4d $\mathcal{N} = 2$ supergravity observables from Nekrasov-like partition functions

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DIAS
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Based on..

- Main conjecture - 2111.06903, implication - 2204.02992
- General rotating BPS black holes in AdS_4 - *[KH, Katmadas, Toldo'18-19]*
- Gravitational building blocks - *[Hosseini, KH, Zaffaroni'19]*
- Higher derivative asymptotically AdS_4 backgrounds - *[Bobev, Charles, KH, Reys'20-21]*
- Supergravity localization - *[KH, Lodato, Reys'18-19], [KH, Reys'21]*

Main message

- Structure of supersymmetric observables in 4d $\mathcal{N}=2$ supergravity in precise analogy with the one in 4d $\mathcal{N}=2$ field theory.

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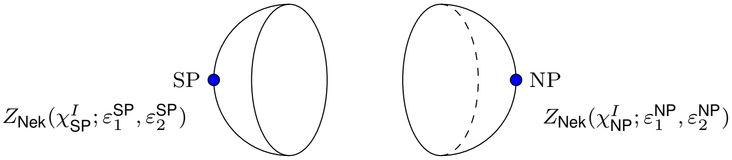
- Nekrasov partition function $\rightarrow$ gravitational Nekrasov-like partition function as a basic building block.
- Agreement with holographically dual results for 3d $\mathcal{N}=2$ SCFTs.

Field theory localization



- Ω-deformation: exact evaluation of the partition function on $\mathbb{C}^2$, Z_{Nek} - $\varepsilon_{1,2}$ deformation parameters, χ^I Coulomb branch parameters, *[Nekrasov'02]*. $\varepsilon_1\varepsilon_2 \log Z_{\mathsf{Nek}}$ - expansion in $\varepsilon_{1,2}$.

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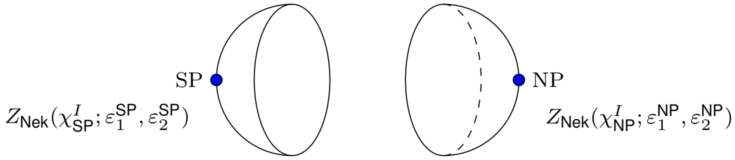

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- "Gluing" copies of Z_{Nek} on fixed points σ to reproduce many localization results, *[Nekrasov'03], [Pestun'07]*.

$$Z = \int \prod_I \mathrm{d}\chi^I \prod_\sigma Z_{\text{Nek}}(\chi^I_\sigma; \varepsilon_1^\sigma, \varepsilon_2^\sigma)$$

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only the ratio ω is physical in sugra (difference with rigid susy).

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- Here: extend to higher derivative $\mathcal{N} = 2$ sugra with $U(1)$ vector multiplets, build intuition with more examples.

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- Higher derivative supergravity formalism
- Formulation of the conjecture
- BPS black holes in Minkowski
- AdS_4 space
- Static/rotating BPS black holes in AdS_4
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Higher derivative supergravity

- The formalism of $4d\ \mathcal{N}=2$ superconformal gravity *[de Wit, van Proeyen et al'80-84]* allows for the construction of large classes of HD terms with $\geq 4\partial$.
- F-terms from (anti-)chiral superspace integrals, correcting the 2∂ *prepotential*, D-terms from full superspace integrals, correcting the 2∂ Kähler potential.

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- A number of different auxiliary multiplets allow for different off-shell formulations.
- Assume (physical) hypermultiplets are decoupled - consider only extra abelian vector multiplets.
- Argue that D-terms vanish on susy backgrounds, consider only F-terms

Bosonic field content and HD invariants

- Weyl multiplet: vielbein ${e_\mu}^a$, auxiliary $\mathrm{U}(1) \times \mathrm{SU}(2)$ R-symmetry gauge fields $A_\mu, {\mathcal{V}_\mu}^{ij}$, auxiliary tensor $T^{\pm}_{ab}$, auxiliary scalar D.

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- Aux. hypermultiplet: four real scalars ${A_i}^\alpha$, gauging of a $\mathrm{U}(1)$ subgroup of the $\mathrm{SU}(2)_R$ via the combination $g_I W^I_\mu$, constant FI parameters g_I. Limit to ungauged sugra: $g_I = 0$.

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- Two different 4∂ F-terms: the Weyl^2 *[Bergshoeff, de Roo, de Wit'81]* and the T-$\log$ *[Butter, de Wit, Kuzenko, Lodato'13]* invariants. Defined via composite chiral multiplets with lowest components $A_{\mathbb{W}}$ and $A_{\mathbb{T}}$.

