INSTITIÚID ÁRD-LÉINN BHAILE ÁTHA CLIATH

SCOIL NA FISICE COSMAÍ

Dublin Institute for Advanced Studies

SCHOOL OF COSMIC PHYSICS

GEOPHYSICAL MEMOIRS NO. 2, PART 1

MEASUREMENTS OF GRAVITY IN IRELAND

PENDULUM OBSERVATIONS AT DUBLIN, SLIGO, GALWAY AND CORK

BY

A. H. COOK

DEPARTMENT OF GEODESY AND GEOPHYSICS

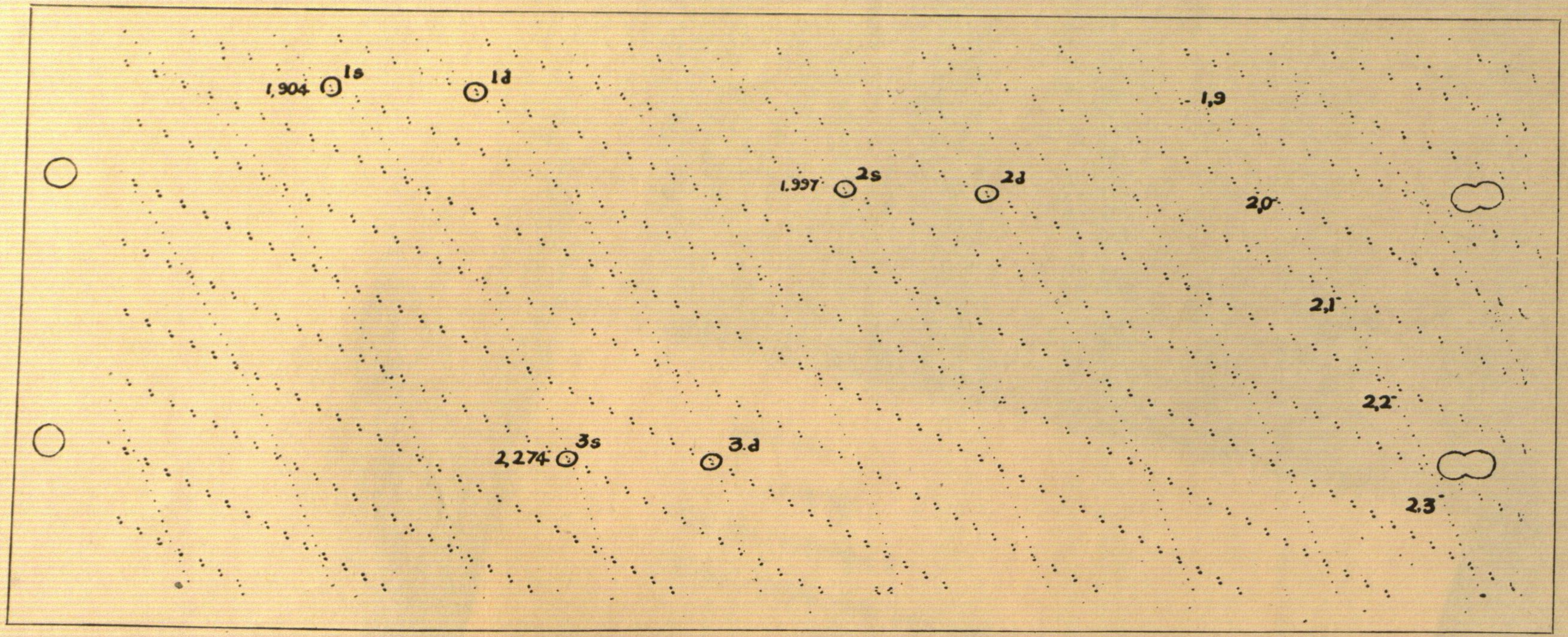
CAMBRIDGE UNIVERSITY

DUBLIN:

1950

Price: 10s. 6d.

EXAMPLE OF PENDULUM RECORD



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PENDULUMS: VI. ***A*** **and** ***B***

Cambridge, 17·13 hrs 20th April, 1949

The paper moved to the left and upwards, the axis of the drum being parallel to the short sides. The figures, *1,904* etc., are the times of the short timing marks in half-seconds; the figures *1,9, 2,0* . . ., are the times of the long timing marks in hundreds of half-seconds.

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CONTENTS

ABSTRACT

The values of gravity at Dunsink Observatory (Dublin) and at Sligo, Galway and Cork, have been compared with the value at Cambridge by pendulum observations. The procedure, the sites and the results are described and the errors which occur are compared with those expected. The pendulum values are combined with gravimeter observations made at the same time, and the *Differences of gravity from Cambridge* found by least squares are :—

Dunsink Observatory	+120·84 mgal.	±0·56
Sligo Courthouse	+197·28 „	±0·82
Galway, University College	+ 96·91 „	±0·64
Cork, University College	− 22·51 „	±0·87

The value of gravity at Cambridge (Pendulum House) has been taken as 981·2650 cm/sec^2.

The discussion of the residuals shows that there are no systematic errors in the corrections made for temperature, pressure and amplitude, but that random changes in the lengths of the pendulums occur between stations.

I. INTRODUCTION

Prior to the measurements now to be described, no observations of gravity had been made in Ireland and there were many reasons why it was desirable to fill this deficiency. The submarine observations of Browne and Cooper (1950) remained incomplete while gravity was unknown on the land nearest to them. Measurements in Wales (Cook and Thirlaway, 1948) had shown that gravity increased towards the Irish Sea, whereas submarine observations in the Irish Sea gave somewhat lower values, and it was of interest to know how the gravity field of Ireland was related to that of Wales and the Irish Sea. The Irish Geological Survey had made a survey of the vertical magnetic force in Ireland and a comparison of the magnetic anomalies, the gravity anomalies and the geological structure might be interesting. In particular it had been suggested that a large magnetic anomaly on Carnsore Point should be investigated and we were anxious to have observations over the granite of the Wicklow Mountains, since gravity measurements over the Dartmoor granite had shewn very large, yet local, negative anomalies (see Bullard and Jolly, 1936). Finally, the absence of observations in Ireland was serious when an attempt was made to use the values of gravity in the British Isles for geodetic purposes.

It was therefore arranged that gravity measurements should be made by members of the Department of Geodesy and Geophysics, Cambridge University, using apparatus and equipment owned by the Department, and that they should be financed by the School of Cosmic Physics in the Dublin Institute for Advanced Studies. The author made pendulum observations at Dunsink Observatory, Sligo, Galway and Cork, and the values of gravity at these places were compared by gravimeter observations, an account of which by Mr. H. I. S. Thirlaway forms Part Two of this Memoir. The observations were made in March and April, 1949.

These measurements were sponsored and financed by the School of Cosmic Physics in the Dublin Institute for Advanced Studies, and in addition we had more particular assistance from many people in Ireland. Professor H. A. Brück, Senior Professor in the School of Cosmic Physics and Director of Dunsink Observatory, made available a laboratory for the measurements there, lent us a wireless set and looked after one set of pendulums which was not taken to the other places. Mr. Bishopp, the Director of the Irish Geological Survey, assisted us in every conceivable way, and put the facilities and staff of his Department at our disposal. Mr. Cunningham, an officer of the Geological Survey, assisted in most of the actual pendulum observations. Finally, we wish to express our gratitude to the Presidents of the University Colleges of Galway and Cork who made available rooms for the pendulum observations, afforded us every help and, with the members of their staffs, made us most welcome.

I wish to thank the Department of Scientific and Industrial Research of Great Britain for a maintenance grant which enabled me to take part in these measurements.

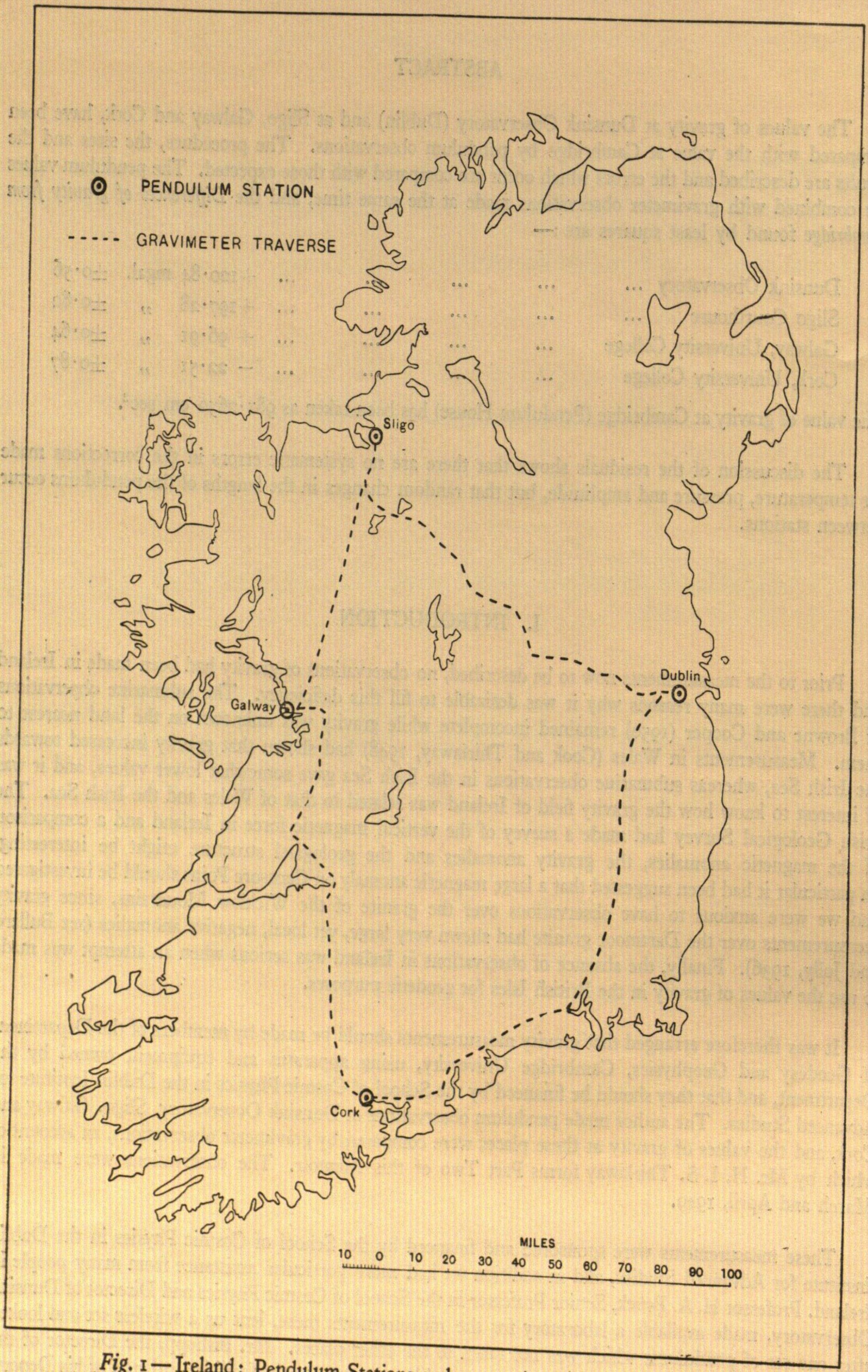


Fig. 1—Ireland: Pendulum Stations and connecting Gravimeter Traverses.

