AD GRAVITAS: An exploration of the Unruh effect in particle physics

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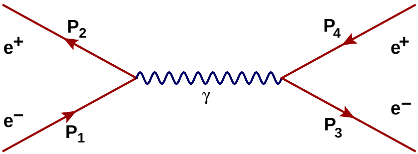
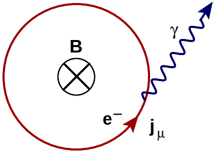
A Timeline of Milestones

Historical Background: The worldline from Schrodinger, Parker, Hawking, and Unruh to spacetime thermality and analogue gravity.
New Results: Applications of thermalized accelerated QED to radiation reaction in high energy channeling radiation experiments (Wistisen et al. 2018, Lynch et al. 2021).
Future Outlooks: Acceleration $\leftrightarrow$ gravitation analogue systems and systematic studies of thermality.



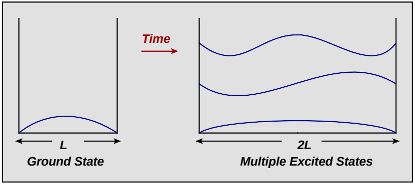


Lets see what "QEDCST" has to say about high energy channeling!

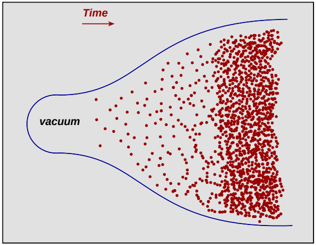
The Schrodinger Paradox

Schrodinger examined the wave equation in an expanding universe and was troubled by particle creation (Schrodinger 1939); a variant of Olbers paradox.

$$i\partial_t \psi(x,t) = \left[\frac{-\nabla^2}{2m} + V(x,t) \right] \psi(x,t)$$



Excitation of energy levels by the expanding box.

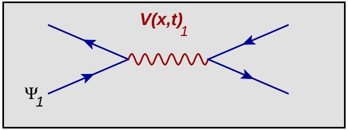


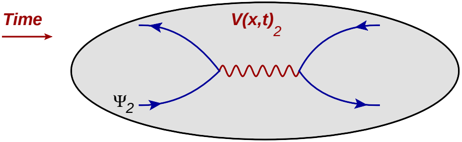
Runaway particle creation.

This paradox was resolved by Leonard Parker in the late 60's via the use of the Bogoliubov transformation.

The solutions to the wave equation forms a complete and orthonormal set. We also have creation and annihilation operators (Bogoliubov 1958).

$$\left[\Box + m^2 + V(x,t)\right]\psi(x,t) = 0$$





Vacuum 1

$$\hat{\psi}_1 = \sum \hat{a}_k f_+ + \hat{a}_k^\dagger f_-$$

$$\hat{a} = \alpha^* \hat{b} + \beta \hat{b}^\dagger$$

$$\hat{a}^\dagger = \alpha \hat{b}^\dagger + \beta^* \hat{b}$$

$$\langle 0 | \hat{b}^\dagger \hat{b} | 0 \rangle_1 = |\beta|^2$$

Vacuum 2

$$\hat{\psi}_2 = \sum \hat{b}_\kappa g_+ + \hat{b}_\kappa^\dagger g_-$$

$$\hat{b} = \alpha \hat{a} - \beta \hat{a}^\dagger$$

$$\hat{b}^\dagger = \alpha^* \hat{a}^\dagger - \beta^* \hat{a}$$

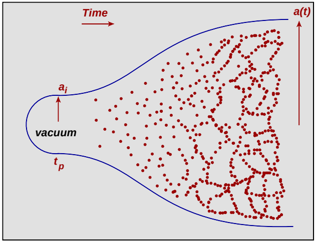
$$\langle 0 | \hat{a}^\dagger \hat{a} | 0 \rangle_2 = |\beta|^2$$

The computation of things like the number operator, $N(\omega) = |\beta|^2$, lies at the foundation of quantum field theory in curved spacetime!

The Birth of Quantum Field Theory in Curved Spacetime

For his PhD thesis, Leonard Parker analyzed particle creation by an expanding universe; the Parker effect. This was accomplished by using the Bogoliubov transformation. (Parker 1966, 1976).

Parker in Valencia, Spain.



Particles created by the expansion the universe.

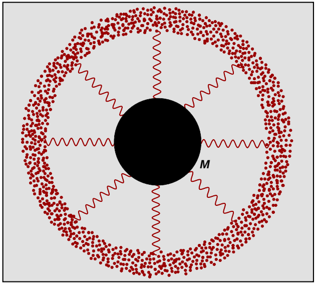
He found that conformally invariant wave equations, i.e. photons, yield no particle production! The initial result was not known to be thermal (1966). Later (1976), found the distribution of particles created depends on the initial and final scale factors, a_i and $a(t)$, and the Planck time, t_p.

$$N(\omega) = \frac{1}{e^{4\pi\omega t_p a(t)/a_i} - 1}$$

Hawking Effect

Utilizing the same Bogoliubov technique, Stephen Hawking analyzed the particles created by a spherically symmetric, pressureless, and gravitational collapsing ball of dust. (Hawking 1974).

