A topologically massive double copy

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Gravitational phenomena

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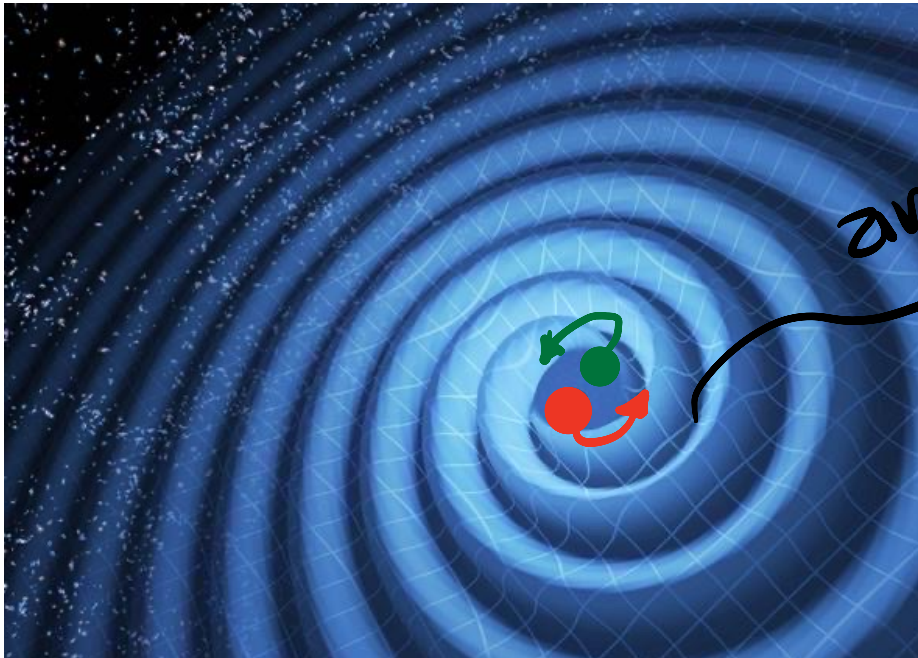