II. THE SITES OF THE PENDULUM OBSERVATIONS

The places where pendulum observations were made and the lines of gravimeter observations that connect them are shown in *Fig.* 1

The plans of the sites of the pendulum observations are shown in *Fig.* 2.

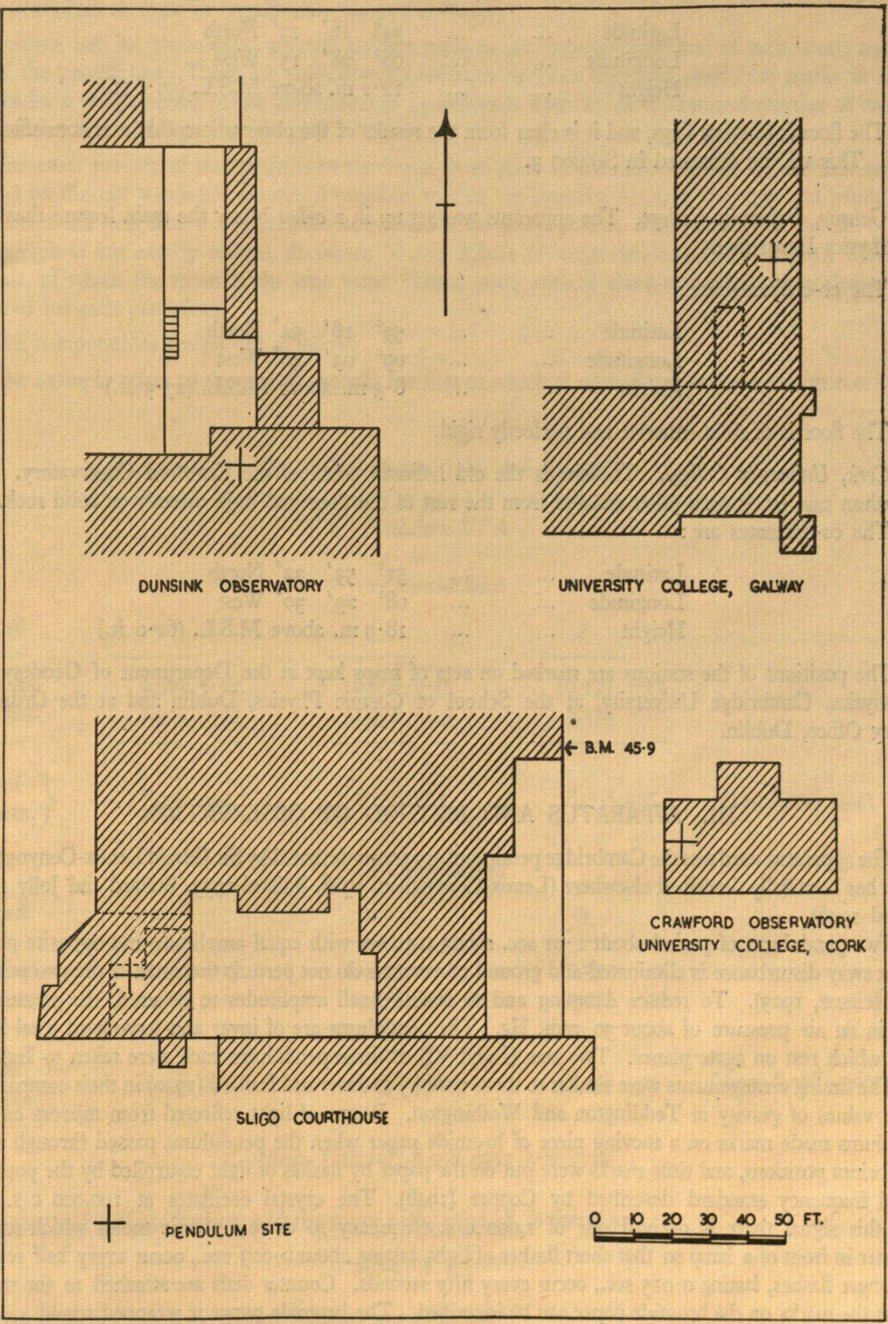


Fig. 2—Plans of Sites of Pendulum Stations.

Dunsink Observatory, Dublin, about four miles North-west of Dublin, has been chosen as the gravity base station for Ireland. The pendulum station was in the small laboratory in the basement, on the floor below the meridian circle, from which it is about 15 yds. North and 10 yds. West. The co-ordinates, very nearly those of the centre of the meridian circle, are:

Latitude	...	...	53° 23′ 13·1″ North
Longitude	...	...	6° 20′ 16·5″ West
Height	...	...	80·8 m. above M.S.L.* (265 ft.)

* The Irish Ordnance datum is about 8 ft. below Mean Sea Level.

The floor on which the pendulum apparatus stood is composed of wooden blocks set in pitch on a solid concrete base. The behaviour of the pendulum shows that this was a sufficiently rigid base (see Section 3).

Sligo Courthouse. The apparatus was set up in an old muniment room. The co-ordinates are:

Latitude	... ...	54° 16′ 12″ North
Longitude	... ...	08° 28′ 15″ West
Height	... ...	12·1 m. above M.S.L. (39·6 ft.)

The floor is of stone flags, and it is clear from the results of the observations that it is not sufficiently rigid. This will be discussed in Section 3.

Galway, University College. The apparatus was set up in a cellar below the main lecture theatre of the Physics Department.

The co-ordinates are:

Latitude	... ...	53° 16′ 34″ North
Longitude	... ...	09° 03′ 40″ West
Height	... ...	8·3 m. above M.S.L. (27·3 ft.)

The floor is of solid concrete and perfectly rigid.

Cork, University College. The site is the old heliostat room of the Crawford Observatory. The pendulum case stood on a pillar isolated from the rest of the floor and built directly on solid rock. The co-ordinates are:

Latitude	... ...	51° 53′ 32″ North
Longitude	... ...	08° 29′ 30″ West
Height	... ...	18·9 m. above M.S.L. (62·0 ft.)

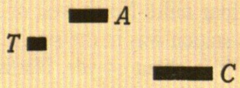
The positions of the stations are marked on sets of maps kept at the Department of Geodesy and Geophysics, Cambridge University, at the School of Cosmic Physics, Dublin and at the Ordnance Survey Office, Dublin.

III. APPARATUS AND METHOD OF OBSERVATION

The apparatus used was the Cambridge pendulum apparatus designed by Sir Gerald Lenox-Conyngham, which has been fully described elsewhere (Lenox-Conyngham 1928, Bullard 1933, Bullard and Jolly 1936, Bullard 1936).

Two pendulums of period about 1·01 sec. swing together with equal amplitude and opposite phase, so that sway disturbance is eliminated and ground movements do not perturb the mean of the two periods (see Meinesz, 1923). To reduce damping and so enable small amplitudes to be used, the pendulums wing in an air pressure of about 30 mm. Hg. The pendulums are of invar and have hard steel knife edges which rest on agate planes. Two sets (I and VI) of three pendulums each were taken to Ireland.

The timing arrangements were similar to those used by Browne and Bullard (1940) in their comparison of the values of gravity at Teddington and Washington. Beams of light reflected from mirrors on the pendulums made marks on a moving piece of bromide paper when the pendulums passed through their equilibrium positions, and time marks were put on the paper by flashes of light controlled by the portable crystal frequency standard described by Cooper (1948). The crystal oscillates at 100,000 c/s. and from this signal there is derived one of 1,000 c/s. frequency to drive a phonic motor which rotates a shutter in front of a lamp so that short flashes of light, lasting about 0·003 sec., occur every half second and longer flashes, lasting 0·015 sec., occur every fifty seconds. Counter dials are attached to the motor so that the marks on the bromide paper can be identified. The bromide paper is wrapped round a drum which rotates and moves parallel to its axis so that the pendulum and time-marks form spiral patterns. The record is arranged as follows:



A is a mark made by pendulum *A*, and *C* one made by pendulum *C*, while *T* is a half-second timing mark.* The pendulum marks are alternately sharp and diffuse, the former being made when the light

* The paper would be rotating from right to left about an axis parallel to the length of the page and would be moving slowly upwards—compare the Frontispiece.

beam from the pendulum is moving in the opposite direction to the paper, and the latter being made when it is moving in the same direction. An annotated example of a record is given in the Frontispiece. The interval between a pendulum mark and a time mark is measured with a travelling microscope, and as the speed of the paper is known from the interval between successive time marks, the time of the passage of the pendulum through its equilibrium position is found.

Records last for about eight minutes and are made at the beginning and end of each hour's oscillation of the pendulums. There are sufficient coincidences between time and pendulum marks in eight minutes for a rough period of the pendulums to be deduced, from which the integral number of swings between the first and last records may be found.

The exact periods of the pendulums are found from pairs of coincidences, one on the first record and one on the last which are chosen, if possible, so that the intervals between the time and pendulum marks are small (less than 0·015 sec.) for both pendulums of the pair on both records. If the slit of the chronograph is not exactly central, the sharp (*s*) and diffuse (*d*) coincidences occur at slightly different intervals, of which the mean is the true value. Three pairs each of sharp and diffuse coincidences are measured for each pendulum.

The computations are set out as below.

The example refers to the pair of records the first of which is reproduced in the Frontispiece.

PENDULUM HOUSE, 20th APRIL 1949

Pendulum VI A

s-Coincidences

Record	1	2	3	
18·20 hrs.	9,963 −*18*	10,056 −*15*	10,333 −*23*	
17·13 hrs.	1,904 −*12*	1,997 − *9*	2,274 −*18*	
Interval in ½-secs.	8,058·994	8,058·994	8,058·995	Mean 8,058·9943

d-Coincidences

Record	1	2	3	
18·20 hrs.	9,964 −*15*	10,057 −*10*	10,334 −*21*	
17·13 hrs.	1,905 −*12*	1,998 − *6*	2,275 −*18*	
Interval in ½-secs.	8,058·997	8,058·996	8,058·997	Mean 8,058·9967

Mean Interval: 8,058·9955 ½-seconds

There are 7972 half oscillations, and the mean half period, *S*, is therefore:

0·505,4563·2 seconds.

The headings and data in this example are as follows.

The times, 17·13 and 18·20 under "Record" identify the record by the time at which it was started.