Hawking at the Perimeter Institute.



Hawking radiation emitted by a black hole.

The distribution of particles created depends on the surface gravity, g,

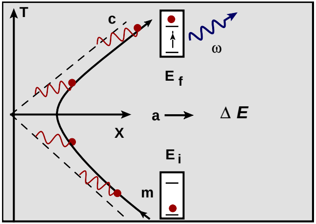
$$N(\omega) = \frac{1}{e^{2\pi\omega/g} - 1}.$$

The temperature is given by $T = \frac{\hbar c^3}{8\pi G k_B M}$. Note k_B = thermodynamics!

Unruh Effect

By taking the near horizon limit of the Schwarzschild metric, Bill Unruh analyzed the interactions of an *Unruh-DeWitt detector* with Hawking radiation. This limit yields the Rindler spacetime associated with uniform acceleration (Unruh 1976).

Unruh and company at the Perimeter Institute.



Unruh-DeWitt detector transition.

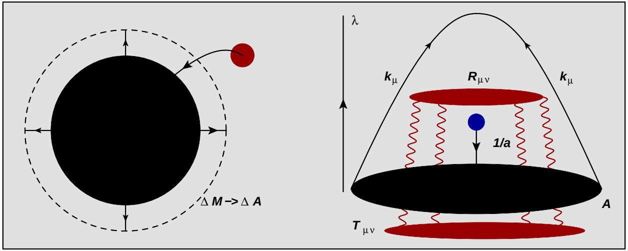
The distribution of particles created depends on the acceleration, a,

$$N(\omega) = \frac{1}{e^{2\pi\omega/a} - 1}.$$

This built on the work of Stephen Fulling (1973) and Paul Davies (1975); now known as the Fulling-Davies-Unruh temperature.

The Thermodynamics of Spacetime

Building upon these results, we can ask about the implications of the thermodynamics. (Bekenstein 1972 & Jacobson 1995).



Entropy-Area Law

$$dS = dQ/T$$

$$T = 1/(8\pi M)$$

$$S_{BH} = A/4$$

Rindler Horizon

$$dQ \sim \int T_{\mu\nu} k^\mu k^\nu d\lambda dA$$

$$dA \sim \int R_{\mu\nu} k^\mu k^\nu d\lambda dA$$

$$R_{\mu\nu} - 1/2 R g_{\mu\nu} + \Lambda g_{\mu\nu} = 4\pi G T_{\mu\nu}$$

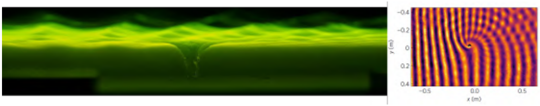
The Einstein equation is a thermodynamic equation of state!

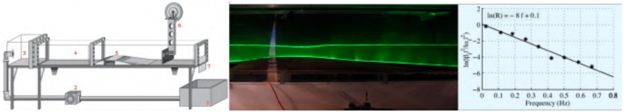
Analogue Gravity: Now A Thriving Community!

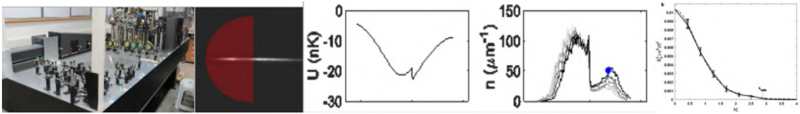
Experimental Black-Hole Evaporation?

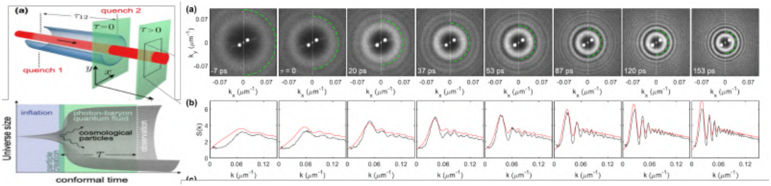
W. G. Unruh
Department of Physics, University of British Columbia, Vancouver, British Columbia V6T 2A6, Canada

(Received 8 December 1980)





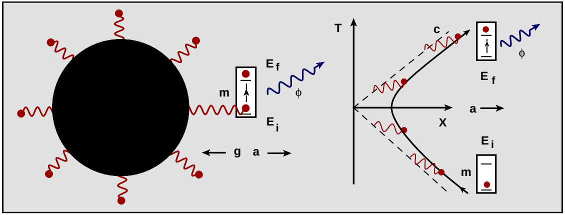




BUT CAN WE FIND A FUNDAMENTAL SYSTEM?

Unruh Effect and the Unruh-DeWitt Detector

Unruh-DeWitt detectors, or two level systems, provide a way to analyze radiation emission from uniformly accelerated sources. They can also be used to model elementary particles (Cozzella et al. 2020).



