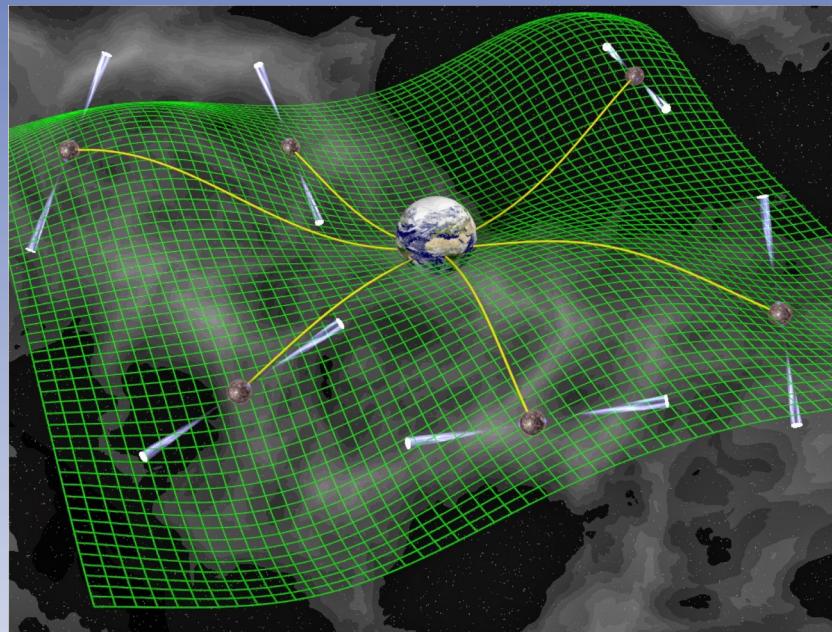


# Impact of planetary ephemerides on gravitational wave searches with Pulsar Timing Arrays

Aurélien Chalumeau (APC/USN/LPC2E)

G. Theureau (USN/LPC2E), S. Babak (APC), A. Petiteau (APC), L. Guillemot (LPC2E), S. Chen (LPC2E)

Collaborations with A. Fienga (Géoazur), M. Vallisneri (JPL/Caltech)



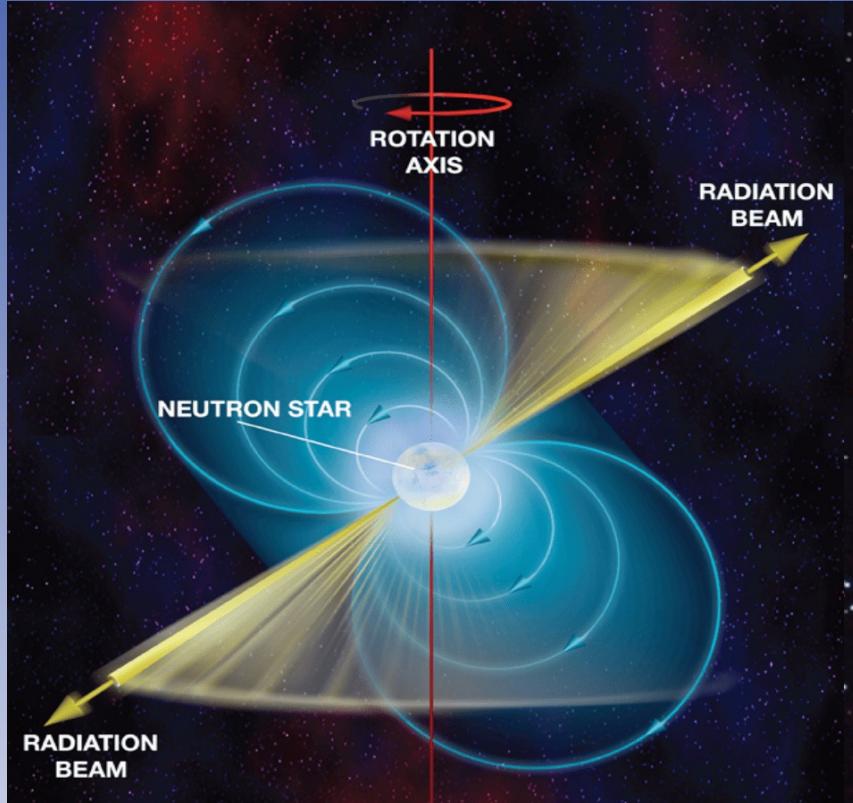
YERAC Dublin, 27/08/2019



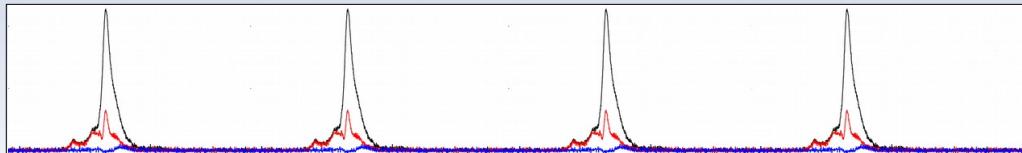
# Pulsars

*Fast spinning, highly magnetized compact objects*

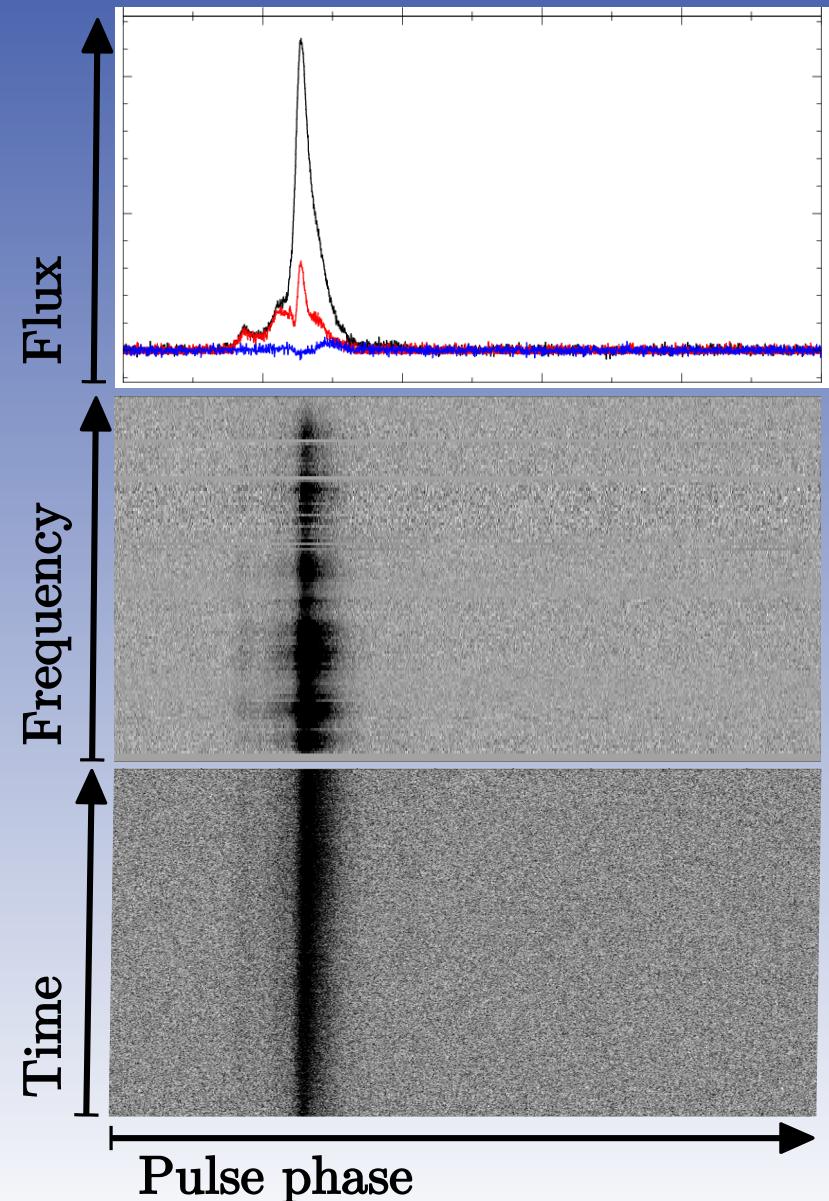
Credit : B. Saxton.