1, 2, 3 under "*s*-Coincidences" refer to the sharp coincidences marked 1, 2, 3 on the records, and similarly for the *d*-Coincidences. The four or five-figure numbers under these headings are the numbers of the time marks at the particular coincidences; the figures in *Italics* are the intervals in half milli-seconds between the time marks and the pendulum marks. The sum of these figures is therefore the time in half-seconds at which the pendulum passed through its central position. From them, the interval between the passages on the two records is found, and hence the period of the pendulum.

The periods so found must be corrected to standard pressure (0 mm. Hg.), standard temperature (0°C) and zero amplitude, and the corrections are the following:

For correction to 0°C.,

$$-\delta S = 2{\cdot}6 \times 10^{-7} \times \theta \text{ sec.}$$

where δS is the correction to the half-period, and θ is the temperature in degrees Centigrade (Bullard and Jolly 1936).

For correction to zero pressure,

$$-\delta S = \frac{606 \times 10^{-7}\, p}{(760 + 2{\cdot}79\,\theta)} + \frac{91 \times 10^{-7}\, p^{\frac{1}{2}}}{(760 + 2{\cdot}79\,\theta)^{\frac{1}{2}}}$$

in which p is the pressure in millimetres of mercury (Mace, 1939).

For correction to zero amplitude,

$$-\delta S = 2{\cdot}0\, a^2 \times 10^{-7}$$

where a is the double amplitude measured in centimetres on a scale 1 metre from the pendulums.

A correction has also to be applied for the departure of the output of the crystal frequency standard from the nominal 100,000 c/s. The departure is measured by comparing the first harmonic of this output with the 200,000 c/s. carrier signal of the B.B.C. Light Programme from Droitwich, a transmission which was received clearly all over Ireland. Reception was in fact better on the West coast of Ireland than in Cambridge, no doubt because of the absence of local artificial background noise. A small signal from the crystal frequency standard was injected by a very loose coupling into the aerial of a wireless receiver tuned to the Light Programme and beats between the two signals were heard as variations in the amplitude of the wireless programme. The number of beats in fifty seconds is equal to the difference in frequency in parts in 10^7, and the counts of beats could easily be made correctly to one in fifty seconds. The half periods of the pendulums are about half a second, and therefore if n beats are counted in fifty seconds the correction to the half period is $(n/2) \times 10^{-7}$ seconds. The frequency of the crystal output is greater than 100,000 c/s. and this correction has therefore to be subtracted from the measured period.

A further correction has to be made because the B.B.C. carrier deviates slightly from the nominal 200,000 c/s.; these deviations are measured by the Royal Observatory Greenwich, from whom we obtained them.

The temperature, pressure and arcs are noted at the beginning and the end of each swing, and the mean values are used in making the corrections. Beats are counted two or three times during the swing.

The corrections for pendulum VI A for the swing from 17·13 to 18·20 on 20 April, 1949, at Cambridge, are given as an example.

PENDULUM VI A

Uncorrected period:			0·505,4563·2 sec.
Temperature:	Beginning	15·6°C.	
	End	15·8	
	Mean	15·7	
	Correction		−40·8
Pressure:	Beginning	22·0 mm.	
	End	28·0	
	Mean	25·0	
	Correction		−34·9
Arc:	Beginning	3·15 cm.	
	End	2·75	
	Mean	2·95	
	Correction		−17·4
Crystal Deviation Correction			−33·8
B.B.C. Deviation Correction			negligible
Fully Corrected Period:			0·505,4436·3 sec.

IV. RESULTS OF THE PENDULUM OBSERVATIONS AND DISCUSSION OF ERRORS

The complete results are set out in *Tables* 1 to 5, which show:

The fully corrected periods of the pendulums at each station in chronological order, the differences between the periods of each pair, and the last figures of the means of each pair;

The mean periods, means of means, and sums of squares of residuals ($\Sigma\varepsilon^2$) of periods, means, and differences, for each set;

The pressures, temperatures, and arcs, and their mean values.

The expected errors of the observations will first be discussed, and those that actually occur will then be compared with them.

Errors of measurement of the coincidence intervals

These errors arise because of the uncertainty of the measurement of the interval between time and pendulum marks and because of irregularities in the speed of the recording paper. The latter is of very slight importance, for the speed can be determined at the time of coincidence by measuring the interval between adjacent time marks. Throughout the measurements the speed was very nearly 66 mm./sec. and rarely differed by more than 1 mm./sec. from this value. The largest interval between a time and a pendulum interval that can be measured with the reading microscope is 0·015 sec., and for this interval to be in error by 0·0005 sec. the paper speed would have to be in error by 2 mm./sec. The uniformity of the speed is checked by measuring it at a number of different places on the record.

The reading microscope has graduations at intervals corresponding to 0·001 sec. (with paper speed 66 mm./sec.) and intervals are estimated to the nearest 0·0005 sec. Since the pendulums are swung for 4,000 sec. the corresponding uncertainty in the half-period is $0{\cdot}62\times10^{-7}$ sec., corresponding to an uncertainty in gravity of about 1/4 mgal.

The standard deviation of the measured period has been found from the residuals of the measured coincidence intervals. The observations used were all those at Dunsink and the final ones at Cambridge. The standard deviation of a single measurement of the interval between a time mark and a pendulum mark is found to correspond to the following uncertainties in the half period:

$$0{\cdot}67\times10^{-7}\text{ sec. for a sharp mark}$$
$$2{\cdot}00\times10^{-7}\text{ sec. for a diffuse mark.}$$

Each estimate is based on 112 degrees of freedom.

It is rather surprising that the standard deviation of a "sharp" interval is less than $\sqrt{2}$ times the graduation interval of the microscope.

The final mean coincidence interval is the mean of three "sharp" and three "diffuse" intervals, and therefore the corresponding standard deviation of the half period is:

$$\left[\tfrac{1}{6}\{(0{\cdot}67)^2+(2{\cdot}00)^2\}\right]^{\frac{1}{2}}\times10^{-7}\text{ sec.}=0{\cdot}87\times10^{-7}\text{ sec.}$$

This uncertainty includes any error due to fluctuating paper speed, since it is determined from separate measurements of the same interval.

Arc Correction

The correction is $-2a^2\times10^{-7}$ sec. where a is the double amplitude in centimetres of the light spot one metre from the pendulums.

The variance of δS, corresponding to a variance of a, is $8a^2\,\overline{\delta a^2}$.*

Representative values are 3·0 cm. for a, and $2\times(0{\cdot}05)^2$ cm.^{2} for $\overline{\delta a^2}$.

The expected standard deviation of δS is therefore about $0{\cdot}60\times10^{-7}$ sec.

The mean correction is also in error because the amplitudes are not measured at the same instants as the coincidences from which the periods are determined.

If the amplitude at time t is a_0e^{-kt}, the mean correction is

$$-\frac{a_0^2}{t_2-t_1}\cdot\frac{1}{k}\left(e^{-2kt_1}-e^{-2kt_2}\right)\times10^{-7}\text{ sec.}$$

*The bar denotes average.

Table 1

PENDULUM PERIODS AT CAMBRIDGE

Corrected for B.B.C. Rate

(*a*) 1st Series

	Pressure	Temperature	Arcs A	Arcs B/C	I A	I B	I C	$A-B$	$A-C$	$\frac{A+B}{2}$	$\frac{A+C}{2}$
	mm.	°C.	cm.	cm.	sec.			10^{-7} sec.		10^{-7} sec.	
	38·95	17·85	3·35	3·35	·506,2291·7		2303·9		−12·2		97·8
	39·85	18·1	3·48	3·48	2291·1		2301·9		−10·8		96·5
	21·5	17·8	2·85	2·90	2294·1	2324·6		−30·5		09·4	
	34·1	20·15	3·08	3·08	2294·5	2324·8		−30·3		09·7	
	24·8	20·9	3·23	3·23	2291·3	2320·6		−29·3		06·5	
Mean	30·3	20·6	3·14	3·16	·506,2292·5	2323·3	2302·9	−30·0	−11·5	08·5	97·2
$\Sigma\epsilon^2$					10·60	11·23	2·0	0·83	0·98	6·25	0·85
	Pressure	Temperature	Arcs A	Arcs B/C	VI A	VI B	VI C	$A-B$	$A-C$	$\frac{A+B}{2}$	$\frac{A+C}{2}$
	24·8	15·7	3·35	3·33	·505,4432·5		4435·1		−2·6		33·8
	24·8	15·9	3·25	3·33	4429·3		4438·1		−8·8		33·7
	22·9	16·9	3·28	3·25	4428·0	4452·1		−24·1		40·1	
	27·5	17·2	3·28	3·20	4434·2	4450·5		−16·3		42·4	
Mean	25·1	16·4	3·29	3·28	4431·0	4451·3	4436·6	−20·2	−5·7	41·3	33·8
$\Sigma\epsilon^2$					24·38	1·28	4·50	30·42	19·22	2·63	—

	Standard Deviation	Degrees of Freedom
var (S) = 4·9 ;	2·21 × 10^{-7} sec.	12
var $(\bar{S})$ = 2·88 ;	1·70	5
var (ΔS) = 10·93 ;	3·31	5

Table 1

(*b*) 2nd Series

	Pressure	Temperature	Arcs A	Arcs B/C	I A	I B	I C	$A-B$	$A-C$	$\frac{A+B}{2}$	$\frac{A+C}{2}$
	mm.	°C.	cm.	cm.	sec.			10^{-7} sec.		10^{-7} sec.	
	30·8	16·75	2·75	2·75	·506,2295·0		2301·3		−6·3		98·2
	33·3	17·35	2·68	2·68	2297·5		2303·5		−6·0		00·4
	30·8	16·8	3·05	3·05	2298·6	2326·8		−28·2		12·7	
	23·5	16·95	3·08	3·08	2297·0	2327·2		−30·2		12·1	
Mean	29·6	16·96	2·89	2·89	2297·0	2327·0	2302·4	−29·2	−6·2	12·4	99·3
$\Sigma\epsilon^2$					6·81	·08	2·42	2·0	·04	0·18	2·64
	Pressure	Temperature	Arcs A	Arcs B/C	VI A	VI B	VI C	$A-B$	$A-C$	$\frac{A+B}{2}$	$\frac{A+C}{2}$
	22·5	15·75	2·95	2·95	·505,4434·0		4436·1		−2·1		35·1
	36·0	15·75	2·95	2·95	4439·7		4437·2		+2·5		38·5
	22·3	15·4	2·98	2·98	4433·4	4450·4		−17·0		41·9	
	25·0	15·7	2·95	2·95	4436·3	4450·1		−13·8		43·2	
Mean	26·7	15·65	2·96	2·96	4435·9	4450·3	4436·7	−15·4	+0·2	42·6	36·8
$\Sigma\epsilon^2$					24·42	0·04	0·40	5·12	10·58	0·84	5·78

	Standard Deviation	Degrees of Freedom
var (S) = 3·42 ;	1·85 × 10^{-7} sec.	10
var $(\bar{S})$ = 2·36 ;	1·54	4
var (ΔS) = 4·74 ;	2·18	4