Unruh-DeWitt detector transitions accompanied by the emission of a particle.

For massless scalar fields, we have the action $\hat{S} = q \int d^4x \hat{\phi}(x)\hat{m}(\tau)$.

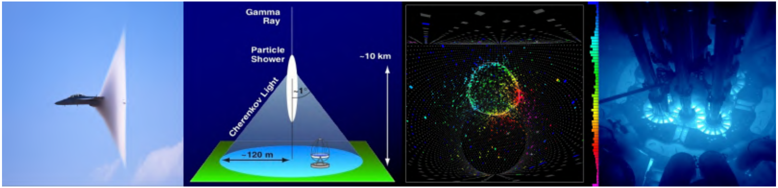
$$\mathcal{P} = |\langle\phi| \otimes \langle E_f|\hat{S}|E_i\rangle \otimes |0\rangle|^2.$$

The response function, $\Gamma = \frac{d\mathcal{P}}{d\tau}$, is thermalized at $T_{FDU} = \frac{a}{2\pi}$.

$$\Gamma = q^2 \int d\tau e^{-i\Delta E\tau} G[x', x] = \frac{q^2}{2\pi} \frac{\Delta E}{e^{2\pi\Delta E/a} - 1}$$

Example: Cherenkov Radiation

Charged particles traveling faster than the speed of light in an indexed medium will emit “superluminal booms” known as Cherenkov radiation (Cherenkov 1934).



Cherenkov emission forms a well defined ring or cone.

The emission spectrum, $\frac{d\Gamma}{d\omega}$, is determined by the Frank-Tamm formula (Frank & Tamm 1937),

$$\boxed{\frac{d\Gamma}{d\omega} = \alpha\beta \sin^2(\theta_c) \qquad \cos(\theta_c) = \frac{1}{n\beta}}.$$

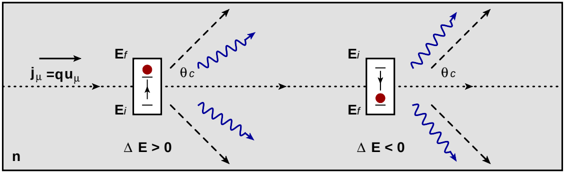
Example: Cherenkov Radiation

The QED current interaction is given by $\hat{S} = q \int d^4x \hat{A}^\mu(x) \hat{j}_\mu(\tau)$. The QED response function is given by (Lynch et al. 2019),

$$\Gamma = q^2 \int d\tau e^{-i\Delta E \tau} u_\mu u_\nu \langle 0| \hat{A}^{\dagger\nu}(x') \hat{A}^\mu(x) |0\rangle .$$

For trajectories with superluminal constant velocity, $u^\mu = (\gamma, 0, 0, \gamma\beta)$, in an indexed medium, we obtain the Frank-Tamm formula.

$$\frac{d\Gamma}{d\omega} = \alpha\beta \sin^2(\theta_c) \qquad \cos(\theta_c) = \frac{1}{n\beta} + \frac{\Delta E}{n\beta\omega\gamma}$$



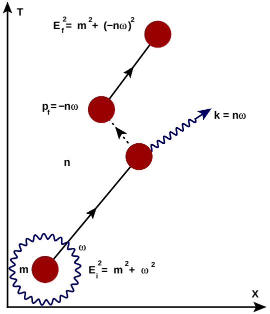
Cherenkov emission from an Unruh-DeWitt detector yields the anomalous Doppler effect.

Example: Incorporating Radiation Reaction

MASS RENORMALIZATION: The energy gap is defined as the difference between the initial and final electron energy, $\Delta E = E_f - E_i$.

$$\Delta E = \sqrt{m^2 + (-n\omega)^2} - \sqrt{m^2 + \omega^2} \approx \frac{\omega^2(n^2 - 1)}{2m}$$

This reproduces the known recoil correction to Cherenkov emission identically! (Sokolov 1940 & Tsytovich 1963)



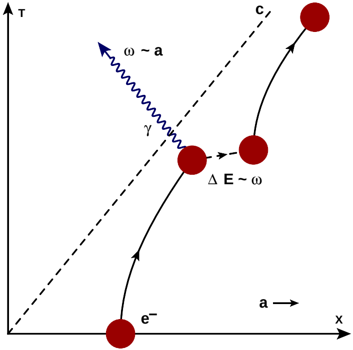
Diagramatics of mass renormalization in an optical medium.

Radiation Reaction:

1. The photon builds up around the particle.
2. The rest energy now includes the photon energy.
3. The electron-photon "atom" decays.
4. The recoil momentum is enhanced by the index.
5. *In vacuum, no mass renormalization!*

Incorporating the radiation reaction force on the electron in the Lorentz force equation (Dirac 1938).

$$m\frac{du^\mu}{d\tau} = qF^{\mu\nu}u_\nu + \frac{2}{3}\alpha\left[\frac{d^2u^\mu}{d\tau^2} + a^2u^\mu\right]$$



Spacetime diagram of recoil.

