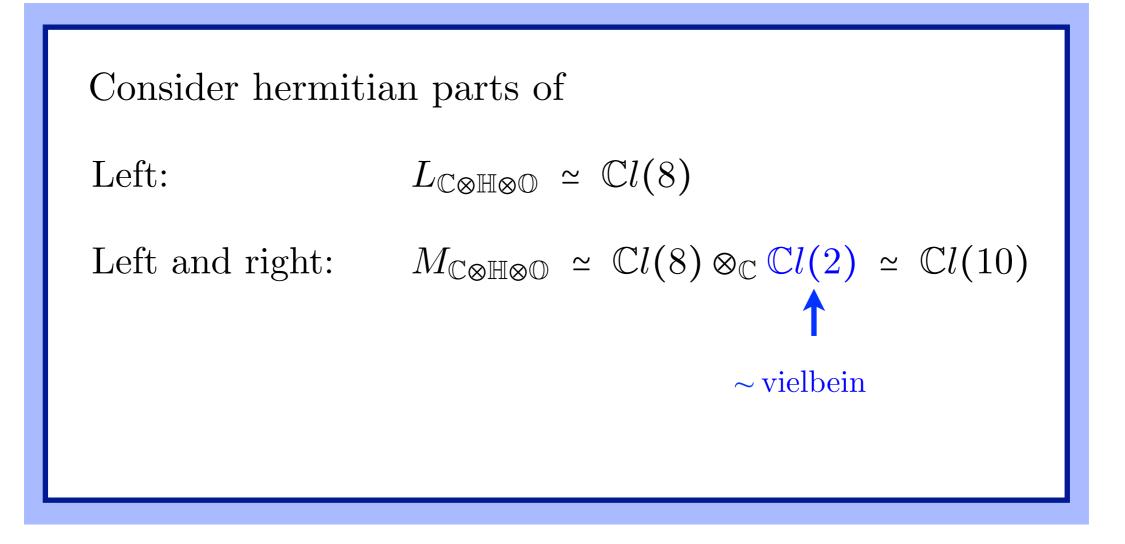
Left:  $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq \mathbb{C}l(8)$ 

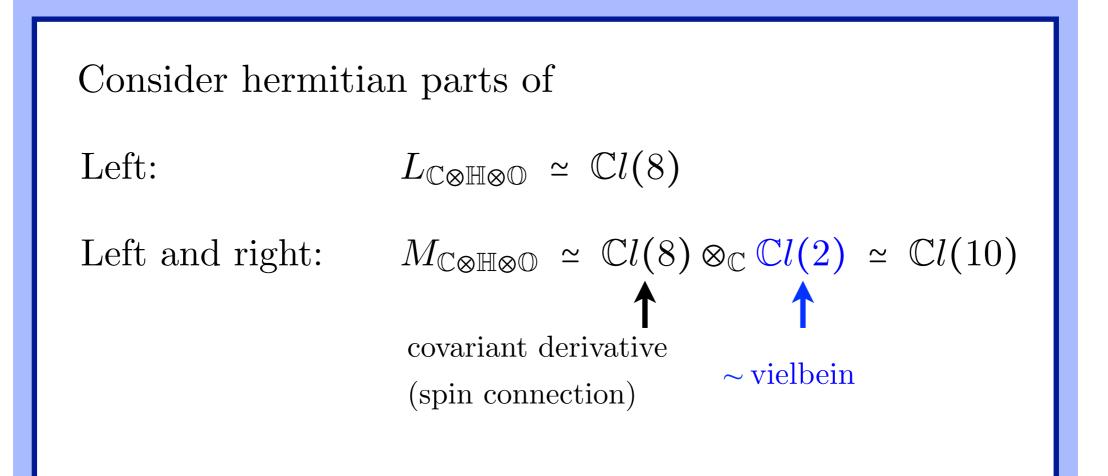
Left and right:  $M_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq \mathbb{C}l(8) \otimes_{\mathbb{C}} \mathbb{C}l(2)$ 