TABLE 2

PENDULUM PERIODS AT DUNSINK

Corrected for B.B.C. Rate

(*a*) 1st SERIES

	Pressure	Temperature	Arcs A	Arcs B/C	I A	I B	I C	$A-B$	$A-C$	$\frac{A+B}{2}$	$\frac{A+C}{2}$
	mm.	°C.	cm.	cm.	sec.			10^{-7} sec.		10^{-7} sec.	
	39·3	10·45	3·2	3·2	·506,1975·2		1990·9		−15·7		83·1
	34·8	11·05	3·1	3·18	1982·1		1989·4		− 7·3		85·8
	31·5	10·2	3·13	3·05	1984·7	2012·2		−27·5		98·5	
	30·5	10·65	3·13	3·03	1982·4	2012·1		−29·7		97·3	
Mean	34·0	10·59	3·14	3·12	1981·1	2012·2	1990·2	−28·6	−11·5	97·9	84·5
$\Sigma\epsilon^2$					50·45	—	0·84	2·42	35·28	0·72	3·65
	Pressure	Temperature	Arcs A	Arcs B/C	VI A	VI B	VI C	$A-B$	$A-C$	$\frac{A+B}{2}$	$\frac{A+C}{2}$
	31·3	11·45	3·0	3·08	·505,4123·9		4126·7		−2·8		25·3
	27·3	11·55	2·9	3·0	4123·6		4128·6		−5·0		26·1
	29·8	9·7	2·93	2·93	4119·2	4139·7		−20·5		29·5	
	24·5	10·15	2·9	2·95	4112·5	4140·1		−27·6		26·3	
	33·3	10·85	2·85	2·93	4121·1		4129·7		−8·6		25·3
	33·3	11·0	2·85	2·90	4120·6		4129·5		−8·9		25·1
Mean	30·0	10·78	2·91	2·97	4120·1	4139·9	4128·6	−24·0	−6·3	27·9	25·5
$\Sigma\epsilon^2$					86·51	0·08	5·63	25·21	22·39	5·12	0·60

		Standard Deviation	Degrees of Freedom
var (S)	= 9·97;	3·16 × 10^{-7} sec.	14
var ($\bar{S}$)	= 1·86;	1·36	6
var (ΔS)	= 13·45;	3·67	6

TABLE 2

(*b*) 2nd SERIES

	Pressure	Temperature	Arcs A	Arcs B/C	I A	I B	I C	$A-B$	$A-C$	$\frac{A+B}{2}$	$\frac{A+C}{2}$
	mm.	°C.	cm.	cm.	sec.			10^{-7} sec.		10^{-7} sec.	
	29·3	8·5	2·93	2·88	·506,1981·8		1987·3		−5·5		84·6
	34·3	10·4	2·85	2·78	1984·2	2011·5		−27·3		97·9	
	29·0	10·55	2·88	2·85	1983·2	2009·3		−26·1		96·3	
Mean	31·6	9·63	2·89	2·84	1983·1	2010·4	1987·3	−26·7	−5·5	97·1	84·6
$\Sigma\epsilon^2$					2·91	2·42		0·72		1·28	
	Pressure	Temperature	Arcs A	Arcs B/C	VI A	VI B	VI C	$A-B$	$A-C$	$\frac{A+B}{2}$	$\frac{A+C}{2}$
	26·8	10·8	2·78	2·78	·505,4121·0		4124·6		−3·6		22·8
	30·5	11·15	2·70	2·70	4120·9		4125·4		−4·5		23·2
	34·8	10·55	2·8	2·8	4121·1	4136·0		−14·9		28·6	
	33·3	11·0	2·8	2·8	4119·3	4137·9		−18·6		28·6	
	29·8	11·85	2·8	2·8	4121·5		4122·6		−1·1		22·1
	30·0	12·0	2·8	2·8	4120·8		4124·2		−3·4		22·5
Mean	31·3	11·23	2·78	2·58	4120·8	4137·0	4124·2	−16·8	−3·2	28·6	22·7
$\Sigma\epsilon^2$					2·88	2·20	4·16	6·84	6·30	—	0·56

		Standard Deviation	Degrees of Freedom
var (S)	= 1·01;	1·0 × 10^{-7} sec.	12
var ($\bar{S}$)	= 0·34;	0·6	5
var (ΔS)	= 2·50;	1·6	5

Table 3

PENDULUM PERIODS AT SLIGO

Corrected for B.B.C. Rate

	Pressure	Temperature	Arcs A	Arcs B/C	VI A	VI B	VI C	$A-B$	$A-C$	$\frac{A+B}{2}$	$\frac{A+C}{2}$
	mm.	°C.	cm.	cm.		sec.		10^{-7} sec.		10^{-7} sec.	
	29·3	8·75	2·98	2·90	·505,3926·4		3935·1		− 8·7		30·8
	28·8	9·0	2·95	2·93	3940·3		3921·9		+18·4		31·1
	31·0	8·1	2·9	2·95	3940·4	3927·8		12·6		34·1	
	28·3	8·5	2·93	2·93	3932·5	3934·2		−1·7		33·4	
Mean	29·3	8·6	2·94	2·93	3934·9	3931·0	3928·5	5·5	4·9	33·8	31·0
$\Sigma\epsilon^2$					136·82	20·48	87·12	102·25	367·21	0·25	0·05

		Standard Deviation	Degrees of Freedom
var (S)	$= 48·84$;	$7·0 \times 10^{-7}$ sec.	5
var $(\bar{S})$	$= 0·15$;	0·4	2
var (ΔS)	$= 234·7$;	15·3	2

Table 4

PENDULUM PERIODS AT GALWAY

Corrected for B.B.C. Rate

	Pressure	Temperature	Arcs A	Arcs B/C	VI A	VI B	VI C	$A-B$	$A-C$	$\frac{A+B}{2}$	$\frac{A+C}{2}$
	mm.	°C.	cm.	cm.		sec.		10^{-7} sec.		10^{-7} sec.	
	25·8	12·1	2·65	2·65	·505,4184·5		4187·0		−2·5		85·8
	30·5	12·05	2·63	2·65	4180·6	4203·9		−23·3		92·3	
	30·8	12·45	2·65	2·60	4182·4	4205·5		−23·1		94·0	
Mean	29·0	12·0	2·64	2·63	4182·5	4204·7	4187·0	−23·2	−2·5	93·2	85·8
$\Sigma\epsilon^2$					7·62	1·28		0·02		1·45	—

		Standard Deviation	Degrees of Freedom
var (S)	$= 3·10$;	$1·76 \times 10^{-7}$ sec.	3
var $(\bar{S})$	$= 2·64$;	1·62	1
var (ΔS)	$= 0·02$;	0·14	1

Table 5

PENDULUM PERIODS AT CORK

Corrected for B.B.C. Rate

	Pressure	Temperature	Arcs A	Arcs B/C	VI A	VI B	VI C	$A-B$	$A-C$	$\frac{A+B}{2}$	$\frac{A+C}{2}$
	mm.	°C.	cm.	cm.		sec.		10^{-7} sec.		10^{-7} sec.	
	30·8	12·05	3·03	3·03	·505,4490·4		4497·4		−7·0		93·9
	25·0	12·7	2·98	3·0	4494·7		4492·5		+2·2		93·6
	29·5	10·8	3·1	3·1	4492·0	4508·4		−16·4		00·2	
	26·0	11·5	3·1	3·1	4491·5	4508·8		−17·3		00·2	
	33·5	12·85	3·0	3·1	4490·9		4492·2		−1·3		91·6
Mean	29·0	12·0	3·04	3·07	4491·9	4508·6	4494·0	−16·9	−2·0	00·2	93·0
$\Sigma\epsilon^2$					11·26	·08	17·05	0·40	43·13	0	3·13

		Standard Deviation	Degrees of Freedom
var (S)	$= 4·25$;	$2·06 \times 10^{-7}$ sec.	7
var $(\bar{S})$	$= 1·24$;	1·11	3
var (ΔS)	$= 14·48$;	3·80	3

For variations of t_1 and t_2,

$$10^7 \cdot \delta S/\delta t_1 = -\frac{2a_1^2}{t_2 - t_1} + \frac{a_1^2 - a_2^2}{k(t_2 - t_1)^2}$$

$$10^7 \cdot \delta S/\delta t_2 = \frac{2a_2^2}{t_2 - t_1} - \frac{a_1^2 - a_2^2}{k(t_2 - t_1)^2}.$$

Representative values are: $a_1 = 3 \cdot 2$ cm., $a_2 = 2 \cdot 8$ cm., $t_2 - t_1 =$ about 1 hr.

Then $10^7 \cdot \delta S/\delta t_1$ and $10^7 \cdot \delta S/\delta t_2$ are both about $-1 \cdot 2$ sec. per hour, and since the standard deviations of t_1 and t_2 are about $0 \cdot 1$ hr., the standard deviation of δS from this cause is $0 \cdot 17 \times 10^{-7}$ sec.

The total standard deviation of the period arising from causes which affect each pendulum of a pair independently is therefore composed as follows:

Timing error :	$0 \cdot 87 \times 10^{-7}$ sec.
Arc: Reading error :	$0 \cdot 60$
Averaging error :	$0 \cdot 17$
Total :	$1 \cdot 08 \times 10^{-7}$ sec.

Temperature and Pressure Corrections

Uncertainties in these corrections, and in the correction for the rate of the crystal frequency standard (see next section) affect the separate periods and the mean period of two pendulums swinging together, but not the difference, since the corrections are the same for both pendulums.

Temperature. The temperature correction is $-2 \cdot 6 \times 10^{-7}$ sec. per deg. C. The temperature can be read to $0 \cdot 1°$ C., and if the temperature is changing, the mean can be estimated with this accuracy so that the correction to the period has a standard deviation of $0 \cdot 26 \times 10^{-7}$ sec.

This may be an underestimate since there is some doubt whether, when the temperature is changing, that read on the thermometer in the bob of the dummy pendulum is the same as that of the stems of the oscillating pendulums.

Pressure. The manometer readings can be made with an accuracy of about $0 \cdot 5$ mm. Since the actual pressure is the sum of two readings it has a standard deviation of $0 \cdot 7$ mm. Hg. and the standard deviation of the mean pressure during a swing is $0 \cdot 5$ mm. Hg. It is possible that sticking of the mercury increases this value.

We have:

$$10^7 \cdot \delta S = \frac{606\,p}{760 + 2 \cdot 79\,\theta} + \frac{91\,p^{\frac{1}{2}}}{(760 + 2 \cdot 79\,\theta)^{\frac{1}{2}}}$$

Put $760 + 2 \cdot 79\,\theta = p_0$

The effect of variation of the temperature, θ, is very slight. Then

$$10^7 \cdot (\delta S/\delta p) = 606/p_0 + 45 \cdot 5(pp_0)^{\frac{1}{2}}$$

Representative values are:

$$\theta = 10°\text{C.},\ p_0 = 788 \text{ mm.},\ p = 30 \text{ mm.},\ 10^7 \cdot (\delta S/\delta p) = 1 \cdot 07 \text{ sec./mm.}$$

The standard deviation of p being $0 \cdot 5$ mm., that of S is $0 \cdot 54 \times 10^{-7}$ sec.