THE PARADOX:

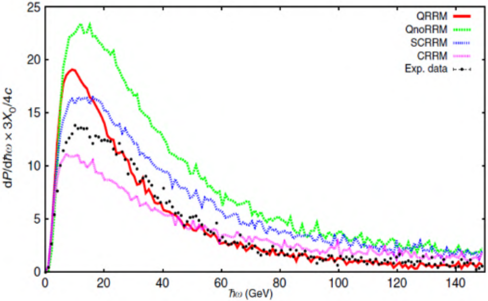
1) Acceleration ⇒ Energy.
2) Energy ⇒ Recoil.
3) Recoil ⇒ Acceleration.

The full equation of motion with recoil still remains an open problem!

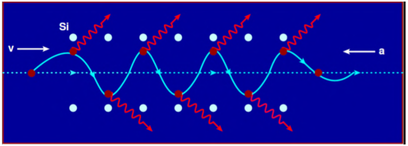
Models based on Lorentz force + variants are used in the experiment.

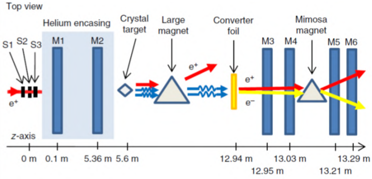
Channeling Radiation Experiment + Radiation Reaction

178.2 GeV positrons fired into samples of amorphous and single crystal silicon at NA63-CERN. A first observation of quantum radiation reaction in channeling radiation. (Tobias N. Wistisen, Antonino Di Piazza, Helge V. Knudsen & Ulrik I. Uggerhoj, Nature Communications, 2018).



The power spectrum for the 3.8 mm sample high energy channeling experiment compared to the standard radiation reaction models.





Diagramatics of channeling radiation and the NA63 experiment.

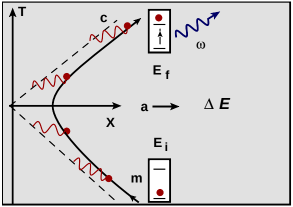
Larmor Power Spectrum

The QED current interaction is given by $\hat{S} = q\int d^4x \hat{A}^\mu(x)\hat{j}_\mu(\tau)$. The QED response function is given by (Lynch et al., NJP 2019, arXiv:1901.00855),

$$\Gamma = q^2 \int d\tau e^{-i\Delta E\tau} u_\mu u_\nu G^{\mu\nu}[x', x].$$

The uniformly accelerated trajectory $u^\mu = (\sinh(a\tau), 0, 0, \cosh(a\tau))$, yields a power spectrum, $\frac{d\mathcal{S}}{d\omega} = \frac{d(\Gamma\omega)}{d\omega}$, given by,

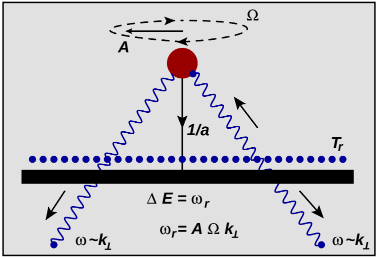
$$\frac{d\mathcal{S}}{d\omega} = -i\frac{2}{3}\alpha\frac{\omega^2}{a}e^{-2\pi\Delta E/a}\Bigg[(2\gamma^2-1)H^{(2)}_{-\frac{2i\Delta E}{a}}\left(-\frac{2i\omega\gamma}{a}\right) -\frac{1}{2}\left(H^{(2)}_{-\frac{2i\Delta E}{a}+2}\left(-\frac{2i\omega\gamma}{a}\right)+H^{(2)}_{\frac{-2i\Delta E}{a}-2}\left(-\frac{2i\omega\gamma}{a}\right)\right)\Bigg].$$



Spacetime diagram of Larmor radiation with energy transition.

The Rindler Frame Processes

A Rindler frame analysis provides us with an explicit dependence on the Unruh bath. (Cozzella et al., PRL 2017, Paithankar et al. PRD 2020)



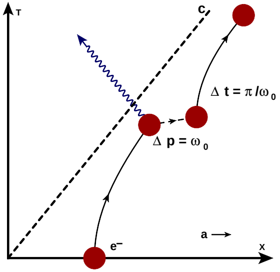
1. $\mathcal{P}^r_{abs} \sim \frac{1}{e^{\frac{\omega_r}{T_r}}-1}$ and $\mathcal{P}^r_{emi} \sim 1 + \frac{1}{e^{\frac{\omega_r}{T_r}}-1} \Rightarrow \mathcal{P}^r_{tot} \sim \sinh(\pi\omega_r/a)\coth(\omega_r/(2T_r))e^{\frac{\pi\omega_r}{a}}$
2. $\mathcal{P}^m_{\downarrow} = \mathcal{P}^m_{\uparrow} e^{\Delta E/T_{FDU}} \Rightarrow \mathcal{P}^m_{tot} \sim 1 + e^{\Delta E/T_{FDU}}$
3. $\mathcal{P}^m_{tot} = \mathcal{P}^r_{tot}$

$$\boxed{1 + e^{\Delta E/T_{FDU}} = 2\sinh(\pi\omega_r/a)\coth(\omega_r/(2T_r))e^{\frac{\pi\omega_r}{a}}}$$

By leaving the temperature of the Rindler bath arbitrary, we can measure, and thus confirm, the presence of the Fulling-Davies-Unruh temperature!