*Emitting a pulsated signal*



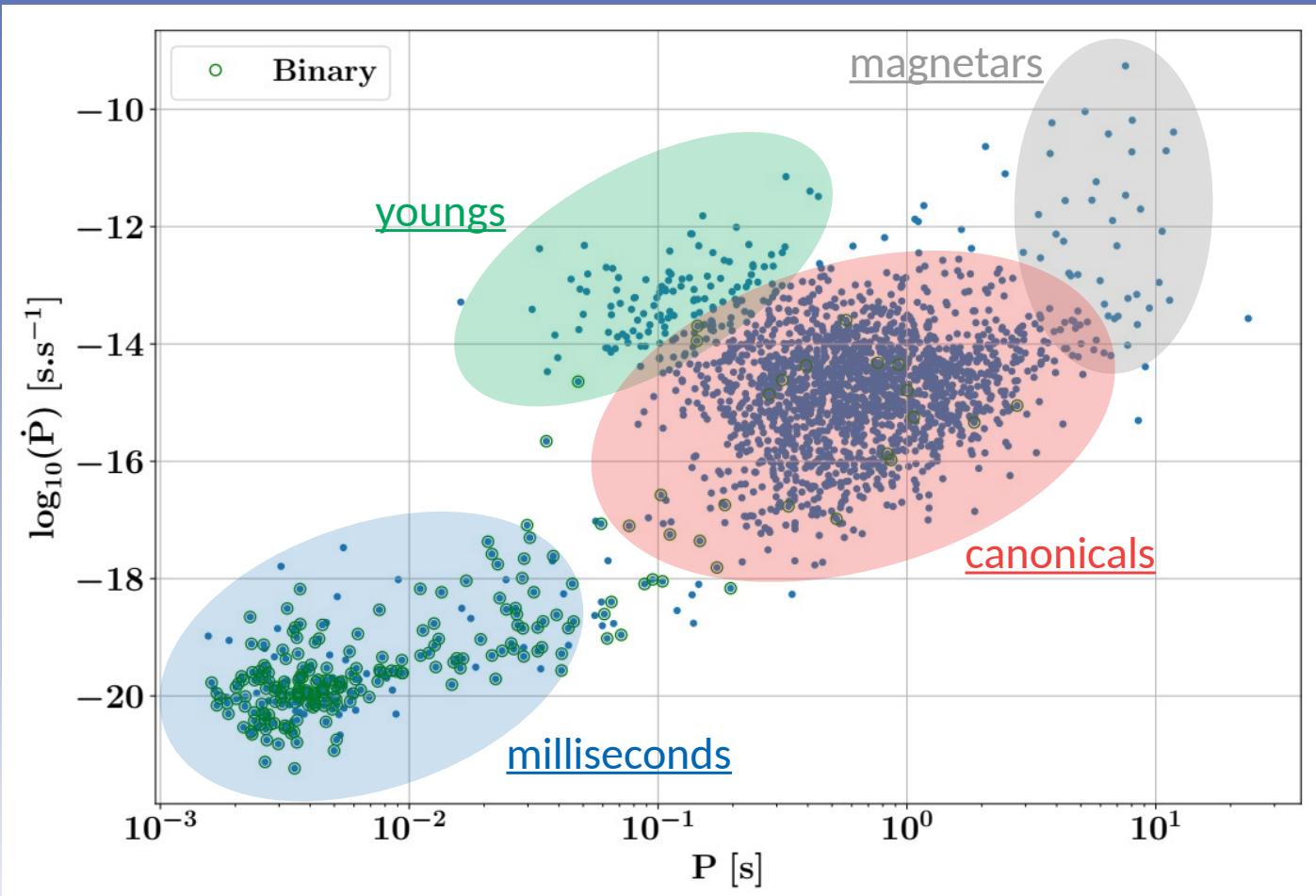
*Mostly observed in radio freq.*



# Pulsars

A full zoology !

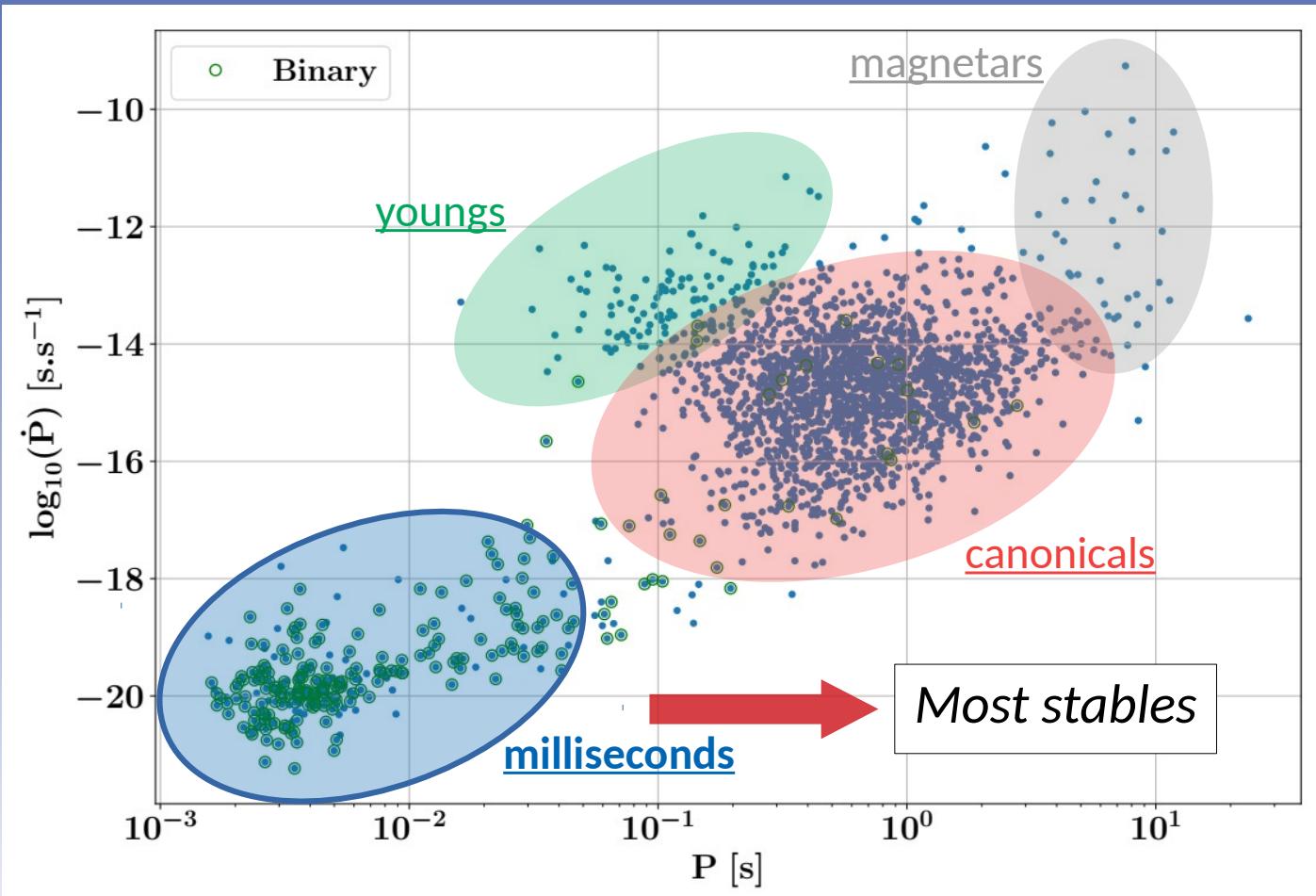
- Canonical pulsars
- Millisecond pulsars
- Magnetars
- Young pulsars
- X-ray pulsar
- Gamma-ray pulsar
- Spiders
- .....