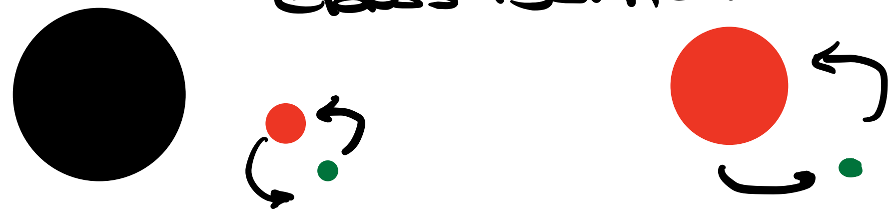
and

Beyond GR

massive gravity

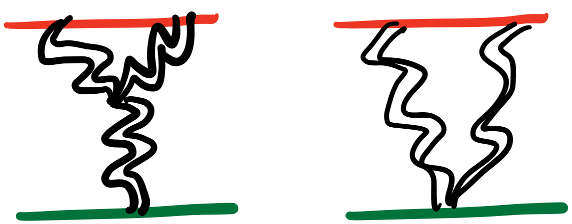
scalar-tensor theories

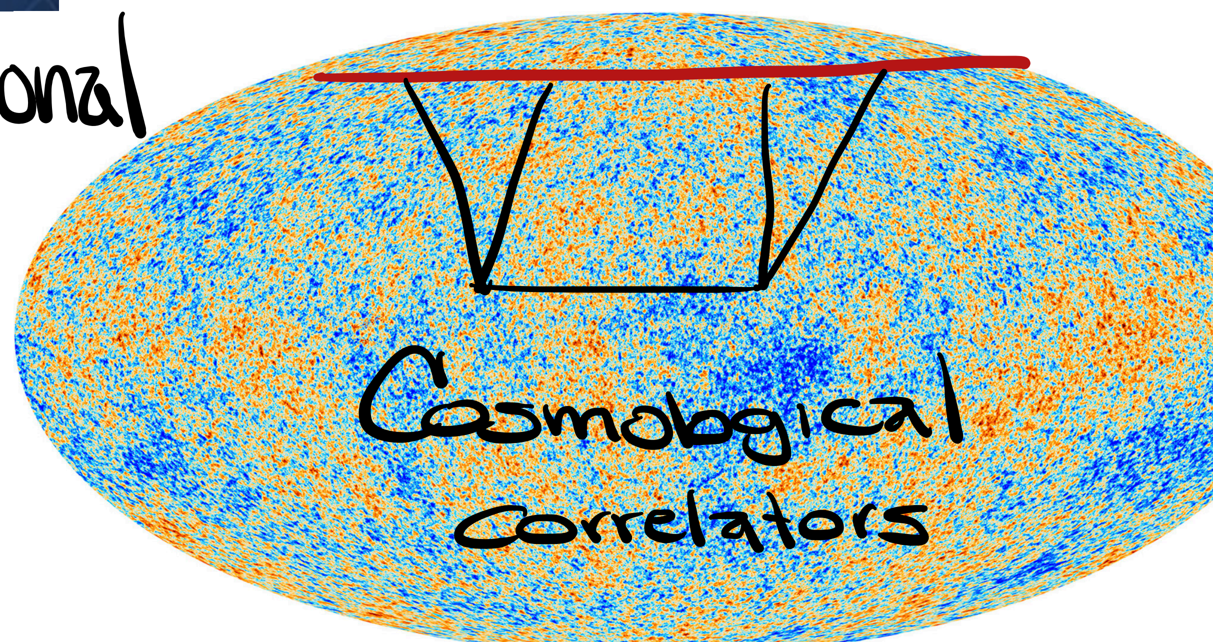
Gauss-Bonnet



Gravitational waves

in GR

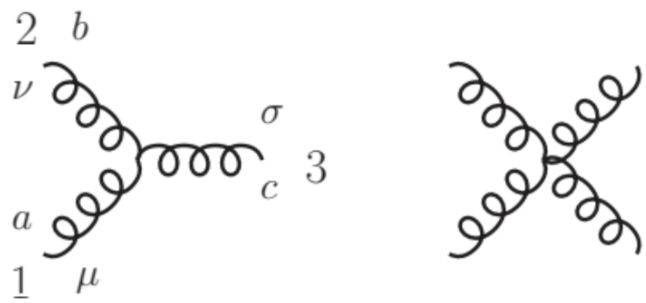




Cosmological correlators

S-matrix Computation: Gravity is Hard

Yang-Mills



$$\Gamma^{abc}_{\mu\nu\sigma} = g\ f^{abc}\left(\eta_{\mu\nu}(p_1 - p_2)_\sigma + \text{cyclic perm.}\right)$$

Gravity

$$\Gamma_{\mu\alpha,\nu\beta,\sigma\gamma} =$$
$$i\frac{\kappa}{2}\text{Sym}\left(-\frac{1}{2}P_3(p_1\cdot p_2\eta_{\mu\alpha}\eta_{\nu\beta}\eta_{\sigma\gamma}) - \frac{1}{2}P_6(p_{1\nu}p_{2\beta}\eta_{\mu\alpha}\eta_{\sigma\gamma}) + 9\ \text{terms}\right)$$

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On-shell:

Yang-Mills

$$\Gamma^{abc}_{\mu\nu\sigma} = g\ f^{abc}\left(\eta_{\mu\nu}(k-p)_\sigma + \text{cyclic perm.}\right)$$

Gravity

$$\Gamma_{\mu\alpha,\nu\beta,\sigma\gamma} = -i\frac{\kappa}{2}\left(\eta_{\mu\nu}(k-p)_\sigma + \text{cyclic perm.}\right)\left(\eta_{\alpha\beta}(k-p)_\gamma + \text{cyclic perm.}\right)$$

Related through color-kinematics duality

Yang-Mills^2=Gravity

- Solve hard problems by starting with simpler ones:
  Gravitational radiation Goldberger, Ridgway, Prabhu, Thompson Li, Shen
  Conservative 2-body Hamiltonian Bern, Cheung, Luna, Roiban, Shen, Solon, Zeng,...
- Explore range of applicability → understand origin of duality