Comparison to Data

$$\frac{d\mathcal{S}}{d\omega} = -i\frac{2}{3}\alpha\frac{\omega^2}{a}\left[(2\gamma^2-1)H^{(2)}_{-\frac{2i\Delta E}{a}}\left(-\frac{2i\omega\gamma}{a}\right) - \frac{1}{2}\left(H^{(2)}_{-\frac{2i\Delta E}{a}+2}\left(-\frac{2i\omega\gamma}{a}\right) + H^{(2)}_{\frac{-2i\Delta E}{a}-2}\left(-\frac{2i\omega\gamma}{a}\right)\right)\right] \times 2\sinh(\pi\omega_r/a)\coth(\omega_r/(2T_r))e^{\frac{\pi\omega_r}{a}}$$



Recoil Acceleration:

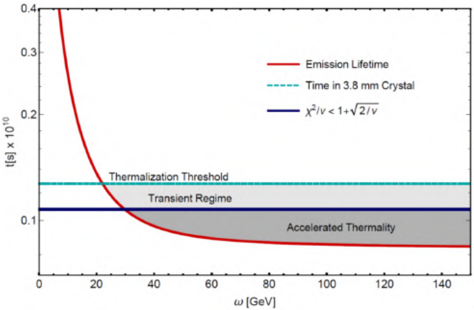
$$a = \frac{1}{m}\frac{\Delta p}{\Delta t} = \frac{\omega_0^2}{\pi m}.$$

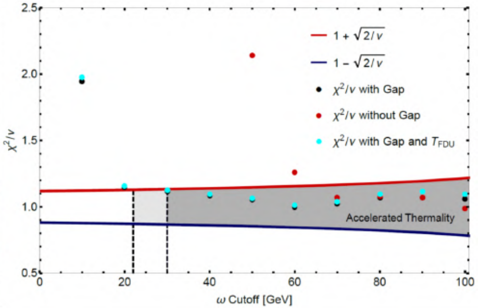
Best Fit Parametrization:

1. Scale factor s.
2. Acceleration $\tilde{a}$.
3. Temperature T_r.
4. $\Delta E = \sqrt{m^2+\omega^2} - m \sim \omega^2/(2m)$
   $\Delta E = a_0 + a_1\omega + a_2\omega^2 + a_3\omega^3$
   a_0 encodes the channeling oscillation, a_1 the Rindler term, and a_2 the recoil.
5. Best fit and χ^2/ν.

Thermalization and χ^2/ν for the 3.8 mm Sample

For the Unruh effect to manifest, the energy gap, ΔE, must have time to thermalize. Thermalization time, $t(\omega) = \frac{1}{\int_0^\omega \frac{d\mathcal{S}}{d\omega'} \frac{1}{\omega'} d\omega'}$.





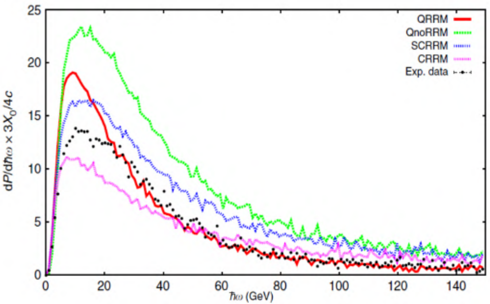
(Left) The time to traverse the crystal and the emission lifetime (IN THE LAB FRAME) for the first cutoff which yields a $\chi^2/\nu \lesssim 1$. (Right) The best fit χ^2/ν for each cutoff.

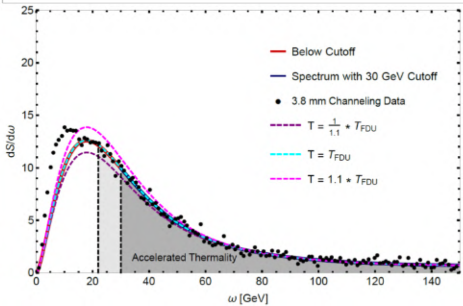
1. Historically speaking, the thermalization time has ruined ALL previous attempts!
2. *Above* ~ 22 *GeV the system has time to thermalize*
3. The χ^2/ν rapidly converges to 1 below the thermalization threshold.

Now that we have evidence of thermality, lets look at power spectrum for the 30 GeV cutoff.

Comparison of Theory to the Data + χ^2/ν Analysis

The best fit power spectrum with the first $\chi^2/\nu \sim 1$.