HD Lagrangian

- HD invariants encoded in the holomorphic prepotential

$$F(X^I; A_{\mathbb{W}}, A_{\mathbb{T}}) := \sum_{m,n=0}^{\infty} F^{(m,n)}(X^I)\,(A_{\mathbb{W}})^m\,(A_{\mathbb{T}})^n\,. \qquad (1)$$

- Lagrangian specified by the choice for $F(X^I; A_{\mathbb{W}}, A_{\mathbb{T}})$ and gauging g_I - 4∂ theory *off-shell*, an infinite derivative expansion *on-shell* ($A_{\mathbb{W},\mathbb{T}} \sim 2\partial$).

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- Higher order terms $F^{(m,n)}(X^I)$: homogeneous of degree $2(1-m-n)$ ($A_{\mathbb{W},\mathbb{T}}$ of weight 2), giving rise to terms with $2(1+m+n)$ derivatives.

Conjecture, part I: the on-shell action

- On-shell action,

$$\mathcal{F}(M_4,\chi^I,\omega) = \sum_{\sigma\in M_4} s_{(\sigma)}\,\mathcal{B}(\kappa^{-1}\,X^I_{(\sigma)}(\chi^I,\omega),\omega_{(\sigma)}(\omega))\ ,$$
$$\mathcal{B}(X^I,\omega) := \frac{4i\pi^2\,F(X^I;(1-\omega)^2,(1+\omega)^2)}{\omega}\ , \tag{2}$$

with $s_{(\sigma)} = \pm 1$ aligned with the chirality of the Killing spinors at each fixed point σ.

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- Additional constraint $\lambda^{M_4}(g_I, \chi^I, \omega) = 0$, restoring the correct number of Coulomb branch parameters (one aux. v.m.).

Conjecture, part I: the entropy function

- For black hole solutions, χ^I conjugate to q_I, ω conjugate to $\mathcal{J}$:

$$\mathcal{I}(M_4, \chi^I, \omega, q_I, \mathcal{J}) = -\mathcal{F}(M_4, \chi^I, \omega) - \frac{8i\pi^2}{\kappa^2}(\chi^I q_I - \omega \mathcal{J}) \ , \quad (3)$$

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- Recover the BH entropy via extremization,

$$S_{\text{BH}}(M_4, q_I, \mathcal{J}) = \mathcal{I}(M_4, \chi^I\Big|_{\text{crit.}}, \omega\Big|_{\text{crit.}}, q_I, \mathcal{J}) \in \mathbb{R} \, , \tag{4}$$

with a resulting constraint

$$\hat{\lambda}^{M_4}(g_I, q_I, \mathcal{J}) := \text{Im}\left(\mathcal{I}(M_4, \chi^I\Big|_{\text{crit.}}, \omega\Big|_{\text{crit.}}, q_I, \mathcal{J})\right) = 0 \, .$$

Conjecture, part II: the partition function

- A gravitational Nekrasov partition function

$$Z^{\text{sugra}}_{\text{Nek}}(X^I,\omega) := \exp\left(-\frac{4i\pi^2\, F(\kappa^{-1}\, X^I; (1-\omega)^2, (1+\omega)^2)}{\omega}\right) ,$$

corrected in a UV complete theory

$$Z_{\text{Nek}}(X^I,\omega) := Z^{\text{sugra}}_{\text{Nek}}(X^I,\omega)\, Z^{\text{UV}}_{\text{Nek}}(X^I,\omega) \,.$$

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- Grand-canonical partition function via gluing rules

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- Microcanonical partition function / Quantum entropy function

$$Z(M_4,q_I,\mathcal{J}) := \int \mathrm{d}\chi^I \mathrm{d}\omega\, \delta(\lambda(g_I,\chi^I,\omega))\, e^{-\frac{8i\pi^2}{\kappa^2}(\chi^I q_I-\omega\mathcal{J})}\, Z(M_4,\chi^I,\omega) \;.$$

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BPS black holes in Minkowski

- 2∂ ungauged supergravity,

$$F_{2\partial} = -\frac{1}{6}\, c_{ijk}\, \frac{X^i X^j X^k}{X^0}\ , \qquad g_I = 0\ , \quad I = \{0, i\}\ .$$