The BCJ double copy

Bern, Carrasco, Johansson (2008)

$$A_\mu^a : \quad \mathcal{A}_{YM} = \sum_{i \,\epsilon\, \text{trivalent}} \frac{c_i n_i}{d_i}$$

$$h_{\mu\nu},\ \phi,\ B_{\mu\nu} : \quad \mathcal{M}_G = \sum_{i \,\epsilon\, \text{trivalent}} \frac{n_i n_i}{d_i}$$

$$\phi^{a\,a'} : \quad \mathcal{A}_{\phi^3} = \sum_{i \,\epsilon\, \text{trivalent}} \frac{c_i c_i}{d_i}$$

Jacobi relations for color-kinematics duality

$$c_i + c_j + c_k = 0$$

$$n_i + n_j + n_k = 0$$

Amplitudes and numerators invariant under generalized gauge transformations

$$n_{12} \to n_{12} + s_{12}\Delta, \quad n_{13} \to n_{13} + s_{13}\Delta, \quad n_{14} \to n_{14} + s_{14}\Delta$$

BCJ for the 4-point Amplitude YM^2=Gravity

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$$\mathcal{A}_{YM} = \sum_{i\,\epsilon\,\text{trivalent}} \frac{c_i n_i}{d_i} \qquad \mathcal{M}_G = \sum_{i\,\epsilon\,\text{trivalent}} \frac{n_i n_i}{d_i}$$

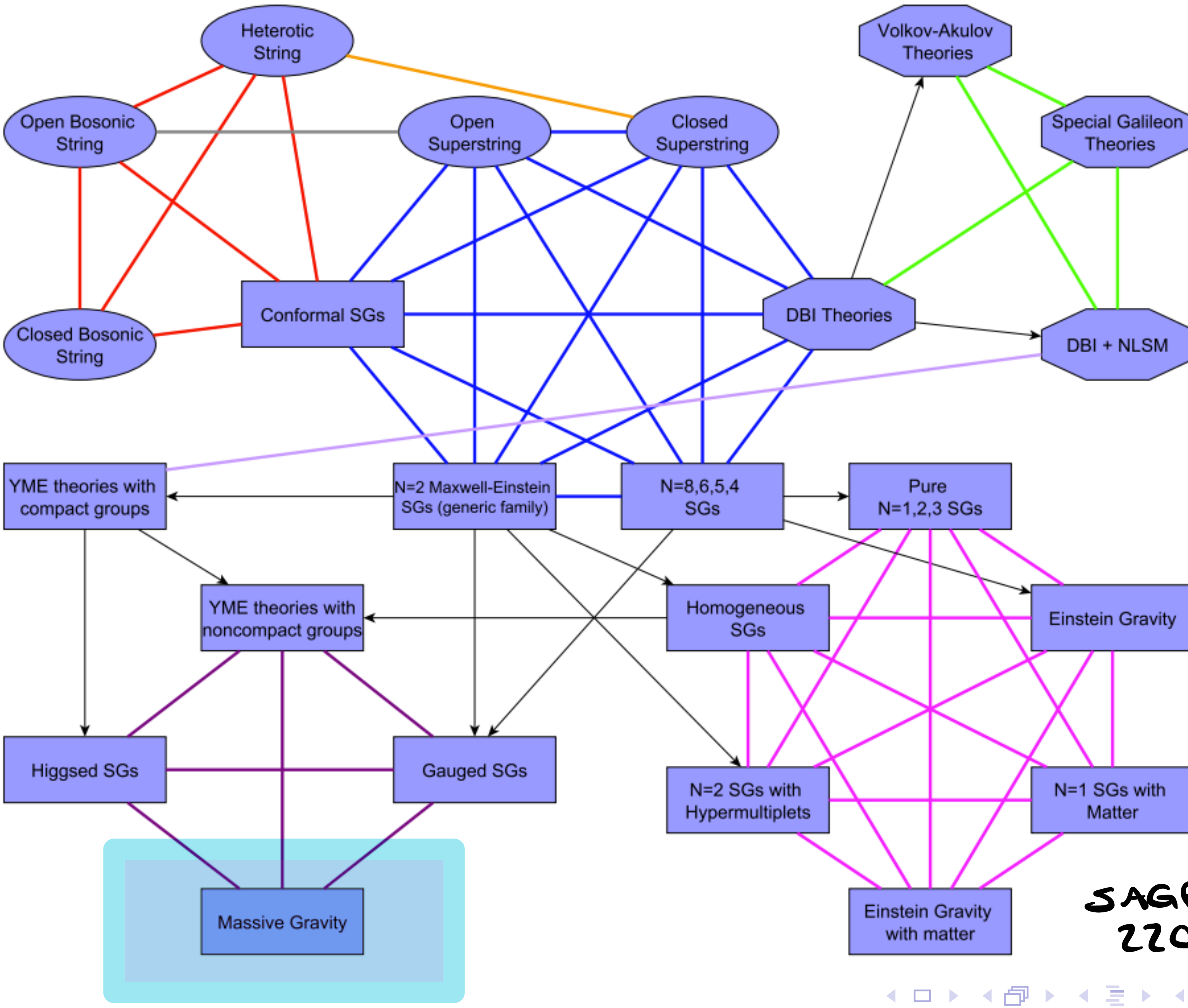
$$d_i = s\ , \qquad c_i = -2 f^{abc} f^{cde}\ ,$$