# Pulsars

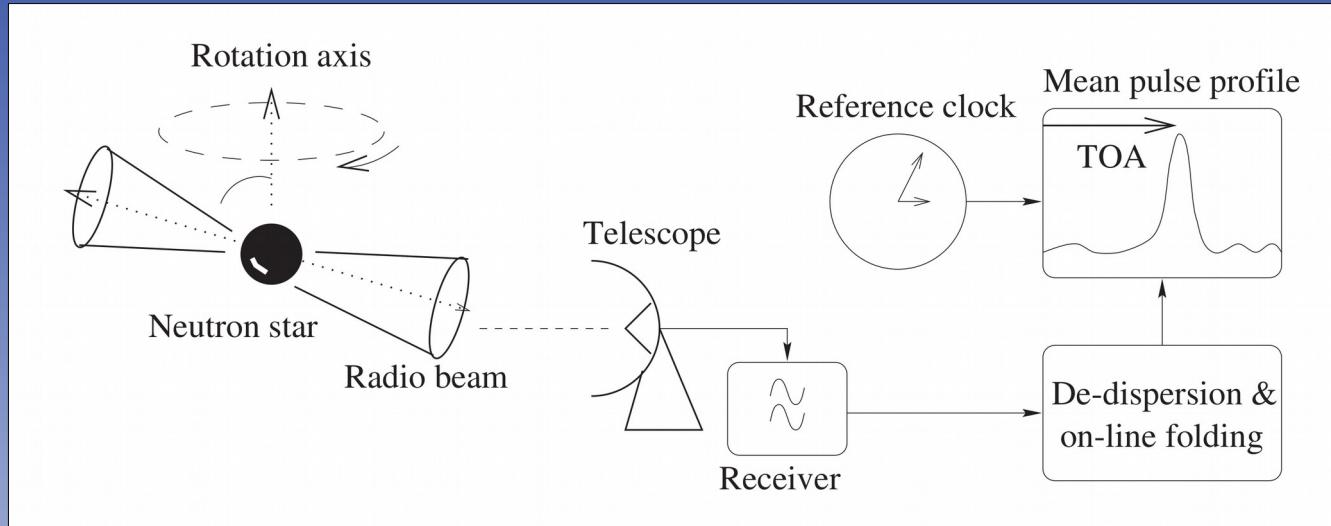
A full zoology !

- Canonical pulsars
- **Millisecond pulsars**
- Magnetars
- Young pulsars
- X-ray pulsar
- Gamma-ray pulsar
- Spiders
- .....

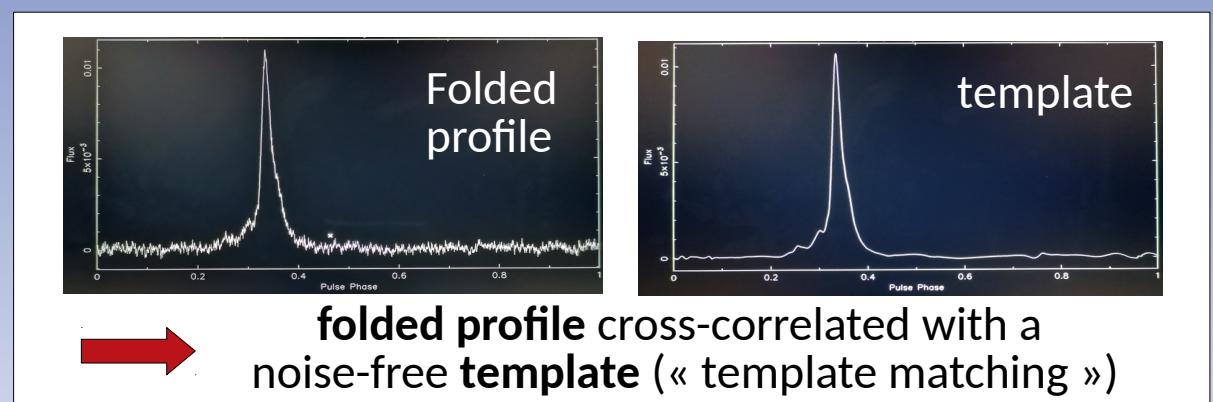
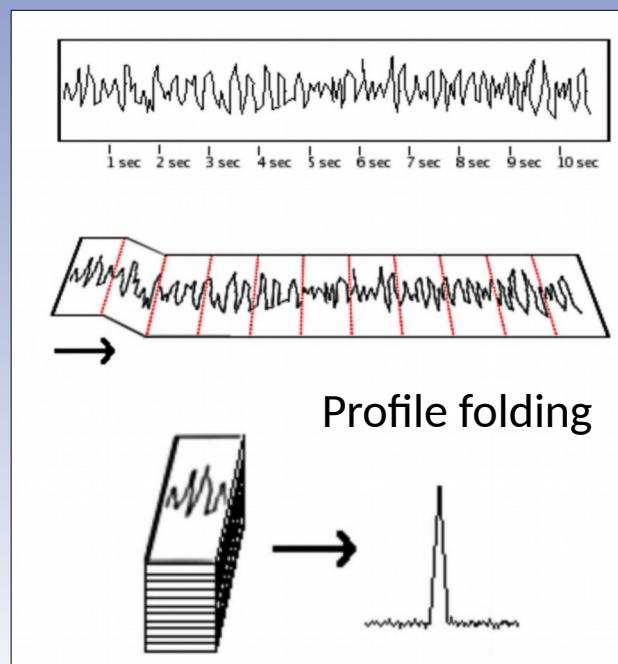


# Pulsar timing

## Determinate times of arrival (TOAs)



cf. Lorimer & Kramer 2005



$$\sigma_{\text{TOA}} \propto \frac{P}{S_{\text{PSR}}} \frac{T_{\text{sys}}}{\Delta f}$$

Choose good the pulsars  
Observe in a large bandwidth

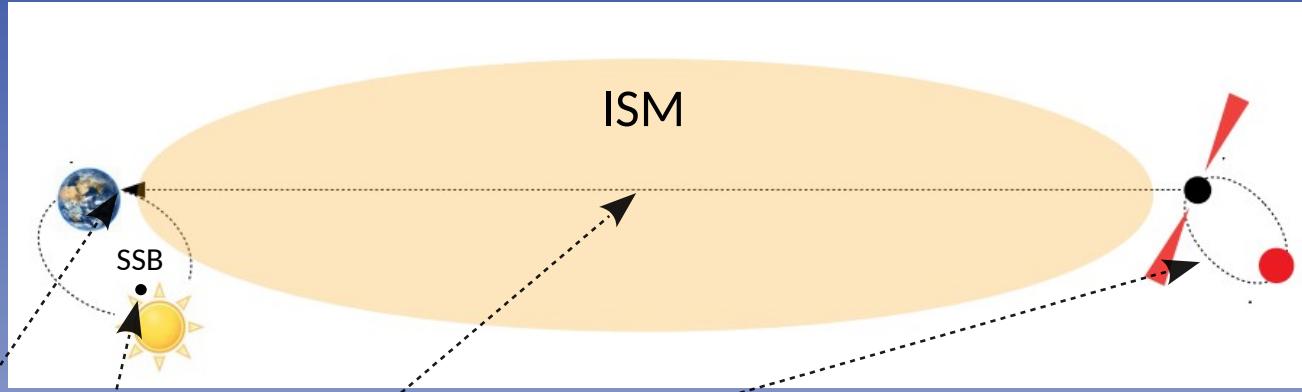
Good receiver

$\sigma_{\text{TOA}} \sim 200 \text{ ns}$

cf. McKee

# Pulsar timing

Build a timing model and get residuals



$$t_e^{psr} = t_a^{obs} - \Delta_{\odot} - \Delta_{ISM} - \Delta_B$$

$t_e^{psr}$ : Time of emission from the center of the pulsar

$t_a^{obs}$ : TOA at observatory time after clock correction

$\Delta_{\odot}$ : Transformation to the solar system barycenter (SSB)