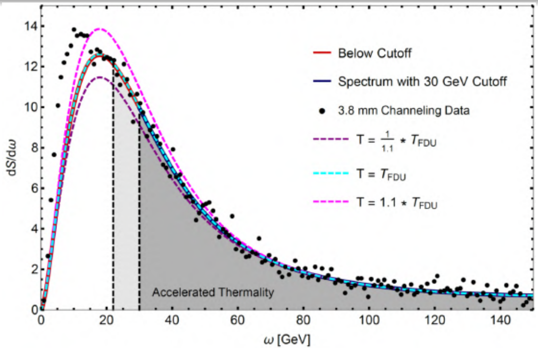


LEFT:Standard models of radiation reaction. RIGHT:Theory which incorporates the Unruh effect.

Model	QRRM	QnoRRm	SCRRM	CRRM	AQED
χ^2/ν	47.4	187.7	50.2	22.4	1.12

1. The low energy portion of the power spectrum is not described by the theory (potentially pure channeling radiation).
2. *Are we able to witness the onset of thermality!?!*
3. *A potential window into the information loss paradox!*

Parameter Analysis of Best Fit



	s	$\tilde{a}$ [PeV]	a_0 [GeV]	a_1	a_2 [GeV^{-1}]	a_3 [GeV^{-2}]
Measured	12.93	7.939	0.00197	0.0120	846.2	0.4691
Expected	-	~ 6.24	$\Omega \lesssim .004$	0.011	978.2	-

$$\tilde{a}_{RR} = \frac{\omega_{100}^2}{\pi m} = 6.24\ PeV$$

$$a_1 = A_{Si} a_0 = .011$$

$$\Delta E \sim \Omega + \Omega A\omega + \frac{\omega^2}{2m}$$

$$T_{FDU} = 1.80 \pm .51\ PeV$$

$$T_R = 1.96 \pm .49\ PeV$$

$$T_R = T_{FDU}[1.09 \pm .41]$$

$$T_{RR}^{150} = 2.23\ PeV$$

This measurement realizes the experimental proposal put forth by Cozzella et al. (2017) for directly measuring T_{FDU}.

With temperature $T = \frac{a}{2\pi} = \frac{\alpha}{2\pi} E^n$, we can compute the entropy and area change.

$$dS = dE'/T$$

$$ds = \frac{2\pi m}{\alpha} \frac{dE}{E^{n+1}}$$

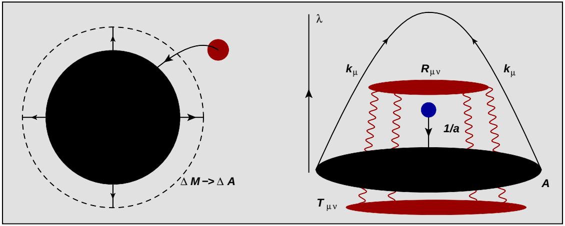
$$\Delta S = \frac{2\pi m}{\alpha n} \left[\frac{1}{E_f^n} - \frac{1}{E_i^n} \right]$$

$$\Delta A = 8\pi \int dy^2 \int_0^\infty dv v T_{\mu\nu} k^\mu k^\nu$$

$$T_{\mu\nu} k^\mu k^\nu = \frac{\Delta E}{\gamma} \delta(v - 1/a) \delta^2(y)$$

$$\Delta A = \frac{8\pi m \Delta E}{\alpha E^{n+1}}$$

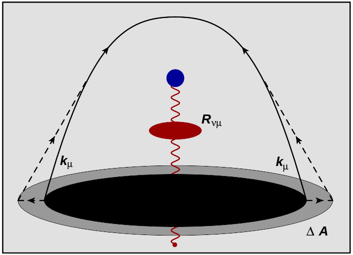
$$\boxed{\frac{\Delta A}{\Delta S} = 4\ell_p^2 \frac{n \Delta E}{E_i^{n+1}} \left[\frac{1}{E_f^n} - \frac{1}{E_i^n} \right]^{-1}}$$



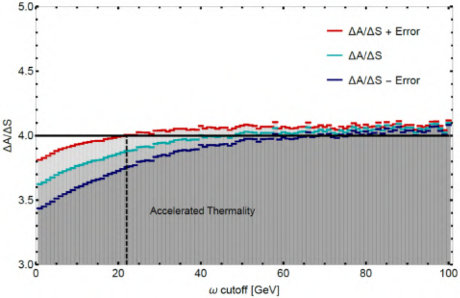
Thermalized emission into the Rindler horizon.

The positron radiates into the Rindler horizon and changes the area in accordance with the 2nd law of thermodynamics. (Bekenstein 1972, Hawking 1974, and Bianchi & Satz 2013). With temperature $T_{RR} = \frac{\omega_0'^2 E^2}{2m^3\pi^2}$, we have

$$\boxed{\frac{\Delta A}{\Delta S} = 8\ell_p^2 \frac{\Delta E}{E_i^3}\left[\frac{1}{(E_i - \Delta E)^2} - \frac{1}{E_i^2}\right]^{-1}}.$$



Rindler horizon area change from geodesic deviation.