Throughout all the observations except the first at Cambridge, the pressure changed at the rate of 10 mm. Hg. in an hour, very nearly, and we must therefore consider the error which may arise because the pressure is not read at the same time as those coincidences occur, the intervals between which are used to find the period.

Let $p = p' + rt$

where p' is a constant, and r is the rate of increase of pressure, and put

$$p_1 = p' + rt_1,\ p_2 = p' + rt_2.$$

Then

$$10^7 \cdot \delta S = \frac{606\,p'}{p_0} + \frac{606}{p_0} \cdot \frac{r\,(t_1 + t_2)}{2} + \frac{2}{3} \cdot \frac{91}{p_0^{\frac{1}{2}}} \cdot \frac{1}{r\,(t_2 - t_1)} \{p_2^{3/2} - p_1^{3/2}\}$$

and

$$10^7 \cdot \frac{\delta S}{\delta t_1} = \frac{606\, r}{2\, p_0} - \frac{91}{p_0^{\frac{1}{2}}} \cdot \frac{p_1^{\frac{1}{2}}}{t_2 - t_1} + \frac{2}{3} \cdot \frac{91}{p_0^{\frac{1}{2}}} \cdot \frac{p_2^{3/2} - p_1^{3/2}}{r(t_2 - t_1)^2},$$

$$10^7 \cdot \frac{\delta S}{\delta t_2} = \frac{606\, r}{2\, p_0} + \frac{91}{p_0^{\frac{1}{2}}} \cdot \frac{p_2^{\frac{1}{2}}}{t_2 - t_1} - \frac{2}{3} \cdot \frac{91}{p_0^{\frac{1}{2}}} \cdot \frac{p_2^{3/2} - p_1^{3/2}}{r(t_2 - t_1)^2}.$$

The variance of S is

$$\overline{\delta t^2} \left\{ \left(\frac{\delta s}{\delta t_1}\right)^2 + \left(\frac{\delta s}{\delta t_2}\right)^2 \right\}$$

where $\overline{\delta t^2}$ is the variance of t_1 or t_2.

Representative values are:

$$p_0 = 788 \text{ mm.},\ p_1 = 25 \text{ mm.},\ p_2 = 35 \text{ mm.},\ t_2 - t_1 = 1 \text{ hr.},\ r = 10 \text{ mm./hr.}$$

Then:

$$10^7 \cdot (\delta s/\delta t_1) = 6 \cdot 0 \text{ sec.}$$
$$10^7 \cdot (\delta s/\delta t_2) = 3 \cdot 0 \text{ sec.}$$

The variance of t_1 or t_2 is about $0 \cdot 01$ (hr.)^2, and therefore

$$10^{14} \text{ var } (S) = 0 \cdot 01\ (36 + 9);$$

the standard deviation of S is $0 \cdot 67 \times 10^{-7}$ sec.

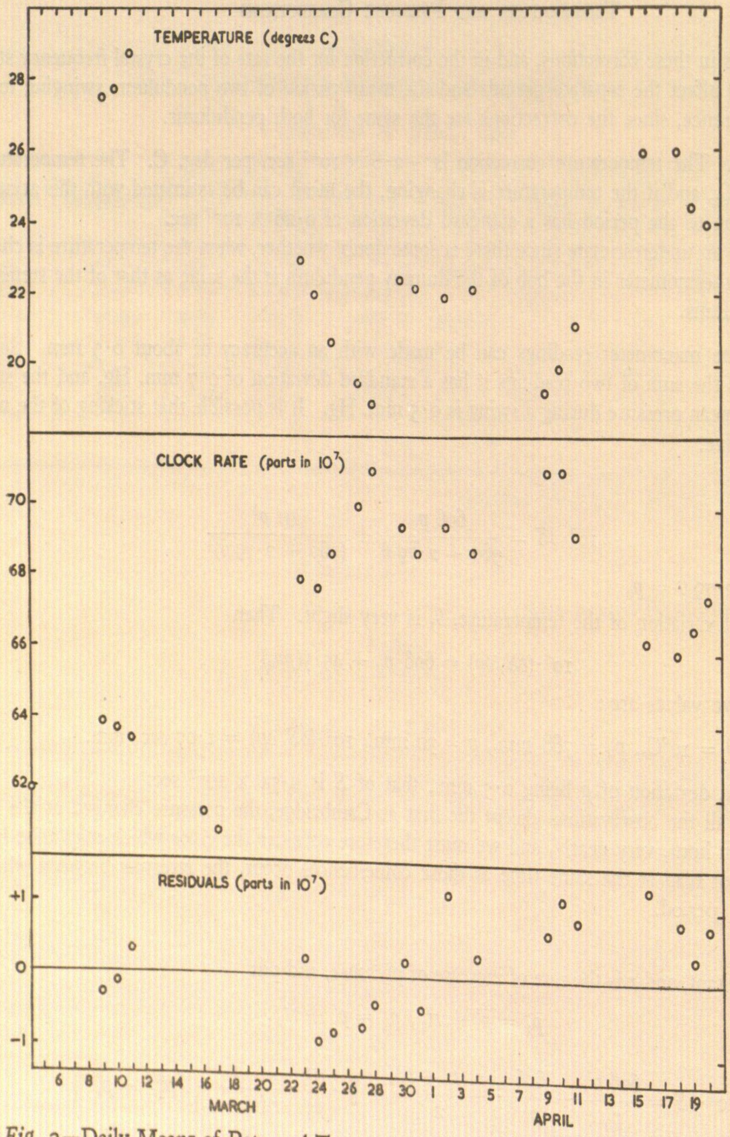


Fig. 3—Daily Means of Rate and Temperature of Crystal Frequency Standard; Departures of Rate from the Regression on Temperature.

There is also a systematic error because the correction used is that corresponding to the mean pressure instead of the mean of the corrections corresponding to the initial and final pressures, but it is only about $0{\cdot}06 \times 10^{-7}$ sec., and will be neglected.

Behaviour of the Crystal Frequency Standard

Since the frequency of the standard varies slightly with temperature, the temperature of the crystal enclosure was read every time the frequency was measured, so that the temperature coefficient of frequency and the standard deviation of an estimate of frequency can both be estimated.

The day-to-day variations of rate (number of beats in 50 sec.) and temperature are shown in *Fig.* 3, and a scatter-diagram of rate against temperature is given in *Fig.* 4.

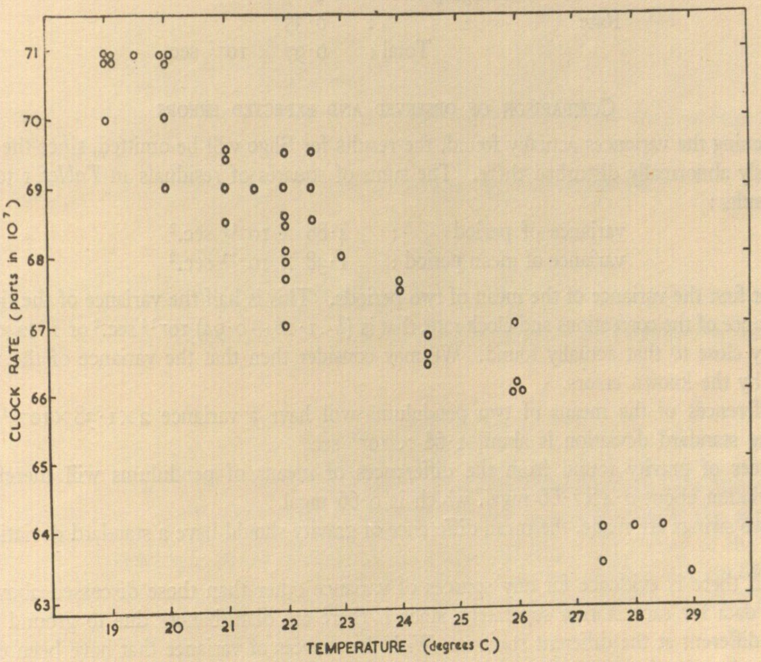


Fig. 4—Dependence of Clock Rate on Temperature.

We denote the temperature by θ (deg. C.) and the rate by R.

The following statistics are found from the data:

mean θ	22·06 deg.C.
mean R	68·15
var. θ	7·60 (deg.)^2
var. R	4·78
cov. $(R.\theta)$	−5·60

The coefficient of correlation is 0·93, and the regression of rate on temperature is

$$R - 68{\cdot}2 = -0{\cdot}737\ (\theta - 22{\cdot}1)$$

The residuals* from this line are shown on *Fig.* 3. From them the standard deviation σ of a single count of rate is found to be:

$$0{\cdot}7 \text{ parts in } 10^7 \text{ of the B.B.C. frequency.}$$

The variance of the temperature coefficient is

$$\sigma^2/(\text{var } \theta) = 0{\cdot}06.$$

The temperature coefficient of the crystal frequency is therefore

$$-0{\cdot}74 \pm 0{\cdot}25 \text{ parts in } 10^7 \text{ per deg. C.}$$

Examination of the day-to-day variation of the residuals shows that the mean frequency of the crystal changed by about 1 part in 10^7 between the 4th and 9th April. The mean residual up to the 4th April

*Corrections for the deviation of the B.B.C. frequency from the nominal 200 Kc/s have been applied.

is $-0{\cdot}44 \times 10^{-7}$ (17 negative, 5 positive residuals) with a standard deviation of $0{\cdot}12 \times 10^{-7}$, and the mean residual after the 9th April is $+0{\cdot}6 \times 10^{-7}$ (18 positive residuals) with a standard deviation of $0{\cdot}17 \times 10^{-7}$. The difference, $1{\cdot}04 \times 10^{-7}$, has a standard deviation of about $0{\cdot}2 \times 10^{-7}$, and is therefore quite definite.

Since the standard deviation of an estimate of the rate is $0{\cdot}7$ parts in 10^7, the standard deviation of the correction to the half-period of the pendulum is $0{\cdot}35 \times 10^{-7}$ sec.

The *total standard deviation of the mean period* due to causes which affect both pendulums of a pair equally is therefore composed as follows:

Temperature	:	$0{\cdot}26 \times 10^{-7}$ sec.
Pressure: Reading error	:	$0{\cdot}54$
Averaging error	:	$0{\cdot}67$
Rate	:	$0{\cdot}35$
Total	:	$0{\cdot}97 \times 10^{-7}$ sec.