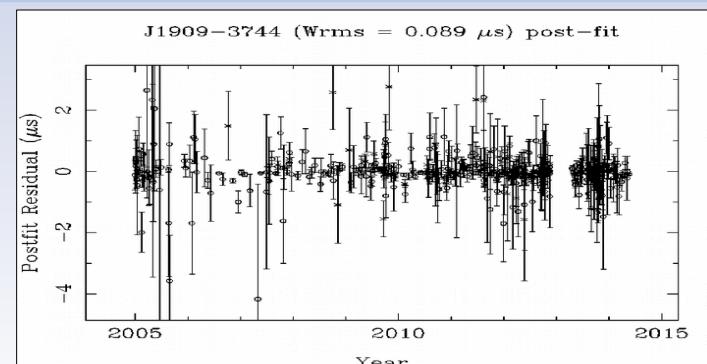
$\Delta_{ISM}$ : Transformation to the binary barycenter  
(dispersion from the ISM)

$\Delta_B$ : Transformation to the pulsar proper time of emission

**Timing model**

- Rotational params
- Astrometric params
- Orbital params
- ISM effects
- Clock correction
- Transformation to the SSB

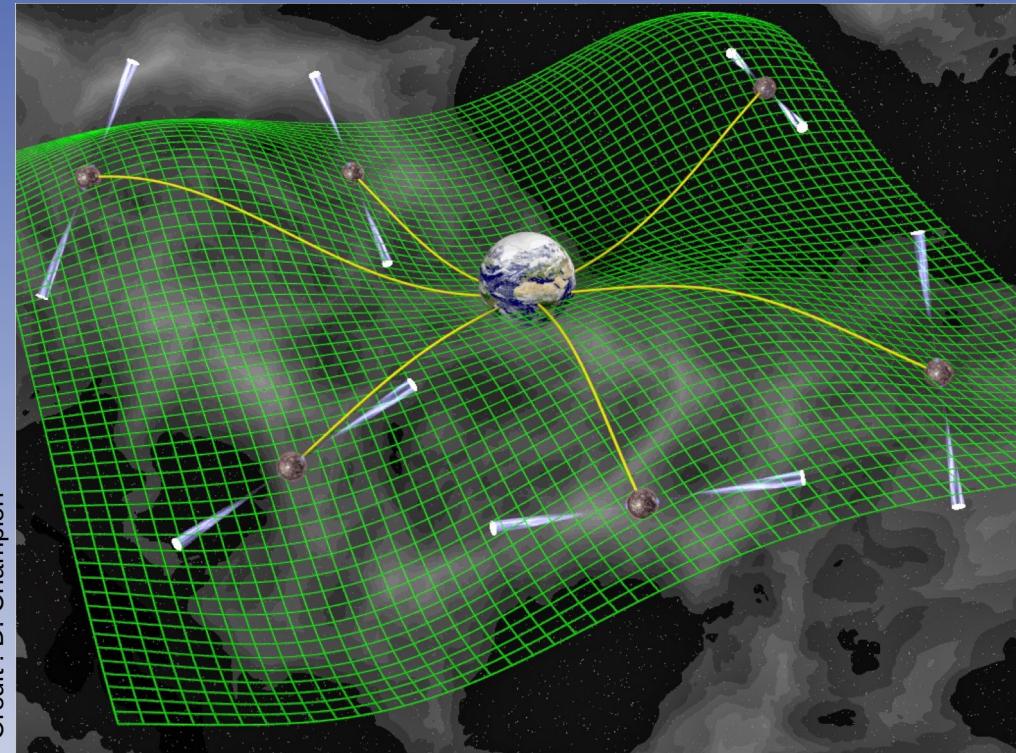
**TOAs – predicted TOAs from timing model**  
=  
**Residuals**



# Pulsar Timing Arrays

Probe very low-frequency gravitational waves effects in a combined set of residuals of a full array of pulsars !

PTA\_NRAO\_Outreach\_animation

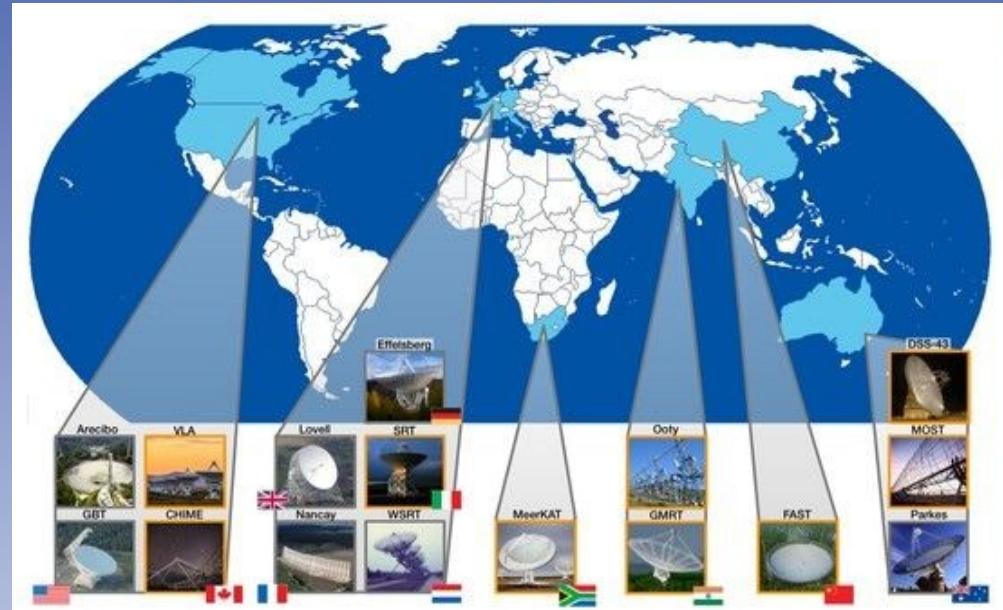


Credit : D. Champion

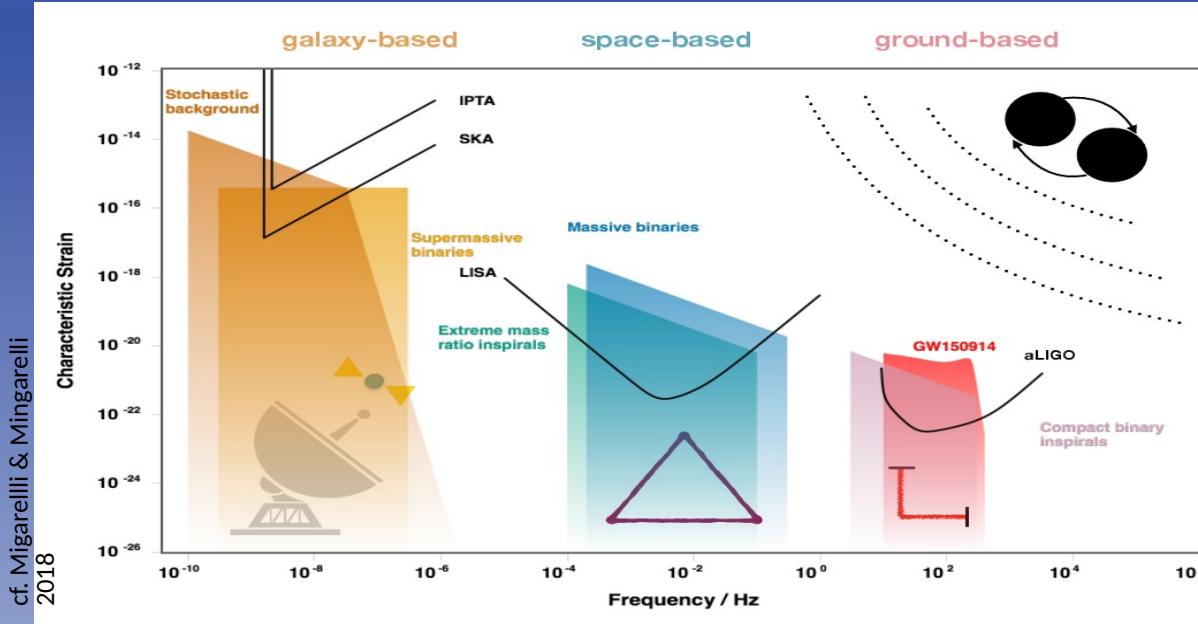


SMBHB  
(stochastic background +  
single sources)