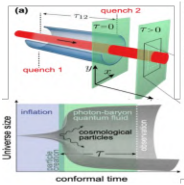
The Bekenstein-Hawking area-entropy ratio.

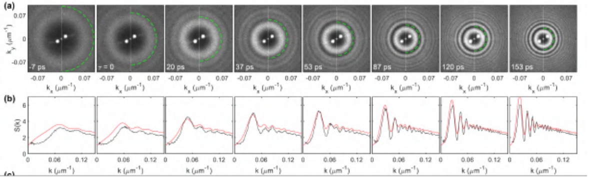
1. The area-entropy ratio does indeed converge, $\frac{\Delta A}{\Delta S} \to 4\ell_p^2$.
2. Note the same $\sim$ 22 GeV threshold!

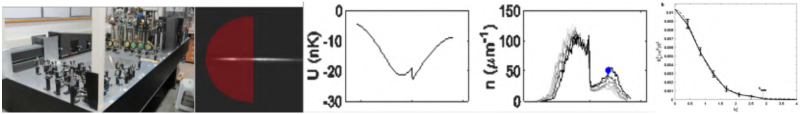
Analogue Gravity: Now an Accelerating Community!

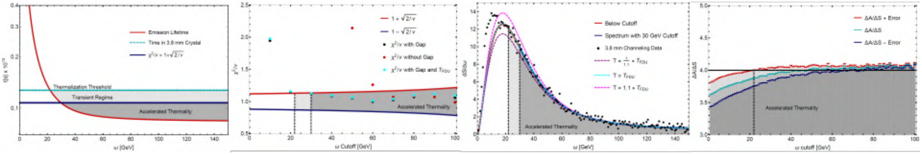
Experimental Black-Hole Evaporation?

W. G. Unruh
Department of Physics, University of British Columbia, Vancouver, British Columbia V6T 2A6, Canada
(Received 8 December 1980)





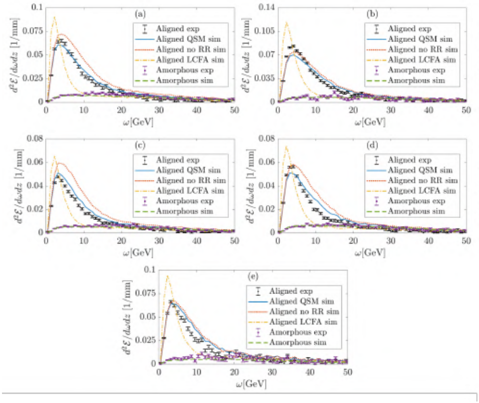


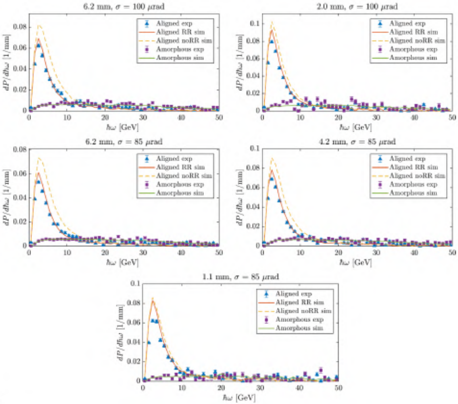


The Parker, Hawking, and Unruh effects are now being explored experimentally!

Future Analysis

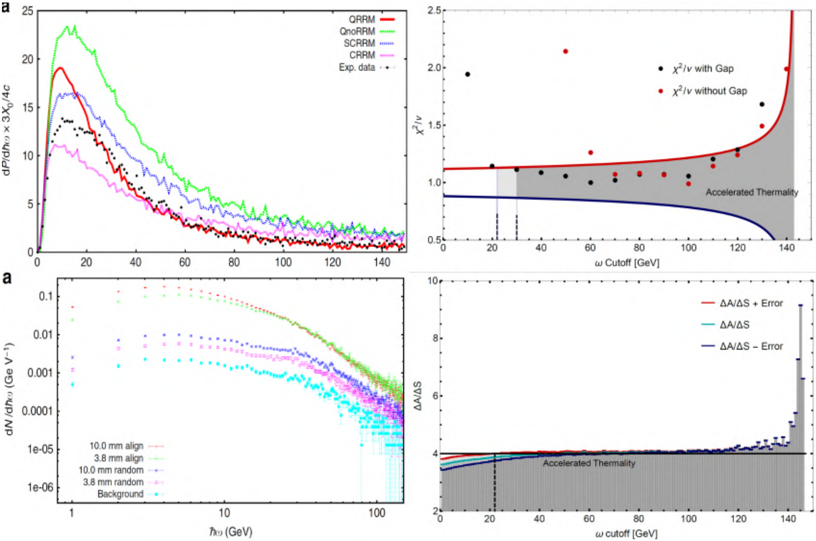
A wide range of new data samples are already available from NA63 (T. N. Wistisen et al. 2019, C. F. Neilson et al. 2020).