Comparison of observed and expected errors

In discussing the variances actually found, the results for Sligo will be omitted, since the pendulums were obviously abnormally disturbed there. The sums of squares of residuals in *Tables* 1 to 5 give the following results:

variance of period	:	$4{\cdot}88 \times 10^{-14}$ sec.^{2}
variance of mean period	:	$1{\cdot}38 \times 10^{-14}$ sec.^{2}

Consider first the variance of the mean of two periods. This is half the variance of the timing errors plus the variance of the corrections and clock rate, that is $(\frac{1}{2}\,.1{\cdot}16 + 0{\cdot}94)\,10^{-14}$ sec.^{2} or $1{\cdot}52 \times 10^{-14}$ sec.^{2}, which is very close to that actually found. We may consider then that the variance of the mean is accounted for by the known errors.

The differences of the means of two pendulums will have a variance $2 \times 1{\cdot}38 \times 10^{-14}$ sec.^{2}; the corresponding standard deviation is about $1{\cdot}66 \times 10^{-7}$ sec.

Differences of gravity found from the differences of means of pendulums will therefore have a standard deviation about $0{\cdot}4 \times 1{\cdot}66$ mgal. which is $0{\cdot}66$ mgal.

When two swings are made, the mean difference of gravity should have a standard deviation of about $0{\cdot}47$ mgal.

To see if there is evidence for any sources of variance other than those discussed above, we shall examine the data for each station separately, since if there are disturbances due to ground movements they will be different at the different stations. With the sources of variance that have been enumerated, the expected values for the variances of a period, a mean, and a difference are: $1{\cdot}98 \times 10^{-14}$ sec.^{2}, $1{\cdot}52 \times 10^{-14}$ sec.^{2}, $2{\cdot}00 \times 10^{-14}$ sec.^{2} respectively.

The values actually found at each station (excluding Sligo) are shown in *Table* 6. For each the parameter z, that is $\frac{1}{2}$ log (observed variance/expected variance) is given, as well as the number of degrees of freedom, ν, on which the observed variance is based. From tables of z, such as those given by Fisher (1946), the probability $P(z, \nu)$ of z exceeding the observed value in a normal population is found, and is entered in the table. Only those entries will be further discussed for which $P(z, \nu)$ is less than 5%, since the differences of all the others from the expected values can be explained as sampling fluctuations.

Table 6

Comparison of observed and expected variances

Station	*Variable*	*Variance*	z	ν	$P(z, \nu)$
Cambridge ...	Period ...	$4{\cdot}00 \times 10^{-14}$ sec.	$0{\cdot}35$	22	$> 1\%$
	Mean ...	$2{\cdot}14$	$0{\cdot}20$	9	$>> 5\%$
	Difference ...	$7{\cdot}70$	$0{\cdot}66$	9	$> 1\%$
Dunsink ...	Period ...	$6{\cdot}07$	$0{\cdot}56$	26	$< 0{\cdot}1\%$
	Mean ...	$1{\cdot}09$	$0{\cdot}14$	11	$>> 5\%$
	Difference ...	$9{\cdot}01$	$0{\cdot}73$	11	$< 1\%$
Galway ...	Period ...	$2{\cdot}97$	$0{\cdot}20$	3	all much
	Mean ...	$1{\cdot}45$	$0{\cdot}00$	1	greater
	Difference ...	$0{\cdot}02$	$2{\cdot}3$	1	than 5%
Cork ...	Period ...	$4{\cdot}06$	$0{\cdot}36$	7	both much
	Mean ...	$1{\cdot}04$	$0{\cdot}16$	3	greater than 5%
	Difference ...	$14{\cdot}48$	$0{\cdot}97$	3	$> 5\%$

The variances which are significantly different from the expected values at the 5% level are those of the period and difference at Cambridge and Dunsink.

Considering Cambridge first, the differences between the observed and expected values are:

period	:	$2\cdot02 \times 10^{-14}$ sec.
mean	:	$0\cdot62$,, ,,
difference	:	$5\cdot62$,, ,,

These are all explicable if we assume that there is an additional source of random variance affecting the individual periods of amount about $2\cdot0\times10^{-14}$ sec.^{2}.

Then the variances of the period, mean, and difference as estimated and observed would be:

		Observed	*Estimated*
Period	:	$4\cdot00 \times 10^{-14}$ sec.	$3\cdot98 \times 10^{-14}$ sec.
Mean	:	$2\cdot14$,, ,,	$2\cdot52$,, ,,
Difference	:	$7\cdot70$,, ,,	$6\cdot00$,, ,,

Examination of the individual periods (*Table* 1, a, b) shows that most of this extra variance can be attributed to Pendulum VI *A*.

At Dunsink, the differences between the observed and expected values are:

Period	...	$4\cdot09 \times 10^{-14}$ sec.
Difference	...	$6\cdot93$,, ,,

The variance of the mean is less than that expected.

Again, inspection of the individual periods suggests that Pendulum VI *A* is subject to greater variation than any of the others.

The Sligo results

The characteristic of these observations, that the periods and differences are very disturbed, whereas the means vary no more than at other stations, shows that the support on which the apparatus stood suffered random movements during the oscillations of the pendulums. The theory of the behaviour of two pendulums swinging in antiphase under these conditions has been worked out by Meinesz (1923) and we shall apply it to determine the magnitude of the ground movement which we shall compare with the movements Meinesz found in the Netherlands.

The following is an outline of the essential points of Meinesz's treatment.

Let ω be the speed of the pendulum, θ the angle variable, and let F represent the effect of disturbing moments, M, i.e. $F = M/I$ where I is the moment of inertia about the point of support.

The pendulum equation,

$$\ddot{\theta} + \omega^2\theta = F,$$

is transformed by the substitution

$$q = \theta + i\omega\dot{\theta},$$

to

$$\dot{q} - i\omega q = i\,F/\omega.$$

Hence

$$q = q_0\, e^{i\omega t} + \frac{i}{\omega}\cdot e^{i\omega t}\int_0^t Fe^{-i\omega t}dt.$$

We put

$$\delta q = \frac{i}{\omega}\cdot\int_0^t Fe^{i\omega t}dt.$$

When $F = 0$ (an undisturbed pendulum),

$$q = q_0 e^{i\omega t},$$

so that

$$\theta = \tfrac{1}{2}\, q_0 e^{-i\omega t}.$$

Meinesz shows that the disturbance $\delta\tau$ to the half-period S is given by:

$$a \sin\left(\frac{t}{S}\,\omega\cdot\delta\tau\right) = \text{component of } \delta q \text{ perpendicular to } q_0;$$

a is the amplitude of the vector q i.e. $|\, q_0 \,|$.

If the horizontal acceleration of the ground is $\ddot{x}$, the disturbing moment is

$$m h \ddot{x},$$

where m is the mass of the pendulum, and h is the distance between the centre of gravity and centre of rotation.

Hence

$$F = \ddot{x}/l,$$

where l is the length of the equivalent simple pendulum.

Therefore

$$\delta q = \frac{i}{\omega l} \int_0^t \ddot{x}\, e^{-i\omega t}\, dt.$$

Now we assume $\ddot{x}$ is random, and hence the integral is a random vector.
Hence we may write:

$$\left| \int_0^t \ddot{x}\, e^{-i\omega t}\, dt \right| = G\sqrt{t},$$

in which G depends only on $\ddot{x}$, and is, in fact $(\overline{\ddot{x}^2}/2)^{\frac{1}{2}}$ where $\overline{\ddot{x}^2}$ is the mean square value of $\ddot{x}$.

Since the direction of δq is random, the projection on any axis is

$$\frac{G}{\omega l}\sqrt{t/2},$$

i.e.

$$\frac{\omega G}{g}\sqrt{t/2}.$$

For two pendulums, we write

$$P = q_2/q_1 = \frac{a_2}{a_1} \cdot e^{i(\phi_2 - \phi_1)} \text{ or } p.e^{i(\phi_2 - \phi_1)}.$$

Hence

$$\frac{\delta P}{P} = \frac{q_2 - q_1}{q_2\, q_1} \cdot \delta q = \delta p/p - \frac{i\omega t}{S}(\delta\tau_2 - \delta\tau_1).$$

The modulus of $(q_2 - q_1)$ is $[p^2 + 1 - 2p\cos(\phi_2 - \phi_1)]^{\frac{1}{2}}$ and the projection of δq on the imaginary axis is

$$\frac{\omega G}{g}\sqrt{t/2}.$$

Hence, the root mean square value of the difference of periods is given by:

$$[\delta(\tau_2 - \tau_1)] = \frac{G S}{a_2\, g\sqrt{2t}} \cdot (p^2 + 1 - 2p\cos\overline{\phi_2 - \phi_1})^{\frac{1}{2}}.$$

Now $a_1 = a_2$, so $p = 1$ and $\phi_2 = \pi + \phi_1$ by the conditions of the experiment and therefore

$$[\delta(\tau_2 - \tau_1)] = \frac{2\, G S}{a\, g\sqrt{2t}}.$$

This is the formula we use to find the magnitude of the soil movements, that is, G.

It may be shown that the mean period of the two pendulums is unaffected by the accelerations if the phases differ by π; this is so at Sligo.

To find the value of $[\delta(\tau_2 - \tau_1)]$ at Sligo we compare the observed values at this place with the standard provided by the preceding and succeeding places.
These differences are:

	VI $(A-B)$	VI $(A-C)$
Cambridge ...	$-20{\cdot}2 \times 10^{-7}$ sec.	$-5{\cdot}7 \times 10^{-7}$ sec.
Dunsink ...	$-24{\cdot}0$	$-6{\cdot}3$
Sligo	—	—
Galway ...	$-23{\cdot}2$	$-2{\cdot}5$
Cork	$-16{\cdot}9$	$-2{\cdot}0$
Dunsink ...	$-16{\cdot}8$	$-3{\cdot}2$
Cambridge ...	$-15{\cdot}4$	$+0{\cdot}8$

The standard values we shall take are:

$$-22{\cdot}5 \text{ for } (A-B)$$
$$-\ 4{\cdot}0 \text{ for } (A-C).$$

The observed differences are:

$$A-B \quad \ldots \quad +12{\cdot}6,\ -1{\cdot}7$$
$$A-C \quad \ldots \quad -\ 8{\cdot}7,\ +18{\cdot}4,$$

and therefore the departures from the standards are:

$$A-B \quad \ldots \quad +35{\cdot}1,\ +20{\cdot}8$$
$$A-C \quad \ldots \quad -\ 4{\cdot}7,\ +22{\cdot}4.$$

The mean square value is $546{\cdot}9 \times 10^{-14}$ sec.^{2}

From this we subtract the variance of a difference found at the other places, $8{\cdot}8 \times 10^{-14}$ sec. and so find

$$[\delta(\tau_2 - \tau_1)] = 23{\cdot}2 \times 10^{-7} \text{ sec.}$$

In the formula

$$G = \frac{a \cdot g\sqrt{2t} \cdot [\delta(\tau_2 - \tau_1)]}{2\,S},$$

$$g = 981 \text{ cm/sec.}^2,\ a = 0{\cdot}015 \text{ rad.},\ t = 4{,}000 \text{ sec.},\ S = \tfrac{1}{2} \text{ sec.}$$

and therefore

$$G = 15{\cdot}4 \times 10^{-4} \text{ cm. sec.}^{-2}$$

This is a value very similar to those found by Meinesz which range up to 22×10^{-4} cm. sec.$^{-2}$

In discussing why such ground movements should occur, it is useful to recall that Meinesz found that values of G increased towards the sea coast, and that they could be quite large even though the ground locally was very stable, as at Schoorl ($G=19 \times 10^{-4}$), Helder ($G=15 \times 10^{-4}$) and Ameland ($G = 21 \times 10^{-4}$). Mr. Cunningham, of the Irish Geological Survey informs me that the Sligo station is on about forty feet of boulder clay overlying limestone which may be cavernous. The station is only about one mile from the head of Sligo Bay, so that the value of G is in accord with Meinesz's results. At the same time, it must be remembered that the floor, being of flagstones on an unknown foundation, is under suspicion, although Mr. Cunningham has examined it after the disturbances were noticed, and believes it to be quite rigid.