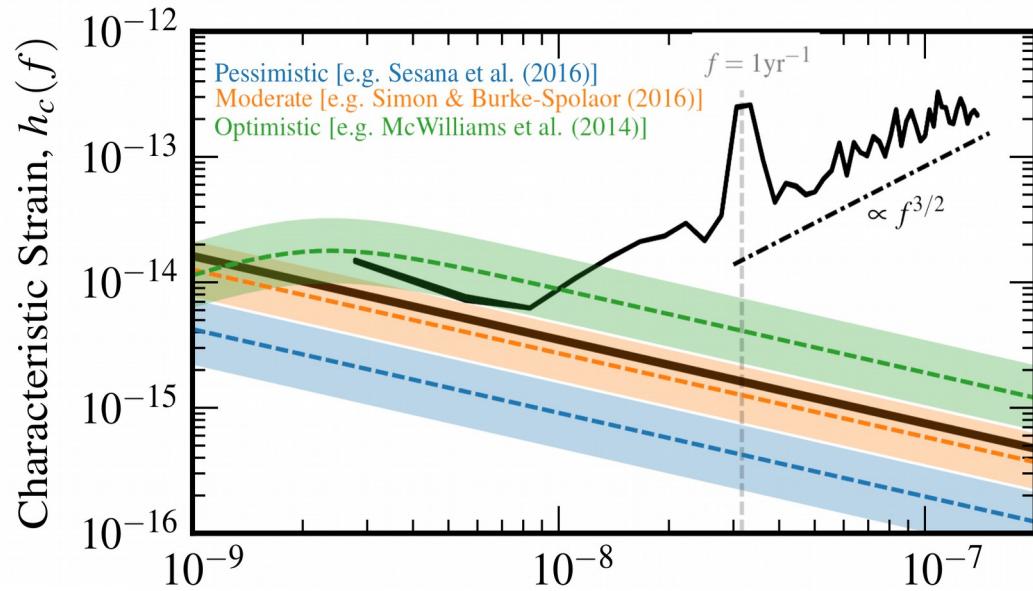
Cosmic strings,  
primordial GWs, ...



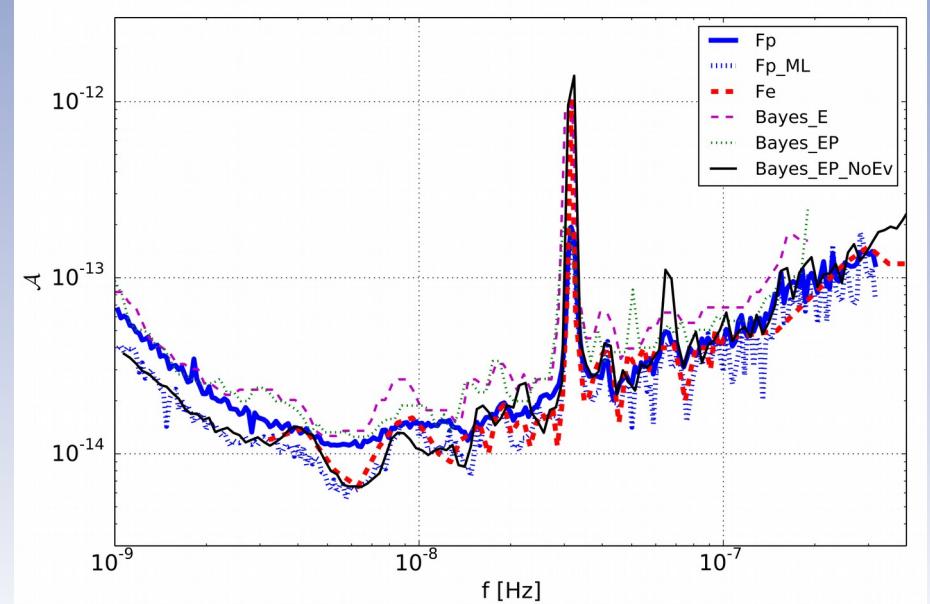
# Pulsar Timing Arrays



## Gravitational wave background (GWB) upper-limits

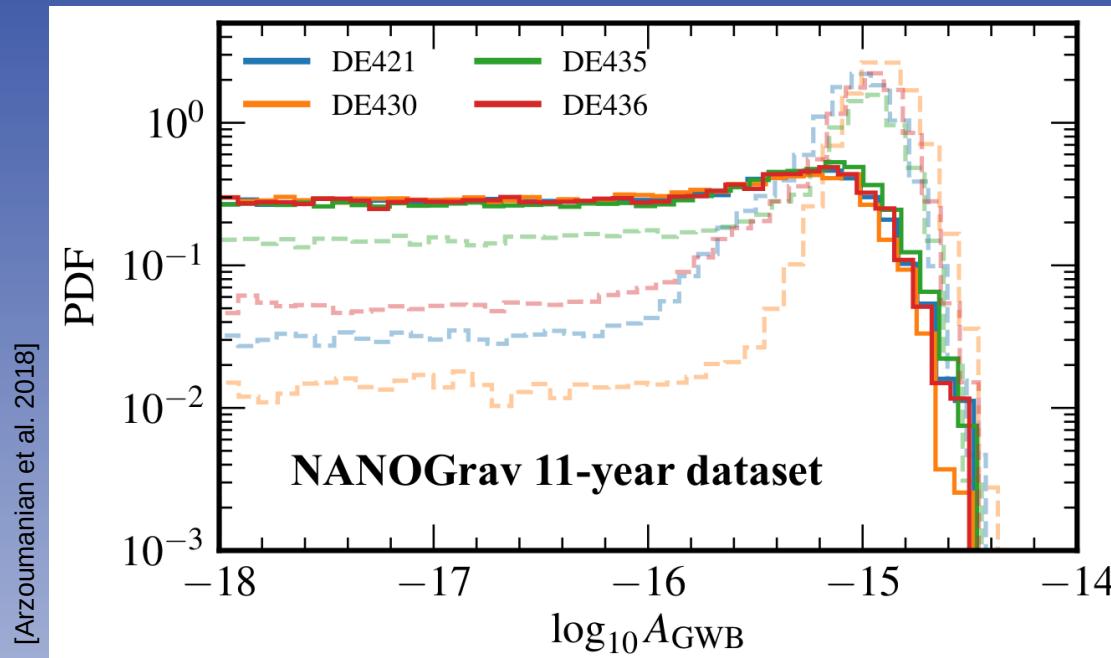


## Individual sources upper-limits



# Our motivations

## GWB Amplitude PDF vs. Solar-system ephemerides (SSEs)



**Problem : GWB results seem dependent of the chosen ephemeris model !**

Possible to fix it by modelling some SSE parameters

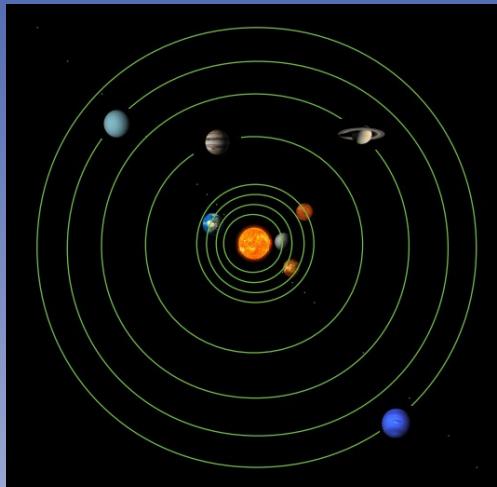
→ BAYSEPHEM model (11 params) to « unfix » ephemeris parameters