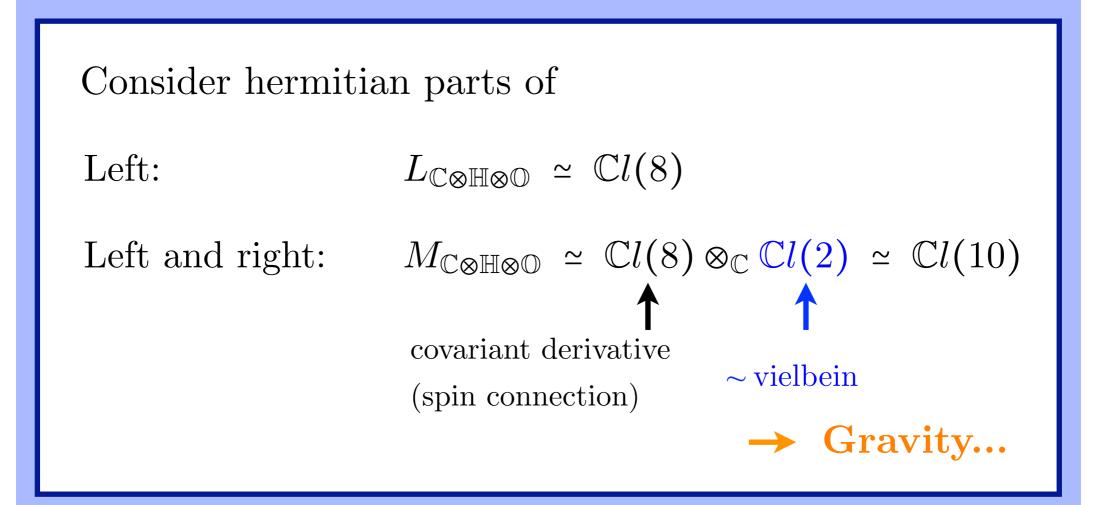
Consider hermitian parts of

Left:  $L_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq \mathbb{C}l(8)$ 

Left and right:  $M_{\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}} \simeq \mathbb{C}l(8) \otimes_{\mathbb{C}} \mathbb{C}l(2) \simeq \mathbb{C}l(10)$ 







## Summary



### Action of $\ell_{sm}$ on $\mathcal{H}_{16}(\mathbb{C})$

$$\begin{split} \delta b &= \ell_{sm} b + b \, \ell_{sm}^{\dagger} & \text{diagonal} \\ \delta f_0 &= \ell_{sm} s f_0 s^* + s f_0 s^* \ell_{sm} + h.c. & \text{outer off-diagonal} \\ \delta f_+ &= \ell_{sm} (s S^* f_+ s S + s^* S^* f_+ s^* S) \\ &+ (s S^* f_+ s S + s^* S^* f_+ s^* S) \ell_{sm}^{\dagger *} + h.c. & \text{inner off-diagonal} \end{split}$$

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$\left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_2 \\ \left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_2 \\ \left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_2 \end{array}$
$( \  u_e, \ e \ )_L \ ( \  u_\mu, \ \mu \ )_L \ ( \  u_\tau, \  au \ )_L$	$ \begin{array}{c} \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_2 \\ \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_2 \\ \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_2 \end{array} \\ \end{array} $
$egin{array}{c} u_R \ c_R \ t_R \end{array}$	$ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \end{array} $
$d_R \ s_R \ b_R$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$ u_{eR} $ $ u_{\mu R} $ $ u_{ au R}$	$\begin{array}{c} (\underline{1}, \underline{1}, 0)_{2} \\ (\underline{1}, \underline{1}, 0)_{2} \\ (\underline{1}, \underline{1}, 0)_{2} \end{array}$
$e_R \ \mu_R \  au_R$	$( \underline{1}, \underline{1}, -1 )_2$ $( \underline{1}, \underline{1}, -1 )_2$ $( \underline{1}, \underline{1}, -1 )_2$
$p_{\mu} \ p_{\mu}^{\prime}$	$(\underline{1}, \underline{1}, 0)_4$ $(\underline{1}, \underline{1}, 0)_4$
$egin{array}{c} G_\mu \ W_\mu \ B_\mu \end{array}$	$(\underline{8}, \underline{1}, 0)_4$ $(\underline{1}, \underline{3}, 0)_4$ $(\underline{1}, \underline{1}, 0)_4$

$232~\mathbb{R}$	$\mathcal{H}_{16}$	$_{\mathrm{S}}(\mathbb{C})$	
$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^*S \mathcal{H}_{16}(\mathbb{C}) sS^*$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$

### $\mathcal{H}_{16}(\mathbb{C})$

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$s^{\star}S \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$

#### "Peirce decomposition"

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$ \left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_{2} \\ \left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_{2} \\ \left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_{2} \end{array} $
$( \  u_e, \ e \ )_L \ ( \  u_\mu, \ \mu \ )_L \ ( \  u_\tau, \  au \ )_L$	$ \begin{array}{c} \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_2 \\ \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_2 \\ \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_2 \end{array} \end{array} $
$egin{array}{l} u_R \ c_R \ t_R \end{array}$	$ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \end{array} $
$d_R \ s_R \ b_R$	$ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ -\frac{1}{3} \end{array} \right)_{2} \\ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ -\frac{1}{3} \end{array} \right)_{2} \\ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ -\frac{1}{3} \end{array} \right)_{2} \end{array} \right. $
$ u_{eR} $ $ u_{\mu R} $ $ u_{ au R}$	$(\underline{1}, \underline{1}, 0)_2$ $(\underline{1}, \underline{1}, 0)_2$ $(\underline{1}, \underline{1}, 0)_2$
$e_R \ \mu_R \  au_R$	$(\underline{1}, \underline{1}, -1)_2 (\underline{1}, \underline{1}, -1)_2 (\underline{1}, \underline{1}, -1)_2$
$p_{\mu} \ p'_{\mu}$	$(\underline{1}, \underline{1}, 0)_4 \\ (\underline{1}, \underline{1}, 0)_4$
$egin{array}{c} G_\mu \ W_\mu \ B_\mu \end{array}$	$(\underline{8}, \underline{1}, 0)_4$ $(\underline{1}, \underline{3}, 0)_4$ $(\underline{1}, \underline{1}, 0)_4$