$$n_i = \frac{i}{2}\Big\{ \left[(\epsilon_1\cdot\epsilon_2)p_1^{\mu} + 2(\epsilon_1\cdot p_2)\epsilon_2^{\mu} - (1\leftrightarrow 2)\right]\times \\ \left[(\epsilon_3\cdot\epsilon_4)p_{3\mu} + 2(\epsilon_3\cdot p_4)\epsilon_{4\mu} - (3\leftrightarrow 4)\right] \\ + s\left[(\epsilon_1\cdot\epsilon_3)(\epsilon_2\cdot\epsilon_4) - (\epsilon_1\cdot\epsilon_4)(\epsilon_2\cdot\epsilon_3)\right]\Big\}$$

$c_i + c_j + c_k = 0$ and $n_i + n_j + n_k = 0$

Necessary for linearized gauge and diffeomorphism invariance

Web of double copy constructible theories



$=(YM)^2$

SAGEX REVIEW
2203.13013

BCJ Double Copy

CK dual

$$\mathcal{A}_{YM} = \sum_i \frac{c_i n_i}{s_i + m^2} = c^T D^{-1} n \qquad \mathcal{M}_G = \sum_i \frac{n_i n_i}{s_i + m^2} = n^T D^{-1} n$$

Color factors satisfy Jacobi relations: $Mc = 0$

Color-kinematics duality $\Rightarrow$ $Mn = 0$

For $Mn \neq 0$ use generalized gauge transformations:

$$A^{YM} \to A^{YM}$$

$n \to n + \Delta n$ s.t. $c^T D^{-1} \Delta n = 0$ $\quad Mn \to Mn + MDM^T v = 0$

$$\to \Delta n = DM^T v$$

BCJ Double Copy

CK dual

$$\mathcal{A}_{YM} = \sum_i \frac{c_i n_i}{s_i + m^2} = c^T D^{-1} n \qquad \mathcal{M}_G = \sum_i \frac{\tilde{n}_i \tilde{n}_i}{s_i + m^2} = \tilde{n}^T D^{-1} \tilde{n}$$

Color factors satisfy Jacobi relations: $Mc = 0$

Color-kinematics duality $\Rightarrow$ $M\tilde{n} = Mn + MDM^T v = 0$

MDM^T ↑ Invertible $\Rightarrow$ CK duality for any theory!

$$\mathcal{M}_G = n^T D^{-1} n + (Mn)^T (MDM^T)^{-1} (Mn)$$

↑ Unphysical poles!