V. THE VALUES OF GRAVITY IN IRELAND

Calculation of differences of gravity from the pendulum observations

Since if there are movements of the ground they do not disturb the mean period of two similar pendulums swinging in antiphase (Meinesz, 1923), and since we have seen that the observed variance of the mean period can be explained entirely by the known sources of variance, whereas the variance of an individual period is sometimes significantly greater than that calculated from known errors, differences of gravity will be calculated from the mean periods of pairs of pendulums swinging together. The results in this form are collected in *Table* 7. The periods at Cambridge and Dunsink are the means of those found on the first and second visits.

The formula for the difference of gravity, δg, corresponding to the change of period δS, is

$$\frac{\delta g}{g} = 2 \cdot \frac{\delta S}{S}.$$

This formula is approximate, but is in error by $3 \cdot 10^{-4}$ at most.

The factor, $2g/S$, differs for the two sets, I and VI.

The sum of squares of residuals is $1{\cdot}777$ (mgal.)^2 and there are 6 degrees of freedom; the standard deviation of a single difference of gravity is therefore $0{\cdot}54$ mgal.

This value agrees with that found in Section IV from the known sources of variance or from the scatter of the separate observations.

TABLE 7

DIFFERENCES OF GRAVITY FROM CAMBRIDGE

Station	Pendulums	Mean period	Difference from Cambridge	Difference of Gravity	Mean	Residuals
		sec.	10^{-7} sec.	mgal.	mgal.	mgal.
Cambridge ...	I A, B	0·506,2310·5				
	I A. C	2298·3				
	VI A, B	0·505,4442·0				
	VI A, C	4435·3				
Dunsink ...	I A, B	0·506,1997·5	−313·0	121·35	121·40	−0·05
	I A, C	1984·6	−313·7	121·62		+0·22
	VI A, B	0·505,4128·3	−313·7	121·82		+0·42
	VI A, B	4124·1	−311·2	120·81		−0·59
Sligo ...	VI A, B	0·505,3933·8	−508·2	197·34	196·59	+0·75
	VI A, C	3931·0	−504·3	195·83		−0·76
Galway ...	VI A, B	0·505,4193·2	−248·8	96·61	96·75	−0·14
	VI A, C	4185·8	−249·5	96·89		+0·14
Cork ...	VI A, B	0·505,4500·2	+58·2	−22·60	−22·51	−0·11
	VI A, C	4493·0	+57·7	−22·41		+0·10

COMBINATION OF PENDULUM AND GRAVIMETER OBSERVATIONS

While the pendulum observations were being made, Mr. Thirlaway* carried out gravimeter observations to compare the values of gravity at the pendulum stations, with the following results:

Dunsink — Sligo	...	− 77·40 mgal.	± 0·56
Sligo — Galway	...	99·72 mgal.	± 0·43
Galway — Cork	...	118·86 mgal.	± 0·47
Cork — Dunsink	...	−142·90 mgal.	± 0·50

The places at which the gravimeter observations were made are not the same as the pendulum stations, and the differences given include corrections for the differences in height between the gravimeter and pendulum sites. The standard deviations are found from the scatter of observations at intermediate stations.

The gravimeter and pendulum observations will now be combined to give the best values of gravity (relative to Cambridge) at the pendulum stations. The two sets of observations are not however strictly comparable, because the calibration of the gravimeter may be in error, and hence in a least squares adjustment the calibration factor should be included as one of the unknowns. Suppose that the factor is $(1 + k)$ times the value used in deducing the above results. Then a gravimeter observation between stations A and B gives the following observation equation:

$$g_A - g_B = \delta_{AB}\,(1 + k)$$

where δ_{AB} is the observed difference of gravity using the trial calibration factor, and g_A and g_B are the values of gravity at the two stations.

Hence, the observation equation for g_A, g_B, and k is:

$$g_A - g_B - k.\delta_{AB} = \delta_{AB}.$$

In the least squares adjustment the observations are given weights proportional to the squares of their standard deviations.

The gravimeter observations all have standard deviations of about 0·5 mgal., and will be given equal weight. The standard deviation of a difference of gravity found from two swings of a single pair of pendulums has been seen to be about 0·5 mgal. also; since the differences of gravity at Sligo, Galway

* The results of these measurements will be published in Part 2 of this Memoir.

and Cork are found from two pairs of pendulums they have a standard deviation of about 0·35 mgal., and are given twice the weight of a gravimeter observation, and similarly, the difference at Dunsink is found from four pairs of pendulums, and is therefore given four times the weight of a gravimeter observation.

If we put g_o for the value of gravity at Cambridge, g_d at Dunsink, g_s at Sligo, g_g at Galway, and g_c at Cork, then the observation equations are those shown in *Table* 8.

TABLE 8

OBSERVATION EQUATIONS

Comparison	*Value*	*Weight*	*Trial*	*O − T*	*Calculated*	*O − C*	χ^2
g_d-g_o	121·40	4	121·40	0	120·96	+0·44	3·1
g_s-g_o	196·59	2	196·59	0	196·97	−0·38	1·2
g_g-g_o	96·75	2	96·75	0	96·84	−0·09	0·06
g_c-g_o	−22·51	2	−22·51	0	−22·30	−0·21	0·4
$g_d-g_s+77{\cdot}40k$	−77·40	1	−75·19	−2·21	−76·04	−1·36	5·9
$g_d-g_c-142{\cdot}90k$	142·90	1	143·91	−1·01	143·20	−0·30	0·5
$g_s-g_g-99{\cdot}72k$	99·72	1	99·84	−0·12	100·09	−0·37	0·6
$g_g-g_c-118{\cdot}86k$	118·86	1	119·26	−0·40	119·10	−0·24	0·2

Sum 12·0

The trial value of k is zero; the units are milligals.

The normal equations for k and the corrections, x_n, to the trial values $g_n - g_o$ are:

$$
\begin{array}{rrrrrcr}
6x_d & -x_s & & -x_c & -65{\cdot}50k & = & -3{\cdot}22 \\
-x_d & +4x_s & -x_g & & -177{\cdot}12k & = & 2{\cdot}09 \\
 & -x_s & +4x_g & -x_c & -19{\cdot}14k & = & -0{\cdot}28 \\
-x_d & & -x_g & +4x_c & +261{\cdot}76k & = & 1{\cdot}41 \\
-65{\cdot}50x_d & -177{\cdot}12x_s & -19{\cdot}14x_g & +261{\cdot}76x_c & +50438{\cdot}9k & = & 32{\cdot}7
\end{array}
$$

The solutions of these equations are:

$x_d = -0{\cdot}44$ mgal., $x_s = +0{\cdot}38$ mgal., $x_g = +0{\cdot}09$ mgal., $x_c = +0{\cdot}21$ mgal., $k = +0{\cdot}04\%$.

There are three degrees of freedom, so that the standard deviation of an observation of unit weight is:

$$\tfrac{1}{3}\sum Wt\times(O-C)^2 = 1{\cdot}13 \text{ (mgal.)}^2$$

Hence, by Jeffreys' (1939) procedure, we find the standard deviations of the unknowns to be:

$\sigma(d) = 0{\cdot}46$ mgal., $\sigma(s) = 0{\cdot}87$ mgal., $\sigma(g) = 0{\cdot}76$ mgal., $\sigma(c) = 0{\cdot}94$ mgal., $\sigma(k) = 0{\cdot}45\%$.

χ^2 in the last column is $\sum(O-C)^2/\sigma^2$, in which the σ is the value for the observation in question, that is 0.25 mgl. for the Dunsink observation, 0·35 mgl. for the other pendulum observations, and the values provided by Mr. Thirlaway for the gravimeter observations.

The value of χ^2, 12·0, is much too large on three degrees of freedom, for the probability of it being exceeded with a normal population is about 1 per cent., but almost half the total value of χ^2 is contributed by the gravimeter observation between Dunsink and Sligo, and all the other observations appear quite satisfactory. There is nothing to suggest that the observations have been weighted incorrectly. The value at Cork is the only one that lies outside the extremes of the separate pendulum results.

The combined results of the pendulum and gravimeter observations are therefore the following differences of gravity from the value $g = 981{\cdot}265$ cm/sec.² at the Pendulum House, Cambridge:

Dunsink	...	+120·96 mgal.	±0·46
Sligo	...	+196·97 mgal.	±0·87
Galway	...	+ 96·84 mgal.	±0·76
Cork	...	− 22·30 mgal.	±0·94

The correction to the calibration of the gravimeter is +0·04 ±0·45*.

* In calculating the gravimeter results the calibration factor was taken to be 0·09% greater than that given by Cook (1950).

VI. DISCUSSION OF RESULTS

The particular interest of these observations is that the gravimeter results, obtained at the same time as the pendulum observations, enable the pendulum results to be compared among themselves, and hence enable sources of variance, other than those which appear in the scatter of the results at a single station, to be detected. That there are additional sources is evident from the large contributions to χ^2 provided by the Dunsink observations. They might be changes in the lengths of the pendulums, or they might be systematic errors in the corrections, and both these possibilities will be examined by reducing all periods to Cambridge by the adjusted differences of gravity found in the last section, and examining the departures from the mean periods for dependence on time or the pressure, temperature or arc.

The mean periods reduced to Cambridge are shown in *Tables* 9 and 10; the residuals are given and the value of χ^2, which is the sum of the squares of the quotients of each residual by the standard deviation. The standard deviation is $(4{\cdot}88/n)^{\frac{1}{2}}$, where $4{\cdot}88 \times 10^{-14}$ sec.^{2} is the variance of a single period as found from the scatter of all the separate observations (Section IV) and n is the number of observations combined into the mean period (see *Tables* 1 to 5).