→ GWB constraint gets robust against SSE errors

→ But modelling SSE errors can absorb some of the GWB signal

# Solar system ephemerides

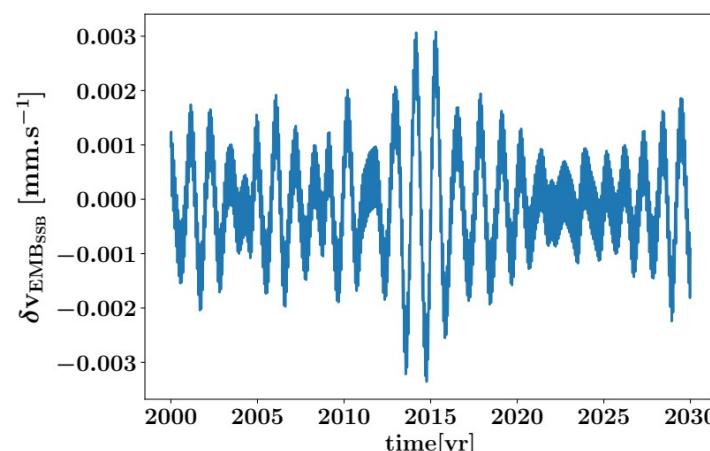
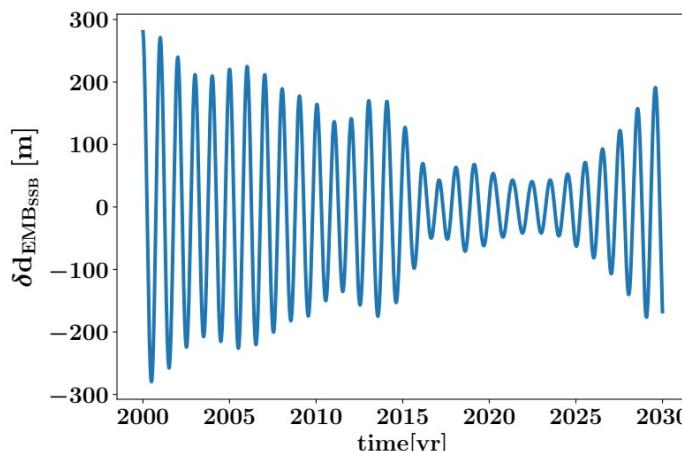
*Pos. & vel. predicted from numerical integration of eq. of motions fitted to the observational data*



Various solutions given by various collaborations !  
JPL, IMCCE (INPOP),...

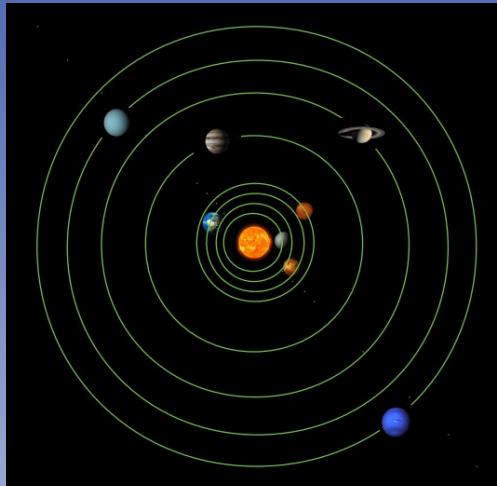
- Different data
- Different uncertainties
- Different models (i.e. eq. & params)

Difference of Earth-Moon barycenter (EMB) position (left) & velocity (right) w.r.t. SSB frame between by INPOP17a & JPL DE436



# Solar system ephemerides

*Pos. & vel. predicted from numerical integration of eq. of motions fitted to the observational data*

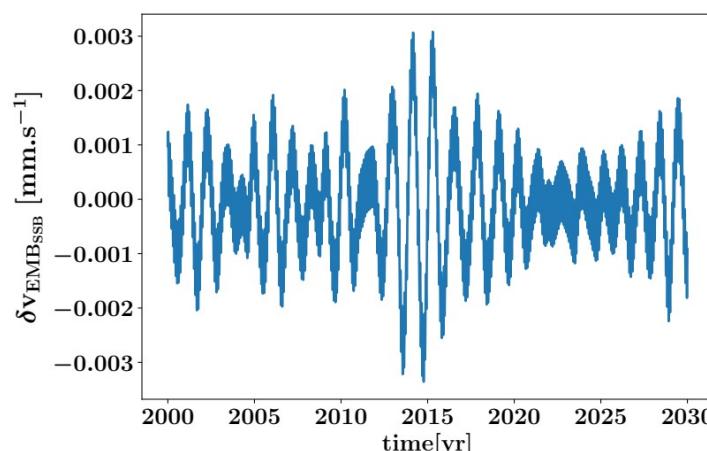
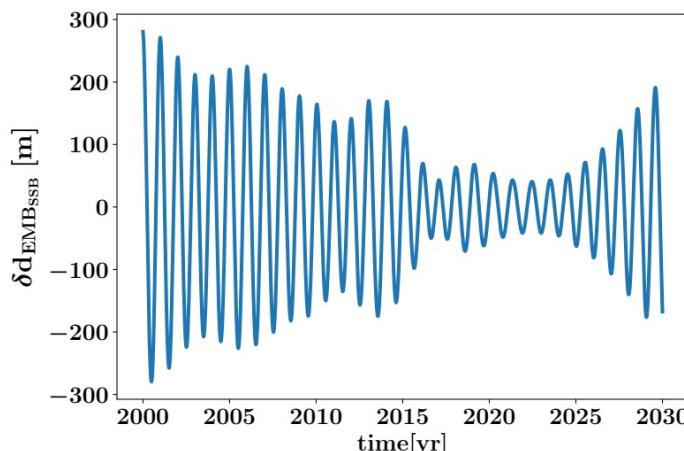


Various solutions given by various collaborations !  
JPL, IMCCE (INPOP),...



Work with INPOP data (cf. A. Fienga)

Difference of Earth-Moon barycenter (EMB) position (left) & velocity (right) w.r.t. SSB frame between by INPOP17a & JPL DE436



# A first approach – SSE perturbation

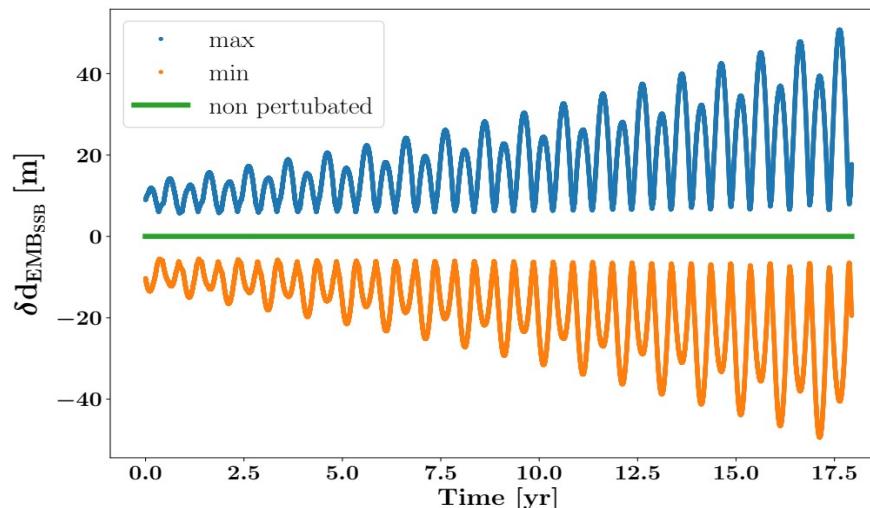
INPOP data (cf. A. Fienga)

- Covariance matrix of orbital parameters
- Linear model coefficients

→ Linear approximation + perturbation of planet positions  
(EMB & 4 outer planets) at initial conditions

$$\vec{x}_{new}(\vec{\theta}) = \vec{x}_{ref}(\vec{\theta}) + \frac{\partial \vec{x}_{ref}(\vec{\theta})}{\partial \vec{\theta}} \delta \vec{\theta}$$

Max & min values of EMB position errors after perturbations at  $1\sigma$  (2000 perturb.)