Previously observed in:

1701.02519 Generalized D.C. — Bern, Carrasco, Chen, Johansson, Roiban

Massive case:

Johnson-Engelbrecht, Jones, Paranjape

Momeni, Rombouts, Tolley

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D=4 special mass spectrum L. A. Johnson, C. R. T. Jones, S. Paranjape; 2020

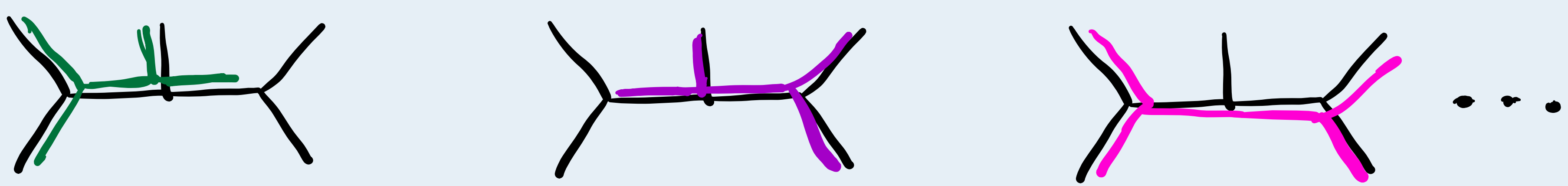
$$\mathcal{A}_{YM} = \sum_{i \,\epsilon\, \text{s, t, u}} \frac{c_i n_i}{s_i + m_i^2}$$

Reduce rank of MDMᵀ

Spectral condition at 4pt: MDMᵀ $= m_1^2 + m_2^2 + m_3^2 + m_4^2 - m_s^2 - m_t^2 - m_u^2 = 0$

Implies massive BCJ relations

At higher points, impose for all sub-4pt graphs



Satisfied by Kaluza Klein theories

Avoiding unphysical poles

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$m=0 \xrightarrow{\text{SSB}} m\neq 0$

MDM^T has minimal rank

$\Rightarrow$ 4 Spt BCJ relations

- SSB in Supergravities
  Chiodaroli, Gunaydin, Johansson, Roiban
- Kaluza-Klein theories
  Johnson-Engelbrecht, Jones, Paranjape; Momeni, Rombutis, Tolley; Li, Hang, He
- $(\phi^a)^3$ $U(N)\times G$
  MCG, Liang, Trodden

- Mass-deformed minimal $(DF)^2$
  Johansson, Mogull, Teng; Menezes

3D Kinematics

$\det(MDM^T) \propto \det(p_i \cdot p_j), \quad i,j<5$

$\rightarrow \det(MDM^T)^{3d} = 0$

Only 1 Spt "BCJ" relation

No spurious poles + correct factorization

Topologically massive theories

MCG, Momeni, Rombutis
Burger, Emond, Moynihan
Hang, He, Shen

Avoiding spurious poles in $D = 3$

$$\mathcal{M}_G = n^T D^{-1} n + (Mn)^T (MDM^T)^{-1} (Mn)$$

$$\det(MDM^T)\big|_{3d} = 0 \rightarrow (MDM^T)\cdot e_0 = 0$$

Invert in space orthogonal to e_0:

$Mn + (MDM^T)v = 0$ need Mn in that space

$\rightarrow Mn \cdot e_0 = 0 \leftarrow$ 1 "BCJ relation"

- Factorizes correctly when taking $s_{ij} \rightarrow 0$.
- Residue of the spurious pole is zero.

2107.00611
MCG, Momeni, Rumbutis

DC @ 3, 4, 5 pt

Topologically massive theories

PARITY BROKEN

Topologically Massive Gravity — 1 dof

$$S_{TMG}=\frac{1}{\kappa^2}\int d^3x\sqrt{-g}\left(-R-\frac{1}{2m}\varepsilon^{\mu\nu\rho}\left(\Gamma^{\alpha}_{\mu\sigma}\partial_\nu\Gamma^{\sigma}_{\alpha\rho}+\frac{2}{3}\Gamma^{\alpha}_{\mu\sigma}\Gamma^{\sigma}_{\nu\beta}\Gamma^{\beta}_{\rho\alpha}\right)\right)$$