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- Half-BPS flow between asymptotic Minkowski and $\mathrm{AdS}_2 \times \mathrm{S}^2$ near-horizon (NH) geometry,

$$\mathrm{d}s^2 = v_1\, \mathrm{d}s^2_{AdS_2} + v_2\, \mathrm{d}s^2_{S^2}\ .$$

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- Fixed points of the canonical isometry: centre of AdS_2 and SP/NP of the sphere

$$\xi = -\partial_\tau + \partial_\varphi ,$$

$$\Rightarrow \omega_{\mathrm{SP}} = \omega_{\mathrm{NP}} = \omega = -1 , \qquad s_{\mathrm{SP}} = -s_{\mathrm{NP}} = 1 .$$

BPS black holes in Minkowski: attractor mechanism

- Scalars fixed at the horizon, *[Ferrara, Kallosh'96]*

$$\frac{1}{2}\left(e^{i\alpha}X^I + e^{-i\alpha}\bar{X}^I\right) = p^I \ , \qquad \frac{1}{2}\left(e^{i\alpha}F_I + e^{-i\alpha}\bar{F}_I\right) = q_I \ .$$

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- In a mixed ensemble, *[Ooguri, Strominger, Vafa'04]*,

$$e^{i\alpha}X^I = p^I + \frac{i}{\pi}\,\phi^I \;,$$

ϕ^I conjugate to q_I,

$$\mathcal{F}_{\text{OSV}}(\phi^I, p^I) = \frac{i\pi}{2\,G_N^{(4)}}\left(F_{2\partial}(p^I + \frac{i}{\pi}\,\phi^I) - F_{2\partial}(p^I - \frac{i}{\pi}\,\phi^I)\right) \;,$$

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Attractor mechanism from gluing

- Gluing rule: 2 fixed points with constraint $\omega = -1$, $s_{(1,2)} = \pm 1$,

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- Precise match with OSV form upon $\phi^I = -i\pi \, \chi^I$.

Higher derivative generalization

- HD version of the Bertotti-Robinson, *[Cardoso, de Wit, Mohaupt'98-99]*. Full HD on-shell action,

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- Relation with the (unrefined) topological string (OSV conjecture) - infer the explicit form of the $\mathbb{W}$ tower $F^{(m,0)}$:

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- Part II of the conjecture - agreement with *[Denef, Moore'07]* and sugra localization *[Dabholkar, Gomes, Murthy'10-11]*.:

$$Z(p^I, q_I) := \int \left(\prod_{I=0}^{n_V} \mathrm{d}\chi^I \right)\, e^{-\mathcal{F}(\chi^I, p^I) - \frac{8i\pi^2}{\kappa^2}\,\chi^I q_I}\, Z^{\text{UV}}(\chi^I, p^I) \; .$$

AdS_4 space

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- A half-BPS generalization with running scalars, radial flow with gradually shrinking S^3 slices in the bulk. *[Freedman, Pufu'13]*

AdS_4 space

- 2∂ gauged supergravity, from 11d on S^7

$$F_{2\partial} = -2i\sqrt{X^0 X^1 X^2 X^3}\,, \qquad g_I = 1\,, \forall I.$$

- Fully BPS (Euclidean) AdS_4 vacuum, choose round S^3 boundary,

$$X^I = \frac{1}{4}\,, \forall I\,.$$

- A half-BPS generalization with running scalars, radial flow with gradually shrinking S^3 slices in the bulk. *[Freedman, Pufu'13]*

- Single fixed point: centre of AdS_4

$$\xi = \partial_\tau + \partial_\varphi\,, \quad \Rightarrow \quad \omega = 1\,, \quad s = 1\,.$$

Holographic (squashed) sphere

- On-shell action from "gluing", $X^I = 2\chi^I$

$$\mathcal{F}(S^3, \chi^I, \omega = 1) = 4i\pi^2 \, F(2\kappa^{-1} \, \chi^I; 0, 4) \ .$$

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- Agreement with 4∂ minimal sugra results in *[Bobev, Charles, KH, Reys'20-21]*.

Holographic bootstrap

- Exact results for round/squashed sphere of ABJM theory from susy localization - *[Fuji et al'11], [Marino, Putrov'11], [Nosaka'15], [Hatsuda'16], [Chester et al'21]*.