# A first approach – SSE perturbation

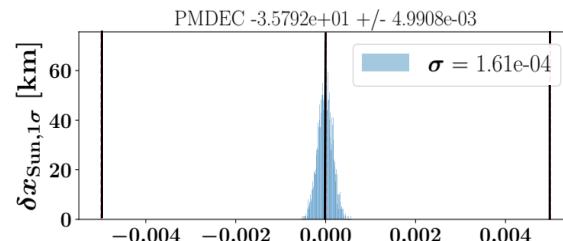
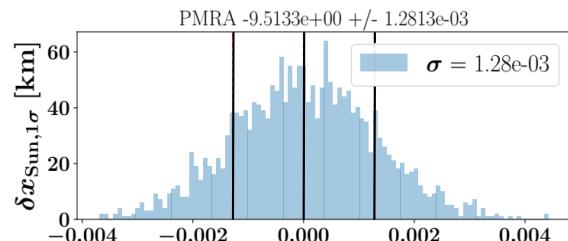
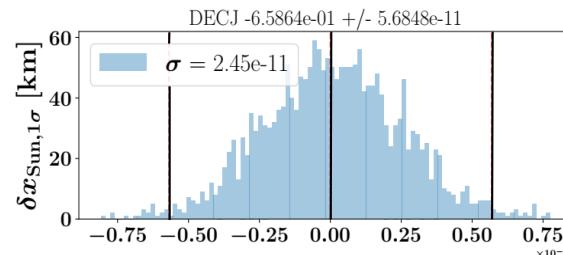
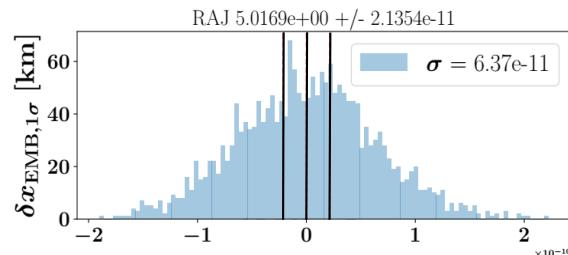
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Post-fit astrometric parameters of J1909-3744  
after perturbations at  $1\sigma$  vs. Non perturbed (2000 perturb.)



# A first approach – SSE perturbation

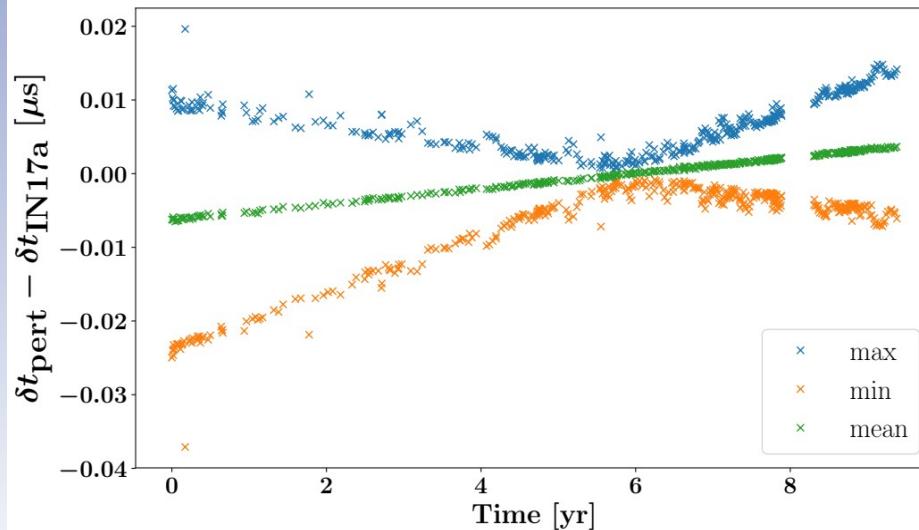
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$$\vec{x}_{new}(\vec{\theta}) = \vec{x}_{ref}(\vec{\theta}) + \frac{\partial \vec{x}_{ref}(\vec{\theta})}{\partial \vec{\theta}} \delta \vec{\theta}$$

Max & min values of post-fit residuals of J1909-3744 after perturbations at  $1\sigma$  (2000 perturb.)



# Model SSE uncertainties as gaussian process

*Description of SSE noise as Gaussian process in residuals with the PTA analysis Bayesian framework software : enterprise (cf. M. Vallisneri)*

## Method(s)

Marginalize over the SSE parameters  
(orbital elements)



Perform a GW searches

Sample for SSE parameter uncertainties & GW parameters together

Get SSE uncertainties  
params by marginalizing  
over other signals

Get constraints on priors  
that take into account  
possible systematic biases

Add SSE errors into a full  
noise model using  
dipolar correlations

# Conclusions

- SSE studies very important to increase PTA sensibility
- Need to take into account ephemeris uncertainties in GW analysis
- Possible to model ephem uncertainties as Gaussian process

Next steps :

- Study systematics between SSEs (importance of probing several ephemeris solutions)
- Find an optimal model for ephemeris uncertainties which takes into account dipolar spatial correlations

*Thank you for your attention*

# Annexe 1

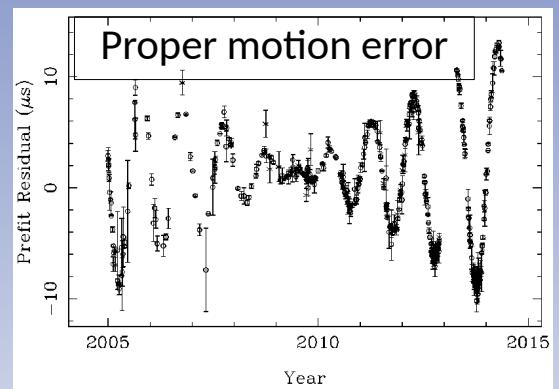
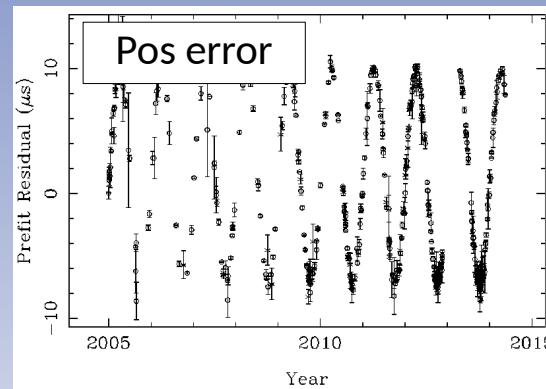
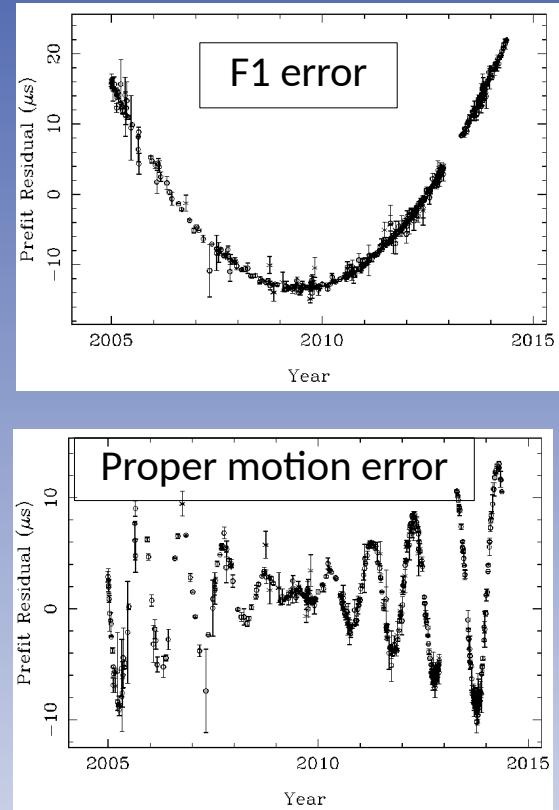
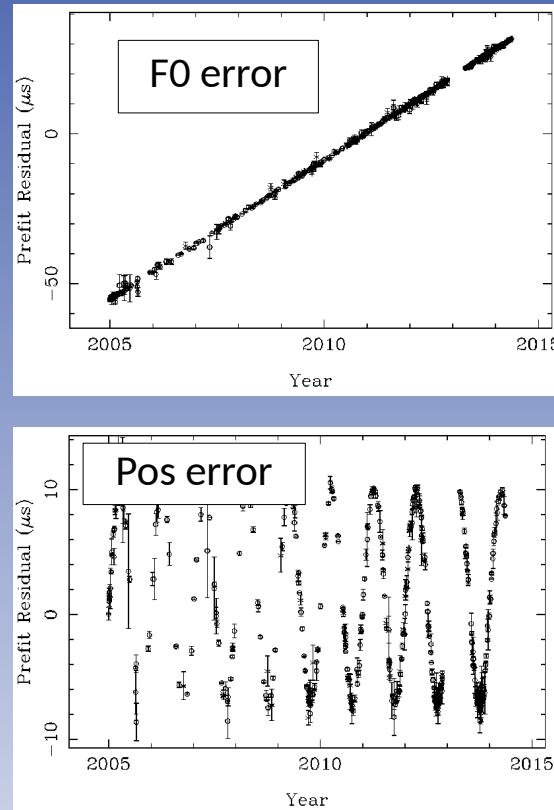
## A timing model