Topologically Massive Yang-Mills — 1 dof

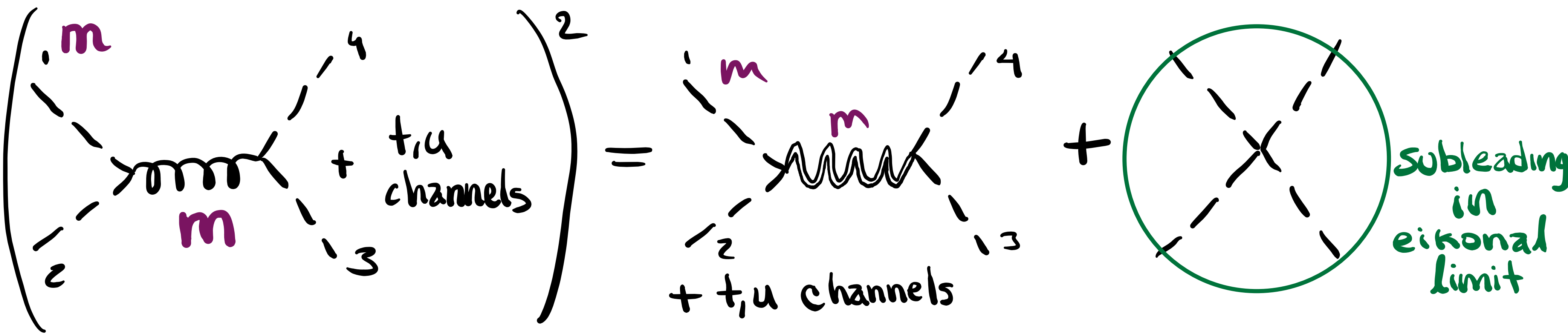
$$S_{TMYM}=\int d^3x\left(-\frac{1}{4}F^{a\mu\nu}F_{a\mu\nu}+\varepsilon_{\mu\nu\rho}\frac{m}{12}\left(6A^{a\mu}\partial^\nu A^\rho_a+g\sqrt{2}f_{abc}A^{a\mu}A^{b\nu}A^{c\rho}\right)\right)$$

$1\otimes 1=1$ ✓

Including matter

$(TMYM)^2 = TMG +$ ~~?~~

for matter with $T_\mu{}^\mu = 0$



2112.08401
MCG, Momeni, Rumbotis

DC in eikonal limit

Also observed in:
Burger, Emond, Moynihan

Eikonal limit

$-t \ll s \sim -u$

$m^2 \ll s$

In 4d C → S takes

$A_n^{YM} \longrightarrow \mathcal{M}_n^{Gravity}$

$$i\mathcal{M} = 2s \int db\, e^{-ibq} \left(e^{i\delta} - 1 \right)$$

$$\delta = \frac{1}{2s} \int \frac{dk_y}{2\pi} \mathcal{M}^{tree}(s, t = -k_y^2)\, e^{-ibk_y}$$

Here $A_{eik}^{TME} = -\frac{Q^2 s\left(1 - i\frac{m}{\sqrt{-t}}\right)}{t - m^2} \xrightarrow[Q^2 \to s(\,)]{} \neq \mathcal{M}_{eik}^{TMG}$

Massive D.C. artifact

$$-i\mathcal{M} \xrightarrow{\text{eikonal}} \frac{n_t^2}{t - m^2} - \frac{(n_s + n_t + n_u)^2}{m^2}$$

Need info. Outside of eikonal lim.

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Point-particle propagating in background:

$$ds^2 = -2du\,dv + \kappa f(y)\delta(u)du^2 + dy^2$$

$$\mathcal{M}^{p.p.}_{eik.} = \int \frac{dy}{2\pi}\, e^{-iqy - \frac{is}{4}\kappa f(y)} = \delta(q) + \frac{\mathcal{M}_{eik}}{2\pi s}$$

Boundary conditions on $g_{\mu\nu}$ = Choice of $i\epsilon$ prescription in phase shift

Which b.c. / $i\epsilon$ prescription → D.C. in coordinate space?

- Useful for time delay computation

Edelstein, Giribert, Gomez, Kiricarslan, Leoni, Tekin; 2016

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Kerr-Schild metric

$$h_{\mu\nu} = k_\mu k_\nu \phi \qquad k_\mu dx^\mu = du$$

★ Special choice of b.c.