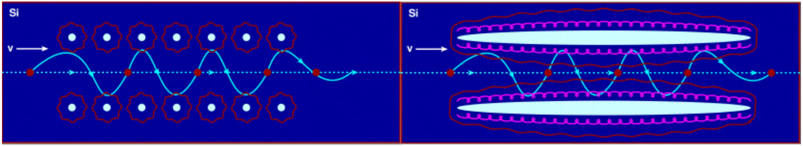
40, 50, and 80 GeV positrons in silicon and diamond with 1.1, 2.0, 4.2, and 6.2 mm thickness.

High Frequency Tail Anomaly



A clear deviation from the standard thermal picture is present in the high energy tail of the spectrum.

Potential Regime of Euler-Heisenberg Dispersion



Vacuum polarization due to the boosted electric field of the crystalline atomic sites may provide an explanation. Ongoing project with Uwe Fischer at SNU.

$$\omega = k\left[1 - \ell_\star^2 k^2\right].$$

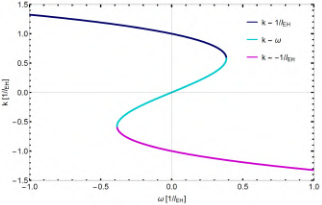
Here $\ell_\star^2 = \frac{2}{30375}\left(\frac{\bar\kappa}{\pi}\right)^2 \frac{\alpha^3}{m_e^2}\left(\frac{\mathcal{E}\gamma}{\mathcal{E}_c}\right)^4$. The critical field $\mathcal{E}_c = 1.3 \times 10^{18}$ V/m. This yields the following indices of refraction,

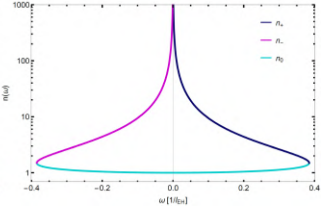
$$n_0 = \frac{1}{1 - \frac{4}{3}\sin^2\left[\frac{1}{3}\sin^{-1}\left[\frac{3\sqrt{3}}{2}\ell_\star\omega\right]\right]}$$

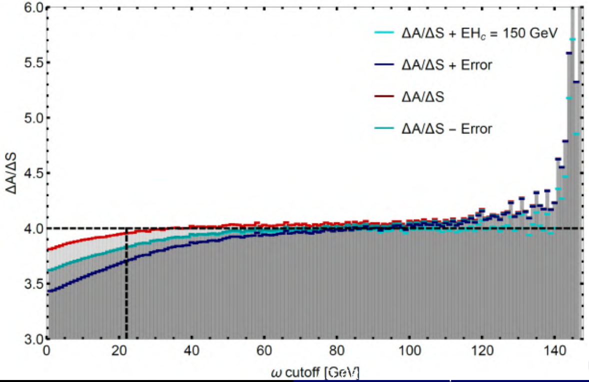
$$n_\pm = \frac{1}{\frac{2}{3}\sin^2\left[\frac{1}{3}\sin^{-1}\left[\frac{3\sqrt{3}}{2}\ell_\star\omega\right]\right] \pm \frac{1}{\sqrt{3}}\sin\left[\frac{2}{3}\sin^{-1}\left[\frac{3\sqrt{3}}{2}\ell_\star\omega\right]\right]}.$$

Potential Regime of Euler-Heisenberg Dispersion

The EH dispersion gives a hard cutoff of $\omega_\star = \frac{2}{3\sqrt{3}}\frac{1}{\ell_\star}$. Here we use $\omega_\star = 150$ GeV and $\ell_\star^{-1} = 386$ GeV. This corresponds to an interaction distance of $\sim A_{Si}/4$.







Conclusions

1. **Radiation Reaction:** Measurement of the recoil acceleration, $a = \frac{\omega^2}{\pi m}$, along with the best fit parameter $a_2 = 846\ GeV^{-2} \sim \frac{1}{2m} = 979\ GeV^{-1}$ confirms the presence of recoil.
2. **The Unruh Effect:** The power spectrum obeys detailed balance at $T_{FDU} = \frac{a}{2\pi}$, has a covariant description which relies explicitly on photon exchange with the thermal Rindler bath at T_{FDU}, and yields a $\chi^2/\nu \sim 1$ beyond the thermalization threshold.
3. **Bekenstein-Hawking Area-Entropy Law:** The area-entropy ratio converges to $4\ell_p^2$ beyond the same thermalization threshold.
4. **Acceleration-Induced Thermality:** The chi-squared statistic and confirmation of the BH-AE law provides experimental evidence for the first observation of the accelerated thermality *in a non-analog system!*
5. **Future:** NA63 already has more data sets out. Different beam energy and crystal samples may enable a systematic study of the acceleration-induced thermality. *"The concepts studied at NA63 even apply in a gravitational analogue – Hawking radiation from black holes – which remains to be detected."*

Thank you!