c.f. Desvignes et al. 2016

PSR name	J1909–3744
MJD range	53368–56794
Number of TOAs	425
rms timing residual ( $\mu$ s)	0.13
Reference epoch (MJD)	55000
Measured parameters	
Right ascension, $\alpha$	19:09:47.433 5737(7)
Declination, $\delta$	−37:44:14.515 61(3)
Proper motion in $\alpha$ (mas yr <sup>−1</sup> )	−9.519(3)
Proper motion in $\delta$ (mas yr <sup>−1</sup> )	−35.775(10)
Period, $P$ (ms)	2.947 108 069 766 629(7)
Period derivative, $\dot{P}$ ( $\times 10^{-20}$ )	1.402 518(14)
Parallax, $\pi$ (mas)	0.87(2)
DM (cm <sup>−3</sup> pc)	10.3925(4)
DM1 (cm <sup>−3</sup> pc yr <sup>−1</sup> )	−0.000 32(3)
DM2 (cm <sup>−3</sup> pc yr <sup>−2</sup> )	0.000 04(1)
Orbital period, $P_b$ (d)	1.533 449 474 329(13)
Epoch of periastron, $T_0$ (MJD)	53 114.72(4)
Projected semimajor axis, $x$ (lt-s)	1.897 990 99(6)
Longitude of periastron, $\omega_0$ (deg)	180(9)
Orbital eccentricity, $e$	1.22(11) $\times 10^{-7}$
$\kappa = e \times \sin \omega_0$	−2.3(1900) $\times 10^{-10}$
$\eta = e \times \cos \omega_0$	−1.22(11) $\times 10^{-7}$
Time of asc. node (MJD)	53 113.950 741 990(10)
Orbital period derivative, $\dot{P}_b$	5.03(5) $\times 10^{-13}$
First derivative of $x$ , $\dot{x}$	0.6(17) $\times 10^{-16}$
Sine of inclination angle, $\sin i$	0.997 71(13)
Companion mass, $m_c$ (M <sub>⊕</sub> )	0.213(3)

- Astrometric parameters
- Rotational parameters
- Dispersion measure
- Orbital parameters
- + solar system ephemerides and clock correction

## Effect of param errors on residuals



# Annexe 2

## Transformation to the SSB

*From the topocentric to the quasi-inertial solar system barycenter frame*

$$\Delta_{\odot} = \Delta_A + \Delta_{R\odot} + \Delta_p + \Delta_{D\odot} + \Delta_{E\odot} + \Delta_{S\odot}$$

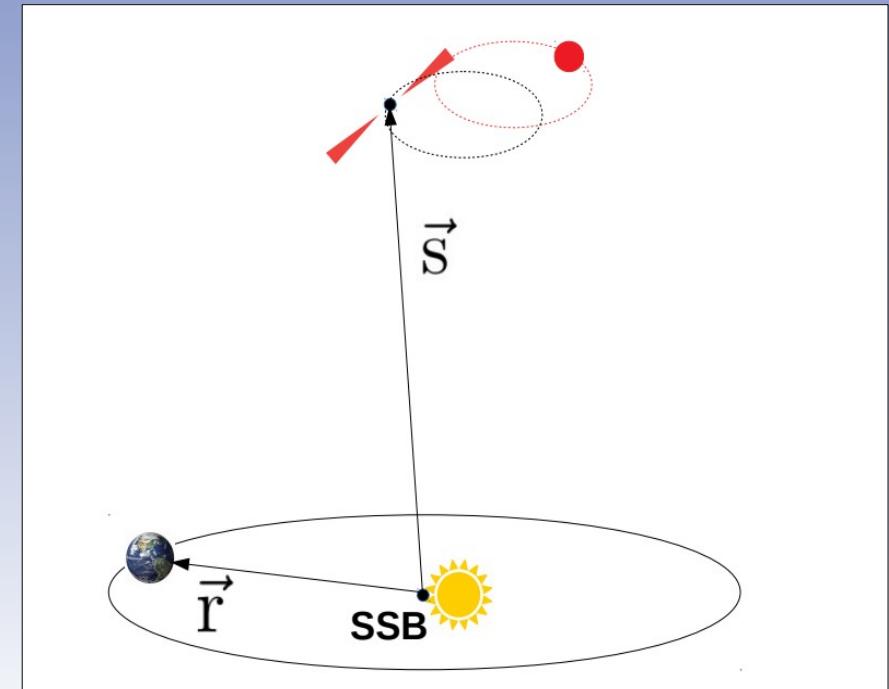
Atmospheric delay      Parallax      Einstein delay

Rømer delay      Solar-system dispersion      Shapiro

Dominant term

$$\Delta_{R\odot} = -\frac{1}{c} \vec{r} \cdot \hat{s}$$

Orbits of solar system bodies needed !



# Annexe 3

## Gaussian processes

« Collection of random variables such that every finite collection of these has a multivariate normal distribution »

Set of basis functions  
« design matrix »

$$\sum_{\mu} \phi_{\mu}(x) w_{\mu}$$

Weights : Gaussian random variables  
« parameter errors »

$$\omega_{\mu}^0$$

Mean vector

$$\Sigma_{\mu\nu}$$

Covariance matrix

### Signals modelization

Approx. to the dominant term : Rømer delay

$$\text{EMB displacement} \quad \delta t_{R\odot} = -\frac{\delta \mathbf{x} \cdot \hat{\mathbf{n}}_p}{c} \quad \text{Pulsar position}$$

Residuals represented as Gaussian processes

$$y(\theta) = \sum_{(A)} y^{(A)}(\theta^{(A)}) + \epsilon$$

Gaussian Process with  
hyp. Params.  $\Theta$

Measurement noise  
vector

$$\mathcal{N}(\mu_{\epsilon}, N)$$

[Van Haasteren & Vallisneri 2014]

# Annexe 4

Marginal likelihood of data (n TOAs) with GP noise description subject to Gaussian measurement noise  $\varepsilon_i$  with covariance matrix  $N$

$$\log p(y|\theta, \text{GP}) = -\frac{1}{2}y^T(N + \sum_{(A)} K^{(A)})^{-1}y - \frac{n}{2}\log(2\pi) - \frac{1}{2}\log \det \left( N + \sum_{(A)} K^{(A)} \right)$$

Gaussian process covariance matrices

Gaussian process « duality »

$$p(y_i|w_\mu, \text{GP}) = \frac{e^{-\frac{1}{2} \sum_{i,j} (y_i - \sum_\mu \phi_\mu(x_i) w_\mu)(N_{ij})^{-1} (y_j - \sum_\mu \phi_\mu(x_j) w_\nu)}}{\sqrt{(2\pi)^n \det N}} \times \frac{e^{-\frac{1}{2} \sum_{\mu\nu} w_\mu (\Sigma_{\mu\nu})^{-1} w_\nu}}{\sqrt{(2\pi)^m \det \Sigma}}$$
$$p(y_i|\text{GP}) = \frac{e^{-\frac{1}{2} \sum_{i,j} y_i (N_{ij} + K_{ij})^{-1} y_j}}{\sqrt{(2\pi)^n \det(N + K)}}, \quad \text{with} \quad K_{ij} = k(x_i, x_j) = \sum_{\mu\nu} \phi_\mu(x_i) \Sigma_{\mu\nu} \phi_\nu(x_j),$$

Marginal likelihood of the data given  
the Gaussian process