Table 9

Mean periods of set I reduced to Cambridge

Station	*A*	*B*	*C*	*Residuals* *A*	*B*	*C*
	sec.			10^{-7} sec.		
Cambridge (1) ...	0·506,2292·5	2323·3	2302·9	−1·5	−0·5	+1·6
Dunsink (1) ...	2292·3	2323·4	2301·4	−1·3	−0·4	+0·1
Dunsink (2) ...	2294·3	2321·5	2298·5	+0·3	−2·3	−2·8
Cambridge (2) ...	2297·0	2327·0	2302·4	+3·0	+3·2	+1·1
Mean ...	2294·0	2323·8	2301·3			

$\chi^2 = 20{\cdot}8$ on 9 degrees of freedom

The probability of a larger value occurring by chance is about 1%, and accordingly a real additional source of variance appears to exist.

Table 10

Mean periods of set VI reduced to Cambridge

Station	*A*	*B*	*C*	*Residuals* *A*	*B*	*C*
	sec.			10^{-7} sec.		
Cambridge (1) ...	·505,4431·0	4451·3	4436·6	−1·9	+0·2	−0·4
Dunsink (1) ...	4431·6	4451·4	4440·1	−1·3	+0·3	+3·1
Galway ...	4431·9	4454·1	4436·4	−1·0	+3·0	−0·6
Cork ...	4434·5	4451·2	4436·6	+1·6	+0·2	−0·4
Dunsink (2) ...	4432·3	4448·5	4435·7	−0·6	−2·6	−1·3
Cambridge (2) ...	4435·9	4450·3	4436·7	+3·0	−0·8	−0·3
Mean ...	4432·9	4451·1	4437·0			

$\chi^2 = 32{\cdot}4$ on 15 degrees of freedom

The probability of a larger value occurring by chance is less than 1%, so that again a real additional source of variance must be considered established.

It is interesting that the residuals for both sets are about the same, although Set I was not being used whilst Set VI was taken to Sligo, Galway and Cork.

This additional variation appears to be due to real changes in the pendulum periods, as for example between the second visits to Dunsink and Cambridge, but before this is taken as definite, possible correlations with temperature, pressure and arc must be examined.

We want to eliminate so far as possible other sources of variance in making this comparison, and we therefore use the mean period of a pair of pendulums.

Since the pressures, temperature and arc are usually much the same for both swings of the same pair of pendulums, the means for the two swings are used. The data are given in *Tables* 11 and 12. $\overline{S}$ is the mean period from two swings, $\overline{P}$, $\overline{\theta}$ are the mean pressure and temperature respectively, and $\overline{A^2}$ is the mean square arc.

Table 11

Correlation of residuals with Pressure, temperature and arc for Set I

Station	S	Residual	$\overline{P}$	$\overline{\theta}$	$\overline{A^2}$
A and B					
	10^{-7} sec.	10^{-7} sec.	mm.	°C	cm.²
Cambridge (1) ...	08·5	−1·1	26·8	19·6	9·4
Dunsink (1) ...	09·1	−0·5	31·0	10·4	9·5
Dunsink (2) ...	08·3	−1·3	31·7	10·5	8·1
Cambridge (2) ...	12·4	+2·8	27·2	16·9	9·4
Mean ...	09·6				
A and C					
Cambridge (1) ...	97·2	+0·2	39·4	18·0	11·7
Dunsink (1) ...	95·7	−1·3	37·1	10·6	10·0
Dunsink (2) ...	95·8	−1·2	29·3	8·5	8·5
Cambridge (2) ...	99·3	+2·3	32·1	17·1	7·3
Mean ...	97·0				

Table 12

Correlation of residuals with Pressure, Temperature and arc for Set VI

Station	S	Residual	$\overline{P}$	$\overline{\theta}$	$\overline{A^2}$
A and B					
	10^{-7} sec.	10^{-7} sec.	mm.	°C.	cm.²
Cambridge (1) ...	41·3	−0·1	25·2	17·0	10·6
Dunsink (1) ...	39·4	−2·0	27·2	9·9	8·6
Sligo ...	41·0	−0·4	29·7	8·3	8·6
Galway ...	42·6	+1·2	30·7	12·3	8·9
Cork ...	42·8	+1·4	27·8	11·2	9·6
Dunsink (2) ...	40·1	−1·3	34·1	10·8	7·8
Cambridge (2) ...	42·6	−1·2	34·2	15·6	8·8
Mean ...	41·4				
A and C					
Cambridge (1) ...	33·8	−2·0	24·8	15·8	11·0
Dunsink (1) ...	37·0	+1·2	31·3	11·2	8·6
Sligo ...	38·2	+2·4	29·1	8·9	8·7
Galway ...	35·2	−0·6	25·8	12·1	7·0
Cork ...	35·6	−0·2	29·8	12·5	9·2
Dunsink (2) ...	34·2	−1·6	29·3	11·5	7·7
Cambridge (2) ...	36·8	+1·0	29·3	15·8	8·7
Mean ...	35·8				

Scatter diagrams of the residuals against pressure, temperature, and arc show that there is no correlation with pressure or temperature, but that the dependence on arc requires investigation.

From the data we find

$$\begin{aligned} \text{var.}\ (\overline{S}) &= 2\cdot 20 \times 10^{-14}\ \text{sec}^2. \\ \text{var.}\ (\overline{A^2}) &= 1\cdot 44\ \text{cm}^4. \\ \text{cov.}\ (\overline{S.A^2}) &= -0\cdot 20 \times 10^{-7}\ \text{sec. cm}^2. \end{aligned}$$

The correlation coefficient is then $-0\cdot 115$.

To see if the association is significant, we calculate the ratio of the probability that the correlation coefficient is zero to the probability that it is some other value. According to Jeffreys (1939) this ratio is

$$\left(\frac{2n-1}{\pi}\right)^{\frac{1}{2}} (1 - r^2)^{\frac{1}{2}(n-3)}$$

n is 22, and the ratio is therefore:

$$\left(\frac{43}{\pi}\right)^{\frac{1}{2}} [1 - (0\cdot 115)^2]^{19/2} = 2\cdot 8.$$

The evidence is therefore, that the association is not significant.

These data therefore establish that there are no significant errors in the pressure, temperature and arc corrections, and hence, that all the additional variance that is found must be due to changes in the pendulum periods between one station and the next. To assist in elucidating whether these changes are random or systematic, *Tables* 13 and 14 contain the history of the two sets of pendulums as the series of measurements of the periods at Cambridge.

Table 13

History of set I

Date	A	B	C	Changes A	Changes B	Changes C
		sec.			10^{-7} sec.	
May, 1937	0·506,2289·6		2279·2			
Dec./Jan. 1938/39 ...	2303·8	2305·8	2309·6	+14·2		+30·4
July, 1939 ...	2297·3	2303·0	2287·2	− 6·5	− 2·8	−22·4
August, 1945 ...	2299·2	2315·2	2298·8	+ 1·9	+12·2	+11·6
April, 1947 ...	2301·8	2312·9	2285·2	+ 2·6	− 2·3	−13·6
May, 1947	2312·2	2315·5	2305·9	+10·4	+ 2·6	+20·7
June, 1948 ...	2296·1	2322·0	2308·4	−16·1	+ 6·8	+ 2·5
March, 1949 ...	2292·5	2323·1	2302·9	− 3·6	+ 1·1	− 5·5
April, 1949 ...	2297·0	2327·0	2302·4	+ 4·5	+ 3·9	− 0·5

There is nothing very definite about all this; both in the long term history and in the short term history in Ireland, random changes and systematic changes both occur. Pendulum IB has lengthened consistently, though by a jump between Dunsink and Cambridge, and VIB has shortened consistently, but in Ireland has behaved erratically. It seems likely that most of the changes are random, and that the appearance of a trend is due to chance.

The variance of the mean periods given in *Tables* 11 and 12 is $2\cdot 20 \times 10^{-14}$ sec^2. The values entered in these tables are the means derived from two swings, and their variance should be half that found for a mean in Section IV, that is $\frac{1}{2}(1\cdot 38) \times 10^{-14}$ sec^2 or $0\cdot 69 \times 10^{-14}$ sec^2. The difference, $1\cdot 5 \times 10^{-14}$ sec^2., represents the variance of changes in the period of pendulums between stations; the actual numerical value is rather too high, for it contains part of the variance of the gravimeter results. Clearly though, the pendulum observations were given too large a weight in the adjustment of Section V, and it will be

Table 14

History of set VI

Date	*A*	*B*	*C*	*Changes* *A*	*Changes* *B*	*Changes* *C*
		sec.			10^{-7} sec.	
May, 1937 ...	0·505,4408·9		4412·2			
				+2·8		+9·4
Dec./Jan. 1938/39 ...	4411·7	4457·0	4421·6			
				−1·0		−3·6
July, 1939 ...	4410·7		4418·0			
		Pendulums gilded				
May, 1947	4437·4	4457·3	4442·1			
				−3·5	−3·3	+2·2
June, 1948 ...	4433·9	4454·0	4444·3			
				−2·9	−2·7	−7·7
March, 1949 ...	4431·0	4451·3	4436·6			
				+4·9	−1·0	+0·1
April, 1949 ...	4435·9	4450·3	4436·7			

more nearly correct to multiply the standard deviations of the pendulum results by $\sqrt{2}$, and to give the pendulum results the same weight as the gravimeter results.

The results have accordingly been readjusted, the pendulum value at Dunsink being given weight 2 and all the others weight 1, and the results are then as follows.

Differences from Cambridge:

Dunsink	...	+120·84 mgal.	±0·56
Sligo	...	+197·28 „	±0·82
Galway	...	+ 96·91 „	±0·64
Cork	...	− 22·51 „	±0·87

Correction to gravimeter calibration: − 0·08% ±0·42%.

(see footnote p. 23).

The actual values are only very slightly changed from those given in Section V; the standard deviations are slightly less, and χ^2 is about 11 instead of about 12, so that these results are slightly better.

The changes in the values of gravity produce some small changes in the mean periods reduced to Cambridge given in *Tables* 9 to 12, but they do not seriously alter the conclusions reached on the basis of that material; in particular, the correlation coefficient of the period with the arc is very slightly increased. No further alteration in the weighting of the observations in the adjustment is therefore required, and the above results are our final values.

SUMMARY

The following results have been obtained about the sources of variance in pendulum observations:

1. It has been shown (Section IV) that known sources of variance - timing errors and correction errors, can account entirely for the variance of the mean period of two pendulums swinging together at a single station when the variance is determined from the scatter of the separate results for the same pair of pendulums or from the scatter of the results for different pairs.

2. Additional errors have been found to affect the individual periods and their differences at a single station.

3. By comparing results at different stations, it has been seen that changes of the pendulums between stations contribute to the variance an amount roughly equal to that contributed by the known sources of variance. This has been allowed for by multiplying the standard deviation of a pendulum result by $\sqrt{2}$; evidently if the accuracy of pendulum observations is to be increased by a factor of two or three or more it is to the identification and removal of this variation that most attention must be paid.

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