### Covariant derivative

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^*S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$

 $sS \mathcal{H}_{16}(\mathbb{C}) sS$ 

 $sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$ 

 $s^*S \mathcal{H}_{16}(\mathbb{C}) sS$ 

 $s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$ 

 $s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$ 

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2$
$( \  u_e, \ e \ )_L \ ( \  u_{\mu}, \ \mu \ )_L \ ( \  u_{\tau}, \  au \ )_L$	$ \begin{pmatrix} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{pmatrix}_{2} \\ ( \ \underline{1}, \ \underline{2}, \ -\frac{1}{2} )_{2} \\ ( \ \underline{1}, \ \underline{2}, \ -\frac{1}{2} )_{2} \end{cases} $
$egin{array}{c} u_R \ c_R \ t_R \end{array}$	$ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \end{array} $
$d_R \ s_R \ b_R$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$ u_{eR} $ $ u_{\mu R} $ $ u_{ au R}$	$\begin{array}{c} (\underline{1}, \underline{1}, 0)_{2} \\ (\underline{1}, \underline{1}, 0)_{2} \\ (\underline{1}, \underline{1}, 0)_{2} \end{array}$
$e_R \ \mu_R \  au_R$	$(\underline{1}, \underline{1}, -1)_2$ $(\underline{1}, \underline{1}, -1)_2$ $(\underline{1}, \underline{1}, -1)_2$
$p_{\mu} \ p'_{\mu}$	$( \underline{1}, \underline{1}, 0 )_4$ $( \underline{1}, \underline{1}, 0 )_4$
$egin{array}{c} G_\mu \ W_\mu \ B_\mu \end{array}$	$( \underline{8}, \ \underline{1}, \ 0 \ )_4$ $( \ \underline{1}, \ \underline{3}, \ 0 \ )_4$ $( \ \underline{1}, \ \underline{1}, \ 0 \ )_4$

# $sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$ $sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$ $sS \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$ $sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$ $sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$ $sS^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$ $s^{\star}S \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$ $s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$ $s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$

 $s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$ 

### Fermions

 $s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$ 

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$ \left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_{2} \\ \left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_{2} \\ \left( \begin{array}{ccc} \underline{3}, \ \underline{2}, \ \frac{1}{6} \end{array} \right)_{2} \end{array} $
$( \  u_e, \ e \ )_L \ ( \  u_\mu, \ \mu \ )_L \ ( \  u_\tau, \  au \ )_L$	$ \begin{array}{c} \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_2 \\ \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_2 \\ \left( \begin{array}{c} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{array} \right)_2 \end{array} \end{array} $
$egin{array}{l} u_R \ c_R \ t_R \end{array}$	$ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{ccc} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \end{array} $
$d_R \ s_R \ b_R$	$ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ -\frac{1}{3} \end{array} \right)_{2} \\ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ -\frac{1}{3} \end{array} \right)_{2} \\ \left( \begin{array}{c} \underline{3}, \ \underline{1}, \ -\frac{1}{3} \end{array} \right)_{2} \end{array} \right. $
$ u_{eR} $ $ u_{\mu R} $ $ u_{ au R}$	$(\underline{1}, \underline{1}, 0)_2$ $(\underline{1}, \underline{1}, 0)_2$ $(\underline{1}, \underline{1}, 0)_2$
$e_R \ \mu_R \  au_R$	$(\underline{1}, \underline{1}, -1)_2 (\underline{1}, \underline{1}, -1)_2 (\underline{1}, \underline{1}, -1)_2$
$p_{\mu} \ p'_{\mu}$	$(\underline{1}, \underline{1}, 0)_4 \\ (\underline{1}, \underline{1}, 0)_4$
$egin{array}{c} G_\mu \ W_\mu \ B_\mu \end{array}$	$(\underline{8}, \underline{1}, 0)_4$ $(\underline{1}, \underline{3}, 0)_4$ $(\underline{1}, \underline{1}, 0)_4$

### Covariant derivative

$sS \mathcal{H}_{16}(\mathbb{C}) sS$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ sS^{\star}$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$	$sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$
$s^*S \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$
$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$	$s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$