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- Exact results for round/squashed sphere of ABJM theory from susy localization - *[Fuji et al'11], [Marino, Putrov'11], [Nosaka'15], [Hatsuda'16], [Chester et al'21]*.
- Match with supergravity conjecture fixes the higher derivative prepotential *uniquely*:

$$F = -2i\sqrt{X^0X^1X^2X^3}\,\sum_{n=0}^{\infty} f_n \left(\frac{k_{\mathbb{W}}(X)A_{\mathbb{W}} + k_{\mathbb{T}}(X)A_{\mathbb{T}}}{64\,X^0X^1X^2X^3}\right)^n \ ,$$

$$k_{\mathbb{W}}(X) = -2\sum_{I<J} X^I X^J \ , \quad k_{\mathbb{T}}(X) = \sum_I (X^I)^2 + ...,$$

$$\frac{2\pi f_n}{(8\pi G_N)^{2(1-n)}} = \left(\frac{(2n-5)!!\,3}{n!\,(6k)^n}\right)\,\frac{\sqrt{2k}}{3}\,(N - \frac{k}{24})^{3/2-n}.$$

Airy function

- Supergravity prediction generalizes available matrix model results, complete perturbative answer (field theory parametrization $\chi^I = \frac{1}{2}\Delta_i$):

$$Z_{S^3}(b;\Delta_i) \simeq \exp\left(-\frac{2}{3}\, C_{S^3}^{-1/2}(N - \frac{k}{24} - B_{S^3})^{3/2}\right) \,,$$

$$C_{S^3} = \frac{2(b+b^{-1})^{-4}}{\pi^2 k \prod_i \Delta_i}\,, B_{S^3} = \frac{1}{48k\prod_i \Delta_i}\left(k_{\mathbb{T}}(\Delta) + \frac{(b-b^{-1})^2}{(b+b^{-1})^2}\, k_{\mathbb{W}}(\Delta)\right),$$

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- Subleading corrections interpreted as the UV completion. Single fixed point = Airy function.

Black holes in AdS_4: twisted branch

- Rotating black holes, susy with a twist, NH: $AdS_2 \times_w S^2$, *[Cacciatori, Klemm'09], [KH, Katmadas, Toldo'18]*. 2 fixed points, *[Hosseini, KH, Zaffaroni'19]*, $g_I \chi^I = 1$, $\sum_I p^I = -1$,

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- Prediction for the partition function (topologically twisted index):

$$Z_{\mathsf{TTI}}(\mathfrak{n}_i, \omega, \Delta_i) \simeq \mathsf{Ai}[C_+^{-1/3}(N - B_+^0)] \times \mathsf{Bi}[C_-^{-1/3}(N - B_-^1)] \; ,$$

$$C_\pm = \frac{2\,\omega^2}{\pi^2 k \prod_i (\Delta_i \pm \omega \mathfrak{n}_i)} \; ,$$

$$B_\pm^s = \frac{k}{24} + \frac{(\omega + (-1)^s)^2 k_{\mathbb{T}}(\Delta \pm \omega \mathfrak{n}_i) + (\omega - (-1)^s)^2 k_{\mathbb{W}}(\Delta \pm \omega \mathfrak{n}_i)}{48 k \prod_i (\Delta_i \pm \omega \mathfrak{n}_i)} \; .$$

Black holes in AdS_4: static limit of twisted branch

- Asymptotic expansion of the Airy functions,

$$\text{Ai/Bi}(z) \sim \frac{e^{\mp 2/3\, z^{3/2}}}{2\sqrt{\pi} z^{1/4}} \left[\sum_{n=0}^{\infty} \frac{(\mp 1)^n 3^n \Gamma(n+\frac{5}{6})\Gamma(n+\frac{1}{6})}{2\pi n! 4^n z^{3n/2}} \right] .$$

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- Precise agreement with numerical matrix model result in *[Bobev, Hong, Reys'22]*.

Black holes in AdS_4: non-twisted branch

- Kerr-Newman-like black holes, no twist, NH: $\mathrm{AdS}_2 \times_w \mathrm{S}^2$, *[KH, Katmadas, Toldo'19]*. 2 fixed points, *[Hosseini, KH, Zaffaroni'19]*, $\sum_I \chi^I = 1 + \omega$, $\sum_I p^I = 0$,

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- Admit Cardy limit $\omega \to 0$ (subleading magnetic charges):

$$-\log Z_{SCI}^{\mathrm{Cardy}} \simeq \frac{2\pi\sqrt{2k\Delta_1\Delta_2\Delta_3\Delta_4}}{3\omega}\, N_{k,\Delta}^{3/2} + \frac{1}{2}\log N_{k,\Delta} \ , \ .$$

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- Sugra observables closely follow from the structure of susy field theory observables.

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- Use the conjecture to prove AdS/CFT for supersymmetric observables? Lessons for non-susy quantum gravity? ...

Míle buíochas!