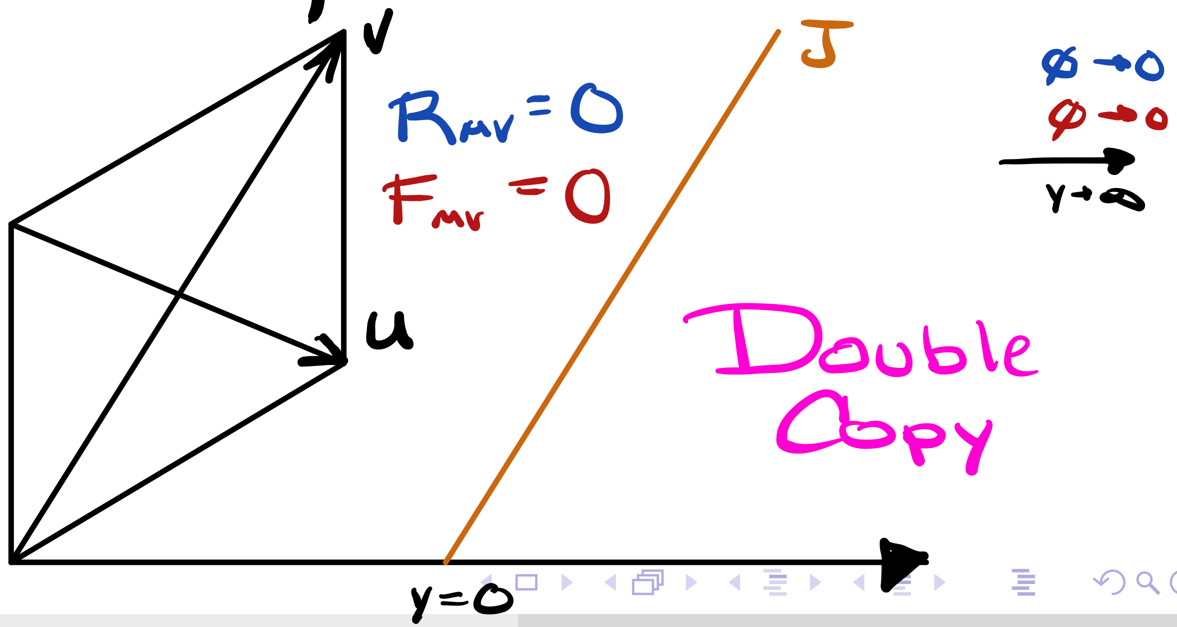
$$\phi = \frac{\kappa}{2}\frac{2E}{m}\left(e^{-my}\theta(y) + (1-my)\theta(-y)\right)$$

$$A_\mu^a = c^a k_\mu \phi$$

$$\phi = g\frac{Q}{m}\left(e^{-my}\theta(y) + \theta(-y)\right)$$

$$\phi^{a\tilde{a}} = c^a c^{\tilde{a}} \phi$$

$$\phi = \frac{\lambda}{2m} e^{-m|y|}$$



Cotton Double Copy

Shockwaves satisfy: $C_{uu} = \frac{m}{2} \frac{{}^*F_u \, {}^*F_u}{\phi}$ Why?

Look at linearized e.o.m plane waves

$$\varepsilon_{\mu\alpha\beta} \nabla^\alpha \left(R_\nu{}^\beta - \frac{1}{4} g_\nu{}^\beta R\right) \overset{\text{lin. eom}}{=} \frac{m}{2} \varepsilon_{\mathcal{C}\nu\,\alpha\beta} F^{\alpha\beta}$$

$$C^{lin}_{\mu\nu} \propto \frac{\nabla^\lambda F^{lin}_{\lambda(\nu} \, \varepsilon_{\mu)\rho\gamma} F^{lin\,\rho\gamma}}{e^{ip\cdot x}}$$

$\mathcal{O}(p^3)$ $\quad$ $\mathcal{O}(p^2)$ $\mathcal{O}(p)$

Conclusions and Future Directions

$(\text{TMYM})^2 = \text{TMG}$

Robust evidence: Amplitudes + Classical

Open Questions:

- Coupling to non-eikonalized matter? (Anyons)
- Type D? (Squashed AdS)
- Origin via dimensional reduction?

Conclusions and future directions