 $sS \mathcal{H}_{16}(\mathbb{C}) sS$ 

 $sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$ 

 $s^*S \mathcal{H}_{16}(\mathbb{C}) sS$ 

 $s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS$ 

 $s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$ 

$(\ u,\ d\ )_L$ $(\ c,\ s\ )_L$ $(\ t,\ b\ )_L$	$ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2 \ ig( \ \underline{3}, \ \underline{2}, \ \frac{1}{6} \ ig)_2$
$( \  u_e, \ e \ )_L \ ( \  u_{\mu}, \ \mu \ )_L \ ( \  u_{\tau}, \  au \ )_L$	$ \begin{pmatrix} \underline{1}, \ \underline{2}, \ -\frac{1}{2} \end{pmatrix}_{2} \\ ( \ \underline{1}, \ \underline{2}, \ -\frac{1}{2} )_{2} \\ ( \ \underline{1}, \ \underline{2}, \ -\frac{1}{2} )_{2} \end{cases} $
$egin{array}{c} u_R \ c_R \ t_R \end{array}$	$ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \\ \left(\begin{array}{c} \underline{3}, \ \underline{1}, \ \frac{2}{3} \end{array}\right)_{2} \end{array} $
$d_R \ s_R \ b_R$	$egin{array}{llllllllllllllllllllllllllllllllllll$
$ u_{eR} $ $ u_{\mu R} $ $ u_{ au R}$	$\begin{array}{c} (\underline{1}, \underline{1}, 0)_{2} \\ (\underline{1}, \underline{1}, 0)_{2} \\ (\underline{1}, \underline{1}, 0)_{2} \end{array}$
$e_R \ \mu_R \  au_R$	$(\underline{1}, \underline{1}, -1)_2$ $(\underline{1}, \underline{1}, -1)_2$ $(\underline{1}, \underline{1}, -1)_2$
$p_{\mu} \ p'_{\mu}$	$( \underline{1}, \underline{1}, 0 )_4$ $( \underline{1}, \underline{1}, 0 )_4$
$egin{array}{c} G_\mu \ W_\mu \ B_\mu \end{array}$	$( \underline{8}, \ \underline{1}, \ 0 \ )_4$ $( \ \underline{1}, \ \underline{3}, \ 0 \ )_4$ $( \ \underline{1}, \ \underline{1}, \ 0 \ )_4$

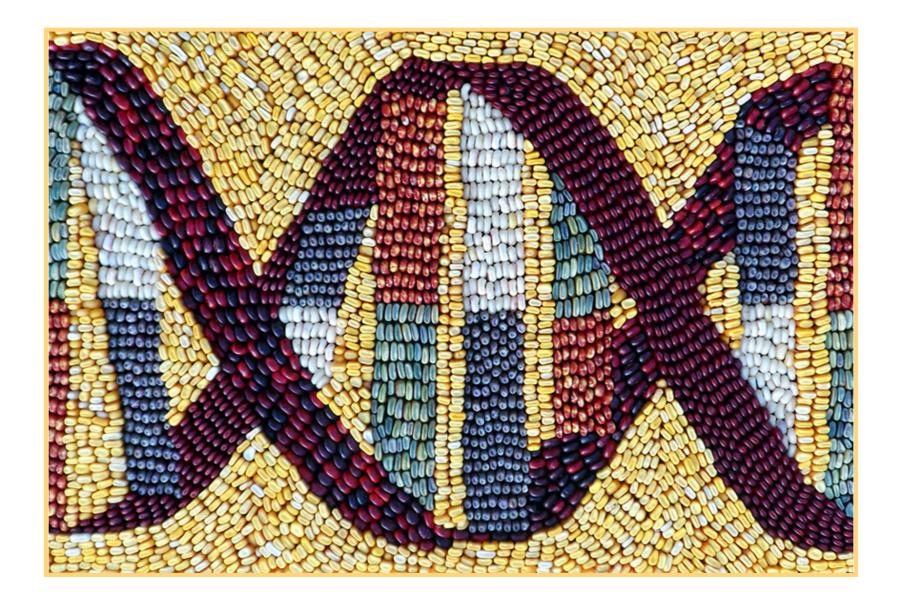
# $sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$ $sS \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$ $sS \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$ $sS^{\star} \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$ $sS^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S$ $sS^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S^{\star}$ $s^{\star}S \ \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$ $s^{\star}S \mathcal{H}_{16}(\mathbb{C}) sS^{\star}$ $s^{\star}S \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$

 $s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) s^{\star}S$ 

### Fermions

 $s^{\star}S^{\star} \mathcal{H}_{16}(\mathbb{C}) \ s^{\star}S^{\star}$ 

### Standard model irreps as an extension of 4-momentum: our first attempt



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